Ch. 7: DARK MATTER

Visible Matter:

V stands for visible.

visible range of electromagnetic radiation: $390 \approx 400 < \lambda < 700$ nm

* If a V-band filter is installed on the telescope photons in the range $500 \text{ nm} < \lambda < 590 \text{ nm}$ pass through the filter. So, a V-band filter lets through the green and yellow wavelengths of light.

color	wavelength (nm)
violet	380 – 450
blue	450 – 495
green	495 – 570
yellow	570 – 590
orange	590 – 620
red	620 – 750

* About 12% of the Sun's luminosity can pass through a V-band filter.

$$L_{\odot,V} = 0.12\, L_{\odot}$$
$$\approx \cancel{4.7 \times 10^{8}}$$
$$4.6 \times 10^{25} \text{ Watts.}$$

* surveys of galaxies show that in the local universe (out to $d \sim 0.1\, d_H = 0.1 \frac{c}{H_0} \approx 450 \text{ Mpc}$) the luminosity density in the V-band is

$$\Psi_V = 1.1 \times 10^{8} L_{\odot,V} \text{ Mpc}^{-3}$$

* To convert a luminosity density into a mass density ρ_* of stars, we need to know the mass-to-light ratio of the stars.

* Spectral type of stars: O, B, A, F, G, K, M

↑ hottest (brightest, most massive)

↑ coolest (dimmest) (least massive)

* It is estimated that the averaged mass-to-light ratio is about $\left\langle \frac{M}{L_V} \right\rangle \approx 4 \frac{M_\odot}{L_{\odot V}}$

⇒ the mass density of stars in the universe today is

$$\rho_{\star 0} = \left\langle \frac{M}{L_V} \right\rangle \Psi_V$$

$$\approx 4 \times 10^{8} M_\odot \, Mpc^{-3}$$

The critical mass density $\rho_{c,0} = \frac{3 H_0^2}{8\pi G} = 8.7 \times 10^{-27} \, kg m^{-3}$

$$= 8.7 \times 10^{-27} \left(\frac{M_\odot \times 10^{-30}}{1.99} \right) \left(\frac{Mpc \times 10^{-22}}{3.09} \right)^{-3}$$

$$= \underbrace{\frac{8.7 \, (3.09)^3}{(1.99)}}_{= 131.95} 10^{\overbrace{-27-30+66}^{+9}}$$

$$\rho_{c0} = 1.32 \times 10^{11} M_\odot \, Mpc^{-3}$$

The present density parameter of stars is

$$\Omega_{\star 0} = \frac{\rho_{\star 0}}{\rho_{c0}} \approx 3 \times 10^{-3} = 0.003$$

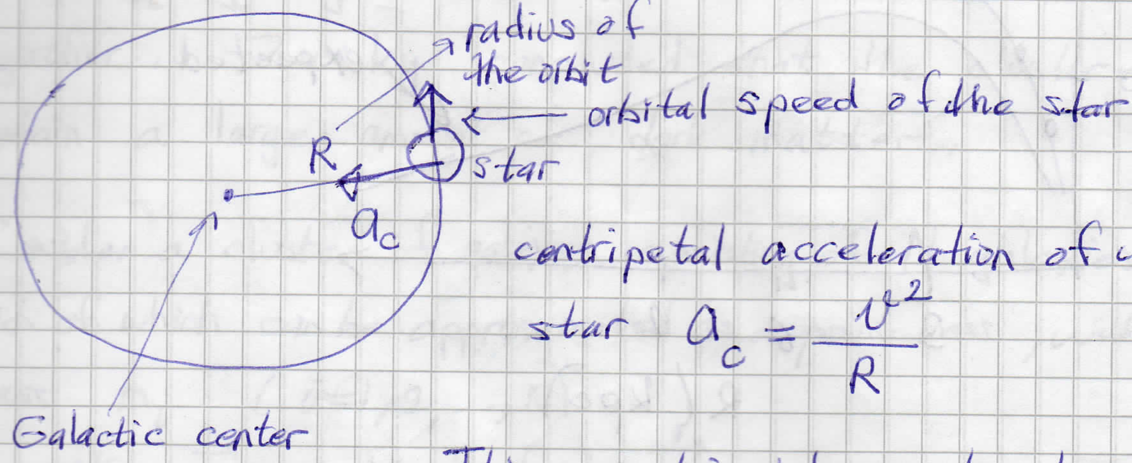
* if contribution from white dwarfs, neutron stars, black holes, brown dwarfs is included we still find

