GM13181

OTOMATİK KONTROL

10. HAFTA

08. 12. 2023

* Diferansiyel Denk. Lineer Sist. Dönüştürme

* Durum Denk. TF dönüştürme

n. Dereceden Adi Dif. Denk. Durum Uzayına Dönüştürme

1) Sistem en yüksek dereceye göre $\frac{d^n z(t)}{dt^n}$ dif. denk. dönüştü...

2) Durum değişkenleri belirlenir

$$x(t) = \left[z(t) \quad \frac{dz(t)}{dt} \quad \frac{d^2 z(t)}{dt^2} \quad \cdots \quad \frac{d^{n-1} z(t)}{dt^{n-1}} \right]$$

3) Kontrol vektörü belirlenir.

$$u(t) = \left[u_1(t) \quad u_2(t) \quad \cdots \quad u_n(t) \right]$$

4) Durum Denklemi Yazılır

$\dot{x}(t) = Ax(t) + Bu(t) \rightarrow$ Lineer

$\dot{x}(t) = f\left(x(t), u(t)\right) \longrightarrow$ nonlineer

5) Çıkış Denklemi Yazılır

$y(t) = Cx(t) + Du(t) \rightarrow$ Lineer

$y(t) = g\left(x(t), u(t)\right) \rightarrow$ Nonlineer

* Dersimize nonlineer konular dahil değildir.

2. Derece Kısmi Diferansiyel Denklemi 1. Derece Lineer Sisteme Çevirme

Ör: $\ddot{y} - 5\dot{y} + 6y = 0$

$$\begin{bmatrix} u \\ v \end{bmatrix}$$

2. dereceden old. için 2 durum değişkeni.

$u = y \qquad v = \dot{y}$

$\dot{u} = \dot{y} = v$

$\dot{u} = v$

$\ddot{y} = 5\dot{y} - 6y$

$\dot{v} = 5v - 6u$

$$\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = [A] \begin{bmatrix} u \\ v \end{bmatrix}$$

$\dot{u} = 0.u + 1.v$

$\dot{v} = -6u + 5v$

$$\boxed{\begin{bmatrix} \dot{u} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix}}$$

$\dot{x} = Ax + Bu \qquad x = \begin{bmatrix} u \\ v \end{bmatrix} \qquad A = \begin{bmatrix} 0 & 1 \\ -6 & 5 \end{bmatrix}$

Ör: $\ddddot{a}(t) - 3\dddot{a}(t) + 2\ddot{a}(t) + 6\dot{a}(t) + 4a(t) = 3b(t) + 5c(t)$

1. $\ddddot{a}(t) = 3\dddot{a}(t) - 2\ddot{a}(t) - 6\dot{a}(t) - 4a(t) + 3b(t) + 5c(t)$

2. $$x(t) = \begin{bmatrix} a(t) \\ \dot{a}(t) \\ \ddot{a}(t) \\ \dddot{a}(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix}$$

3. $$u(t) = \begin{bmatrix} b(t) \\ c(t) \end{bmatrix} = \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix}$$

4. $$\dot{x}(t) = \frac{d}{dt}\left[x(t)\right] = \frac{d}{dt}\begin{bmatrix} a(t) \\ \dot{a}(t) \\ \ddot{a}(t) \\ \dddot{a}(t) \end{bmatrix} = \begin{bmatrix} \dot{a}(t) \\ \ddot{a}(t) \\ \dddot{a}(t) \\ \ddddot{a}(t) \end{bmatrix}$$

$$\begin{bmatrix} \dot{a}(t) \\ \ddot{a}(t) \\ \dddot{a}(t) \\ \ddddot{a}(t) \end{bmatrix} = \begin{bmatrix} 0x_1(t) + x_2(t) + 0x_3(t) + 0x_4(t) + 0u_1(t) + 0u_2(t) \\ x_3(t) \\ x_4(t) \\ 3x_4(t) - 2x_3(t) - 6x_2(t) - 4x_1(t) + 3u_1(t) + 5u_2(t) \end{bmatrix}$$

$$\dot{x} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -6 & -2 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 3 & 5 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

Çıkış Denklemi : Verilen dif. denklemde çıkış ifadesi aşağıdaki şekilde istenirse durum denklemini dönüştürelim.

