Ch. 6: Measuring Cosmological Parameters

If we could determine $a(t)$ from observations, we could use that knowledge to find ε for each component.

Let us do a Taylor series expansion for $a(t)$ around the present moment $t = t_0$.

$$a(t) = a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0} (t - t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0} (t-t_0)^2 + \cdots$$

Keeping the first three terms

$$a(t) \approx a(t_0) + \left.\frac{da}{dt}\right|_{t=t_0} (t-t_0) + \frac{1}{2}\left.\frac{d^2a}{dt^2}\right|_{t=t_0} (t-t_0)^2$$

$$\frac{a(t)}{a(t_0)} \approx 1 + \left.\frac{\dot{a}}{a}\right|_{t=t_0} (t-t_0) + \frac{1}{2}\left.\frac{\ddot{a}}{a}\right|_{t=t_0} (t-t_0)^2 \; ; \; a(t_0) = 1$$

$$\Rightarrow \boxed{a(t) \approx 1 + H_0 (t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2}, \text{ where}$$

$$H_0 \equiv \left.\frac{\dot{a}}{a}\right|_{t=t_0}, \text{ and } q_0 \equiv -\left(\frac{\ddot{a}}{aH^2}\right)_{t=t_0} = -\left(\frac{\ddot{a}a}{\dot{a}^2}\right)_{t=t_0}$$

is the deceleration parameter.

If $q_0 = +$, then $\ddot{a} < 0$ (the universe's expansion is decelerating)

$q_0 = -$, then $\ddot{a} > 0$ (expansion is accelerating)

The recent expansion of the universe can be described in terms of H_0 and q_0.

If our model universe contains N components, each with a different equation of state parameter ω_i ($P_i = \omega_i \mathcal{E}_i$) the accelaration equation can be written

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}(\mathcal{E} + 3P)$$

$$= -\frac{4\pi G}{3c^2}\sum_{i=1}^{N}\mathcal{E}_i(1+3\omega_i)$$

Multiplying this by $-\frac{1}{H^2}$

$$-\frac{\ddot{a}}{aH^2} = \frac{1}{2}\left[\frac{8\pi G}{3c^2H^2}\right]\sum_{i=1}^{N}\mathcal{E}_i(1+3\omega_i)$$

using $\Omega_i = \frac{8\pi G}{3c^2H^2}\mathcal{E}_i = \frac{\mathcal{E}_i}{\mathcal{E}_c} \Rightarrow$

$$-\frac{\ddot{a}}{aH^2} = \frac{1}{2}\sum_{i=1}^{N}\Omega_i(1+3\omega_i)$$

So, at $t = t_0$:

$$q_0 = \frac{1}{2}\sum_i \Omega_{i0}(1+3\omega_i)$$

$$= \frac{1}{2}\Omega_{r0}(1+3\omega_r) + \frac{1}{2}\Omega_{m0}(1+3\omega_m)$$

$$+ \frac{1}{2}\Omega_{\Lambda 0}(1+3\omega_\Lambda);$$

where $\omega_r = \frac{1}{3}$, $\omega_m = 0$, $\omega_\Lambda = -1$

$$\Rightarrow q_0 = \Omega_{r0} + \frac{1}{2}\Omega_{m0} - \Omega_{\Lambda 0}$$

The universe will be accelerating outward if $q_0 < 0$.

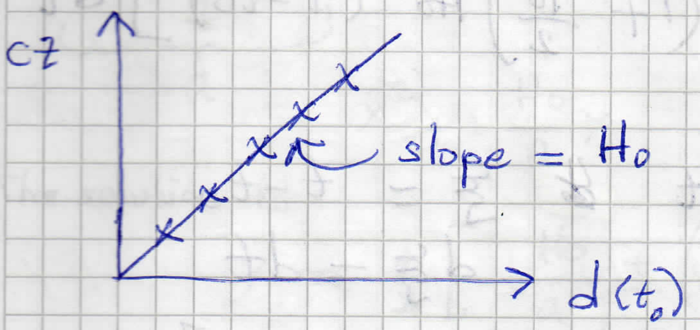
The Benchmark model has $q_0 = \frac{1}{2}(0.31) - 0.69$

