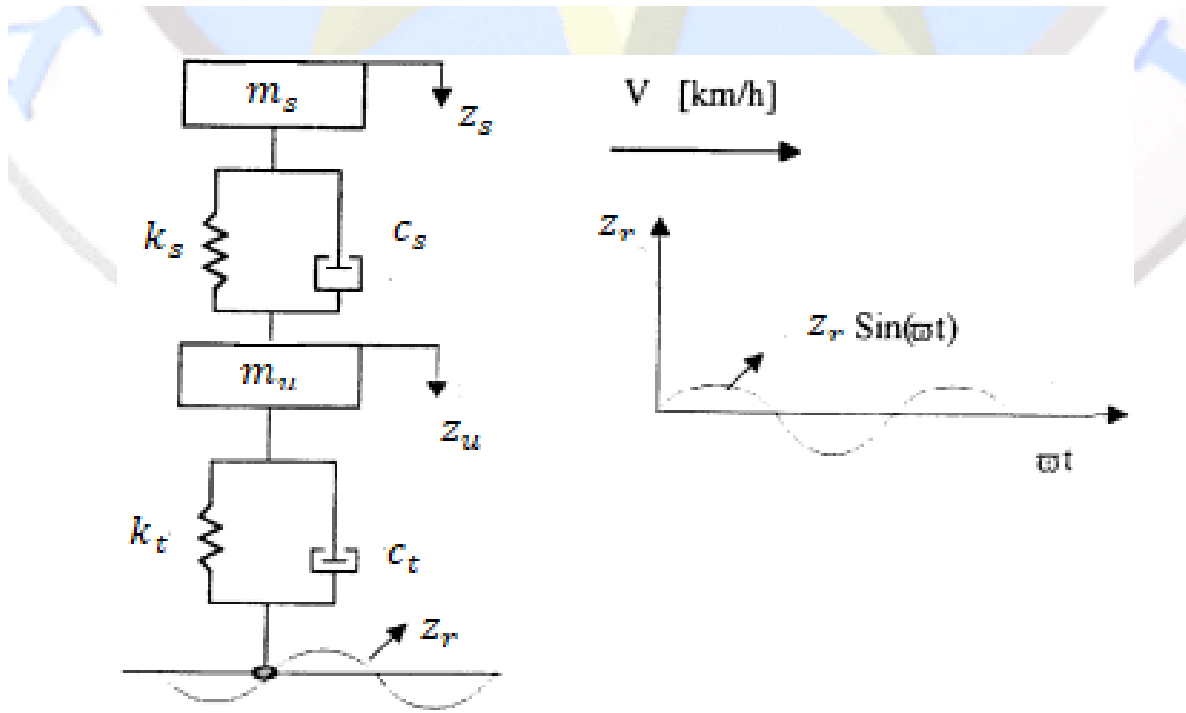




Şekil 2. Çeyrek taşıt modeli

Lagrange Denklemi

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{q}_i} \right) - \frac{\partial E_k}{\partial q_i} + \frac{\partial E_p}{\partial q_i} + \frac{\partial E_c}{\partial \dot{q}_i} = Q_i$$

q_i : Genelleştirilmiş Koordinat

E_k : Kinetik Enerji

E_p : Potansiyel Enerji

E_c : Sönüm Enerjisi Enerji

Q_i : Genelleştirilmiş Kuvvet

$$E_k = \frac{1}{2} m_s (\dot{z}_s)^2 + \frac{1}{2} m_u (\dot{z}_u)^2$$

$$E_p = \frac{1}{2} k_s (z_s - z_u)^2 + \frac{1}{2} k_t (z_u - z_r)^2$$

$$E_c = \frac{1}{2} c_s (\dot{z}_s - \dot{z}_u)^2 + \frac{1}{2} c_t (\dot{z}_u - \dot{z}_r)^2$$

z_s Genelleştirilmiş Koordinatı İçin

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{z}_s} \right) = \frac{d}{dt} \left(\frac{1}{2} (m_s \dot{z}_s) \right) = \frac{d}{dt} (m_s \dot{z}_s) = m_s \ddot{z}_s$$