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} a(t) \\ \ddot{a}(t) + 3\dddot{a}(t) \\ 4b(t) \end{bmatrix}$$

olsun: Buna göre;

$$y(t) = \begin{bmatrix} a(t) \\ \ddot{a}(t) + 3\dddot{a}(t) \\ 4b(t) \end{bmatrix} = \begin{bmatrix} x_1(t) \\ x_3(t) + 3x_4(t) \\ 4u_1(t) \end{bmatrix}$$

$$y(t) = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

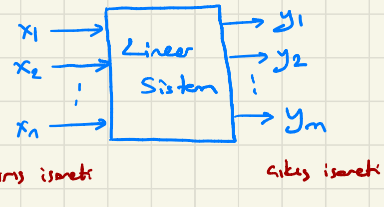
$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -4 & -6 & -2 & 3 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 3 & 5 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{u}$$

$$\underbrace{y(t)}_{\text{Çıkış}} = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix} = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}}_{X} + \underbrace{\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 4 & 0 \end{bmatrix}}_{D} \underbrace{\begin{bmatrix} u_1 \\ u_2 \end{bmatrix}}_{\text{Giriş}}$$

Durum Denklemlerinden Transfer Fonksiyonu Elde Etme



Durum Uzayı

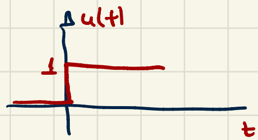
$$\left.\begin{aligned}\dot{x} &= Ax + Bu\\ y &= Cx + Du\end{aligned}\right\} \quad \begin{aligned}[\dot{x}] &= [A][x] + [B][u]\\ [y] &= [C][x] + [D][u]\end{aligned}$$

$$I = \begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

Aşağıdaki işlemler matris boyutunda

Birim matris

$$sX(s) - x(0) = AX(s) + BU(s)$$



$$sX(s) - AX(s) = x(0) + BU(s)$$

$$(sI - A)X(s) = x(0) + BU(s)$$

$$\underbrace{(sI-A)^{-1}(sI-A)}\,X(s) = \underbrace{(sI-A)^{-1}x(0)}_{\phi(t) \quad u(t)=0} + \underbrace{(sI-A)^{-1}BU(s)}_{X(s)\ \text{çözümü}}$$

$$\star \quad \boxed{X(s) = (sI-A)^{-1}BU(s)}$$

Transfer Fonk. Matrisi Çıkış $\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}x_1\\x_3\end{bmatrix}$ olsun. Durum Denklemleri ise $\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$ ise denklem şu şekilde yazılır.

$$\begin{bmatrix}y_1\\y_2\end{bmatrix} = \begin{bmatrix}x_1\\x_3\end{bmatrix} = \underbrace{\begin{bmatrix}1&0&0\\0&0&1\end{bmatrix}}_{C\ \text{matrisi}} \underbrace{\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}}_{X}$$

$$y = Cx$$

$$[Y(s)] = [C][X(s)]$$

$$[Y(s)] = [C]\left[(sI-A)^{-1}B\,U(s)\right]$$

$$\boxed{\frac{Y(s)}{U(s)} = [C]\left[(sI-A)^{-1}B\right] = H(s)}$$

Ör:

$$\dot{x}_1 = -3x_1 + 2x_2 + f_1(t)$$
$$\dot{x}_2 = -4x_1 - 9x_2 + f_1(t) + 3f_2(t)$$

Dif. Denk. $f_1(t)$ ve $f_2(t)$ girişler. $x_1(t)$ çıkış ise

a) $\dfrac{X_1(s)}{F_1(s)} = ?$

b) $\dfrac{X_1(s)}{F_2(s)} = ?$

Çözüm

$$\underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{\dot{x}} = \underbrace{\begin{bmatrix} -3 & +2 \\ -4 & -9 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x} + \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}}_{B} \underbrace{\begin{bmatrix} f_1 \\ f_2 \end{bmatrix}}_{u}$$

x_1 çıkış ise

$y = x_1 \rightarrow$ çıkış

$$\underbrace{y}_{y} = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{x}$$

$$H(s) = C\,(sI-A)^{-1}\,B$$

$$sI-A \Rightarrow \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & 2 \\ -4 & -9 \end{bmatrix} = \begin{array}{c} (-1)^{1+1} \\ \begin{bmatrix} s+3 & -2 \\ 4 & s+9 \end{bmatrix} \end{array}$$

$$(sI-A)^{-1} = \frac{[\mathrm{Adj}(sI-A)]^T}{\det(sI-A)} = \frac{\begin{bmatrix} s+9 & -4 \\ 2 & s+3 \end{bmatrix}^T}{(s+3)(s+9)-(-8)}$$

$$(sI-A)^{-1} = \frac{\begin{bmatrix} s+9 & 2 \\ -4 & s+3 \end{bmatrix}}{s^2+12s+27+8} = \frac{1}{s^2+12s+35}\begin{bmatrix} s+9 & 2 \\ -4 & s+3 \end{bmatrix}$$

