Ch. 8 The Cosmic Microwave Background

The current energy density of the cosmic microwave background is

$$\varepsilon_{\gamma,0} = \alpha T_0^4 = 0.2606 \text{ MeV m}^{-3}$$

whereas the critical energy density is

$$\varepsilon_{c,0} = 4870 \pm 290 \text{ MeV m}^{-3}$$
$$\approx 5000 \text{ MeV m}^{-3}$$

$$\frac{\varepsilon_{\gamma,0}}{\varepsilon_{c,0}} \approx \frac{25\times10^{-2}}{50\times10^{2}} = \frac{1}{20{,}000}$$

The number density of CMB photons is

$$n_{\gamma 0} = \int_0^\infty \frac{8\pi f^2 df}{c^3\left(e^{hf/k_BT}-1\right)} = \frac{30\,\zeta(3)}{\pi^4}\,\frac{\alpha T^3}{k_B} = 0.2436 \times \left(\frac{k_BT}{\hbar c}\right)^3$$

where $\alpha \equiv \dfrac{8\pi^5 k_B^4}{15 h^3 c^3}$, $\zeta(3) = 1.202$

$\alpha = 4.72\times10^{-3}\ \text{MeV}\cdot\text{m}^{-3}\text{K}^{-4}$

For $T = 2.7255$ this gives a present number density

$$n_{\gamma 0} = 410.7 \text{ photons/cm}^3$$

In the Benchmark model, the current energy density of baryons is

$$\varepsilon_{bar,0} = \Omega_{bar,0}\,\varepsilon_{c0}$$
$$= 0.048\,(4870) = 234 \text{ MeV m}^{-3}$$

Thus $\dfrac{\varepsilon_{bar,0}}{\varepsilon_{\gamma,0}} \approx \dfrac{234}{0.26} = 900$

The number density of baryons today

$$n_{bary,0} = \frac{\varepsilon_{bary,0}}{E_{bary}} = \frac{\varepsilon_{bary,0}}{m_{bary}c^2} = \frac{234}{939}\text{ m}^{-3}$$
$$\approx 0.25 \text{ m}^{-3}$$

The ~~baryon~~ photon-to-baryon ration

$$\frac{n_{\gamma 0}}{n_{baryo}} = \frac{4.107 \times 10^{8}\ m^{-3}}{0.25\ m^{-3}} = 16.428 \times 10^{8}$$

$$= 1.6 \times 10^{9}$$

For one baryon there are about 1.6 billion photons in the universe.

Side Remarks: The electromagnetic force, weak force, and strong force become equal in strength and unify to one force governed by a (simple ?) Lie Group. The approximate GUT energy scale is $\Lambda_{GUT} = 1 \times 10^{25}$ eV

These three forces unify with the gravitational force in a so called Theory of Everything at an energy scale assumed to be close to the Planck scale.

$$E_{planck} = 1.22 \times 10^{28}\ eV \quad \#$$

In 1965, Penzias & Wilson discovered the CMB radiation. Later CMB was studied by the COBE satellite (launched in 1989) the Wilkinson Microwave Anisotropy Probe (launched in 2001), and by the Planck satellite (on launched in 2009)

Their findings were:

Result # one: At any angular position (θ, ϕ) on the sky, the spectrum of CMB is very close to an ideal black body spectrum.

Cobe found $\frac{\Delta T}{T} \lesssim 10^{-5}$

Result # 2: The CMB has the dipole distortion in temperature $\frac{\Delta T}{T} = \frac{v}{c}$. Although each point on the sky has a blackbody spectrum, in one half of the sky the spectrum is slightly blueshifted to higher temperatures, and in the other half the spectrum is slightly redshifted to lower temperatures

$$E = N_{\gamma_0} h f = N_{\gamma_0} h \frac{c}{\lambda} = \alpha T_0^4 \Rightarrow \lambda \propto \frac{1}{T_0^4}$$

as $\lambda \rightarrow \underset{(\text{large})}{\text{red}}$, $T_0 \rightarrow$ low

$\lambda \rightarrow \underset{(\text{small})}{\text{blue}}$, $T_0 \rightarrow$ large

This dipole distortion is a simple Doppler shift caused by the motion of the satellite relative to a frame in which the CMB is isotropic.

