Soru 1 : 1) $e^{yz} + \sin(\pi yz) - xyz = 0 \qquad (x,y,z) = (e,1,1)$

$$\frac{\partial x}{\partial z} = -\frac{F_z}{F_x} = \cancel{-}\,\frac{ye^{yz} + \pi y\cos(\pi yz) - xy}{\cancel{-}yz}$$

$$(e,1,1) \longrightarrow \quad = \frac{e - \pi - e}{1} = \underline{-\pi}_{\,//} \qquad \boxed{\text{C şıkkı}}$$

2) $$\frac{\partial y}{\partial x} = -\frac{F_x}{F_y} = \cancel{-}\,\frac{\cancel{-}yz}{ze^{yz} + \pi z\cos(\pi yz) - xz}$$

$$(e,1,1) \longrightarrow \quad = \frac{1}{e - \pi - e} = \underline{-\frac{1}{\pi}}_{\,//} \qquad \boxed{\text{A şıkkı}}$$

3) $e^{xz} - \sin(\pi xz) - yxz = 0 \qquad (1,e,1)$

$$\frac{\partial x}{\partial y} = -\frac{F_y}{F_x} = \cancel{-}\,\frac{\cancel{-}xz}{ze^{xz} - \pi z\cos(\pi xz) - yz}$$

$$(1,e,1) \longrightarrow \quad = \frac{1}{\cancel{e} + \pi - \cancel{e}} = \underline{\frac{1}{\pi}}_{\,//} \qquad \boxed{\text{B şıkkı}}$$

4) $$\frac{\partial y}{\partial z} = -\frac{F_z}{F_y} = \cancel{-}\,\frac{xe^{xz} - \pi x\cos(\pi xz) - yx}{\cancel{-}xz}$$

$$(1,e,1) \longrightarrow \quad = \frac{\cancel{e} + \pi - \cancel{e}}{1} = \underline{\pi}_{\,//} \qquad \boxed{\text{D şıkkı}}$$

Soru 2:

(1)
$$f(x,y)=\begin{cases}(x^2+y)\cdot\sin\dfrac{1}{x+y} & ,(x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$$

$$f_x(0,0)=\lim_{h\to 0}\frac{f(0+h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{h^{\cancel{2}}\cdot\sin\frac{1}{h}}{\cancel{h}}$$

$$=\lim_{h\to 0} h\cdot\underbrace{\sin\frac{1}{h}}_{-1\leq A\leq 1}=\underline{0}$$

$$f_y(0,0)=\lim_{h\to 0}\frac{f(0,0+h)-f(0,0)}{h}=\lim_{h\to 0}\frac{\cancel{h}\cdot\sin\frac{1}{h}}{\cancel{h}}$$

$$=\lim_{h\to 0}\sin\frac{1}{h}\rightarrow \text{Mevcut değildir.}$$

$\boxed{\text{D şıkkı}}$

(2)
$$f(x,y)=\begin{cases}(x+y^3)\sin\dfrac{1}{x+y^2} & ,(x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$$

$$f_x(0,0)=\lim_{h\to 0}\frac{f(0+h,0)-f(0,0)}{h}=\lim_{h\to 0}\frac{\cancel{h}\cdot\sin\frac{1}{h}}{\cancel{h}}$$

$$=\lim_{h\to 0}\sin\frac{1}{h}\rightarrow \text{Mevcut değildir.}$$

$$f_y(0,0)=\lim_{h\to 0}\frac{f(0,0+h)-f(0,0)}{h}=\lim_{h\to 0}\frac{h^{\cancel{3}}\cdot\sin\frac{1}{h^2}}{\cancel{h}}$$

$$=\lim_{h\to 0}h^2\cdot\underbrace{\sin\frac{1}{h^2}}_{-1\leq\ \leq 1}=0$$

$\boxed{\text{B şıkkı}}$

Soru 3: (1) $f(x,y,z) = x^2+y^4+z^3 \quad (1,1,1)$

I. -6 II. -4 III. 1 IV. 5 V. 7

$\nabla f = \langle 2x, 4y^3, 3z^2 \rangle$

$\nabla f(1,1,1) = \langle 2,4,3 \rangle \quad |\nabla f| = \sqrt{29}$

$-\sqrt{29}$ ile $\sqrt{29}$ arasındaki değerler; II, III, IV

$\boxed{\text{C şıkkı}}$

(2) $f(x,y,z) = x^2+y^3+z^3 \quad (1,1,1)$

I. -6 II. -4 III. 1 IV. 5 V. 7

$\nabla f = \langle 2x, 3y^2, 3z^2 \rangle$

$\nabla f(1,1,1) = \langle 2,3,3 \rangle \quad |\nabla f| = \sqrt{22}$

$-\sqrt{22}$ ile $\sqrt{22}$ arasındaki değerler; II, III $\boxed{\text{D şıkkı}}$

(3) $f(x,y,z) = x^3+y^4+z^4 \quad (1,1,1)$ I. -8, II. -4, III. 1, IV. 5, V. 6

$\nabla f = \langle 3x^2, 4y^3, 4z^3 \rangle$

$\nabla f(1,1,1) = \langle 3,4,4 \rangle \quad |\nabla f| = \sqrt{41}$

$-\sqrt{41}$ ile $\sqrt{41}$ arasındaki değerler; II, III, IV, V $\boxed{\text{B şıkkı}}$