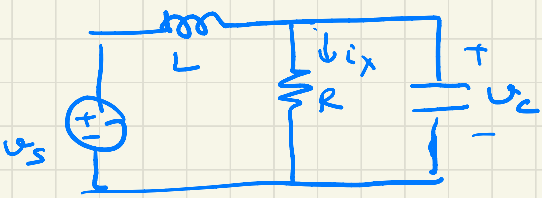
$$H(s) = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \frac{1}{s^2+12s+35} \cdot \begin{bmatrix} s+9 & 2 \\ -4 & s+3 \end{bmatrix} \underbrace{\begin{bmatrix} 1 & 0 \\ 1 & 3 \end{bmatrix}}_{B}$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} s+9+2 & 6 \\ \underset{s-1}{(-4+s+3)} & 3s+9 \end{bmatrix}$$

$$\downarrow F_1 \qquad \downarrow F_2$$

$$H(s) = \frac{1}{s^2+12s+35}\begin{bmatrix} s+11 & 6 \end{bmatrix}$$

$$\frac{X_1(s)}{F_1(s)} = \frac{s+11}{s^2+12s+35} \qquad \frac{X_1(s)}{F_2(s)} = \frac{6}{s^2+12s+35}$$



$$\frac{i_x(s)}{v_s(s)} = ?$$