Result # 3: After the dipole distortion of the CMB is subtracted away, the remaining temperature fluctuations are small:

$$\langle T \rangle = \frac{1}{4\pi} \iint T(\theta, \phi) \sin\theta \, d\theta \, d\phi = 2.7255 K$$

$$\frac{\delta T}{T} = \frac{T(\theta, \phi) - T}{\langle T \rangle}$$

$$\left\langle \left(\frac{\delta T}{T} \right)^2 \right\rangle^{1/2} = 1.1 \times 10^{-5}$$

$$\Rightarrow \delta T_0 = T_0 \times (1.1 \times 10^{-5}) = 2.7255 \times 1.1 \times 10^{-5} K$$
$$= 2.998 \times 10^{-5} K \approx 30 \mu K \Rightarrow$$ the CMB is very closely isotropic.

Recombination and Decoupling:

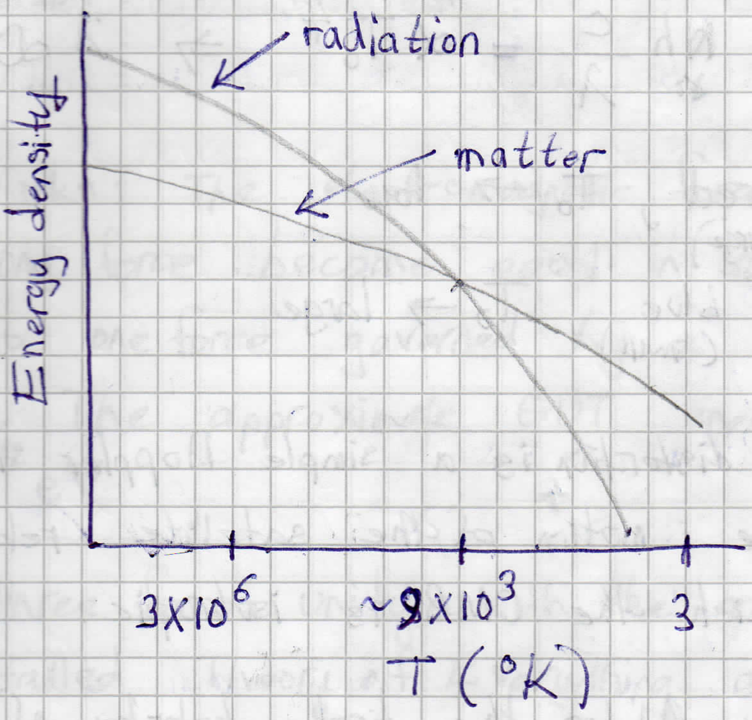
In the very early universe

$$\gamma + \gamma \rightleftarrows \text{particle} + \text{antiparticle}$$

Recall that

$$\varepsilon_m = \varepsilon_{m0}\, a^{-3}$$

$$\varepsilon_r = \varepsilon_{r0}\, a^{-4}$$



The epoch of recombination: The time at which the baryonic component of the universe goes from being ionized to being neutral.

photons + baryons + electrons $\rightarrow$ photons + H atoms + He atoms

He atoms ↑ negligible

$t_{recombination}$ is when $n_p = n_e$

n_p: # density of protons

n_e: # density of electrons $= n_H$

n_H: # density of hydrogen atoms

when $\varepsilon_r = \varepsilon_m$

$$\Rightarrow \frac{\varepsilon_{r0}}{a^4} = \frac{\varepsilon_{m0}}{a^3} \Rightarrow a(t_{rm}) = \frac{\varepsilon_{r0}}{\varepsilon_{m0}} = \frac{\Omega_{r0}}{\Omega_{m0}}$$

$$a_{rm} = \cancel{a(t_{recomb})} = \frac{9 \times 10^{-5}}{0.31} \approx 2.9 \times 10^{-4}$$

We found before that $\cancel{t_{recom} \neq}\ t_{rm} = 50\,000$ yrs.

The epoch of photon decoupling:

Let $\Gamma_{\gamma e}$ = the rate at which photons scatter from electrons.

~~When~~ $H(t) = \frac{\dot{a}}{a}$ = the rate at which the univere expands

When $\Gamma_{\gamma e}$ becomes less than $H(t)$ photons cease to interact with the electrons. This time is called photon decoupling.

$T_{decoup} \simeq 3 \times 10^3 K$

When photons decouple, the universe becomes transparent.

The epoch of last scattering:

This is when a CMB photon underwent its last scattering from an electron.

