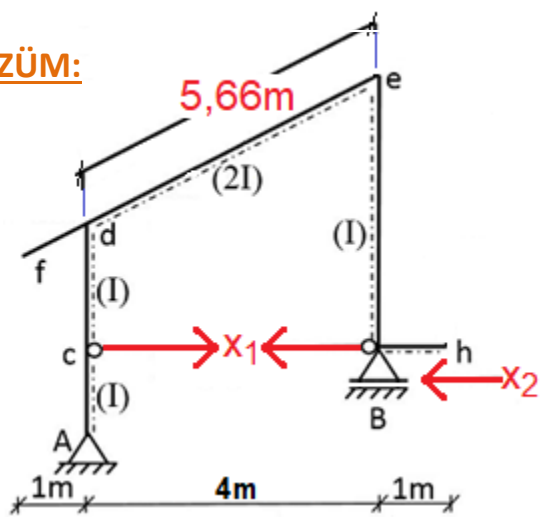


a)



İzostatik Esas Sistem ve Hiperstatik Bilinmeyenler

$$\Sigma M_A = 0 \quad (\curvearrowright +)$$

$$25+20 \times 4 \times 2+50 \times 5-R_{B Y 0} \times 4=0$$

$$R_{BY0} = 435 / 4 = 108,75 \text{ kN}$$

$$\sum M_B = 0 \quad (\curvearrowright +)$$

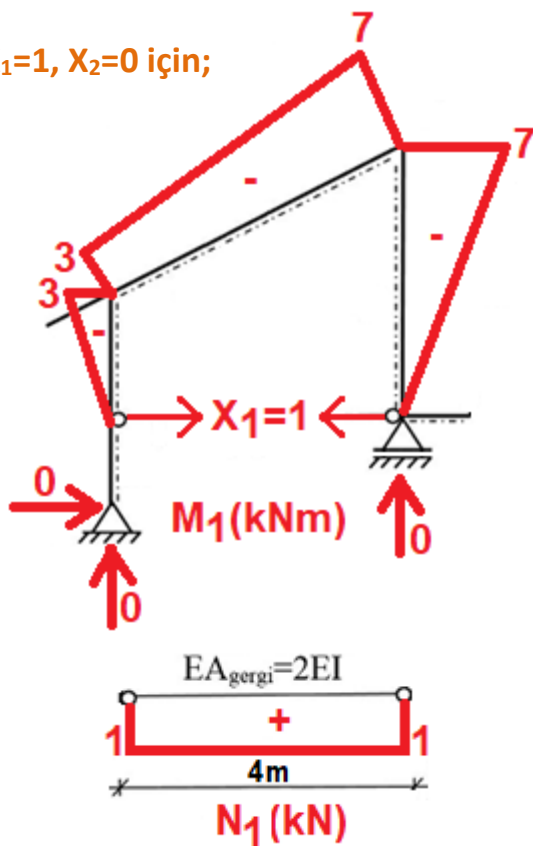
$$25 - 20 \times 4 \times 2 + 50 \times 1 + R_{AY0} \times 4 = 0$$

$$R_{AY0} = 85/4 = 21,25 \text{ kN}$$

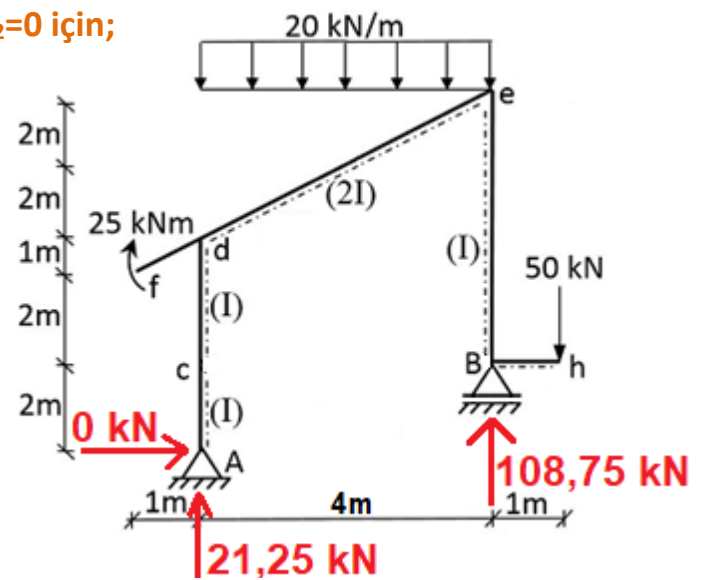
$$\sum F_x = 0 \quad (\rightarrow +)$$

$$R_{AX0} = 0 \text{ kN}$$

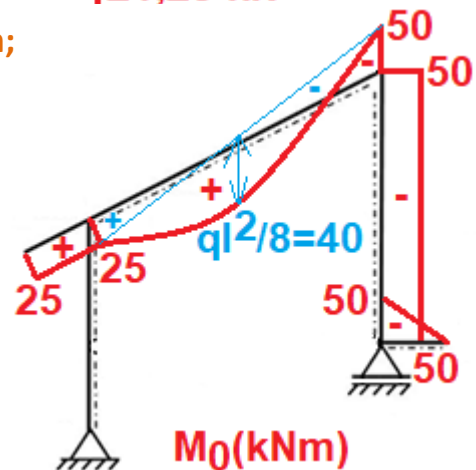
$X_1=1, X_2=0$ için;