$R = 1\,\Omega$ $\quad C = 0.25\,F$ $\quad L = 0.5\,H$

$$\frac{I_x(s)}{V_s(s)} = ?$$

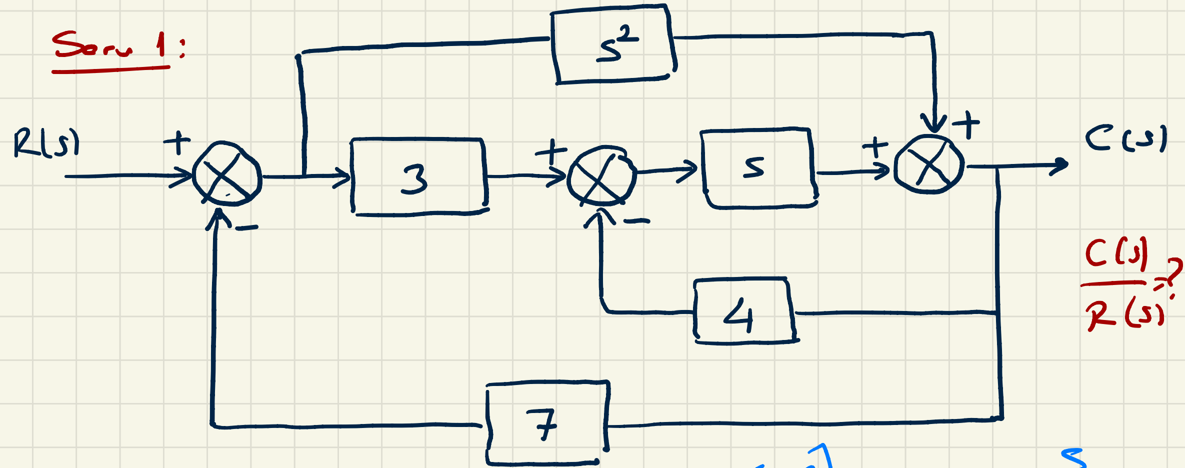
Kontrol

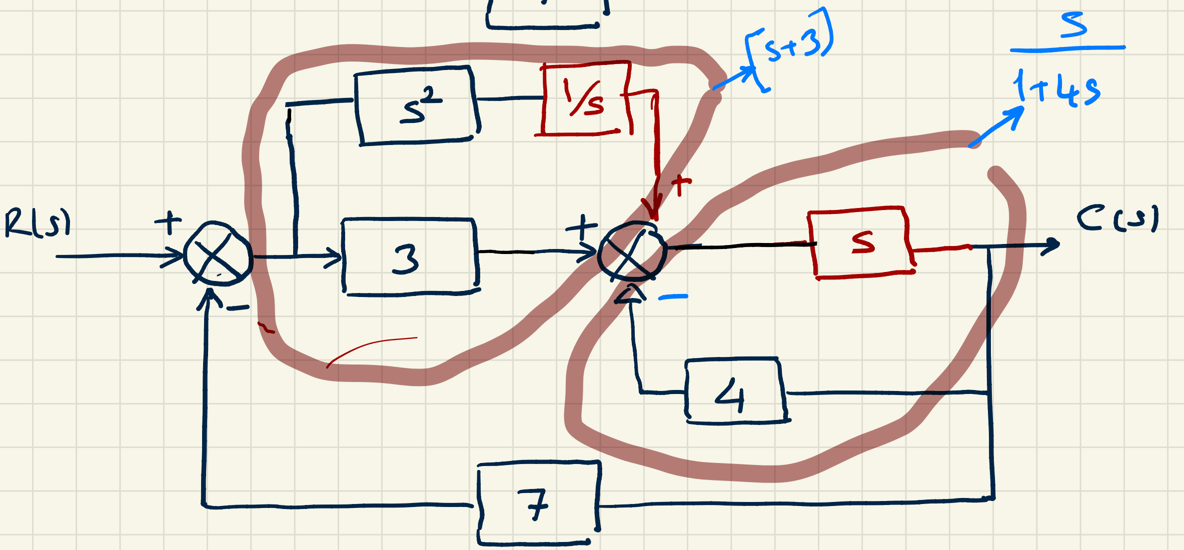
15.12.2023

11. Hafta

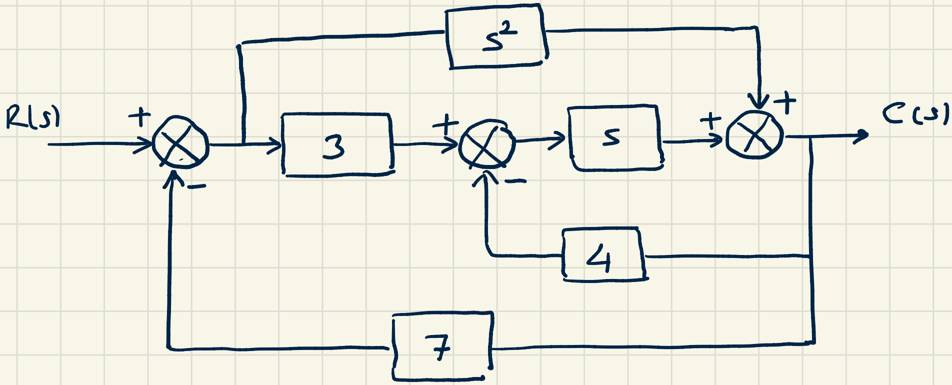
- Vize Sınavı Çözümü
- Durum Denklemleri

Soru 1:



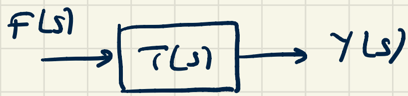


$$\frac{(s+3)\frac{s}{1+4s}}{1+\frac{(s+3)s}{1+4s}\cdot 7} = \frac{s^2+3s}{1+4s+7s^2+21s} = \frac{s^2+3s}{7s^2+25s+1}$$

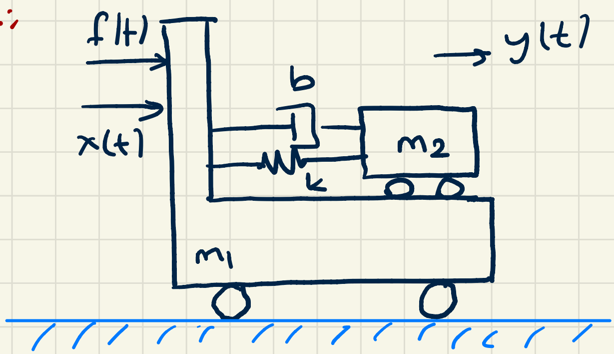


$$\big((R-7C)3-4C\big)s+(R-7C)s^2=C$$

$$\frac{C}{R}=\frac{s^2+3s}{7s^2+25s+1}$$



Soru:

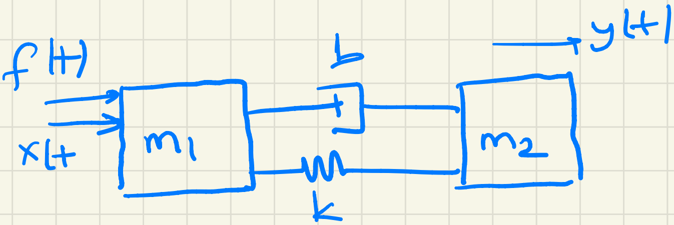


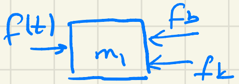
$m_1 = 5\,kg$

$m_2 = 3\,kg$

$b = 4\ Ns/m$

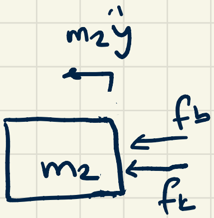
$k = 6\ N/m$





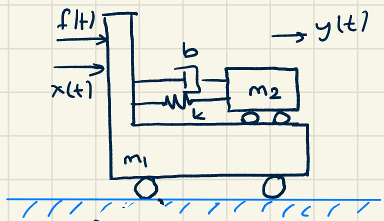
$$f(t) - m_1\ddot{x} - f_b - f_k = 0$$

$$F(s) = m_1 s^2 X(s) + bs\left(X(s) - Y(s)\right) + k\left(X(s) - Y(s)\right)$$

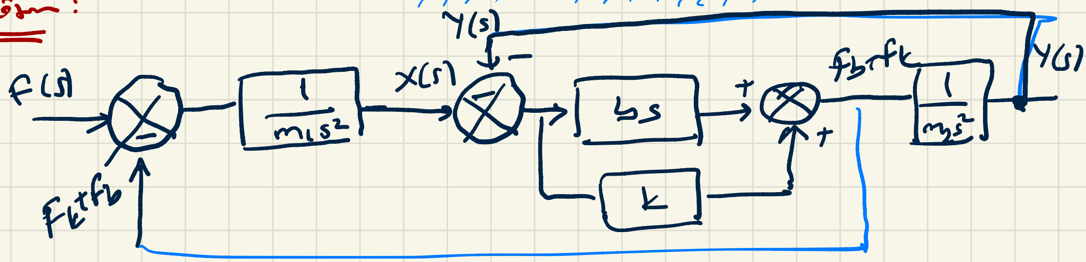


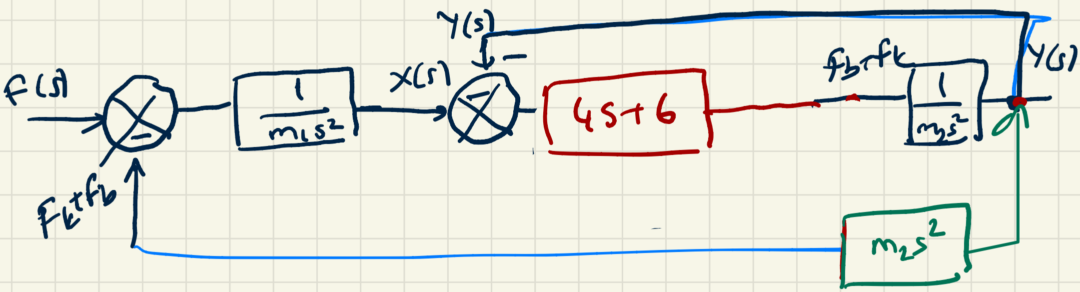
$$m_2\ddot{y} + f_b + f_k = 0$$

$$m_2 s^2 Y(s) + bs\left(Y(s) - X(s)\right) + k\left(Y(s) - X(s)\right) = 0$$

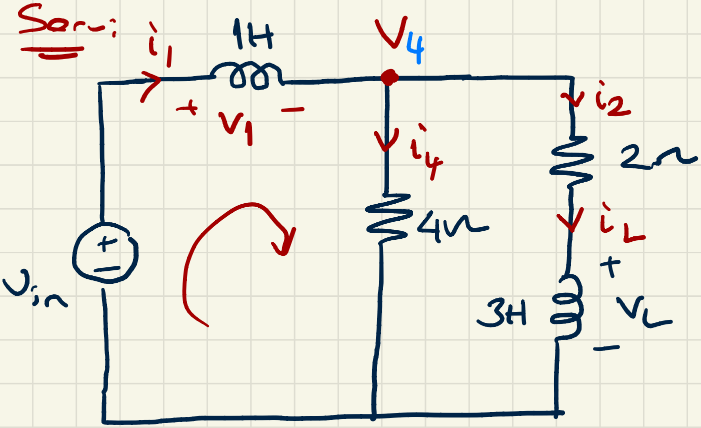


Çözüm :





Soru:



$\underline{i_2 = i_L}$

durum denk.

çıkış $\begin{bmatrix} i_L \\ V_L \end{bmatrix}$

$$V_1 = L_1 \cdot \frac{di_1}{dt} \qquad V_L = L_2 \cdot \frac{di_L}{dt}$$

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} & A & \end{bmatrix} \begin{bmatrix} i_1 \\ i_L \end{bmatrix} + \begin{bmatrix} B \end{bmatrix} v_{in}$$

$$i_1 - i_4 - i_L = 0 \qquad i_4 = i_1 - i_L$$

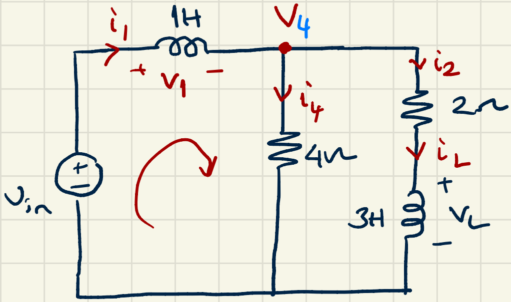
$$-V_{in} + V_1 + V_4 = 0 \Rightarrow V_4 = V_{in} - V_1$$

$$4 i_4 = V_{in} - L_1 \frac{di_1}{dt}$$

$$4(i_1 - i_L) = V_{in} - L_1 \frac{di_1}{dt}$$

$$L_1 \dot{i}_1 = V_{in} - 4 i_1 + 4 i_L \Rightarrow \boxed{\dot{i}_1 = -\frac{4 i_1}{L_1} + \frac{4 i_L}{L_1} + \frac{V_{in}}{L_1}}$$