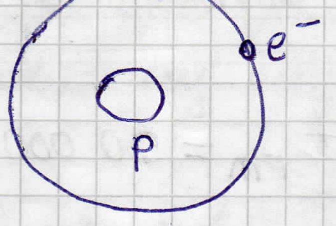
Now the calculations:

Let X = the fractional ionization $= \frac{n_p}{n_p + n_H}$

$$= \begin{cases} 1, & n_H = 0 \ \text{(fully ionized universe)} \\ 0, & n_p = 0 \ \text{(universe consisted of neutral atoms, mainly Hydrogen)} \end{cases}$$

Radiative Recombination:

$$p + e^- \rightarrow H + \gamma$$: Formation of Hydrogen



H-atom

Photoionization:

$$H + \gamma \rightarrow p + e^-$$

energy $= Q = 13.6$ eV

* At the time of recombination the photons, electrons, protons, and hydrogen atoms are in a state of thermal equilibrium.
$\Rightarrow$ they all have the same temperature T,

** Each particle type is in a state of kinetic equilibrium. This means that

$$n_x(p)\,dp = g_x \frac{4\pi}{h^3} \frac{p^2\,dp}{e^{(E-\mu_x)/k_B T} \pm 1},$$

where $n_x =$ number density of x particles

$g_x =$ statistical weight of the x particle

$= 2$ (for photons, electrons, protons, neutrons)

$\mu_x =$ chemical potential of particle x

$+$ sign for Fermions

$-$ " " Bosons

At $t = t_{recomb}$ all the particles had $m_x c^2 \gg k_B T$

$$E_x = \sqrt{m_x^2 c^4 + p^2 c^2} = m_x c^2 \sqrt{1 + \frac{p^2}{m_x^2 c^2}}$$

$$\approx m_x c^2 \left(1 + \frac{1}{2} \frac{p^2}{m_x^2 c^2}\right) = m_x c^2 + \frac{1}{2} \frac{p^2}{m_x}$$

Thus for particles with $(m_x c^2 - \mu_x) \gg k_B T$ $\Rightarrow$

$$n_x(p)dp = g_x \frac{4\pi}{h^3} \exp\left(\frac{-m_x c^2 + \mu_x}{k_B T}\right) \exp\left(-\frac{p^2}{2m_x k_B T}\right) \times p^2 dp$$

$$n_x = \int n_x(p)dp = g_x \exp\left(\frac{-m_x c^2 + \mu_x}{k_B T}\right) \frac{4\pi}{h^3} \int e^{-\frac{p^2}{2m_x k_B T}} p^2 dp$$

$$\int_0^\infty e^{-ax^2} x^2 dx = \frac{\pi^{1/2}}{4a^{3/2}} \quad , x = p \; ; \; a = \frac{1}{2m_x k_B T}$$

$$\Rightarrow n_x = g_x \frac{4\pi}{h^3} \frac{\pi^{1/2}}{4} (2m_x k_B T)^{3/2} \exp(\quad)$$

$$n_x = g_x \left(\frac{m_x k_B T}{2\pi \hbar^2}\right)^{3/2} \exp\left(\frac{-m_x c^2 + \mu_x}{k_B T}\right)$$

~~$\mu_x = 0$, if $x \neq$ photons~~

At $t = t_{recombination}$ $\mu_\gamma = 0$

$$H + \gamma \rightleftharpoons p + e^-$$

$$\Rightarrow \mu_H + \underset{0}{\cancel{\mu_\gamma}} = \mu_p + \mu_{e^-}$$

$$\Rightarrow \frac{n_H}{n_p n_e} = \frac{g_H}{g_p g_e} \left(\frac{m_H}{m_p m_e}\right)^{3/2} \left(\frac{k_B T}{2\pi\hbar^2}\right)^{-3/2} \exp\left[\frac{(m_p + m_e - m_H)c^2}{k_B T}\right]$$

since $m_H \approx m_p$; $(m_p + m_e - m_H)c^2 = Q = 13.6\,eV$

$$g_p = g_e = 2 \quad , \; g_H = \overset{1}{\cancel{g_p}} + \overset{3}{\cancel{g_e}} = 4 \Rightarrow$$

$$\boxed{\frac{n_H}{n_p n_e} = \left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{k_B T}\right)}$$

This is called the Saha Equation

Next, we will convert this to an ~~equation~~ relation between X, T, and $\eta \equiv \frac{n_{bary}}{n_\gamma}$