(4) $f(x,y,z) = x^3+y^4+z^3 \quad (1,1,1)$

I. -8, II. -4, III. 1, IV. 5, V. 6

$\nabla f = \langle 3x^2, 4y^3, 3z^2 \rangle$

$\nabla f(1,1,1) = \langle 3,4,3 \rangle \quad |\nabla f| = \sqrt{34}$

$-\sqrt{34}$ ile $\sqrt{34}$ arasındaki değerler; II, III, IV $\boxed{\text{C şıkkı}}$

Soru 4: ① $x = t^2uv$, $y = u + tv^2$ $(t,u,v) = (1,1,1)$

$\frac{\partial z}{\partial x}(1,2) = 4$ $\frac{\partial z}{\partial y}(1,2) = -1$

$$\frac{\partial z}{\partial t} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial t} = z_x \cdot 2tuv + z_y \cdot v^2$$

$$\frac{\partial z}{\partial t}(1,1,1) = 4 \cdot 2 \cdot 1 \cdot 1 \cdot 1 - 1 \cdot 1 = 8 - 1 = \underbrace{7}_{} \quad \boxed{\text{D şıkkı}}$$

② $$\frac{\partial z}{\partial u} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial u} = z_x \cdot t^2v + z_y \cdot 1$$

$$\frac{\partial z}{\partial u}(1,1,1) = 4 \cdot 1 - 1 \cdot 1 = \underbrace{3}_{} \quad \boxed{\text{B şıkkı}}$$

③ $$\frac{\partial z}{\partial v} = \frac{\partial z}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial z}{\partial y} \cdot \frac{\partial y}{\partial v} = z_x \cdot t^2u + z_y \cdot 2tv$$

$$\frac{\partial z}{\partial v}(1,1,1) = 4 \cdot 1 - 1 \cdot 2 \cdot 1 = \underbrace{2}_{} \quad \boxed{\text{A şıkkı}}$$

Soru 5: 1 $f(x,y) = g(x^2y)$, $g(2)=3$, $g'(2)=1$

$f(1.1, 2.1) \approx f(1,2) + f_x(1,2)(1.1-1) + f_y(1,2)(2.1-2)$

$f_x = 2xyg'(x^2y) \Rightarrow f_x(1,2) = 4g'(2) = 4.1 = 4$

$f_y = x^2 . g'(x^2y) \Rightarrow f_y(1,2) = 1. g'(2) = 1.1 = 1$

$f(1.1, 2.1) \approx 3 + 4.0{,}1 + 1.0{,}1 = \underbrace{3{,}5}_{\shortparallel}$ $\boxed{\text{A şıkkı}}$

2 $f(x,y) = g(x^3y)$ $g(2)=3$, $g'(2)=1$

$f_x = 3x^2yg'(x^3y) \Rightarrow f_x(1,2) = 6$

$f_y = x^3 g'(x^3y) \Rightarrow f_y(1,2) = 1$

$f(1.1, 2.1) \approx 3 + 6.0{,}1 + 1.0{,}1 = \underbrace{3{,}7}_{\shortparallel}$ $\boxed{\text{C şıkkı}}$

3 $f(x,y) = g(xy^2)$ $g(2)=4$, $g'(2)=1$

$f_x = y^2 g'(xy^2) \Rightarrow f_x(2,1) = 1$

$f_y = 2xy g'(xy^2) \Rightarrow f_y(2,1) = 4$

$f(2.1, 1.1) \approx 4 + 1.0{,}1 + 4.0{,}1 = \underbrace{4{,}5}_{\shortparallel}$ $\boxed{\text{E şıkkı}}$

4 $f(x,y) = g(xy^4)$, $g(2)=4$, $g'(2)=1$

$f_x = y^4 g'(xy^4) \Rightarrow f_x(2,1) = 1$

$f_y = 4y^3x\, g'(xy^4) \Rightarrow f_y(2,1) = 4.2 = 8$

$f(2.1, 1.1) \approx 4 + 1.0{,}1 + 8.0{,}1 = \underbrace{4{,}9}_{\shortparallel}$ $\boxed{\text{A şıkkı}}$

Soru 6: 1 $f(x,y) = y\ln x + xe^y$

$h.f_{xx} + x.f_{xy} - f_{yy} = 0$

$f_x = \frac{y}{x} + e^y \quad f_{xy} = \frac{1}{x} + e^y \quad f_{xx} = \frac{-y}{x^2} \quad f_y = \ln x + xe^y$

$f_{yy} = xe^y$

$h.\frac{-y}{x^2} + x.\left(\frac{1}{x} + e^y\right) - xe^y = 0$

$-\frac{y}{x^2}.h + 1 + x\cancel{e^y} - x\cancel{e^y} = 0 \Rightarrow \boxed{h = \frac{x^2}{y}}$ // $\boxed{\text{B şıkkı}}$