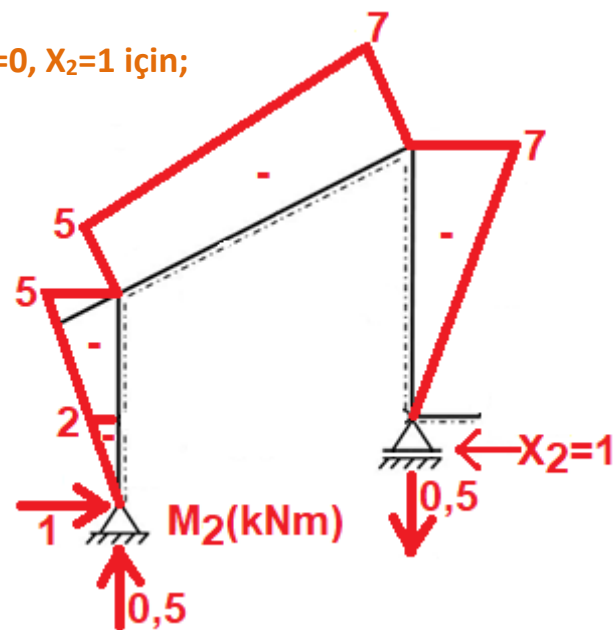
$X_1=X_2=0$ için;



$X_1=X_2=0$ için;



$X_1=0, X_2=1$ için;



Süreklilik Denklemi

$$\delta_{10} + \delta_{11} \times \mathbf{x}_1 + \delta_{12} \times \mathbf{x}_2 = 0$$

$$\delta_{20} + \delta_{21} \times \mathbf{X}_1 + \delta_{22} \times \mathbf{X}_2 = 0$$

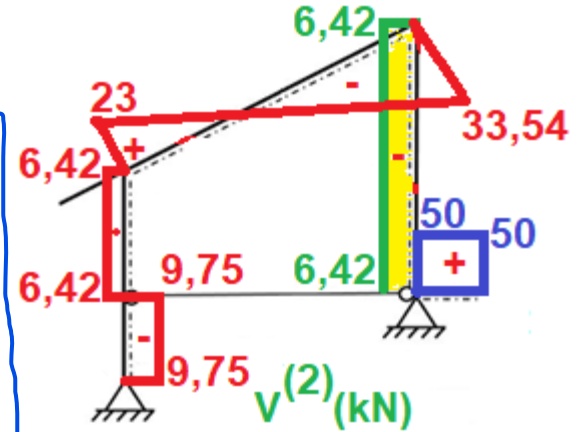
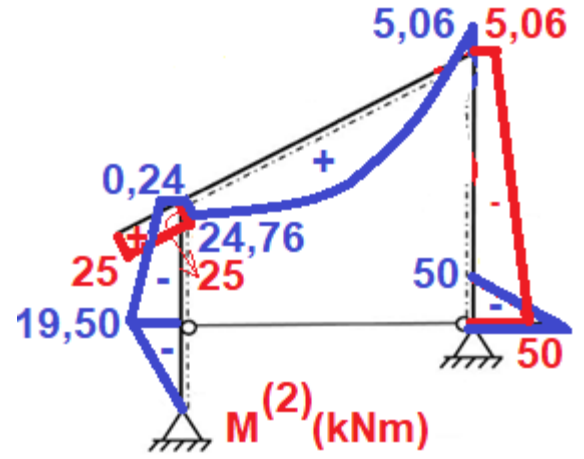
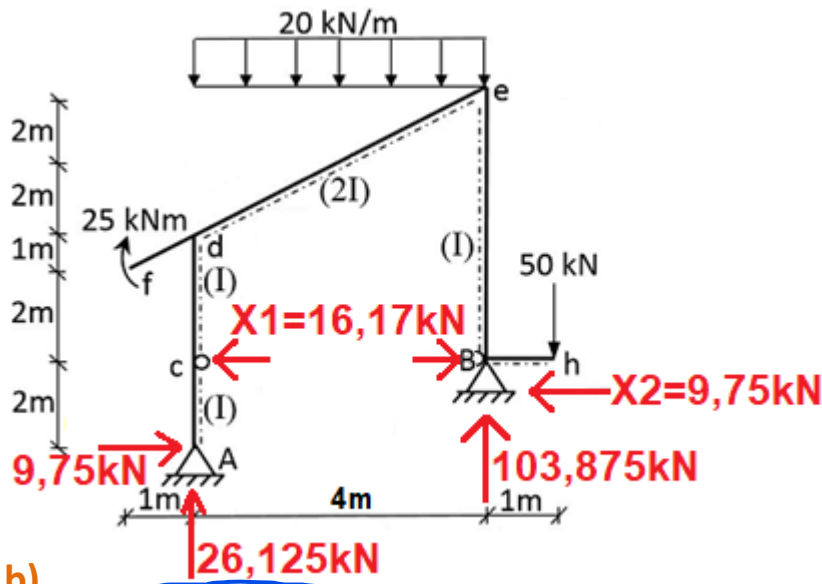
$$\delta_{10}=1095,30 \ ; \ \delta_{11}=199,85 \ ; \ \delta_{12}=\delta_{21}=219,12$$

$$\delta_{20}=1019,83 \quad ; \quad \delta_{22}=258,82$$

$$1095,30 + 199,85 \times X_1 + 219,12 \times X_2 = 0$$

$$1019,83 + 219,12 \times X_1 + 258,82 \times X_2 = 0$$

$X_1 = -16,17 \text{ kN}$; $X_2 = 9,75 \text{ kN}$



b)