$$\frac{\partial E_k}{\partial z_s} = 0$$

$$\frac{\partial E_p}{\partial z_s} = \frac{1}{2} 2k_s(z_s - z_u) = k_s(z_s - z_u)$$

$$\frac{\partial E_c}{\partial \dot{z}_s} = \frac{1}{2} 2c_s(\dot{z}_s - \dot{z}_u) = c_s(\dot{z}_s - \dot{z}_u)$$

$$Q_{z_s} = 0$$

$$\mathbf{m}_s \ddot{\mathbf{z}}_s + \mathbf{k}_s(\mathbf{z}_s - \mathbf{z}_u) + \mathbf{c}_s(\dot{\mathbf{z}}_s - \dot{\mathbf{z}}_u) = \mathbf{0}$$

z_u Genelleştirilmiş Koordinatı İçin

$$\frac{d}{dt} \left(\frac{\partial E_k}{\partial \dot{z}_u} \right) = \frac{d}{dt} \left(\frac{1}{2} (m_u \dot{z}_u) \right) = \frac{d}{dt} (m_u \dot{z}_u) = m_u \ddot{z}_u$$

$$\frac{\partial E_k}{\partial z_u} = 0$$

$$\frac{\partial E_p}{\partial z_u} = -\frac{1}{2} 2k_s(z_s - z_u) + \frac{1}{2} 2k_t(z_u - z_r) = -k_s(z_s - z_u) + k_t(z_u - z_r)$$

$$\frac{\partial E_c}{\partial \dot{z}_u} = -\frac{1}{2} 2c_s(\dot{z}_s - \dot{z}_u) + \frac{1}{2} 2c_t(\dot{z}_u - \dot{z}_r) = -c_s(\dot{z}_s - \dot{z}_u) + c_t(\dot{z}_u - \dot{z}_r)$$

$$Q_{z_u} = 0$$

$$\mathbf{m}_u \ddot{\mathbf{z}}_u + -\mathbf{k}_s(\mathbf{z}_s - \mathbf{z}_u) + \mathbf{k}_t(\mathbf{z}_u - \mathbf{z}_r) - \mathbf{c}_s(\dot{\mathbf{z}}_s - \dot{\mathbf{z}}_u) + \mathbf{c}_t(\dot{\mathbf{z}}_u - \dot{\mathbf{z}}_r) = \mathbf{0}$$

$$m_s\ddot{z}_s + k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u) = 0$$

$$m_u\ddot{z}_u - k_s(z_s - z_u) + k_t(z_u - z_r) - c_s(\dot{z}_s - \dot{z}_u) + c_t(\dot{z}_u - \dot{z}_r) = 0$$

$$\ddot{z}_s = \left(\frac{-1}{m_s}\right) (k_s(z_s - z_u) + c_s(\dot{z}_s - \dot{z}_u))$$

$$\ddot{z}_u = \left(\frac{-1}{m_u}\right) (-k_s(z_s - z_u) + k_t(z_u - z_r) - c_s(\dot{z}_s - \dot{z}_u) + c_t(\dot{z}_u - \dot{z}_r))$$

Durum Uzayı (State Space) Formu

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

Değişken Dönüşümleri

$$x_1 = z_s \rightarrow \dot{x}_1 = \dot{z}_s = x_3$$

$$x_2 = z_u \rightarrow \dot{x}_2 = \dot{z}_u = x_4$$

$$x_3 = \dot{z}_s \rightarrow \dot{x}_3 = \ddot{z}_s$$

$$x_4 = \dot{z}_u \rightarrow \dot{x}_4 = \ddot{z}_u$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} z_s \\ z_u \\ \dot{z}_s \\ \dot{z}_u \end{bmatrix}$$

$$u = \begin{bmatrix} z_r \\ \dot{z}_r \end{bmatrix}$$

$$y = \begin{bmatrix} z_s - z_u \\ z_u - z_r \\ \ddot{z}_s \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -k_s/m_s & k_s/m_s & -c_s/m_s & c_s/m_s \\ k_s/m_u & -(k_s + k_t)/m_u & c_s/m_u & -c_s/m_u \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ k_t/m_u & c_t/m_u \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ -k_s/m_s & k_s/m_s & -c_s/m_s & c_s/m_s \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$