1H



$$V_4 = 2i_L + V_L$$

$$4(i_1 - i_L) = 2i_L + L\frac{di_L}{dt}$$

$$L\frac{di_L}{dt} = 4i_1 - 4i_L - 2i_L$$

$$\frac{3\dot{i}_L}{3} = \frac{4i_1}{3} - \frac{6i_L}{3}$$

$$V_L = 4i_1 - 6i_L$$

$$\boxed{\dot{i}_L = \frac{4}{3}i_1 - 2i_L}$$

$$\dot{i}_1 = -4i_1 + 4i_L + v_{in}$$

$$\boxed{\dot{i}_L = \frac{4}{3}i_1 - 2i_L}$$

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_L \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ \frac{4}{3} & -2 \end{bmatrix} \begin{bmatrix} i_1 \\ i_L \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} v_{in}$$

$$\begin{bmatrix} i_L \\ v_L \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -6 \end{bmatrix} \begin{bmatrix} i_1 \\ i_L \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} v_{in}$$

Durum Denk. Transfer Fonk.

$$\begin{bmatrix} \dot{i}_1 \\ \dot{i}_L \end{bmatrix} = \underbrace{\begin{bmatrix} -4 & 4 \\ \frac{4}{3} & -2 \end{bmatrix}}_{A} \begin{bmatrix} i_1 \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} 1 \\ 0 \end{bmatrix}}_{B} v_{in} \quad \left.\right\} \text{Durum Denklemi}$$

Çıkış sadece v_L olsun

$$v_L = \underbrace{\begin{bmatrix} 4 & -6 \end{bmatrix}}_{C} \begin{bmatrix} i_1 \\ i_L \end{bmatrix} + \underbrace{0}_{D}\, v_{in} \quad \left.\right\} \text{Çıkış Denklemi}$$

$$T(s) = C\left[sI - A\right]^{-1} B + D$$

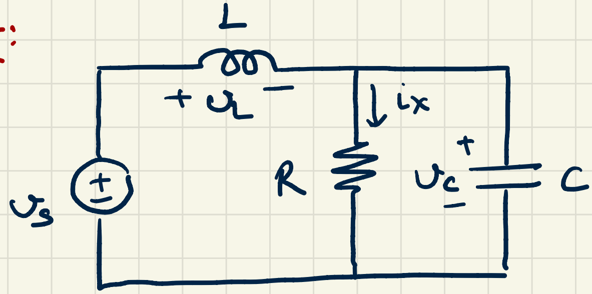
$$\left[sI - A\right] = \begin{pmatrix} s & s \\ s & s \end{pmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 4 \\ \frac{4}{3} & -2 \end{bmatrix} = \begin{bmatrix} s+4 & -4 \\ -\frac{4}{3} & s+2 \end{bmatrix}$$

$$\begin{bmatrix} s+4 & -4 \\ -\frac{4}{3} & s+2 \end{bmatrix}^{-1} = \frac{\operatorname{adj}(sI-A)^T}{\det(sI-A)} = \frac{\begin{bmatrix} (s+2) & \frac{4}{3} \\ 4 & s+4 \end{bmatrix}^T}{(s+4)(s+2) - (-4)\left(-\frac{4}{3}\right)} = \frac{\begin{bmatrix} s+2 & 4 \\ \frac{4}{3} & s+4 \end{bmatrix}}{s^2+6s+8-\frac{16}{3}}$$

$$T(s) = \frac{\begin{bmatrix} 4 & -6 \end{bmatrix} \begin{bmatrix} s+2 & 4 \\ \frac{4}{3} & s+4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2+6s+\frac{8}{3}} = \frac{\begin{bmatrix} 4s+8-8 & 16-6s-24 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}}{s^2+6s+\frac{8}{3}}$$

$$T(s) = \frac{4s}{s^2+6s+\frac{8}{3}}$$

Ör:



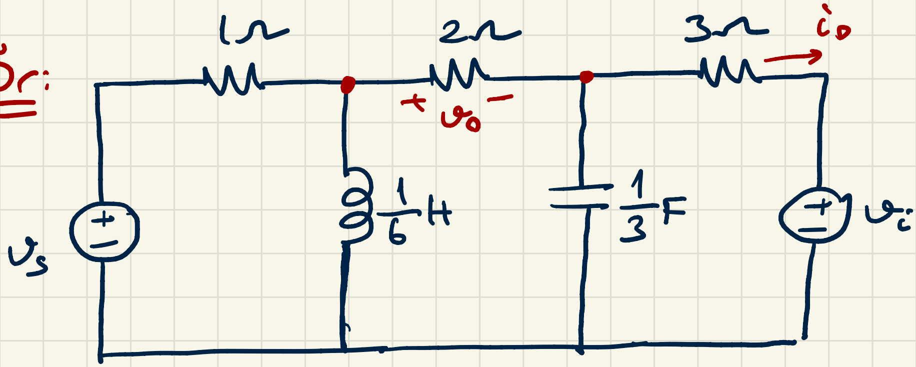
v_s giriş

i_x çıkış.

Durum denk. ve çıkış denk.

Transfer $\dfrac{I_x(s)}{V_s(s)} = ?$

Ör:



Girişler → v_s ve v_i

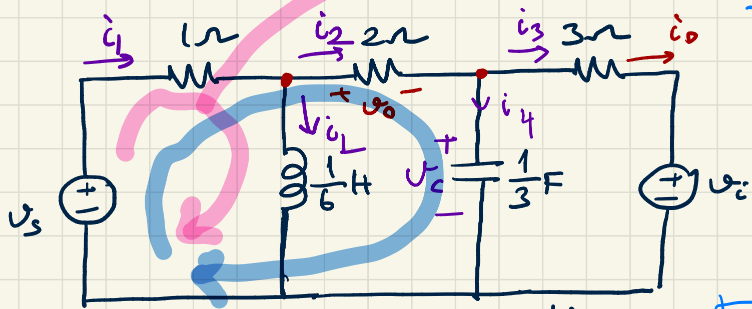
Çıkışlar → v_o ve i_o

Durum denklemlerini ve çıkış matrisini belirle.