Süreklilik Denklemi

$$\delta_{10}^w + \delta_{11} \times X_1 + \delta_{12} \times X_2 = 0 ; \delta_{10}^w = 0$$

$$\delta_{20}^w + \delta_{21} \times X_1 + \delta_{22} \times X_2 = 0 ;$$

$$\delta_{20}^w = -(0,5 \times 0,05 - 0,5 \times 0,02) = -0,015$$

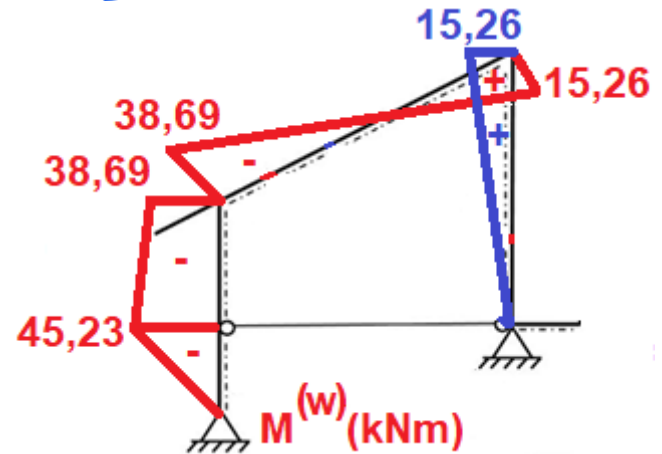
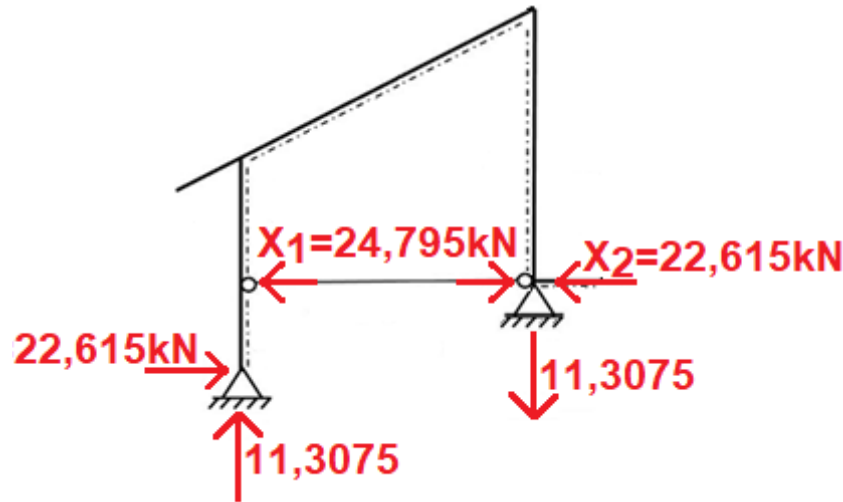
$$\delta_{10}^w = 0 ; \delta_{11} = 199,85 ; \delta_{12} = \delta_{21} = 219,12$$

$$\delta_{20}^w = -0,015 \times 2,8 \times 10^4 = -420 ; \delta_{22} = 258,82$$

$$0 + 199,85 \times X_1 + 219,12 \times X_2 = 0$$

$$-420 + 219,12 \times X_1 + 258,82 \times X_2 = 0$$

$$X_1 = -24,795 \text{ kN} ; X_2 = 22,615 \text{ kN}$$



c)

Süreklilik Denklemi

$$\delta_{1t} + \delta_{11} \times X_1 + \delta_{12} \times X_2 = 0 ; \delta_{1t} = 1 \times 4 \times 10^{-5} \times 15 = 0,00060$$

$$\delta_{2t} + \delta_{21} \times X_1 + \delta_{22} \times X_2 = 0 ; \delta_{2t} = 0$$

$$\delta_{1t} = 0,00060 \times 2,8 \times 10^4 = 16,80 ; \delta_{11} = 199,85 ;$$