$$= -0.535 = -0.54$$

Hubble's work has shown that

$$z = \frac{H_0}{c} d(t) \quad \text{(distant galaxies have higher redshifts)}$$

distance



Recall that the proper distance $d_p(t_0)$ is defined as

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

for light rays

$$\left[\; ds^2 = -c^2 dt^2 + a^2(t) dr^2 = 0 \right.$$

$$\int_0^r dr = d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

also $\quad d_p(t) = \int a(t) dr = a(t) \int dr = a(t) r$

$$d_p(t_0) = a(t_0) r = r$$

comoving coordinate

$$\left. d_p(t_e) = a(t_e) r \; \right]$$

Using $\frac{1}{1+x} = (1+x)^{-1} \approx 1 - x + x^2$, with $x \equiv H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2$

$$\frac{1}{a(t)} \approx 1 - \left(H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2\right) + \left(H_0(t-t_0) - \frac{1}{2} q_0 H_0^2 (t-t_0)^2\right)^2$$

$$\approx 1 - H_0(t-t_0) + \frac{1}{2} q_0 H_0^2 (t-t_0)^2 + H_0^2 (t-t_0)^2 + \text{OT}$$

$$\frac{1}{a(t)} \approx 1 - H_0(t-t_0) + \left(1 + \frac{q_0}{2}\right) H_0^2 (t-t_0)^2$$

$$d_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$\approx c\int_{t_e}^{t_0}\left[1 - H_0(t-t_0) + \left(1+\frac{q_0}{2}\right)H_0^2(t-t_0)^2\right]dt$$

Let $\xi = t - t_0$

$d\xi = dt$

$$\Rightarrow d_p(t_0) \approx c\int_{t_e-t_0}^{0}\left[1 - H_0\xi + \left(1+\frac{q_0}{2}\right)H_0^2\xi^2\right]d\xi$$

$$\approx c\left[\xi - H_0\frac{\xi^2}{2} + \left(1+\frac{q_0}{2}\right)H_0^2\frac{\xi^3}{3}\right]\Bigg|_{-(t_0-t_e)}^{0}$$

$$\approx c\left[0 + (t_0-t_e) + H_0\frac{1}{2}(t_0-t_e)^2 + \underbrace{\text{negligible}}_{\left(1+\frac{q_0}{2}\right)\frac{H_0^2}{3}(t_0-t_e)^3}\right]$$

$$d_p(t_0) \approx c(t_0-t_e) + \frac{cH_0}{2}(t_0-t_e)^2$$

$$\approx \underbrace{c\,t_L}_{\text{what the proper distance would be in a static universe}} + \frac{cH_0}{2}t_L^2$$

correction due to the expansion of the universe

, where $t_L \equiv t_0 - t_e$ is the lookback time.

The light from distant galaxies tells us about the scale factor $a(t_e)$ through

$$z = \frac{1}{a(t_e)} - 1$$

Putting $t = t_e$ in the expansion for $\frac{1}{a(t)}$, we get

$$z \approx 1 - H_0(t_e - t_0) + \left(1+\frac{q_0}{2}\right) H_0^2 (t_e - t_0)^2 - 1$$

$$z \approx H_0(t_0 - t_e) + \left(1+\frac{q_0}{2}\right) H_0^2 (t_0 - t_e)^2$$

$$\underbrace{\left(1+\frac{q_0}{2}\right) H_0^2}_{a} x^2 + H_0 x - z \approx 0 \qquad ; \; x \equiv t_0 - t_e$$

$$x^2 + \frac{H_0}{a} x - \frac{z}{a} \approx 0$$

The solution is

$$x = \frac{1}{2}\left[-\frac{H_0}{a} \pm \left(\left(\frac{H_0}{a}\right)^2 + 4\frac{z}{a}\right)^{1/2}\right]$$