2 $h.f_{yy} + yf_{xy} + xf_{xx} = 0$

$h.xe^y + y\left(\frac{1}{x} + e^y\right) + x.\frac{-y}{x^2} = 0$

$x.e^y.h + \cancel{\frac{y}{x}} + ye^y - \cancel{\frac{y}{x}} = 0$

$h = -\frac{y\cancel{e^y}}{x\cancel{e^y}} \Rightarrow \underline{h = -\frac{y}{x}}$ // $\boxed{\text{A şıkkı}}$

3 $f(x,y) = x\ln y + ye^x$

$f_x = \ln y + ye^x, \; f_{xx} = ye^x, \; f_y = \frac{x}{y} + e^x, \; f_{yy} = -\frac{x}{y^2}, \; f_{xy} = \frac{1}{y} + e^x$

$h.f_{xx} + x\,f_{xy} + yf_{yy} = 0$

$y.e^x.h + x\left(\frac{1}{y} + e^x\right) + y.\frac{-x}{y^2} = 0$

$ye^x.h + \cancel{\frac{x}{y}} + xe^x - \cancel{\frac{x}{y}} = 0 \Rightarrow \underline{h = -\frac{x}{y}}$ // $\boxed{\text{C şıkkı}}$

4 $h.f_{yy} + y\,f_{xy} - f_{xx} = 0$

$\frac{-x}{y^2}h + y.\left(\frac{1}{y} + e^x\right) - ye^x = 0$

$-\frac{x}{y^2}h + 1 + y\cancel{e^x} - y\cancel{e^x} = 0$

$\underline{h = \frac{y^2}{x}}$ // $\boxed{\text{D şıkkı}}$

Soru 7! 1) $f(x,y)=\begin{cases} \dfrac{x^2+y^2}{2-\sqrt{x^2+y^2+4}} & ,(x,y)\neq(0,0) \\ k & ,(x,y)=(0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{2-\sqrt{x^2+y^2+4}}\cdot\frac{(2+\sqrt{x^2+y^2+4})}{(2+\sqrt{x^2+y^2+4})}$$

$$=\lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2+y^2)}\,(2+\sqrt{x^2+y^2+4})}{\underbrace{\cancel{4}-x^2-y^2-\cancel{4}}_{-\cancel{(x^2+y^2)}}} = \underbrace{-4}_{}\ //$$

$\boxed{\text{C şıkkı}}$

2) $f(x,y)=\begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+9}-3} & ,(x,y)\neq(0,0) \\ k & ,(x,y)=(0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+9}-3}\cdot\frac{(\sqrt{x^2+y^2+9}+3)}{(\sqrt{x^2+y^2+9}+3)}$$

$$=\lim_{(x,y)\to(0,0)} \frac{\cancel{(x^2+y^2)}\,(\sqrt{x^2+y^2+9}+3)}{x^2+y^2+\cancel{9}-\cancel{9}} = \underbrace{6}_{}\ //$$

$\boxed{\text{D şıkkı}}$

3) $f(x,y)=\begin{cases} \dfrac{x^2+y^2}{1-\sqrt{x^2+y^2+1}} & ,(x,y)\neq(0,0) \\ k & ,(x,y)=(0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{1-\sqrt{x^2+y^2+1}}\cdot\frac{(1+\sqrt{x^2+y^2+1})}{(1+\sqrt{x^2+y^2+1})}=\lim_{(x,y)\to(0,0)} -(1+\sqrt{x^2+y^2+1})$$

$\boxed{\text{B şıkkı}}$

$$=\underbrace{-2}_{}\ //$$

4) $f(x,y)=\begin{cases} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+16}-4} & ,(x,y)\neq(0,0) \\ k & ,(x,y)=(0,0) \end{cases}$

$$\lim_{(x,y)\to(0,0)} \frac{x^2+y^2}{\sqrt{x^2+y^2+16}-4}\cdot\frac{(\sqrt{x^2+y^2+16}+4)}{(\sqrt{x^2+y^2+16}+4)}=\lim_{(x,y)\to(0,0)}\frac{\cancel{(x^2+y^2)}(\sqrt{x^2+y^2+16}+4)}{\cancel{x^2+y^2}+\cancel{16}-\cancel{16}}$$

$\boxed{\text{E şıkkı}}$

$$=\underbrace{8}_{}\ //$$

Soru8: (1) $f(x,y)=\frac{1}{3}y^3+x^2-2xy-3y$

$f_x=2x-2y=0 \Rightarrow \underline{x=y}$

$f_y=y^2-2x-3=0 \Rightarrow y^2-2y-3=0 \Rightarrow y=-1=x \quad (-1,-1)$
$\qquad\qquad\qquad\qquad\quad \begin{matrix}-3\\+1\end{matrix} \qquad\quad y=3=x \quad (3,3)$

$f_{xx}=2$

$f_{xy}=-2 \qquad \Delta=4-2.2.(-1)>0 \quad (-1,-1)\rightarrow$ semer

$f_{yy}=2y \qquad \Delta=4-2.2.(3)<0\ ,\ f_{xx}>0 \Rightarrow (3,3)\rightarrow$ y. min.

