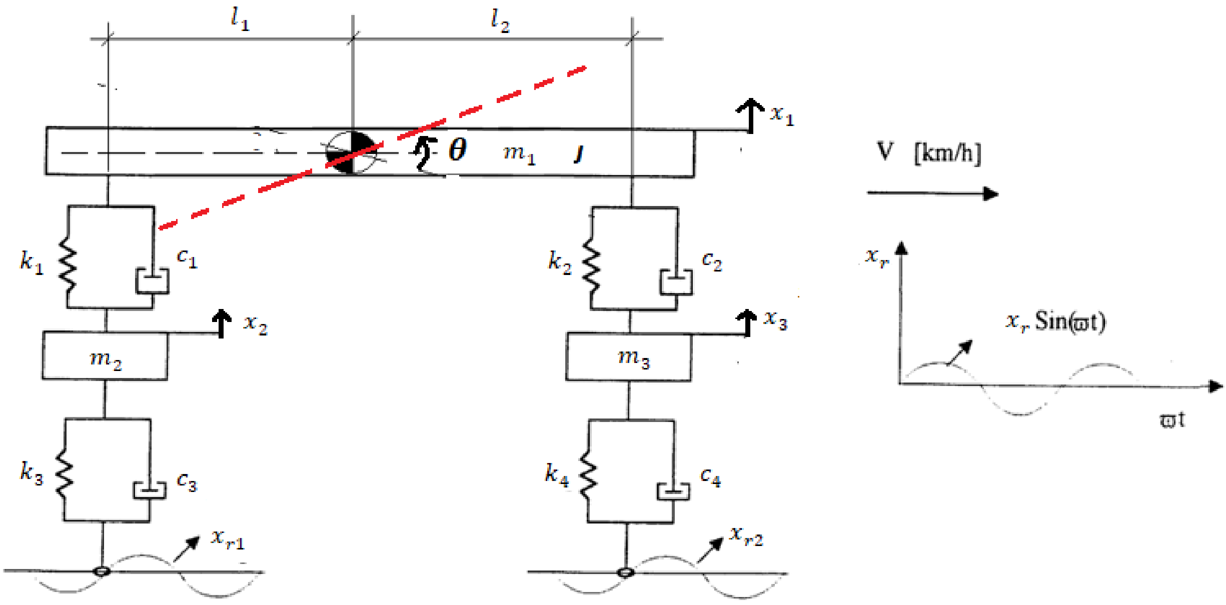


Lagrange Denklemi

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} + \frac{\partial E_c}{\partial \dot{q}_i} = Q_i$$

q_i: Genelleştirilmiş Koordinat

E_k: Kinetik Enerji

E_p: Potansiyel Enerji

E_c: Sönüm Enerjisi

Q_i: Genelleştirilmiş Kuvvet

$$E_k = \frac{1}{2}J\left(\dot{\theta}\right)^2 + \frac{1}{2}m_1(\dot{x}_1)^2 + \frac{1}{2}m_2(\dot{x}_2)^2 + \frac{1}{2}m_3(\dot{x}_3)^2$$

$$E_p = \frac{1}{2}k_1(x_1 - x_2 - l_1\theta)^2 + \frac{1}{2}k_2(x_1 - x_3 + l_2\theta)^2 + \frac{1}{2}k_3(x_2 - x_{r1})^2 + \frac{1}{2}k_4(x_3 - x_{r2})^2$$

$$E_p = \frac{1}{2}c_1\left(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}\right)^2 + \frac{1}{2}c_2\left(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}\right)^2 + \frac{1}{2}c_3(\dot{x}_2 - \dot{x}_{r1})^2 + \frac{1}{2}c_4(\dot{x}_3 - \dot{x}_{r2})^2$$

θ Genelleştirilmiş Koordinatı İçin

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{\theta}}\right) = \frac{d}{dt}\left(\frac{1}{2}2J\dot{\theta}\right) = \frac{d}{dt}\left(J\dot{\theta}\right) = J\ddot{\theta}$$

$$\frac{\partial E_k}{\partial \theta} = 0$$

$$\frac{\partial E_p}{\partial \theta} = -\frac{1}{2}2k_1(x_1 - x_2 - l_1\theta)l_1 + \frac{1}{2}2k_2(x_1 - x_3 + l_2\theta)l_2$$

$$\frac{\partial E_p}{\partial \theta} = -k_1 l_1(x_1 - x_2 - l_1\theta) + k_2 l_2(x_1 - x_3 + l_2\theta)$$

$$\frac{\partial E_c}{\partial \dot{\theta}} = -\frac{1}{2}2c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta})l_1 + \frac{1}{2}2c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta})l_2$$