$$x = \frac{1}{2}\left[-\frac{H_0}{a} \pm \frac{H_0}{a}\left(1 + \underbrace{\frac{a}{H_0^2} 4z}_{=\, y}\right)^{1/2}\right]$$

$$\left(1 + \frac{1}{2}y - \frac{1}{8}y^2 + \cdots\right)$$

$$x \approx \frac{1}{2}\left[\frac{H_0}{a}\cdot\frac{1}{2}y - \frac{1}{8}\frac{H_0}{a}y^2\right]$$

$$\approx \frac{1}{2}\left[\frac{H_0}{2a}\cdot\frac{4a}{H_0^2} z - \frac{1}{8}\frac{H_0}{a}\frac{16a^2}{H_0^4} z^2\right]$$

$$\approx \frac{1}{H_0} z - \frac{1}{H_0^3}\left(1+\frac{q_0}{2}\right) H_0^2 z^2$$

$$\Rightarrow t_L = (t_0 - t_e) \approx \frac{1}{H_0}\left[z - \left(1+\frac{q_0}{2}\right) z^2\right]$$

Substituting this into $d_p(t_0)$ gives

$$d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right) z^2\right] + \frac{cH_0}{2}\frac{1}{H_0^2}\left[z - \left(1+\frac{q_0}{2}\right) z^2\right]^2$$

$$d_p(t_0) \approx \frac{c}{H_0}\left[z - \left(1+\frac{q_0}{2}\right) z^2\right] + \frac{c}{2H_0} z^2 + O(z^3 \ldots)$$

$(z \ll 1)$ $\boxed{d_p(t_0) \approx \frac{c}{H_0} z\left[1 - \left(\frac{1+q_0}{2}\right) z\right]}$; $\left(\frac{1+q_0}{2}\right) z \lll 1$

or $z \lll \left(\frac{2}{1+q_0}\right)$

$$d_p(t_0) \approx \frac{c}{H_0} z - \frac{c}{2H_0}(1+q_0) z^2$$

↑ the Hubble relation

The Hubble relation holds true only in the limit

$$\frac{1}{2}(1+q_0) z \ll 1 \quad \text{or} \quad z \ll \frac{2}{(1+q_0)}$$

If $q_0 > -1$ (in the Benchmark model $q_0 \approx -0.5 > -1$)

then the proper distance to a galaxy of moderate redshift ($z \sim 0.1$ say) would be less than would be predicted ~~by~~ from the linear Hubble relation.

Luminosity Distance: (Do Parallax Distance First.)

Definition: A standard candle is an object whose (absolute) luminosity L is known.

The apparent luminosity or flux is

$$f = \frac{L}{4\pi d_L^2},$$

and d_L is the luminosity distance.

$$d_L = \left(\frac{L}{4\pi f}\right)^{1/2}$$

But in a universe described by the Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a(t)^2 [dr^2 + S_k(r)^2 d\Omega^2]$$

with
$$S_k(r) = \begin{cases} R_0 \sin(r/R_0) & ; \ k=+1 \\ r & ; \ k=0 \\ R_0 \sinh(r/R_0) & ; \ k=-1 \end{cases}$$

So, in a Robertson-Walker universe (~~with~~ possibility of space curvature and expanding) the area $4\pi d_L^2$ in $f = \frac{L}{4\pi d_L^2}$ is replaced by

$$f = \frac{L}{A_p(t_0)} \quad , \quad A_p(t_0) = 4\pi S_k^2(r)$$

(not including the expansion effect).