$$\Omega_{\star 0} < 0.005$$

* Observations show that about 85 % of the baryons in the universe are in the extremely tenuous (thin, diluted) gas of intergalactic space, clusters of galaxies.

Dark Matter in Galaxies:

Suppose that a star is on a circular orbit around the center of its galaxy.



centripetal acceleration of the star $a_c = \frac{v^2}{R}$

This centripetal acceleration is provided by the gravitational attraction of the galaxy

$$a = \frac{GM(R)}{R^2}, \quad \left(F = m_* a = \frac{G m_* M(R)}{R^2} \right)$$

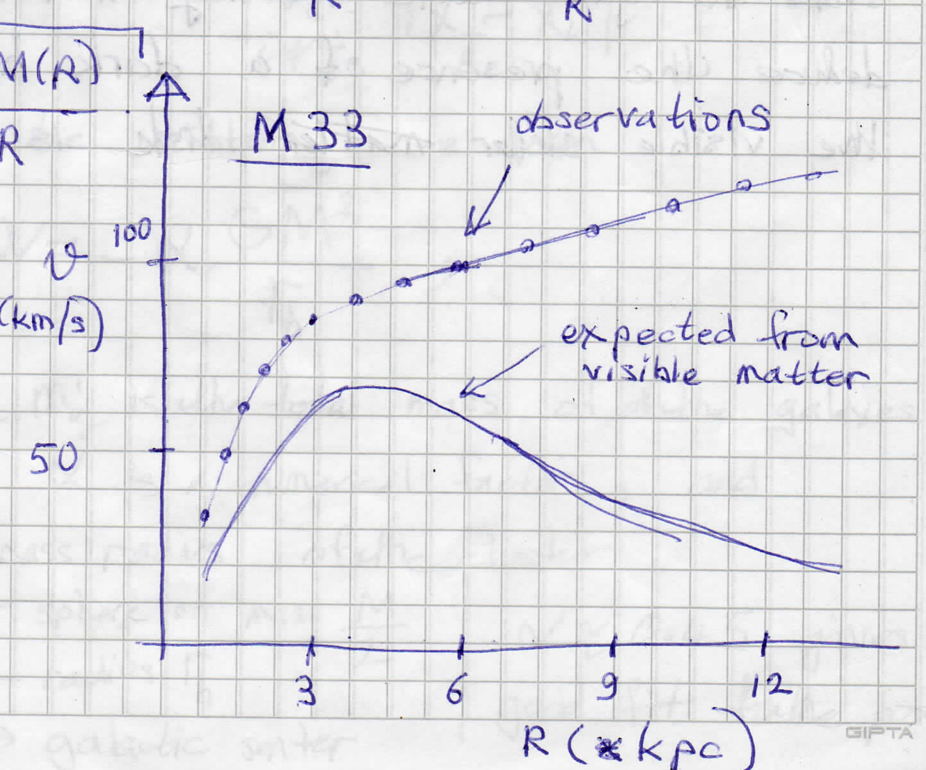
where $M(R)$ is the mass contained within a sphere of radius R centered on the galactic center.