From $X = \frac{n_p}{n_p + n_H} = \frac{n_p}{n_{bary}} = \frac{n_e}{n_{bary}}$

$$\Rightarrow \quad n_p = n_p X + n_H X \quad \Rightarrow \quad n_H = \left(\frac{1-X}{X}\right) n_p$$

We can let $n_p = n_e$ (at $t = t_{recomb}$)

$$\Rightarrow \frac{1-X}{X} = n_p \left(\frac{m_e k_B T}{2\pi \hbar^2}\right)^{-3/2} \exp\left(\frac{Q}{k_B T}\right)$$

$$\eta = \frac{n_{bary}}{n_\gamma} = \frac{n_p + n_H}{n_\gamma} = \frac{n_p}{X} \frac{1}{n_\gamma}$$

Using $n_\gamma = 0.2436 \left(\frac{k_B T}{\hbar c}\right)^3 \Rightarrow$

$$n_p = 0.2436 X \eta \left(\underline{\qquad}\right)^3$$, substituting this

into $\frac{1-X}{X} \Rightarrow$

$$\frac{1-X}{X^2} = 3.84 \eta \left(\frac{k_B T}{m_e c^2}\right)^{3/2} \exp\left(\frac{Q}{k_B T}\right) \equiv S$$

$$\Rightarrow \quad S X^2 + X - 1 = 0$$

$$X = \frac{-1 + \sqrt{1 + 4S}}{2S}$$

If the moment of recombination is defined when $X = \frac{1}{2}$ ($n_p = n_H$), then, assuming

$\eta = 6.1 \times 10^{-10}$ gives

$$k_B T_{recomb} = 0.324\,eV = \frac{Q}{42}$$

or $T_{recomb} = 3760\,K$

In the Benchmark model $t_{recomb} = 250\,000\,yr$

and $z_{recomb} = 1380$

During the epoch of recombination the rate of photon scattering ($\gamma + e^- \rightarrow \gamma + e^-$; Thomson scattering)

The mean free path of a photon, that is, the mean distance it travels before scattering from an electron is

$$\lambda = \frac{1}{n_e \sigma_e} \quad ; (\sigma_e = 6.65 \times 10^{-29}\,m^2)$$

scattering cross section

electron density

The rate at which a photon undergoes scattering is $\Gamma = \frac{c}{\lambda} = n_e \sigma_e c$

When the early universe was ~~fully ionized~~ being neutralized

$$n_e = X n_{bary} = X \frac{n_{bary,0}}{a^3}$$

from $1 + z = \frac{1}{a} \Rightarrow n_e(z) = n_{bary,0} (1+z)^3 X(z)$

$$\Rightarrow \Gamma(z) = X(z)(1+z)^3 n_{bary,0} \sigma_e c \; ; \left(n_{bary} = \frac{0.25}{m^3}\right)$$

$$= 5 \times 10^{-21} s^{-1} X(z)(1+z)^3$$

While recombination is taking place, the universe is matter-dominated.

So, $$\frac{H^2}{H_0^2} = \frac{\Omega_{m,0}}{a^3} = \Omega_{m,0}(1+z)^3$$

Using $\Omega_{m,0} = 0.31 \Rightarrow$

$$H(z) = 1.23 \times 10^{-18} s^{-1} (1+z)^{3/2}$$

Photon decoupling occurs when $\Gamma(z) = H(z)$

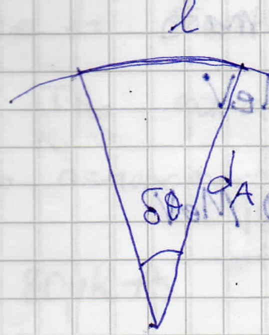
$$\Rightarrow 1 + z_{decoup} = \frac{39.3}{X(z_{dec})^{2/3}}$$

Table 8.1 :

Temperature Fluctuations:

The distortions in the CMB tell us that the universe was not perfectly homogeneous at the time of last scattering ($z_{ls} \approx 1090$)

Let $\delta\theta$ = the angular size of a temperature fluctuation in CMB, l = the physical size of the last scattering surface



d_A = angular-diameter distance

$$\delta\theta = \frac{l}{d_A} \quad \text{or} \quad d_A = \frac{l}{\delta\theta}$$