The expansion of the universe causes the observed flux f of light from a standard candle of redshift z to be decreased by a factor of $(1+z)^{-2}$ ← (see Ryden p. 109)

So, in a spatially curved & expanding universe the flux f is

$$f = \frac{L}{4\pi S_k(r)^2 (1+z)^2} = \frac{L}{4\pi d_L^2}$$

, where $\boxed{d_L = S_k(r)(1+z)}$

Observations show that the universe is nearly flat with a radius of curvature R_0 much larger than the current horizon distance $d_{hor}(t_0)$.

$$d_{hor}(t_0) \ll R_0$$

since $d_p(t_0) < d_{hor}(t_0) \Rightarrow r \ll R_0$

$$\underset{a(t_0)r = r}{\overset{\shortparallel}{d_p(t_0)}}$$

$$\Rightarrow S_k(r) \begin{cases} \approx r & , k=1 \\ = r & , k=0 \\ \approx r & , k=-1 \end{cases}$$

$$\Rightarrow d_L = r(1+z) = d_p(t_0)(1+z)$$

When $z \ll 1$, using $d_p(t_0) \approx \frac{c}{H_0} z \left(1 - \frac{(1+q_0)}{2} z\right)$

$$\Rightarrow \quad d_L \approx \frac{c}{H_0} z \left(1 - \frac{(1+q_0)}{2} z\right)(1+z)$$

$$\approx \frac{c}{H_0} z \left[1 - \frac{(1+q_0)}{2} z + z - \cancel{\frac{(1+q_0)}{2} z^2}\right]$$

negligible

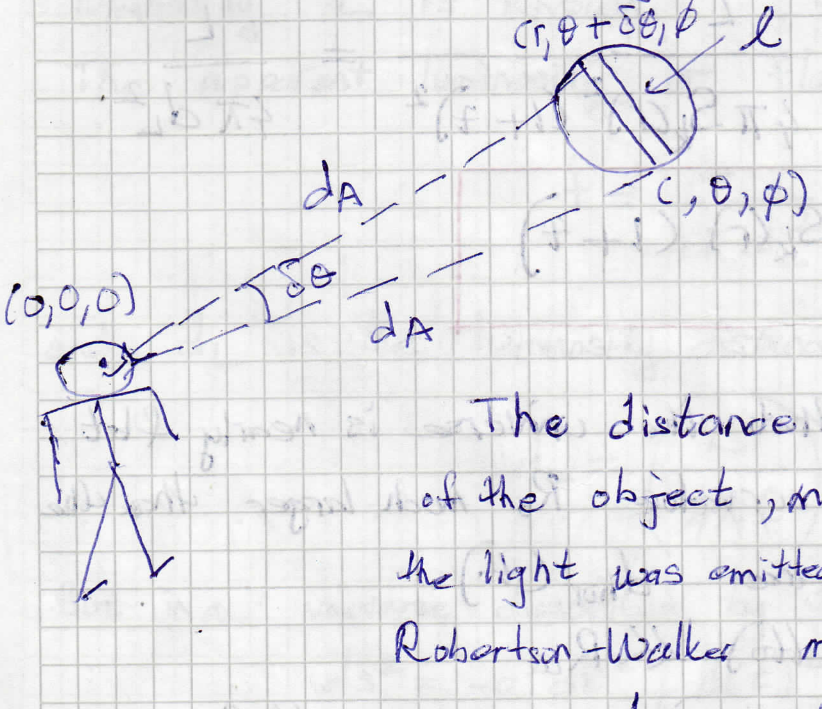
$$\left[1 + \frac{(1-q_0)}{2} z\right]$$

$$\boxed{d_L \approx \frac{c}{H_0} z \left[1 + \frac{(1-q_0)}{2} z\right]} \quad (z \ll 1)$$

(Size)

Angular-Diameter Distance

Suppose that instead of a standard candle, a standard distance (standard yardstick) is used, Suppose a light source of proper length l is aligned perpendicular to the observer's line of sight



$$\delta\theta = \frac{l}{d_A}$$

$$d_A = \frac{l}{\delta\theta}$$

The distance ds between the two ends of the object, measured at time t_e when the light was emitted, can be found from the Robertson-Walker metric

$$ds = a(t_e) S_k(r)\, \delta\theta$$

For a source of diameter l we can set

$$l = ds = a(t_e) S_k(r)\, \delta\theta = \frac{S_k(r)\,\delta\theta}{1+z}$$

, where $1+z = \dfrac{1}{a(t_e)}$ has been used. Thus

$$d_A = \frac{\ell}{\delta\theta} = \frac{S_k(r)}{1+z}$$

Note that for a static and Euclidean universe $(k=0)$
$S_k(r) = S_0(r) = r$ and $z=0 \Rightarrow d_A = r = d_p(t_0)$