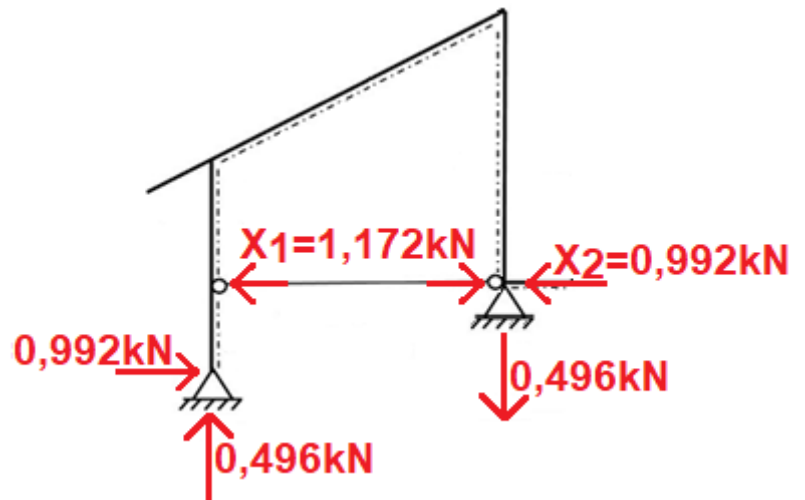
$$\delta_{12} = \delta_{21} = 219,12$$

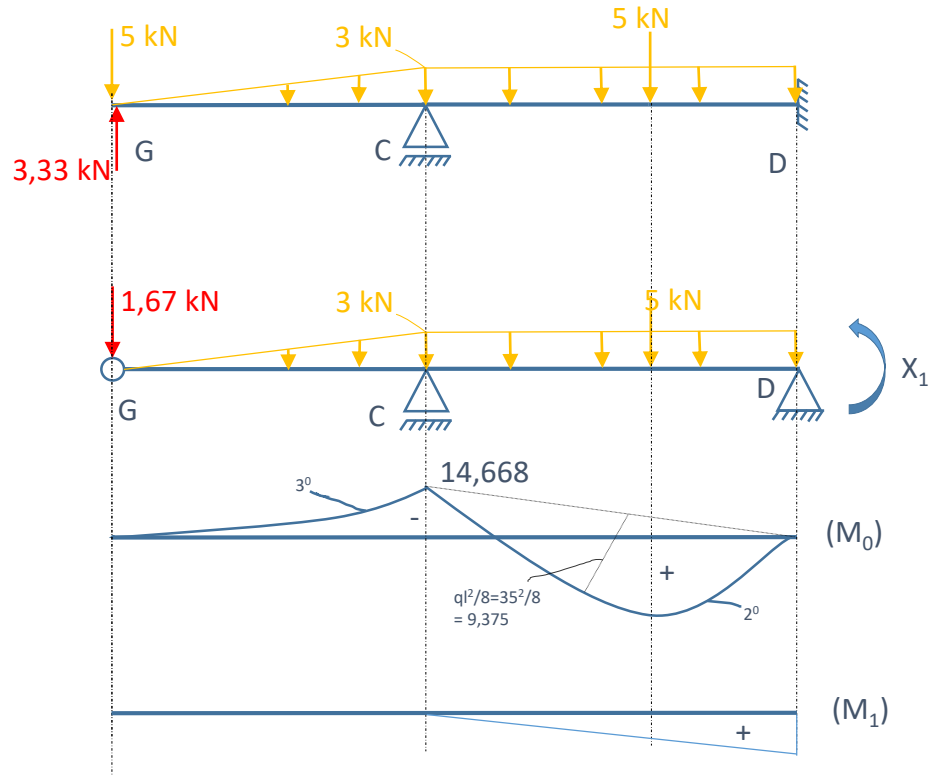
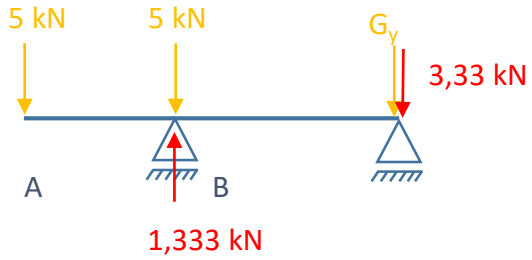
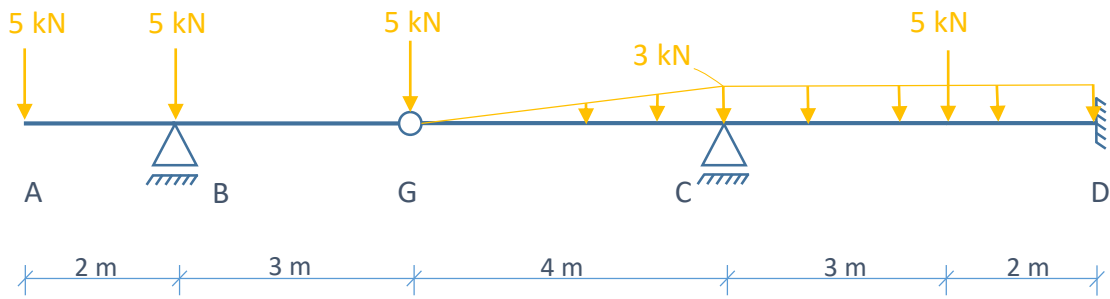
$$\delta_{2t} = 0 ; \delta_{22} = 258,82$$