$$i_c = C \cdot \frac{dv_c}{dt}$$

$$v_L = L \cdot \frac{di_L}{dt}$$

$$\begin{bmatrix} v_c \\ i_L \end{bmatrix}$$



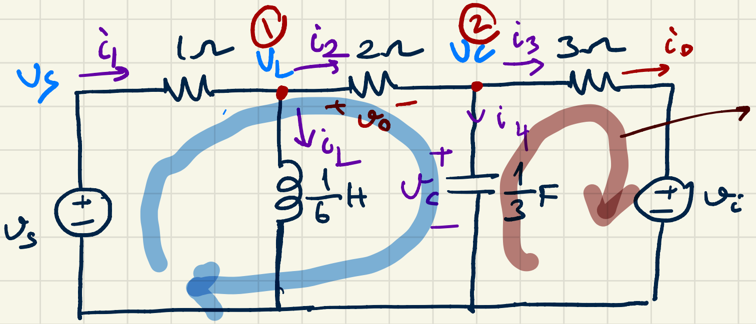
$$-v_s + i_1 1\Omega + v_L = 0$$

$$v_L = v_s - i_1$$

$$L \frac{di_L}{dt} = v_s - i_1$$

$$\boxed{\frac{1}{6} i_L = v_s - i_1}$$

$$i_1 - i_2 - i_L = 0 \qquad i_2 = \frac{v_o}{2}$$



$$\begin{bmatrix} v_c \\ i_L \end{bmatrix}$$

① . $i_1 - i_L - i_2 = 0$

$i_1 = i_L + i_2$

$i_1 = i_L + \frac{v_0}{2}$

$v_0 = (i_1 - i_L)\,2$

$v_0 = \frac{v_L - v_c}{2\Omega}$

$$-v_s + i_1 + v_0 + v_c = 0$$

$$v_s = i_1 + v_0 + v_c$$

$$v_s = i_1 + v_0 + v_c$$

$$v_s = i_1 + 2(i_1 - i_L) + v_c$$

i_1'i v_c ve i_L cinsinden yazalım

$$v_s = 3i_1 - 2i_L + v_c \Rightarrow i_1 = \frac{v_s + 2i_L - v_c}{3}$$

$$-v_c + 3i_0 + v_i = 0 \rightarrow \text{çıkış olur.}$$

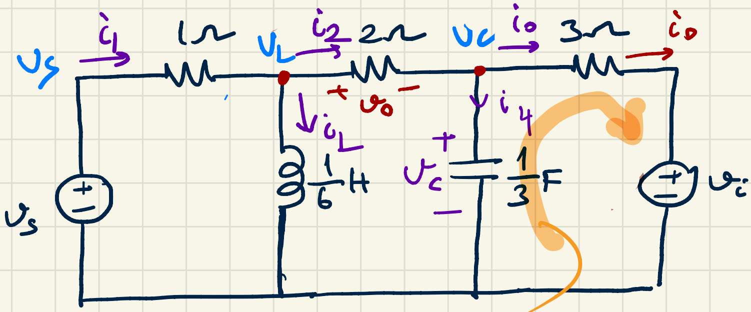
$$v_L = v_s - 1 \cdot i_1$$

Arka sayfadaki: $\frac{1}{6}\dot{i}_L = v_s - i_1$

$$\dot{i}_L = 6v_s - i_1$$

$$\dot{i}_L = 6v_s - \left(\frac{v_s + 2i_L - v_c}{3}\right)$$

$$\dot{i}_L = 6v_s - 2v_s - 4i_L + 2v_c$$



v_c node için KCL

$$i_2 - i_4 - i_o = 0$$

$$\frac{v_o}{2} - \frac{1}{3}\frac{dv_c}{dt} - i_o = 0$$

$$\frac{dv_c}{dt} = \frac{3}{2}v_o - 3i_o$$

Loop?

$$3i_o + v_i - v_c = 0$$

$$i_o = \frac{v_c - v_i}{3}$$

v_L node için KCL

$$i_1 - i_L + \frac{v_o}{2} = 0$$

$$\frac{v_o}{2} = i_L - i_1$$

$$i_1$$

$$\frac{v_s + 2i_L - v_c}{3}$$

$$\frac{v_o}{2} = i_L - \frac{v_s + 2i_L - v_c}{3}$$

$$v_o = -\frac{2}{3}v_s - \frac{4i_L}{3} + \frac{2v_c}{3} + 2i_L$$

$$\boxed{v_o = +\frac{2v_c}{3} - \frac{2i_L}{3} - \frac{2}{3}v_s}$$

$$\dot{v}_c = \frac{3}{2}v_o - 3i_o$$

$$\dot{v}_c = \frac{3}{2}\left(\frac{2v_c}{3} - \frac{2i_L}{3} - \frac{2v_s}{3}\right) - 3\left(\frac{v_c - v_i}{3}\right)$$

$$\dot{v}_C = \frac{3}{2} v_0 - 3 i_0$$

$$\dot{v}_C = \frac{3}{2}\left(\frac{2v_C}{3} - \frac{2i_L}{3} - \frac{2v_s}{3}\right) - 3\left[\frac{v_C - v_i}{3}\right]$$

$$\dot{v}_C = v_C - i_L - v_s - v_C + v_i$$

$$\boxed{\dot{v}_C = -i_L - v_s + v_i}$$

$$\boxed{\dot{i}_L = 6v_s - 2v_s - 4i_L + 2v_C}$$

$$\begin{bmatrix} \dot{v}_C \\ \dot{i}_L \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -1 \\ 2 & -4 \end{bmatrix}}_{A} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} -1 & 1 \\ 4 & 0 \end{bmatrix}}_{B} \begin{bmatrix} v_s \\ v_i \end{bmatrix}$$

$$\begin{bmatrix} v_0 \\ i_0 \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & 0 \end{bmatrix}}_{C} \begin{bmatrix} v_C \\ i_L \end{bmatrix} + \underbrace{\begin{bmatrix} -\frac{2}{3} & 0 \\ 0 & -\frac{1}{3} \end{bmatrix}}_{D} \begin{bmatrix} v_s \\ v_i \end{bmatrix}$$

$$v_0 = +\frac{2v_C}{3} - \frac{2i_L}{3} - \frac{2}{3}v_s \qquad i_0 = \frac{v_C - v_i}{3}$$

$$H(s) = C\,(sI - A)^{-1} B$$

$$(sI-A) = \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} 0 & -1 \\ 2 & -4 \end{bmatrix} = \begin{bmatrix} s & 1 \\ -2 & s+4 \end{bmatrix}$$

$$(sI-A)^{-1} = \frac{Adj(sI-A)}{\det(sI-A)} = \frac{\begin{bmatrix} s+4 & 2 \\ 1 & s \end{bmatrix}^T}{s^2+4s+2}$$

$$(sI-A)^{-1} = \frac{\begin{bmatrix} s+4 & 1 \\ 2 & s \end{bmatrix}}{s^2+4s+2}$$

$$H(s) = C\,(sI-A)^{-1}B =$$

$$H(s) = \frac{\overset{C}{\begin{bmatrix} \frac{2}{3} & -\frac{2}{3} \\ \frac{1}{3} & 0 \end{bmatrix}} \overset{(sI-A)^{-1}}{\begin{bmatrix} s+4 & 1 \\ 2 & s \end{bmatrix}} \overset{B}{\begin{bmatrix} -1 & 1 \\ 4 & 0 \end{bmatrix}}}{s^2+4s+2}$$

Bu işlemler sonucu Transfer fonk. elde edilebilir.