$(3,3)\rightarrow 3+3=\underline{6}$ // $\boxed{\text{E şıkkı}}$

(2) $(-1,-1)\rightarrow -1-1=\underline{-2}$ // $\boxed{\text{C şıkkı}}$

(3) $f(x,y)=\frac{1}{3}y^3-2x^2+4xy-5y$

$f_x=-4x+4y=0 \Rightarrow \underline{x=y}$

$f_y=y^2+4x-5 \Rightarrow y^2+4y-5=0 \Rightarrow y=1=x \quad (1,1)$
$\qquad\qquad\qquad\qquad\quad \begin{matrix}+5\\-1\end{matrix} \qquad\quad y=-5=x \quad (-5,-5)$

$f_{xx}=-4$

$f_{xy}=4 \qquad \Delta=16+4.2.1>0\ ,\ (1,1)\rightarrow$ semer

$f_{yy}=2y \qquad \Delta=16+4.2.(-5)<0\ ,\ f_{xx}<0 \Rightarrow (-5,-5)\rightarrow$ y. max.

$(-5,-5)\rightarrow -5-5=\underline{-10}$ // $\boxed{\text{A şıkkı}}$

(4) $(1,1)\longrightarrow 1+1=\underline{2}$ // $\boxed{\text{D şıkkı}}$

Soru 9 : (1) $z = 3x^2 + x - y^2 + y \quad P(1,1,4)$

$\nabla f = \langle -6x-1, 2y-1, 1 \rangle \quad \nabla f(P) = \langle -7, 1, 1 \rangle$

$-7(x-1) + (y-1) + (z-4) = 0$

$\underbrace{7x - y - z = 2}$ // $\boxed{\text{A şıkkı}}$

(2) $z = 3x^2 - x + y^2 + y \quad P(1,1,4)$

$f: z - 3x^2 + x - y^2 - y = 0$

$\nabla f = \langle -6x+1, -2y-1, 1 \rangle \Rightarrow \nabla f(P) = \langle -5, -3, 1 \rangle$

$-5(x-1) - 3(y-1) + (z-4) = 0$

$\underbrace{5x + 3y - z = 4}$ // $\boxed{\text{C şıkkı}}$

(3) $z = x^2 - x + 3y^2 + y \quad P(1,1,4)$

$f: z - x^2 + x - 3y^2 - y = 0$

$\nabla f = \langle -2x+1, -6y-1, 1 \rangle \Rightarrow \nabla f(P) = \langle -1, -7, 1 \rangle$

$-(x-1) - 7(y-1) + (z-4) = 0$

$\underbrace{x + 7y - z = 4}$ // $\boxed{\text{B şıkkı}}$

(4) $z = x^2 + x + 3y^2 - y \quad P(1,1,4)$

$f: z - x^2 - x - 3y^2 + y = 0$

$\nabla f = \langle -2x-1, -6y+1, 1 \rangle \Rightarrow \nabla f(P) = \langle -3, -5, 1 \rangle$

$-3(x-1) - 5(y-1) + (z-4) = 0$

$\underbrace{3x + 5y - z = 4}$ // $\boxed{\text{D şıkkı}}$

Soru 10: (1) $f(x,y,z) = xy + yz + zx \quad P(2,2,2)$

$\vec{u} = \langle 3,6,2 \rangle \qquad \vec{v} = \frac{\vec{u}}{|\vec{u}|} = \langle \frac{3}{7}, \frac{6}{7}, \frac{2}{7} \rangle$

$\nabla f = \langle y+z, x+z, y+x \rangle \qquad \nabla f(P) = \langle 4,4,4 \rangle$

$D_{\vec{u}} f(P) = \langle \frac{3}{7}, \frac{6}{7}, \frac{2}{7} \rangle \cdot \langle 4,4,4 \rangle = \frac{12+24+8}{7}$

$= \underline{\frac{44}{7}}$ // $\boxed{\text{B şıkkı}}$

(2) $\vec{u} = \langle 3,-6,2 \rangle \qquad \vec{v} = \frac{\vec{u}}{|\vec{u}|} = \langle \frac{3}{7}, -\frac{6}{7}, \frac{2}{7} \rangle$

$D_{\vec{u}} f(P) = \langle \frac{3}{7}, -\frac{6}{7}, \frac{2}{7} \rangle \cdot \langle 4,4,4 \rangle = \frac{12-24+8}{7}$

$= \underline{\frac{-4}{7}}$ // $\boxed{\text{D şıkkı}}$

(3) $\vec{u} = \langle 3,6,-2 \rangle \qquad \vec{v} = \frac{\vec{u}}{|\vec{u}|} = \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle$

$D_{\vec{u}} f(P) = \langle \frac{3}{7}, \frac{6}{7}, -\frac{2}{7} \rangle \cdot \langle 4,4,4 \rangle$