$$16,80 + 199,85 \times X_1 + 219,12 \times X_2 = 0$$

$$0 + 219,12 \times X_1 + 258,82 \times X_2 = 0$$

$$X_1 = -1,172 \text{ kN} ; X_2 = 0,992 \text{ kN}$$



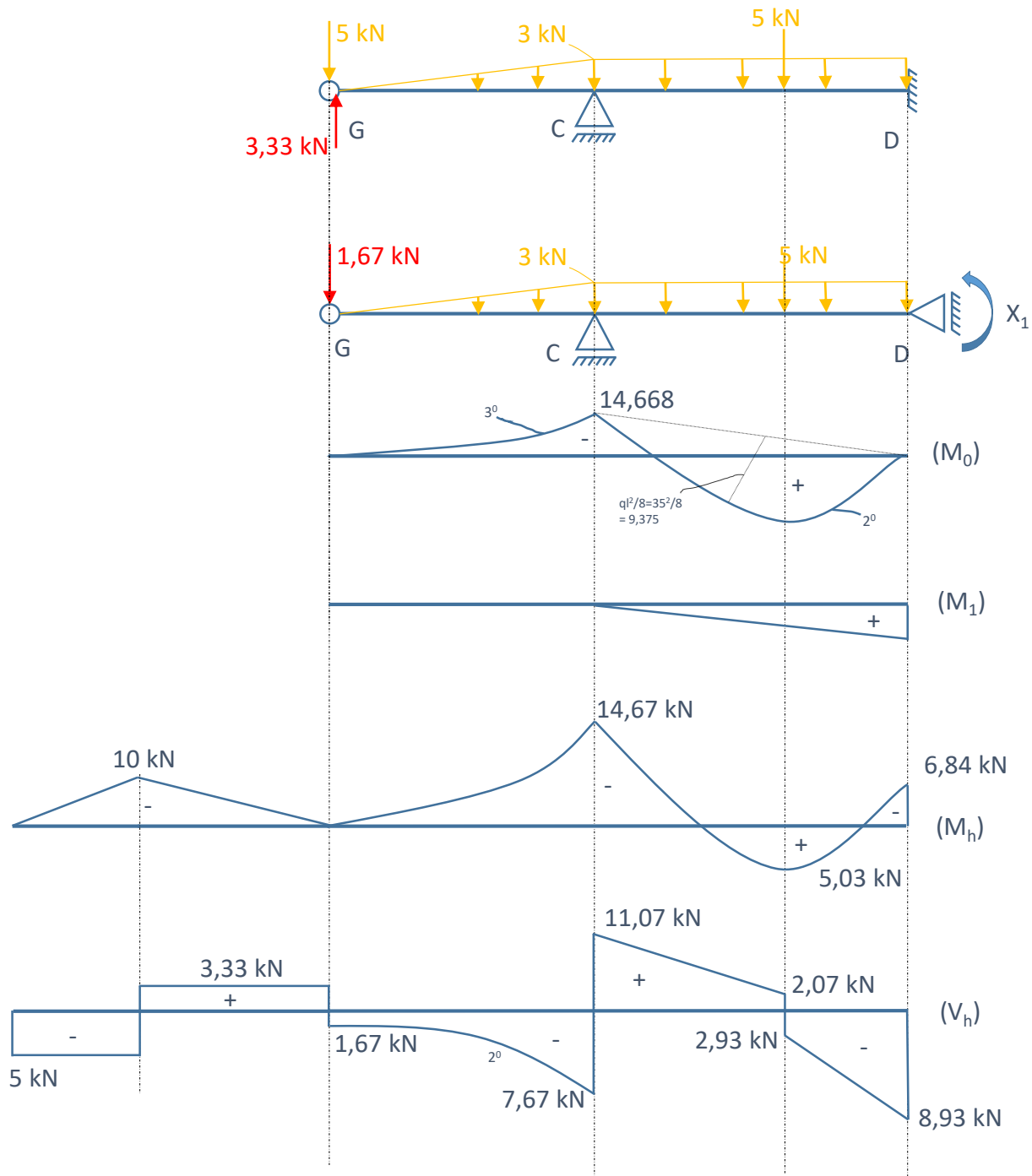


$$\delta_{10} = \delta \frac{MM_0}{EI} dx = \frac{(-14,68) \cdot (1) \cdot 5}{6E(3I)} + \frac{(9,375) \cdot (1) \cdot 5}{3E(3I)} + \frac{6(1) \cdot 5 \cdot (1 + 3/5)}{6E(3I)} = \frac{3,80}{EI}$$

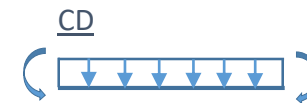
$$\delta_{11} = \delta \frac{MM_0}{EI} dx = \frac{(1) \cdot (1) \cdot 5}{3E(3I)} = \frac{3,80}{EI}$$

$$\delta_{11} \cdot X_1 + \delta_{10} = 0$$

$$X_1 = \frac{-3,80/EI}{5/9EI} = -6,84 \text{ kNm}$$



$$M_C = -14,668 + 0 \cdot (6,84) = -14,668 \text{ kNm}$$



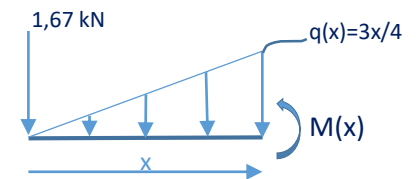
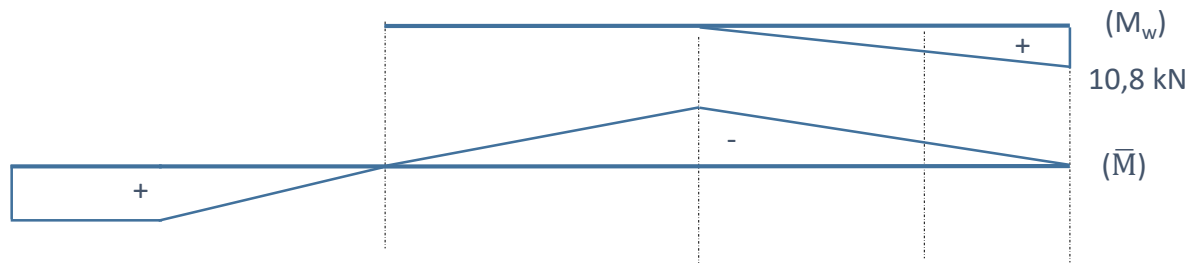
$$V_{CD} = \frac{3 \times 5}{2} + 2 + \frac{(14,67 - 6,84)}{5} = 11,07 \text{ kN}$$

$$V_{CD} = \frac{3 \times 5}{2} + 3 - \frac{(14,67 - 6,84)}{5} = 8,93 \text{ kN}$$

b) $\delta_{11} \cdot X_1 + \delta_{1w} = 0 \longrightarrow \delta_{1w} = -\left(-\frac{1}{5}(0,004) + \frac{1}{5}(0,005)\right) = -0,0002$

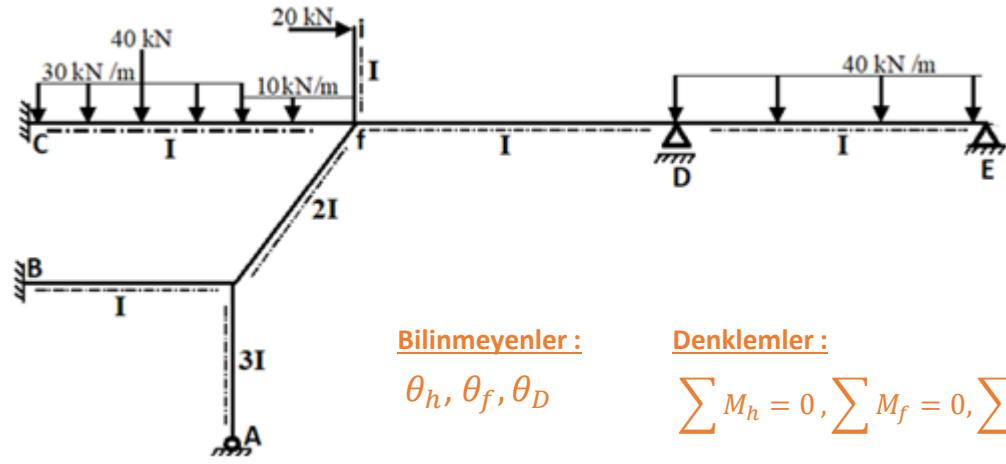
$$\frac{5}{9EI} X_1 = 0,0002 \quad X_1 = 10,8 \text{ kNm}$$