$$\frac{\partial E_c}{\partial \dot{\theta}} = -c_1 l_1\left(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}\right) + c_2 l_2\left(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}\right)$$

$$Q_{z_s} = 0$$

$$\boldsymbol{J\ddot{\theta} - k_1 l_1(x_1 - x_2 - l_1\theta) + k_2 l_2(x_1 - x_3 + l_2\theta)}$$
$$\boldsymbol{-c_1 l_1\left(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}\right) + c_2 l_2\left(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}\right) = 0}$$

$$\boldsymbol{\ddot{\theta} = \left(\frac{-1}{J}\right)\left(-k_1 l_1(x_1 - x_2 - l_1\theta) + k_2 l_2(x_1 - x_3 + l_2\theta) - c_1 l_1\left(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}\right) + c_2 l_2\left(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}\right)\right)}$$

x_1 Genelleştirilmiş Koordinatı İçin

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}_1}\right) = \frac{d}{dt}\left(\frac{1}{2}2m_1\dot{x}_1\right) = \frac{d}{dt}(m_1\dot{x}_1) = m_1\ddot{x}_1$$

$$\frac{\partial E_k}{\partial x_1} = 0$$

$$\frac{\partial E_p}{\partial x_1} = \frac{1}{2}2k_1(x_1 - x_2 - l_1\theta) + \frac{1}{2}2k_2(x_1 - x_3 + l_2\theta)$$

$$\frac{\partial E_p}{\partial x_1} = k_1(x_1 - x_2 - l_1\theta) + k_2(x_1 - x_3 + l_2\theta)$$

$$\frac{\partial E_c}{\partial \dot{x}_1} = \frac{1}{2}2c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}) + \frac{1}{2}2c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta})$$

$$\frac{\partial E_c}{\partial \dot{x}_1} = c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}) + c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta})$$

$$Q_{x_1} = 0$$

$$\boldsymbol{m_1\ddot{x}_1 + k_1(x_1 - x_2 - l_1\theta) + k_2(x_1 - x_3 + l_2\theta)}$$
$$\boldsymbol{+c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}) + c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}) = 0}$$

$$\boldsymbol{\ddot{x}_1 = \left(\frac{-1}{m_1}\right)\left(k_1(x_1 - x_2 - l_1\theta) + k_2(x_1 - x_3 + l_2\theta) + c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}) + c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta})\right)}$$

x_2 Genelleştirilmiş Koordinatı İçin

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}_2}\right) = \frac{d}{dt}\left(\frac{1}{2}2m_2\dot{x}_2\right) = \frac{d}{dt}(m_2\dot{x}_2) = m_2\ddot{x}_2$$

$$\frac{\partial E_k}{\partial x_2} = 0$$

$$\frac{\partial E_p}{\partial x_2} = -\frac{1}{2}2k_1(x_1 - x_2 - l_1\theta) + \frac{1}{2}2k_3(x_2 - x_{r1})$$

$$\frac{\partial E_p}{\partial x_2} = -k_1(x_1 - x_2 - l_1\theta) + k_3(x_2 - x_{r1})$$

$$\frac{\partial E_c}{\partial \dot{x}_2} = -\frac{1}{2}2c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}) + \frac{1}{2}2c_3(\dot{x}_2 - \dot{x}_{r1})$$

$$\frac{\partial E_c}{\partial \dot{x}_2} = -c_1(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}) + c_3(\dot{x}_2 - \dot{x}_{r1})$$

$$Q_{x_2} = 0$$

$$\boldsymbol{m_2\ddot{x}_2 - k_1(x_1 - x_2 - l_1\theta) + k_3(x_2 - x_{r1}) - c_1\left(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}\right) + c_3(\dot{x}_2 - \dot{x}_{r1}) = 0}$$

$$\boldsymbol{\ddot{x}_2 = \left(\frac{-1}{m_2}\right)\left(-k_1(x_1 - x_2 - l_1\theta) + k_3(x_2 - x_{r1}) - c_1\left(\dot{x}_1 - \dot{x}_2 - l_1\dot{\theta}\right) + c_3(\dot{x}_2 - \dot{x}_{r1})\right)}$$

x_3 Genelleştirilmiş Koordinatı İçin

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{x}_3}\right) = \frac{d}{dt}\left(\frac{1}{2}2m_3\dot{x}_3\right) = \frac{d}{dt}(m_3\dot{x}_3) = m_3\ddot{x}_3$$

$$\frac{\partial E_k}{\partial x_3} = 0$$

$$\frac{\partial E_p}{\partial x_3} = -\frac{1}{2}2k_2(x_1 - x_3 + l_2\theta) + \frac{1}{2}2k_4(x_3 - x_{r2})$$