$= \frac{12+24-8}{7} = \frac{28}{7} = \underline{4}$ // $\boxed{\text{A şıkkı}}$

Soru 11: 1 $5x-2y-3z=0$ $\begin{cases} x=3+2t \\ y=1+3t \\ z=2-t \end{cases}$

$$5.(3+2t)-2(1+3t)-3(2-t)=0$$
$$10t-6t+3t=-7$$
$$t=-1$$

$\underline{P(1,-2,3)}$ // $\boxed{\text{B şıkkı}}$

2 $5x-2y-3z=0$ $\begin{cases} x=4+2t \\ y=2+3t \\ z=3-t \end{cases}$

$$5.(4+2t)-2(2+3t)-3(3-t)=0$$
$$10t-6t+3t=-7$$
$$t=-1$$

$\underline{P(2,-1,4)}$ // $\boxed{\text{C şıkkı}}$

3 $5x-2y-3z=0$ $\begin{cases} x=5-t \\ y=3+3t \\ z=3-2t \end{cases}$

$$5.(5-t)-2(3+3t)-3(3-2t)=0$$
$$-5t-6t+6t=+10$$
$$t=2$$

$\underline{P(3,9,-1)}$ // $\boxed{\text{D şıkkı}}$

4 $5x-2y-3z=0$ $\begin{cases} x=6+2t \\ y=4-3t \\ z=4+2t \end{cases}$

$$5(6+2t)-2(4-3t)-3(4+2t)=0$$
$$10t+6t-6t=-10$$
$$t=-1$$

$\underline{P(4,7,2)}$ // $\boxed{\text{A şıkkı}}$

Soru 12 : (1) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{2^{n+2}} = \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{x-2}{2}\right)^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-2|}{2} = \frac{|x-2|}{2} < 1 \Rightarrow -2 < x-2 < 2$

$\underbrace{0 < x < 4}$ //

$a = \frac{1}{4} \cdot \frac{x-2}{2}$

$r = \frac{x-2}{2}$ $\quad S = \frac{a}{1-r} = \frac{\frac{x-2}{8}}{1-\frac{x-2}{2}} = \frac{x-2}{\cancel{8}_4} \cdot \frac{\cancel{2}}{4-x} = \underbrace{\frac{x-2}{16-4x}}$ //

$\boxed{\text{A şıkkı}}$

(2) $\sum_{n=1}^{\infty} \frac{(x-4)^n}{2^{n+2}} = \sum_{n=1}^{\infty} \frac{1}{4}\left(\frac{x-4}{2}\right)^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-4|}{2} = \frac{|x-4|}{2} < 1 \Rightarrow -2 < x-4 < 2$

$\underbrace{2 < x < 6}$ //

$a = \frac{1}{4} \frac{x-4}{2}$

$r = \frac{x-4}{2}$ $\quad S = \frac{a}{1-r} = \frac{\frac{x-4}{8}}{1-\frac{x-4}{2}} = \frac{x-4}{\cancel{8}_4} \cdot \frac{\cancel{2}}{6-x} = \underbrace{\frac{x-4}{24-4x}}$ //

$\boxed{\text{B şıkkı}}$

(3) $\sum_{n=1}^{\infty} \frac{(x-2)^n}{3^{n+2}} = \sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{x-2}{3}\right)^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-2|}{3} = \frac{|x-2|}{3} < 1 \Rightarrow -3 < x-2 < 3$

$\underbrace{-1 < x < 5}$ //

$a = \frac{1}{9} \frac{x-2}{3}$

$r = \frac{x-2}{3}$ $\quad S = \frac{a}{1-r} = \frac{\frac{x-2}{27}}{1-\frac{x-2}{3}} = \frac{x-2}{27} \cdot \frac{3}{5-x} = \underbrace{\frac{x-2}{45-9x}}$ //

$\boxed{\text{C şıkkı}}$

(4) $\sum_{n=1}^{\infty} \frac{(x-4)^n}{3^{n+2}} = \sum_{n=1}^{\infty} \frac{1}{9}\left(\frac{x-4}{3}\right)^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{|x-4|}{3} = \frac{|x-4|}{3} < 1 \Rightarrow -3 < x-4 < 3$

$\underbrace{1 < x < 7}$ //

$a = \frac{1}{9} \cdot \frac{x-4}{3}$ $\quad S = \frac{a}{1-r} = \frac{\frac{x-4}{27}}{1-\frac{x-4}{3}} = \frac{x-4}{27} \cdot \frac{3}{7-x} = \underbrace{\frac{x-4}{63-9x}}$ //

$r = \frac{x-4}{3}$

$\boxed{\text{D şıkkı}}$

Soru 13 : 1 $\vec{r}(t) = \langle 1-2t^2, 4t, 3+2t^2 \rangle \quad 0 \leq t \leq 2$

$\vec{r}\,'(t) = \langle -4t, 4, 4t \rangle$

$$S = \int_0^2 \sqrt{16t^2+16+16t^2}\,dt = \int_0^2 4\sqrt{2t^2+1}\,dt \qquad \boxed{\text{D şıkkı}}$$

//.