c)



$$M(x) = \frac{-x^3}{8} - 1,67x$$

$$\vartheta_A = \int \frac{M_h \bar{M}}{EI} dx = \left(\int_0^4 \left(-\frac{x}{3} \right) \left(-\frac{x^3}{3} - 1,67x \right) dx \right) \frac{1}{2EI} + \left(\frac{4}{3} \right) \cdot \frac{5}{6} [2(-14,67) - 6,84] \cdot \frac{1}{3EI} + \frac{(9,375) \cdot \left(-\frac{4}{3} \right) \cdot 5}{3EI} + \frac{6 \cdot \left(-\frac{4}{3} \right) \cdot 5}{3EI} + \frac{6 \cdot \left(-\frac{4}{3} \right) \cdot 5 \cdot \left(1 + \frac{2}{5} \right)}{6EI} + \frac{(-10) \cdot (1) \cdot 2}{2EI} + \frac{(-10) \cdot (1) \cdot 3}{3EI} = -0,00022 \text{ rad}$$



Bilinmeyenler :

$$\theta_h, \theta_f, \theta_D$$

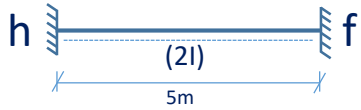
Denklemler :

$$\sum M_h = 0, \sum M_f = 0, \sum M_D = 0$$

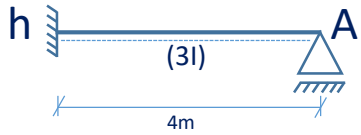
Ankastrelik Uç Momentleri :

$$M_{hf} = M_{hA} = M_{hB} = 0$$

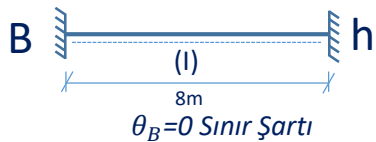
Birim Yer Değiştirme Sabitleri :



$$m_h \theta_h = \frac{4(2EI)}{5} = 1,6EI;$$

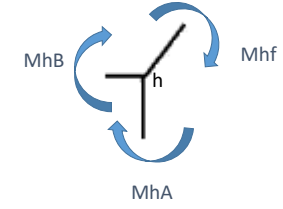


$$m_h \theta_h = \frac{3(3EI)}{4} = 2,25EI;$$



$$m_h \theta_h = \frac{4EI}{8} = 0,5EI;$$

h Düğüm noktası :



$$\sum M_h = M_{hf} + M_{hA} + M_{hB} = 0$$

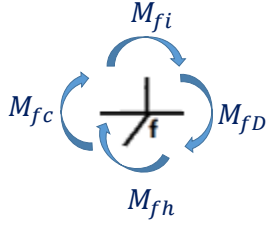
$$\sum M_h = M_{hf} + m_h \theta_h \cdot h_f + m_h \theta_f \cdot \theta_f + M_{hA} + m_h \theta_h \cdot \theta_h + m_h \theta_A \cdot \theta_A + M_{hB} + m_h \theta_h \cdot \theta_h + m_h \theta_B \cdot \theta_B = 0$$

$$m_h \theta_f = \frac{2(2EI)}{5} = 0,8EI;$$

$$m_h \theta_B = \frac{2EI}{8} = 0,25EI$$

$$\sum M_h = 0 = 0 + 1,6EI \cdot \theta_h + 0,8EI \cdot \theta_f + 0 + 2,25EI \cdot \theta_h + 0 + 0 + 0,5EI \cdot \theta_h + 0 = 0 \quad \left. \vphantom{\sum M_h} \right\} \text{ 1. Denklem}$$

f Döğüm noktası :

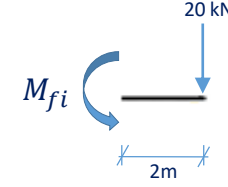


$$\sum M_f = M_{fi} + M_{fD} + M_{fh} + M_{fc} = 0$$

$$M_{fD} = \dot{M}_{fD} + m_f \theta_f \cdot \theta_f + m_f \theta_D \cdot \theta_D$$

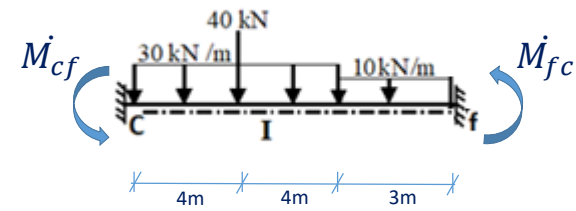
$$M_{fD} = \dot{M}_{fh} + m_f \theta_f \cdot \theta_f + m_f \theta_h \cdot \theta_h$$

$$M_{fD} = \dot{M}_{fc} + m_f \theta_f \cdot \theta_f + m_f \theta_c \cdot \theta_c$$



$$M_{fi} = 40 \text{ kNm}$$

Ankastrelik Uç Momentleri :