$$\frac{\partial E_p}{\partial x_2} = -k_2(x_1 - x_3 + l_2\theta) + k_4(x_3 - x_{r2})$$

$$\frac{\partial E_c}{\partial \dot{x}_3} = -\frac{1}{2}2c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}) + \frac{1}{2}2c_4(\dot{x}_3 - \dot{x}_{r2})$$

$$\frac{\partial E_c}{\partial \dot{x}_3} = -c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}) + c_4(\dot{x}_3 - \dot{x}_{r2})$$

$$Q_{x_3} = 0$$

$$\boldsymbol{m_3\ddot{x}_3 - k_2(x_1 - x_3 + l_2\theta) + k_4(x_3 - x_{r2}) - c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}) + c_4(\dot{x}_3 - \dot{x}_{r2}) = 0}$$

$$\boldsymbol{\ddot{x}_3 = \left(\frac{-1}{m_3}\right)\left(-k_2(x_1 - x_3 + l_2\theta) + k_4(x_3 - x_{r2}) - c_2(\dot{x}_1 - \dot{x}_3 + l_2\dot{\theta}) + c_4(\dot{x}_3 - \dot{x}_{r2})\right)}$$

$\theta \rightarrow \mathrm{u}(1)$

$\dot{\theta} \rightarrow \mathrm{u}(2)$

$x_1 \rightarrow \mathrm{u}(3)$

$\dot{x}_1 \rightarrow \mathrm{u}(4)$

$x_2 \rightarrow \mathrm{u}(5)$

$\dot{x}_2 \rightarrow \mathrm{u}(6)$

$x_3 \rightarrow \mathrm{u}(7)$

$\dot{x}_3 \rightarrow \mathrm{u}(8)$

$x_{r1} \rightarrow \mathrm{u}(9)$

$\dot{x}_{r1} \rightarrow \mathrm{u}(10)$

$x_{r2} \rightarrow \mathrm{u}(11)$

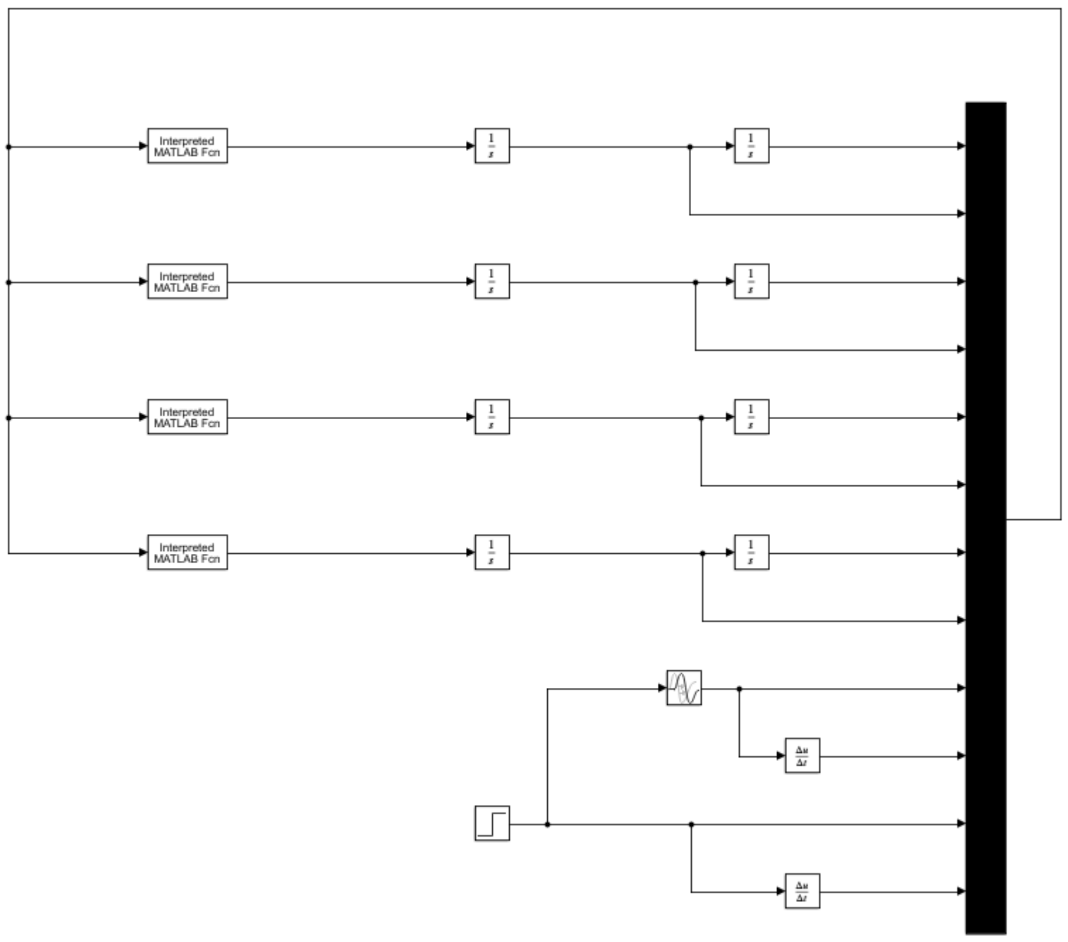
$\dot{x}_{r2} \rightarrow \mathrm{u}(12)$