Since $d_L = S_k(r)(1+z) \Rightarrow$

$$d_A = \frac{d_L}{(1+z)^2} \quad \text{(for all } k\text{)}$$

Since $d_L = d_p(t_0)(1+z)$

$$d_A = \frac{d_p(t_0)}{(1+z)} = d_p(t_e)$$

$$d_A \approx \frac{c}{H_0} z\left(1 - \frac{1+q_0}{2} z\right)\frac{1}{1+z} \approx \left[\frac{c}{H_0} z - \frac{c}{2H_0}(1+q_0) z^2\right] \times (1 - z + z^2)$$

$$\approx \frac{c}{H_0} z - \frac{c}{2H_0}(1+q_0) z^2 - \frac{c}{H_0} z^2$$

$$\boxed{d_A \approx \frac{c}{H_0} z\left(1 - \frac{3+q_0}{2} z\right)}$$

Note that for $z \to 0$, $d_A \approx d_L \approx d_p(t_0) \approx \dfrac{c}{H_0} z$.

For large z:

Recall: $d_L = d_p(t_0)(1+z)$; $d_A = \dfrac{d_p(t_0)}{1+z}$

as $z \to \infty$ (very large, actually)

Also, $d_p(t_0) \xrightarrow[z\to\infty]{} d_{hor}(t_0)$ as $z \to \infty$

$$\Rightarrow \quad d_L \xrightarrow[z\to\infty]{} d_p(t_0) \cdot z = z\, d_{hor}(t_0)$$

$$d_A \xrightarrow[z\to\infty]{} \frac{1}{z} d_p(t_0) = \frac{1}{z} d_{hor}(t_0)$$

Side Remark: Proof that $d_p(t_0) \xrightarrow[z\to\infty]{} d_{hor}(t_0)$

Take $a(t) = \left(\frac{t}{t_0}\right)^p$, where p may be >1, $=1$ or <1

$$d_{hor}(t) = \int_0^t \frac{dt}{a(t)} = t_0^p \int_0^t t^{-p} dt = t_0^p \left. \frac{t^{1-p}}{(1-p)} \right|_0^t$$

$$= \frac{t_0^p}{(1-p)} \left[t^{1-p} - \frac{1}{0^{p-1}} \right] = \begin{cases} \frac{t_0^p}{(1-p)} \cdot t^{1-p} \ ; & p<1 \\ \infty \ ; & p=1 \\ \infty \ ; & p>1 \end{cases}$$

$$d_{hor}(t_0) = \begin{cases} \frac{t_0}{(1-p)} \ ; & p<1 \\ \infty \ ; & p \geqslant 1 \end{cases}$$

$$d_p(t_0) = \int_{t_e}^{t_0} \frac{dt}{a(t)} = \frac{t_0^p}{(1-p)} \left. t^{1-p} \right|_{t_e}^{t_0} = \frac{t_0^p}{(1-p)} \left(t_0^{1-p} - t_e^{1-p} \right)$$

$$= \frac{t_0}{(1-p)} \left[1 - \left(\frac{t_e}{t_0}\right)^{1-p} \right] = \frac{t_0}{(1-p)} \left[1 - a(t_e)^{\frac{1}{p}(1-p)} \right]$$

$$= \frac{t_0}{(1-p)} \left[1 - (1+z)^{-\frac{1}{p}(1-p)} \right] = \frac{t_0}{(1-p)} \left[1 - (1+z)^{1-\frac{1}{p}} \right]$$

$$\xrightarrow[z\to\infty]{} \begin{cases} \frac{t_0}{(1-p)} = d_{hor}(t_0) \ ; & p<1 \\ \frac{t_0}{(p-1)} z^{\frac{p-1}{p}} \to \infty \ ; & p>1 \end{cases}$$