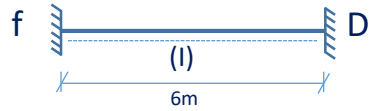
$$\dot{M}_{fD} = \dot{M}_{fh} = 0$$

$$\theta_c = 0 \text{ Sınır Şartı}$$

$$\dot{M}_{fc} = \left(-\frac{10 \cdot 11^2}{12} \right) + \left(-40 \cdot 7 \cdot \left(\frac{4}{11} \right)^2 \right) + \left(-\frac{20 \cdot 8^2}{3} \cdot \frac{8}{11} \cdot \left(1 - 0,75 \cdot \frac{8}{11} \right) \right) = -278,905 \text{ kNm}$$

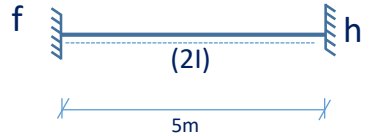
$$\dot{M}_{fc} = \left(\frac{10 \cdot 11^2}{12} \right) + \left(40 \cdot 4 \cdot \left(\frac{7}{11} \right)^2 \right) + \left(\frac{20 \cdot 8^2}{3} \cdot \left(1,5 - 2 \cdot \frac{8}{11} \right) + 0,75 \cdot \left(\frac{8}{11} \right)^2 \right) = 354,277 \text{ kNm}$$

Birim Yer Değıştirme Sabitleri :



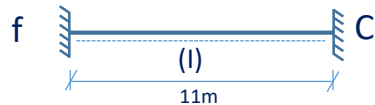
$$m_f \theta_f = \frac{4EI}{6} = 0,667EI ;$$

$$m_f \theta_f = \frac{2EI}{6} = 0,333EI$$



$$m_f \theta_f = \frac{4(2EI)}{6} = 1,6EI ;$$

$$m_f \theta_f = \frac{2(2EI)}{5} = 0,8EI$$



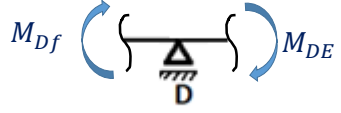
$$m_f \theta_f = \frac{4EI}{11} = 0,364EI ;$$

$$m_f \theta_f = \frac{2EI}{11} = 0,182EI$$

$$\sum M_f = +40 + 0 + 0,667EI \theta_f + 0,333EI \theta_D + 0 + 1,6EI \theta_f + 0,8EI \theta_h - 278,905 + 0,364EI \theta_f + 0 = 0$$

$$\rightarrow \sum M_f = \left(-\frac{238,905}{EI} \right) + 2,631\theta_f + 0,333\theta_D + 0,8\theta_h = 0 \quad \left. \vphantom{\sum M_f} \right\} \text{ 2. Denklem}$$

D Düğüm noktası :

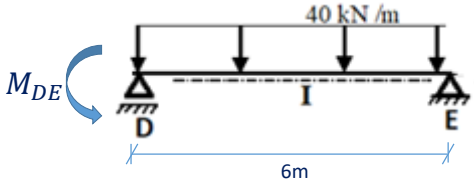


$$\sum M_D = M_{DE} + M_{Df} = 0$$

$$M_{DE} = \dot{M}_{DE} + m_D^{\text{D}_E} \theta_D \cdot \theta_D + m_D^{\text{D}_E} \theta_E \cdot \theta_E$$

$$M_{Df} = \dot{M}_{Df} + m_D^{\text{D}_f} \theta_D \cdot \theta_D + m_D^{\text{D}_f} \theta_f \cdot \theta_f$$

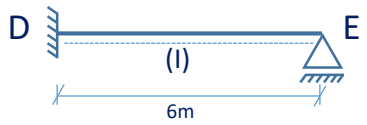
Ankastrelik Uç Momentleri :



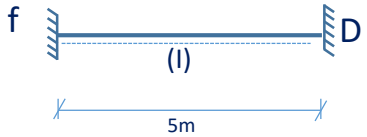
$$\dot{M}_{Df} = 0$$

$$\dot{M}_{DE} = \frac{40 \cdot 6^2}{8} = 180 \text{ kNm}$$

Birim Yer Değiştirme Sabitleri :



$$m_D^{\text{D}_E} \theta_D = \frac{3EI}{6} = 0,5EI ;$$



$$m_D^{\text{D}_f} \theta_D = \frac{4EI}{6} = 0,667EI ;$$

$$m_D^{\text{D}_f} \theta_f = \frac{2EI}{6} = 0,333EI$$

$$\sum M_D = 180 + 0,5EI \theta_D + 0 + 0 + 0,667EI \cdot \theta_D + 0,333EI \cdot \theta_f = 0$$

$$\Rightarrow \sum M_D = \left(\frac{180}{EI} \right) + 1,167\theta_D + 0,333\theta_f = 0 \quad \left. \vphantom{\sum M_D} \right\} \text{ 3. Denklem}$$

$$4,35 \cdot \theta_h + 0,8 \cdot \theta_f$$

$$0,8 \cdot \theta_h + 2,631 \cdot \theta_f + 0,333 \cdot \theta_D = 238,905/EI$$

$$0,333 \cdot \theta_f + 1,167 \cdot \theta_D = -180/EI$$

$$\theta_h = -22,347/EI$$

$$\theta_f = 121,509/EI$$

$$\theta_D = -189,914/EI$$

Çubuk Uç Momentleri:

$$M_{hf} = \dot{M}_{hf} + m_h \theta_h \cdot \theta_h + m_h \theta_f \cdot \theta_f = 0 + 1,6 \cdot (-22,347) + 0,8 \cdot 121,509 = -35,76 + 97,21 = 61,45 \text{ kNm}$$

$$M_{hA} = \dot{M}_{hA} + m_h \theta_h \cdot \theta_h + m_h \theta_A \cdot \theta_A = 0 + 2,25 \cdot (-22,347) + 0 = -50,28 \text{ kNm}$$