2 $\vec{r}(t) = \langle \ln t, t, \frac{t^2}{4} \rangle \quad 1 \leq t \leq 4$

$\vec{r}\,'(t) = \langle \frac{1}{t}, 1, \frac{t}{2} \rangle$

$$S = \int_1^4 \sqrt{\frac{1}{t^2} + 1 + \frac{t^2}{4}}\,dt = \int_1^4 \sqrt{\left(\frac{1}{t} + \frac{t}{2}\right)^2}\,dt$$

$$= \int_1^4 \left(\frac{1}{t} + \frac{t}{2}\right) dt \qquad \boxed{\text{C şıkkı}}$$

//

Soru 14: 1) $(x^2+y^2)\tan(\ln(xy)) + e^{x-y} = f(x,y)$

$$f_x = 2x \cdot \tan(\ln(xy)) + (x^2+y^2)\frac{\frac{y}{xy}}{1+\ln^2(xy)} + e^{x-y}$$

$$f_x(1,1) = \frac{2.1}{1} + 1 = 3$$

$$f_y = 2y \cdot \tan(\ln(xy)) + (x^2+y^2) \cdot \frac{\frac{x}{xy}}{1+\ln^2(xy)} - e^{x-y}$$

$$f_y(1,1) = \frac{2.1}{1} - 1 = 1$$

$$2f_x - f_y = 2.3 - 1 = \underline{5} \quad \boxed{\text{C şıkkı}}$$

2) $2f_x + f_y = 2.3 + 1 = \underline{7}$ $\boxed{\text{E şıkkı}}$

3) $f_x + 3f_y = 3 + 3.1 = \underline{6}$ $\boxed{\text{D şıkkı}}$

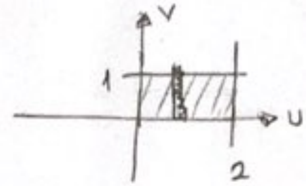
4) $f_x + f_y = 3 + 1 = \underline{4}$ $\boxed{\text{B şıkkı}}$

5) $2f_x - 3f_y = 2.3 - 3 = \underline{3}$ $\boxed{\text{A şıkkı}}$

Soru 15:

(1) $\int_0^1 \int_0^4 \sqrt{y} \cos\sqrt{x} \, dx\,dy \qquad u=\sqrt{x}, \ v=\sqrt{y}$

$\left.\begin{matrix} x=0 & y=0 \\ x=4 & y=1 \end{matrix}\right] \begin{matrix} u=0 & v=0 \\ u=2 & v=1 \end{matrix}$



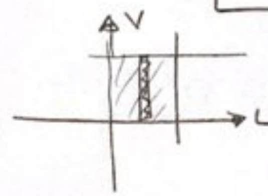
$J = \dfrac{1}{\begin{vmatrix} u_x & u_y \\ v_x & v_y \end{vmatrix}} = \dfrac{1}{\begin{vmatrix} \frac{1}{2\sqrt{x}} & 0 \\ 0 & \frac{1}{2\sqrt{y}} \end{vmatrix}} = 4uv$

$\int_0^2 \int_0^1 v \cdot \cos u \cdot 4uv \, dv\, du$

$= \int_0^2 \int_0^1 4uv^2 \cos u \, dv\,du$

$\boxed{\text{B Şıkkı}}$

(2) $\int_0^4 \int_0^1 \sqrt{y} \cdot \cos\sqrt{y} \, dx\,dy \qquad \begin{matrix} u=x \\ v=\sqrt{y} \end{matrix}$

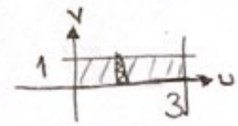


$\left.\begin{matrix} x=0 & y=0 \\ x=1 & y=4 \end{matrix}\right] \begin{matrix} u=0 & v=0 \\ u=1 & v=2 \end{matrix}$

$J = \dfrac{1}{\begin{vmatrix} 1 & 0 \\ 0 & \frac{1}{2\sqrt{y}} \end{vmatrix}} = 2v \qquad \int_0^1 \int_0^2 2v \cdot v \cos v \, dv\,du$

$\boxed{\text{A Şıkkı}}$

(3) $\int_0^9 \int_0^1 \sqrt{y} \sin\sqrt{x} \, dx\,dy \qquad u=\sqrt{y}, \ v=\sqrt{x}$



$\left.\begin{matrix} x=0 & y=0 \\ x=1 & y=9 \end{matrix}\right] \begin{matrix} u=0 & v=0 \\ u=3 & v=1 \end{matrix}$

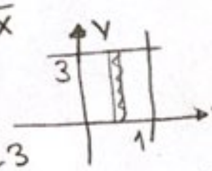
$\int_0^3 \int_0^1 u \cdot \sin v \cdot 4uv \, dv\,du$

$J = \dfrac{1}{\begin{vmatrix} 0 & \frac{1}{2\sqrt{y}} \\ \frac{1}{2\sqrt{x}} & 0 \end{vmatrix}} = -4uv$

$\int_0^3 \int_0^1 4u^2 v \sin v \, dv\,du$

$\boxed{\text{C Şıkkı}}$

(4) $\int_0^9 \int_0^1 \sqrt{x} \sin\sqrt{x} \, dy\,dx \qquad u=y, \ v=\sqrt{x}$



$\left.\begin{matrix} x=0 & y=0 \\ x=9 & y=1 \end{matrix}\right] \begin{matrix} u=0 & v=0 \\ u=1 & v=3 \end{matrix}$

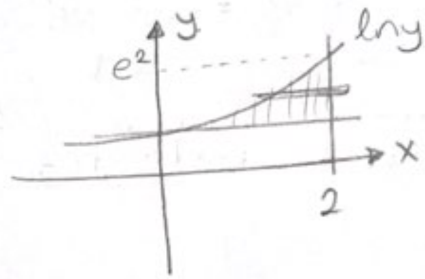
$\int_0^1 \int_0^3 v \sin v \cdot 2v \, dv\,du$