Standard Candles & H_0:

Hubble himself used standard candles to determine H_0:

The recipe:

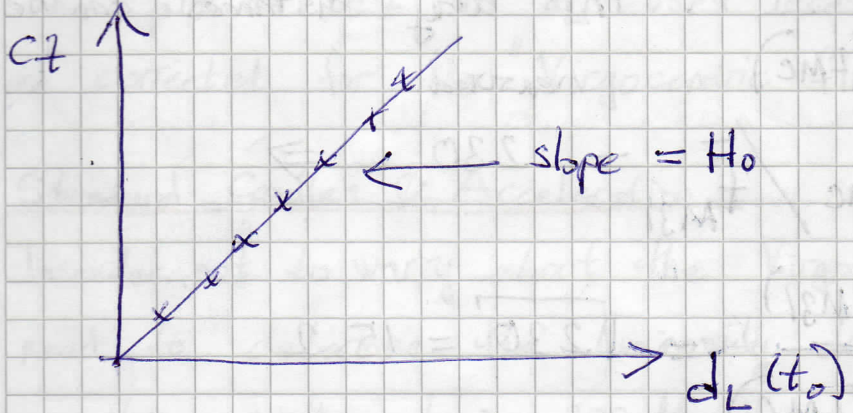
a) Identify a population of standard candles with luminosity L

b) Measure the redshift z and flux f for each standard candle

c) Compute $d_L = \left(\frac{L}{4\pi f}\right)^{1/2}$ for each standard candle

d) Plot cz versus d_L

e) Measure the slope of the cz versus d_L relation when $z \ll 1$; this gives H_0. $d_L \approx \frac{c}{H_0} z \Rightarrow cz \approx H_0 d_L$



For step a) Cepheid variable stars are used. Cepheids are highly luminous supergiant stars, with mean luminosities in the range $\bar{L} = 400 - 40\,000\, L_\odot$ ← luminosity of the Sun

$(L_\odot = 3.846 \times 10^{26}\ W)$

From $L = 4\pi f d_L^2 \Rightarrow \frac{L_1}{L_2} = \frac{d_{L_1}^2}{d_{L_2}^2}$

if $d_{L_1} = 10\, d_{L_2} \Rightarrow \frac{L_1}{L_2} = 100 \Rightarrow L_1 = 100 L_2$

Observed brightness $f = \frac{L}{\text{Area}}$

/levit/

Henrietta Swan Leavitt studied the cepheids in the Small Magellanic Cloud in our galaxy, the Milky Way.

Their brightness pulsate.

She noted that $L \propto$ Period P

She assumed the cepheids in SMC had approximately the same distance.

$$L = \text{Const.}\, P = f\, 4\pi d_L^2$$

So, if $P_{LMC} = P_{M31} \Rightarrow \frac{f_{LMC}\, d_{L\,LMC}^2}{f_{M31}\, d_{L\,M31}^2} = 1$

$$\left(\frac{f_{LMC}}{f_{M31}}\right)^{1/2} = \frac{d_L(M31)}{d_L(LMC)}$$. This way distances can be measured.

Example: $\bar{f}_{LMC} / \bar{f}_{M31} = 230 \Rightarrow$

$$\frac{d_L(M31)}{d_L(LMC)} = \sqrt{230} = 15.2$$

$$\Rightarrow d_L(M31) = 15.2\, d_L(LMC)$$

But, how to measure $d_L(LMC)$? Answer: By measuring the parallax distance.

<u>Note:</u> Within our galaxy, which is not expanding, the parallax, proper, and luminosity distances are identical.

The fluxes (f) and periods of Cepheids can be accurately measured out to luminosity distances $d_L \sim 30$ Mpc.