$$M_{hB} = \dot{M}_{hB} + m_h \theta_h \cdot \theta_h + m_h \theta_B \cdot \theta_B = 0 + 0,5 \cdot (-22,347) + 0 = -11,17 \text{ kNm}$$

$$M_{fi} = 40 \text{ kNm}$$

$$M_{fD} = \dot{M}_{fD} + m_f \theta_f \cdot \theta_f + m_f \theta_D \cdot \theta_D = 0 + 0,667 \cdot (121,509) + 0,333 \cdot (-188,914) = 81,05 - 62,91 = 18,14 \text{ kNm}$$

$$M_{fh} = \dot{M}_{fh} + m_f \theta_f \cdot \theta_f + m_f \theta_h \cdot \theta_h = 0 + 1,6 \cdot (121,509) + 0,8 \cdot (-22,347) = 194,41 - 17,88 = 176,53 \text{ kNm}$$

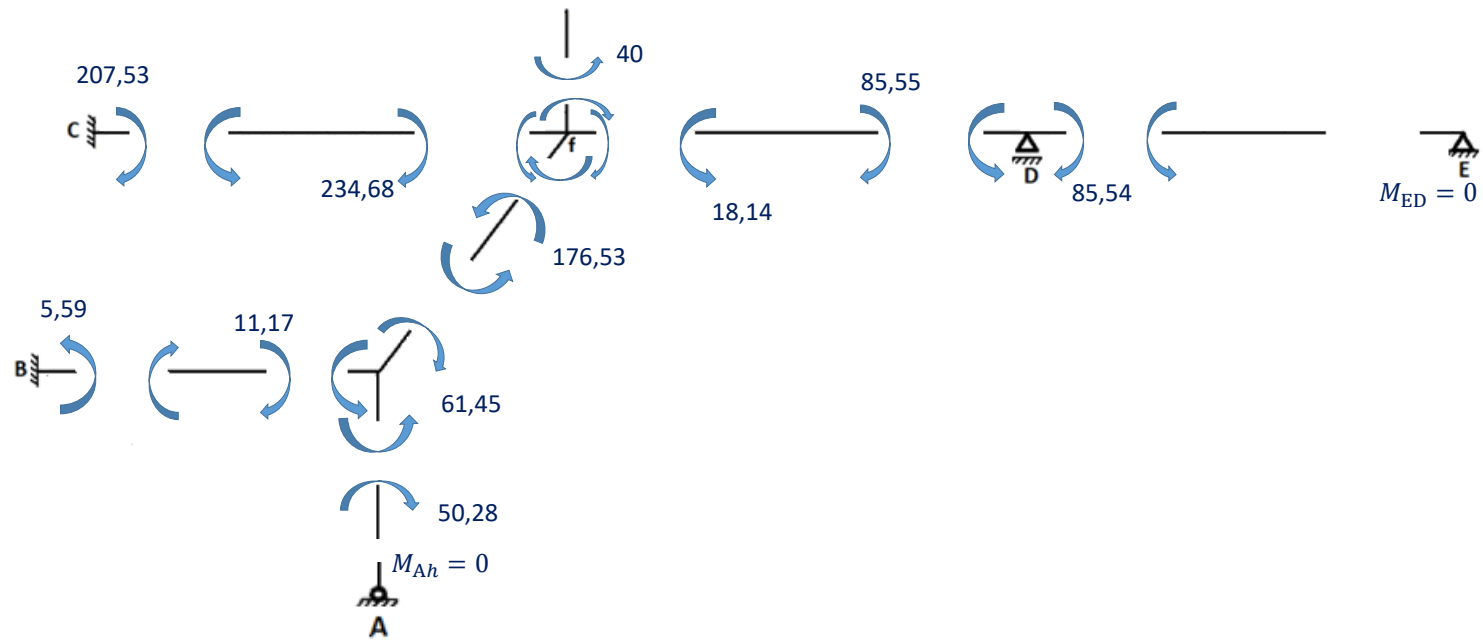
$$M_{fC} = \dot{M}_{fC} + m_f \theta_f \cdot \theta_f + m_f \theta_C \cdot \theta_C = (-278,905) + 0,364 \cdot 121,509 + 0 = (-278,915) + 44,23 = -234,68 \text{ kNm}$$

$$M_{DE} = \dot{M}_{DE} + m_D \theta_D \cdot \theta_D + m_D \theta_E \cdot \theta_E = 180 + 0,5 \cdot (188,914) + 0 = 180 - 94,46 = 85,54 \text{ kNm}$$

$$M_{Df} = \dot{M}_{Df} + m_D \theta_D \cdot \theta_D + m_D \theta_f \cdot \theta_f = 0 + 0,667 \cdot (-188,914) + 0,333 \cdot 121,509 = -126,01 + 40,46 = 85,55 \text{ kNm}$$

$$M_{Bh} = \dot{M}_{Bh} + m_B \theta_B \cdot \theta_B + m_B \theta_h \cdot \theta_h = 0 + 0 + 0,25 \cdot (-22,347) = -5,59 \text{ kNm}$$

$$M_{cf} = \dot{M}_{cf} + m_c \theta_c \cdot \theta_c + m_c \theta_f \cdot \theta_f = 185,417 + 0 + 0,182 \cdot 121,509 + 354,28 + 22,11 = 376,39 \text{ kNm}$$



Düğüm Noktası Kontrolü:

$$\sum M_h = 61,45 - 50,28 - 11,17 = 0 \quad \checkmark$$

$$\sum M_f = 40 + 18,14 + 176,53 - 234,68 \cong 0 \quad \checkmark$$

$$\sum M_D = 85,54 - 85,55 \cong 0 \quad \checkmark$$