From $d_A = \dfrac{d_p(t_0)}{1+z}$, and $d_p(t_0) \xrightarrow[z\to\infty]{} d_{hor}(t_0) \Rightarrow$

$$d_A \approx \frac{d_{hor}(t_0)}{z_{ls}}$$

In the Benchmark model $d_{hor}(t_0) \approx 14\,000$ Mpc $\Rightarrow$

$$d_{A_{ls}} \approx \frac{14\,000 \text{ Mpc}}{1090} = 12.84 \text{ Mpc}$$

So, the physical size of the fluctuations ~~was~~ at the time of last scattering was $l = d_A \cdot \delta\theta = 12.84 \text{ Mpc}\left(\dfrac{\delta\theta}{1\text{rad}}\right)$

$$1 \text{ arcminute} = \frac{1 \text{ degree}}{60} = \frac{1}{60}\,\frac{1 \text{ turn}}{360} = \frac{1 \text{ turn}}{21\,600}$$

$$= \frac{2\pi \text{ rad}}{21600} = \frac{\pi \text{ rad}}{10\,800}$$

$$\Rightarrow 1 \text{ rad} = \left(\frac{10\,800}{\pi}\right) \text{arcminute} = 3437.75 \text{ arcmin}.$$

$$\Rightarrow l = \frac{12.84 \text{ Mpc}}{3437.75}\left(\frac{\delta\theta}{1\text{arcmin}}\right) = 0.003735^{\text{Mpc}}\left(\frac{\delta\theta}{1\text{arcmin}}\right)$$

$$= 3.74 \text{ kpc}\left(\frac{\delta\theta}{1\text{arcmin}}\right)$$

The smallest fluctuations resolved by the Planck satellite have $\delta\theta \approx 5$ arcmin. This corresponds to

$$l_{ls} = 3.74 \text{ kpc} \left(5\right) = 18.7 \text{ kpc}$$

$$l_{today} = (1+z)\, l_{ls} = 1091 \times 18.7 \text{ kpc} = 20401.7 \text{ kpc} \approx 20 \text{ Mpc}$$

A sphere with this radius has a baryon mass

$$M_{bary} = \frac{4\pi}{3}\left(\frac{l_{today}}{\cancel{2}}\right)^3 \cdot n_{bary} \times 939 \text{ MeV}$$

$$= 33.49 \times 10^3 \text{ Mpc}^3 \, \frac{0.25}{m^3} \times 939 \text{ MeV}$$

$$= 8.37 \left(3.09 \times 10^{22} m\right)^3 m^{-3} \times 939 \text{ MeV}$$

$$= 247 \times 10^{66} \times 939 \frac{\text{MeV}}{c^2}$$

$$= 4.15 \times 10^{71} \times 10^{-30} = 4.15 \times 10^{41} \text{ kg}$$

$$\approx 2 \times 10^{11} M_\odot$$

$1 M_\odot = 1.989 \times 10^{30}$ kg

$1 \frac{\text{MeV}}{c^2} = 1.79 \times 10^{-30}$ kg

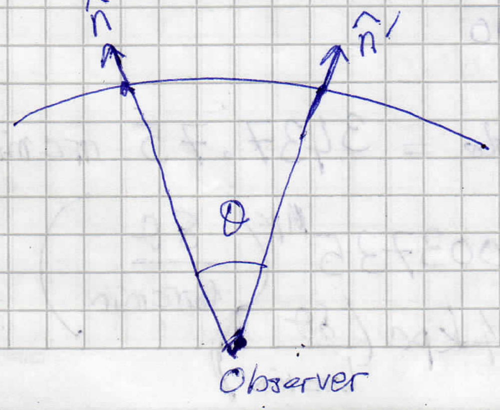
skip

The density fluctuations $\frac{\delta T}{T}$ is defined on the celestial sphere and can be expanded in spherical harmonics:

$$\frac{\delta T}{T}(\theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{+l} a_{lm} Y_{lm}(\theta, \phi)$$

The correlation function $C(\theta)$:

Consider two points on the last scattering surface



$$\hat{n} \cdot \hat{n}' = \cos\theta$$

$$C(\theta) \equiv \left\langle \frac{\delta T}{T}(\hat{n}) \, \frac{\delta T}{T}(\hat{n}') \right\rangle_{\hat{n}\cdot\hat{n}' = \cos\theta}$$

$$C(\theta) = \frac{1}{4\pi} \sum_{l=0}^{\infty} (2l+1)\, C_l\, P_l(\cos\theta),$$