$J = \dfrac{1}{\begin{vmatrix} 0 & 1 \\ \frac{1}{2\sqrt{x}} & 0 \end{vmatrix}} = -2v$

$= \int_0^1 \int_0^3 2v^2 \sin v \, dv\,du$

$\boxed{\text{D Şıkkı}}$

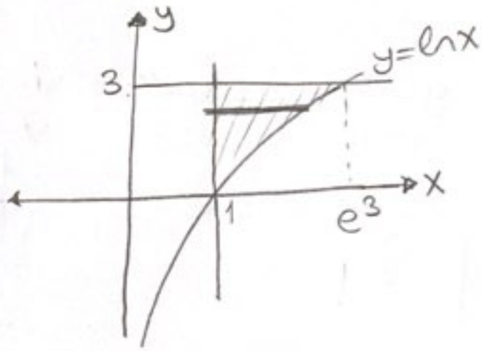
Soru 16: 1) $\int_0^2 \int_1^{e^x} \sin x^3 \, dy\,dx$



$x=0 \quad y=1$
$x=2 \quad y=e^x$

$\int_1^{e^2} \int_{\ln y}^{2} \sin x^3 \, dx\,dy$ $\boxed{\text{B şıkkı}}$

2) $\int_1^{e^3} \int_{\ln x}^{3} \cos x^3 \, dy\,dx$

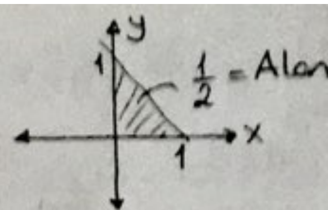


$x=1 \quad y=3$
$x=e^3 \quad y=\ln x$

$\int_0^3 \int_1^{e^y} \cos x^3 \, dx\,dy$ $\boxed{\text{C şıkkı}}$

Soru 17: (1) $x=0, y=0, y+x=1$

$f(x,y)=xy$



$$\text{ort}\,f=\frac{1}{\text{Alan}}\int_0^1\int_0^{1-x} xy\;dydx = \cancel{2}\int_0^1 x\frac{y^2}{\cancel{2}}\Big|_0^{1-x} dx$$

$$=\int_0^1 x(1-x)^2dx = \int_0^1 (x-2x^2+x^3)\,dx = \frac{x^2}{2}-\frac{2}{3}x^3+\frac{x^4}{4}\Big|_0^1$$

$$=\frac{1}{2}-\frac{2}{3}+\frac{1}{4}=\frac{3}{4}-\frac{2}{3}=\underline{\frac{1}{12}}$$ $\boxed{\text{D şıkkı}}$

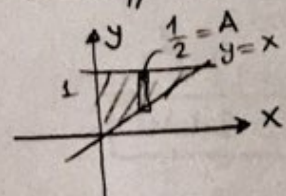
(2) $f(x,y)=y$

$$\text{ort}\,f = 2.\int_0^1\int_0^{1-x} y\;dydx = \cancel{2}\int_0^1 \frac{y^2}{\cancel{2}}\Big|_0^{1-x}dx = \int_0^1 (1-x)^2dx$$

$$=-\frac{(1-x)^3}{3}\Big|_0^1=\underline{\frac{1}{3}}$$ $\boxed{\text{C şıkkı}}$

(3) $x=0, y=1, y=x$

$f(x,y)=xy$



$$\text{ort}\,f=2.\int_0^1\int_x^1 xy\;dydx$$

$$=\cancel{2}\int_0^1 x\frac{y^2}{\cancel{2}}\Big|_x^1dx=\int_0^1(x-x^3)\,dx=\frac{x^2}{2}-\frac{x^4}{4}\Big|_0^1=\frac{1}{2}-\frac{1}{4}=\underline{\frac{1}{4}}$$

$\boxed{\text{A şıkkı}}$

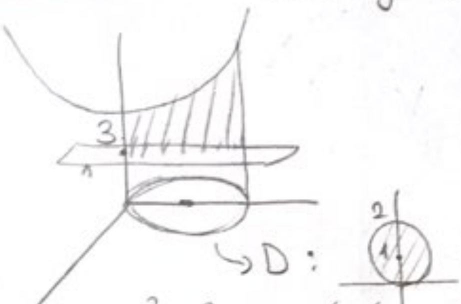
(4) $f(x,y)=x^2y$

$$\text{ort}\,f=2.\int_0^1\int_x^1 x^2y\;dydx=\cancel{2}\int_0^1 x^2\frac{y^2}{\cancel{2}}\Big|_x^1dx$$

$$=\int_0^1(x^2-x^4)\,dx=\frac{x^3}{3}-\frac{x^5}{5}\Big|_0^1=\frac{1}{3}-\frac{1}{5}=\underline{\frac{2}{15}}$$