$$\ddot{\boldsymbol{\theta}} = (-\mathbf{1}/\boldsymbol{J}) * \begin{pmatrix} -\boldsymbol{k1} * \boldsymbol{l1} * \big(\mathrm{u}(3) - \mathrm{u}(5) - \boldsymbol{l1} * \boldsymbol{u}(\mathbf{1})\big) + \boldsymbol{k2} * \boldsymbol{l2} * \big(\mathrm{u}(3) - \mathrm{u}(7) + \boldsymbol{l2} * \boldsymbol{u}(\mathbf{1})\big) \\ -\boldsymbol{c1} * \boldsymbol{l1} * \big(\mathrm{u}(4) - \mathrm{u}(6) - \boldsymbol{l1} * \boldsymbol{u}(\mathbf{2})\big) + \boldsymbol{c2} * \boldsymbol{l2} * \big(\mathrm{u}(4) - \mathrm{u}(8) + \boldsymbol{l2} * \boldsymbol{u}(\mathbf{2})\big) \end{pmatrix}$$

$$\ddot{\boldsymbol{x}}_{\mathbf{1}} = (-\mathbf{1}/\boldsymbol{m1}) * \begin{pmatrix} \boldsymbol{k1} * \big(\mathrm{u}(3) - \mathrm{u}(5) - \boldsymbol{l1} * \boldsymbol{u}(\mathbf{1})\big) + \boldsymbol{k2} * \big(\mathrm{u}(3) - \mathrm{u}(7) + \boldsymbol{l2} * \boldsymbol{u}(\mathbf{1})\big) \\ +\boldsymbol{c1} * \big(\mathrm{u}(4) - \mathrm{u}(6) - \boldsymbol{l1} * \boldsymbol{u}(\mathbf{2})\big) + \boldsymbol{c2} * \big(\mathrm{u}(4) - \mathrm{u}(8) + \boldsymbol{l2} * \boldsymbol{u}(\mathbf{2})\big) \end{pmatrix}$$

$$\ddot{\boldsymbol{x}}_{\mathbf{2}} = (-\mathbf{1}/\boldsymbol{m2}) * \begin{pmatrix} -\boldsymbol{k1} * \big(\mathrm{u}(3) - \mathrm{u}(5) - \boldsymbol{l1} * \boldsymbol{u}(\mathbf{1})\big) + \boldsymbol{k3} * \big(\mathrm{u}(5) - \mathrm{u}(9)\big) \\ -\boldsymbol{c1} * \big(\mathrm{u}(4) - \mathrm{u}(6) - \boldsymbol{l1} * \boldsymbol{u}(\mathbf{2})\big) + \boldsymbol{c3} * \big(\mathrm{u}(6) - \mathrm{u}(10)\big) \end{pmatrix}$$

$$\ddot{\boldsymbol{x}}_{\mathbf{3}} = (-\mathbf{1}/\boldsymbol{m3}) * \begin{pmatrix} -\boldsymbol{k2} * \big(\mathrm{u}(3) - \mathrm{u}(7) + l2 * u(1)\big) + \boldsymbol{k4} * \big(\mathrm{u}(7) - \boldsymbol{u}(\mathbf{11})\big) \\ -\boldsymbol{c2} * \big(\mathrm{u}(4) - \mathrm{u}(8) + l2 * u(2)\big) + \boldsymbol{c4} * (\mathrm{u}(8) - \boldsymbol{u}(\mathbf{12})) \end{pmatrix}$$




```
clear all, clc

J=1000;        %kg*m^2;

m1=500;      %kg
m2=50;       %kg
m3=50;       %kg

k1=17000;      %N/m
k2=15000;      %N/m
k3=220000;      %N/m
k4=220000;      %N/m

c1=500;        %Ns/m
c2=500;        %Ns/m
c3=1000;        %Ns/m
c4=1000;        %Ns/m

l1=1.5;        %m
l2=1.8;        %m

V=50*(1000/3600); %km/h*(1000/3600)=m/s
```