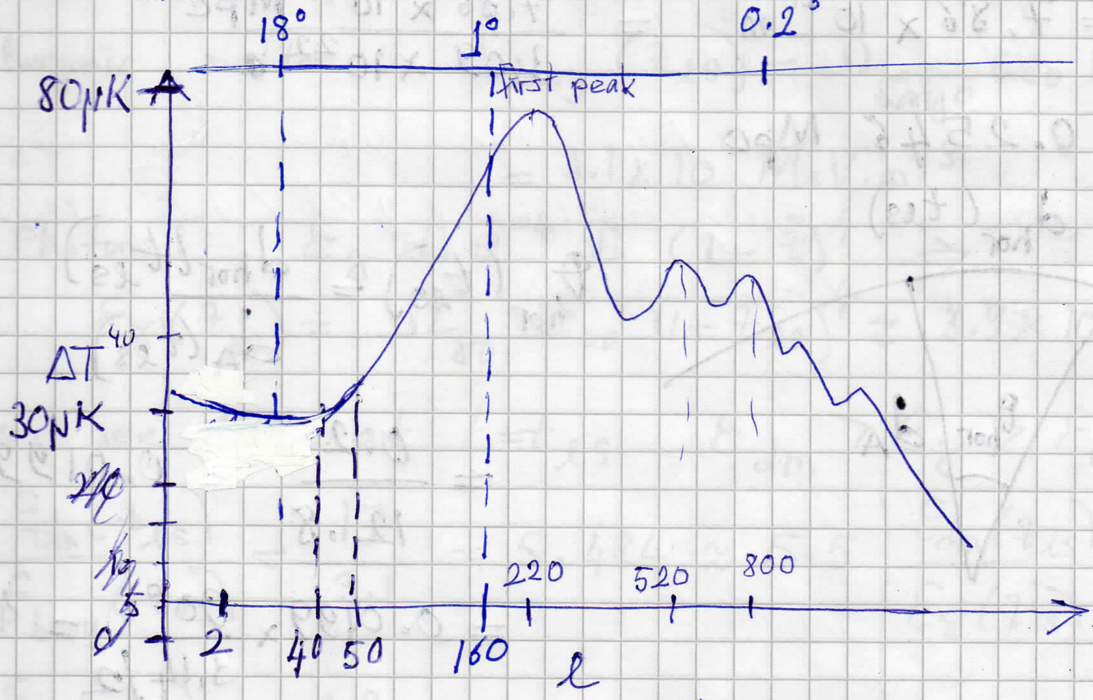
where P_l are the Legendre polynomials:

$P_0(x) = 1$, $P_1(x) = x$, $P_2(x) = \frac{1}{2}(3x^2 - 1)$, ...

* The $l = 0$ (monopole) term vanishes

* The $l = 1$ (dipole) term results from the Doppler shift due to Earth's motion through space.

* The moments with $l \geq 2$ are the most interesting ones to astronomers



$$\Delta T = \left(\frac{l(l+1)}{2\pi} C_l \right)^{1/2} \langle T \rangle$$

What causes the Fluctuations

Recall: $ds^2 = -c^2 dt^2 + a^2(t)\left(dr^2 + S_k\, d\Omega^2\right)$

For light $0 = -c^2 dt^2 + a^2(t)\, dr^2$

$$dr = c\frac{dt}{a(t)} \quad ; \quad r_{horizon} = c\int_0^t \frac{dt}{a(t)}$$

* The distance traveled by light is the horizon distance

$$d_{hor}(t) = a(t)\, r_{hor} = a(t)\, c \int_0^t \frac{dt}{a(t)}$$

At the time of last scattering

$$d_{hor}(t_{ls}) = a(t_{ls})\, c \int_0^{t_{ls}} \frac{dt}{a(t)}$$

For a universe containing just radiation and matter this gives

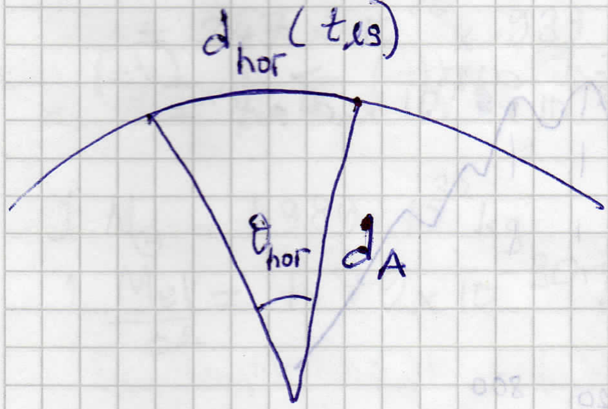
$$d_{hor}(t_{ls}) = 2.24\, c\, t_{ls} =$$