The Hubble Key Project determined that the Cepheid data are best fitted with $H_0 = 75 \pm 8 \, km s^{-1} Mpc^{-1}$

Difficulty with using Cepheids to determine H_0:

Accurately measurable maximum Cepheid distance is $d_L \sim 30 Mpc$. But on such scales which are less than 100 Mpc the universe cannot be assumed to be homogeneous and isotropic.

The Local Group of galaxies, more than 54, whose gravitational center is somewhere between the Milky way and the Andromeda Galaxy (M31), is gravitationally attracted to the Virgo Cluster.

The plot of cz vs d_L uses recession velocities that are corrected for the "Virgocentric flow"

Standard Candles & Acceleration:

In order not to worry about the Virgocentric flow, we need to determine the luminosity distance to standard candles with $d_L > 100 \, Mpc$.

Now,
$$d_L \approx \frac{cz}{H_0} = z \, d_H(t_0) = z \cdot 4500 \, Mpc$$

$$\Rightarrow \quad z4500 > 100 \quad \Rightarrow \quad z45 > 1$$

$$z > \frac{1}{45} \approx \frac{1}{50} \approx 0.02$$

For $z \ll 1 \Rightarrow d_L \approx \frac{c}{H_0} z \left[1 + \left(\frac{1-q_0}{2}\right) z \right]$

* For a standard candle to be seen at $d_L > 1000$ Mpc it must be very luminous.

Recently standard candles are chosen to be Type I supernovae. A supernova may be loosely defined as an exploding star.

Type I supernovae contain no hydrogen absorbtion lines in their spectra

Type II supernovae contain hydrogen " " " .

Type II supernovae are massive stars with $M > 8 M_\odot$

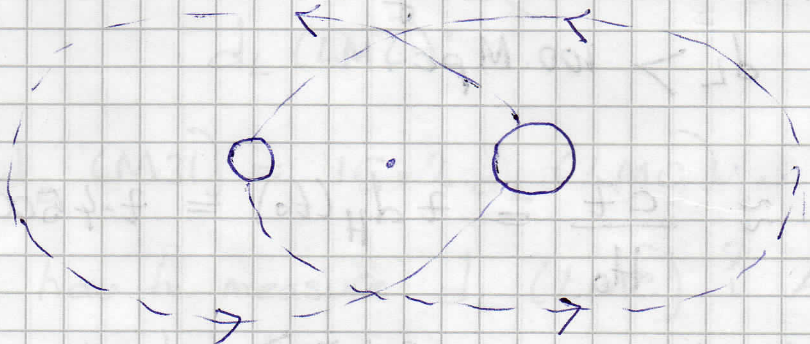
Type I supernovae → Type Ia

Type I supernovae → Type Ib → similar to Type II

A white dwarf star has a mass $M_{WD} < 1.44 M_\odot$ (the Chandrasekhar limit)

it can exceed this mass limit in two steps

i) it can form a binary star system orbiting each other with another WD star



ii) it accreats gas from its ~~a~~ binary ~~stellar~~ companion

When the mass exceeds $1.44 M_\odot$

the WD starts to collapse until its increased density triggers a nuclear fusion reaction. The white dwarf becomes a fusion bomb.

* Type Ia supernovae within our galaxy occur roughly once per century.

They are extraordinarily luminous

At peak brightness $L = 4 \times 10^9 L_\odot$

$$= \underbrace{(4 \times 10^4 L_\odot)}_{\text{brightness of the brightest Cepheid.}} \times 10^5$$

In 1998, two research teams, the Supernova Cosmology Project and the High-z Supernova Search Team published the results of their searches. They concluded that the universe is expanding acceleratingly.