$\boxed{\text{B şıkkı}}$

Soru 18: (1) $x^2+(y-1)^2=1$, $z=3+x^2+y^2$ $z=1$



$$V=\iint_D (3+x^2+y^2-1)\,dx\,dy$$

$x^2+y^2-2y+\cancel{1}=\cancel{1}$

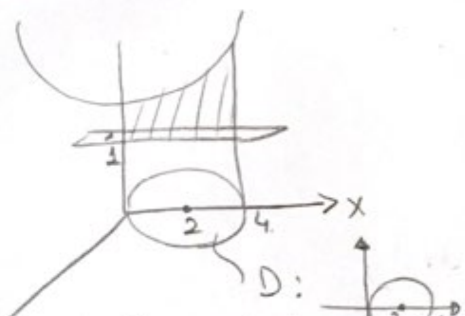
$r^2-2r\sin\theta=0$

$\underline{r=0}$, $\underline{r=2\sin\theta}$

$$V=\int_0^{\pi}\int_0^{2\sin\theta}(2+r^2)\,r\,dr\,d\theta \quad //$$

$\boxed{\text{B Şıkkı}}$

(2) $(x-2)^2+y^2=4$, $z=4+x^2+y^2$, $z=1$



$$V=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{4\cos\theta}(3+r^2)\,r\,dr\,d\theta$$

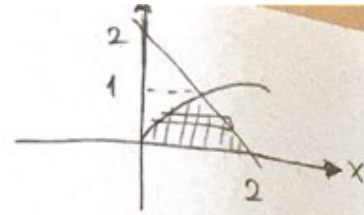
$\boxed{\text{A Şıkkı}}$

$x^2+y^2-4x+\cancel{4}=\cancel{4}$

$r^2-4r\cos\theta=0$

$r=0$, $r=4\cos\theta$

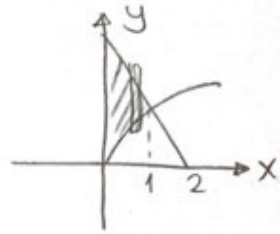
Soru 19: ① $y=\sqrt{x}$, $y+x=2$, $y=0$



$$A=\int_{0}^{1}\int_{y^2}^{2-y} dx\,dy$$

// $\boxed{\text{B şıkkı}}$

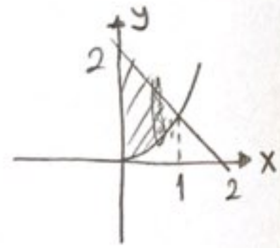
② $y=\sqrt{x}$, $y+x=2$, $x=0$



$$A=\int_{0}^{1}\int_{\sqrt{x}}^{2-x} dy\,dx$$

// $\boxed{\text{A şıkkı}}$

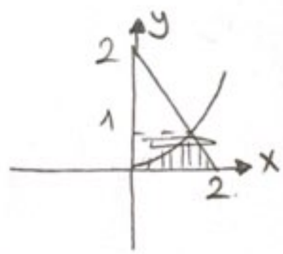
③ $x=\sqrt{y}$, $y+x=2$, $x=0$



$$A=\int_{0}^{1}\int_{x^2}^{2-x} dy\,dx$$

// $\boxed{\text{C şıkkı}}$

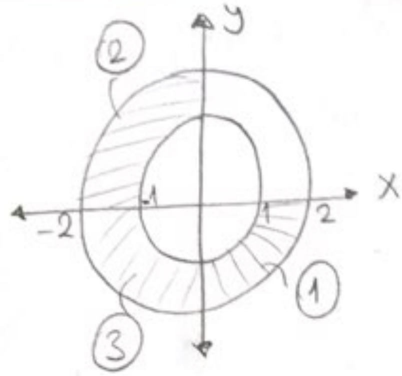
④ $x=\sqrt{y}$, $y+x=2$, $y=0$



$$A=\int_{0}^{1}\int_{\sqrt{y}}^{2-y} dx\,dy$$

// $\boxed{\text{D şıkkı}}$

Soru 20: (1)



$$\int_{-2}^{-1}\int_{0}^{\sqrt{4-y^2}} \sqrt{x^2+y^2}\,dx\,dy + \int_{-1}^{0}\int_{\sqrt{1-y^2}}^{\sqrt{4-y^2}} \sqrt{x^2+y^2}\,dx\,dy$$

$$= \int_{\frac{3\pi}{2}}^{2\pi}\int_{1}^{2} r^2\,dr\,d\theta \quad //$$

$\boxed{\text{B sıkkı}}$

(2)

$$\int_{0}^{1}\int_{-\sqrt{4-y^2}}^{-\sqrt{1-y^2}} \sqrt{x^2+y^2}\,dx\,dy + \int_{1}^{2}\int_{-\sqrt{4-y^2}}^{0} \sqrt{x^2+y^2}\,dx\,dy$$

$$= \int_{\frac{\pi}{2}}^{\pi}\int_{1}^{2} r^2\,dr\,d\theta \quad //$$

$\boxed{\text{C sıkkı}}$

(3)

$$\int_{-2}^{-1}\int_{-\sqrt{4-y^2}}^{0} \sqrt{x^2+y^2}\,dx\,dy + \int_{-1}^{0}\int_{-\sqrt{4-y^2}}^{-\sqrt{1-y^2}} \sqrt{x^2+y^2}\,dx\,dy$$

$$= \int_{\pi}^{\frac{3\pi}{2}}\int_{1}^{2} r^2\,dr\,d\theta \quad //$$

$\boxed{\text{D sıkkı}}$