$$= 2.24 \times 3 \times 10^8\ m/s \cdot 0.37\ Myr$$

$$= 2.49 \times 10^8 \frac{m}{s} \times 10^6 \times 3.16 \times 10^7 s$$

$$= 7.86 \times 10^{21} m = \frac{7.86 \times 10^{21}}{3.09 \times 10^{22}} Mpc$$

$$= 0.2546\ Mpc$$



$$\theta_{hor}(t_{ls}) = \frac{d_{hor}(t_{ls})}{d_A(t_{ls})}$$

$$= \frac{0.25}{12.8} = 0.0199\ rad$$

$$= 0.0199 \times \frac{90^\circ}{3.14/2} = 1.14^\circ$$

$$\approx 1.1^\circ$$

This angle corresponds to a multipole $l \approx 160$

* For $\theta > 4\,\theta_{hor}$ or $l < 40$, namely on large angular scales Δ_T levels of at a nearly constant value $\Delta_T \approx 30\,\mu K$

* On angular scales smaller than θ_{hor} there are peaks & valleys.

The first peak is at $l \approx 220$, the second and third peaks are at $l \approx 520$ and 800.

* Fluctuations on large angular scales with $\theta > \theta_{hor}$ have a different physical cause than fluctuations with $\theta < \theta_{hor}$

* First, let us consider the large scale fluctuations with $\theta > \theta_{hor}$:

Energy densities at $t = t_{ls}$:

Nonbaryonic dark matter: $\varepsilon_{dm} \propto a^{-3} \propto (1+z)^3$.

$$\varepsilon_{dm}(z_{ls}) = \Omega_{dm0}\, \varepsilon_{c0}\, (1+z_{ls})^3 \simeq 1.7 \times 10^{12}\ \mathrm{MeV\,m^{-3}}$$

Baryonic matter: $\varepsilon_{bary}(z_{ls}) = \Omega_{bary0}\, \varepsilon_{c0}\, (1+z_{ls})^3$

$$= 3.1 \times 10^{11}\ \mathrm{MeV \cdot m^{-3}}$$

Photons: $\varepsilon_\gamma \propto a^{-4} \propto (1+z)^4 \Rightarrow$

$$\varepsilon_\gamma(z_{ls}) = \Omega_{\gamma 0}\, \varepsilon_{c0}\, (1+z_{ls})^4 \simeq 3.9 \times 10^{11}\ \mathrm{MeV\,m^{-3}}$$

we see that at $t = t_{ls}$ $\quad \varepsilon_{dm} > \varepsilon_\gamma > \varepsilon_{bary}$

$$\frac{\varepsilon_{dm}(z_{ls})}{\varepsilon_{bary}(z_{ls})} \simeq \frac{17}{3.1} = 5.484 \approx 5.5 \qquad \Bigg| \qquad \frac{\varepsilon_{dm}(z_{ls})}{\varepsilon_\gamma(z_{ls})} = \frac{17}{3.9} \simeq 4.359 = 4.36$$

$$\frac{\varepsilon_\gamma(z_{ls})}{\varepsilon_{bary}(z_{ls})} = \frac{3.9}{3.1} = 1.258 = 1.26$$

Therefore, the gravitational potential at the time of last scattering was dominated by that of dark matter.

The energy density of the dark matter can be written as

$$\varepsilon(\vec{r}) = \bar{\varepsilon} + \delta\varepsilon(\vec{r})$$

local deviation from the mean

$\delta\mathcal{E}$ (or $c^2\,\delta\rho$) gives rise to a spatially varying gravitational potential $\delta\Phi$

$$\nabla^2(\delta\Phi) = 4\pi G(\delta\rho) = \frac{4\pi G}{c^2}\,\delta\mathcal{E}$$

A general relativistic calculation gives

$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\Phi}{c^2}$$

* The creation of temperature fluctuations by variations in the gravitational potential is known as the Sachs-Wolfe effect.
* The region of the Δ_T curve where the temperature fluctuations are nearly constant $(2 < l < 40)$ is known as the Sachs-Wolfe plateau.