For an object of luminosity L at a distance d_L from us, the apparent and absolute magnitudes are defined by

$$m = -2.5 \log \frac{L}{4\pi d_L^2}$$

$$M = -2.5 \log \frac{L}{4\pi (10pc)^2} \leftarrow d_L = 10pc$$

(Absolute magnitude M is defined as the apparent magnitude of an object when seen at a distance of 10 parsecs)

Distance Modulus $= m - M$

$$= 2.5 \left[\cancel{-\log L} + \cancel{\log 4\pi} + \log d_L^2 + \cancel{\log L} \cancel{-\log 4\pi} - \log (10)^2 \right]$$

$\boxed{m - M = 5 \log d_L - 5}$, where d_L is in parsecs.

or $d_L = 10^{\left(\frac{m-M+5}{5}\right)}$ $\rightarrow$ $m - M = 5 \log \left(\frac{d_L}{1pc}\right) - 5$

$\uparrow$ in parsecs.

In cosmology distances are measured in Megaparses.

Then

$$m - M = 5 \log\left(\frac{d_L(\mathrm{Mpc}) \times 10^6}{1\,\mathrm{Mpc}}\right) - 5$$

$$= 5 \log\left(\frac{d_L}{1\,\mathrm{Mpc}}\right) + \underbrace{5 \log 10^6}_{\substack{\shortparallel \\ 30}} - 5$$

$$\boxed{m - M = 5 \log\left(\frac{d_L}{1\,\mathrm{Mpc}}\right) + 25}$$

$$\frac{d_L}{1\,\mathrm{Mpc}} = 10^{\left(\frac{m-M-25}{5}\right)}$$

$m - M$	$d_L\,(\mathrm{Mpc})$
-5	10^{-6}
0	10^{-5}
5	10^{-4}
10	10^{-3}
15	10^{-2}
20	10^{-1}
25	1
30	10
35	10^{2}

When $z \ll 1$ we use

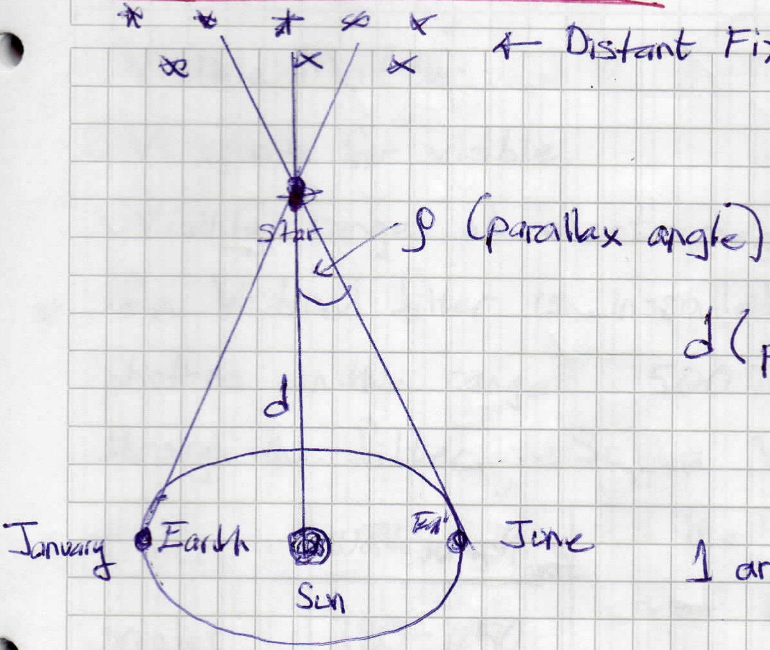
$$d_L \approx \frac{c}{H_0}\left(z + \frac{1-q_0}{2} \cdot z^2\right)$$

$$m - M = 5 \log\left[\frac{\frac{c}{H_0}\left(z + \frac{1-q_0}{2} \cdot z^2\right)}{1\,\mathrm{Mpc}}\right] + 25$$

$$= 5 \log\left(\frac{c}{H_0} z\right) + 5 \log\left(1 + \frac{1-q_0}{2} z\right)$$

$$1\,\mathrm{Mpc} = 3.09 \times 10^{22}\ \mathrm{m}$$

The Parallax Distance



$$d(pc) = \frac{1}{p\,(arcsec)}$$

degree

$$1 \text{ arcsecond} = \frac{1^\circ}{3600}$$

$$\text{arcsecond} = 1''$$