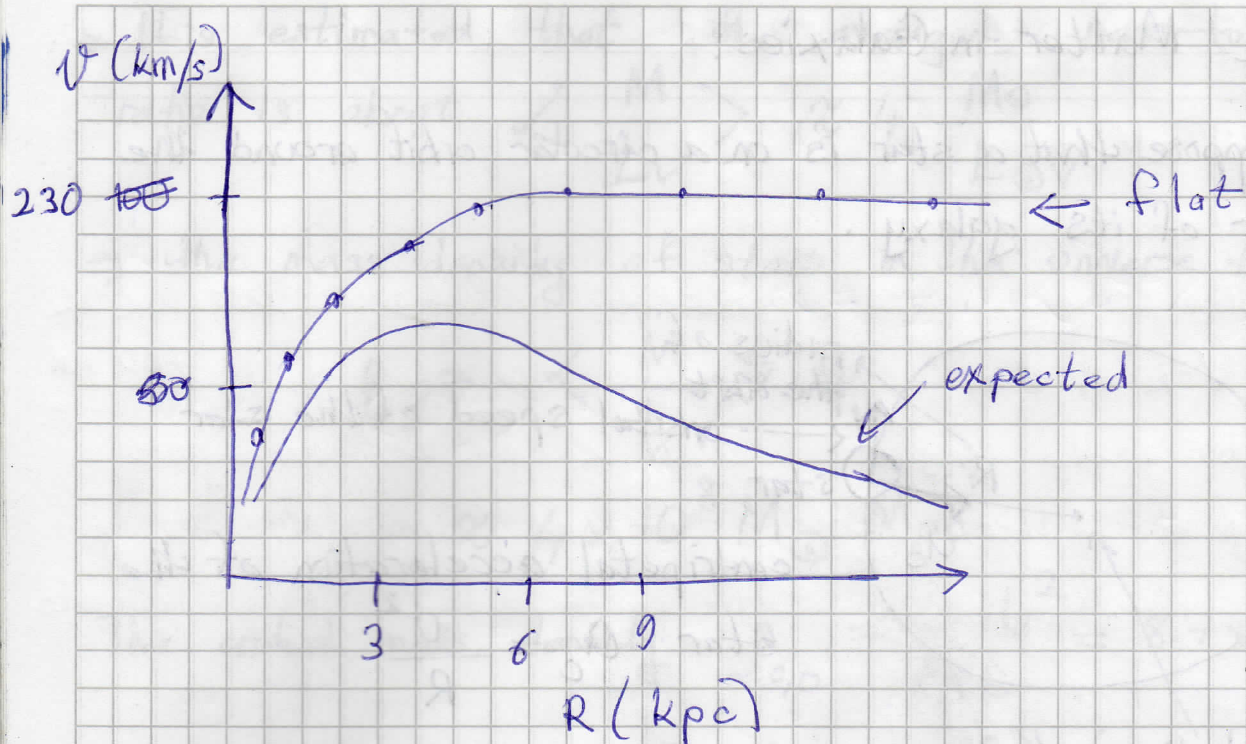
Equation a_c to a: $\frac{v^2}{R} = \frac{GM(R)}{R^2}$

$$\Rightarrow v = \sqrt{\frac{GM(R)}{R}}$$

$v \propto \frac{1}{\sqrt{R}}$

(Keplerian Rotation)





The spiral galaxy rotation curves reveal that the velocity does not follow Kepler's inverse-root law, but stays rather constant after attaining a maximum at about 5 kpc.

* Since the observed orbital speed of the stars and gas at large radii is greater than it would be if stars and gas were the only matter present, we deduce the presence of a dark halo within which the visible stellar disk is embedded.

Dark Matter in Clusters:

In 1930 Fritz Zwicky noted that the stars and gas visible within the galaxies in the Coma cluster did not provide enough gravitational attraction to hold the cluster together. Zwicky concluded that the cluster must contain a large amount of dark matter.

Consider a cluster of galaxies consisting of N galaxies, each of which can be approximated as a point mass, with a mass m_i $(i = 1, 2, \ldots, N)$
a position $\vec{x}_i$, and a velocity $\dot{\vec{x}}_i$

The acceleration of the ith galaxy is given by

$$\ddot{\vec{x}}_i = G \sum_{j \neq i} m_j \frac{\vec{x}_j - \vec{x}_i}{|\vec{x}_j - \vec{x}_i|^3}$$

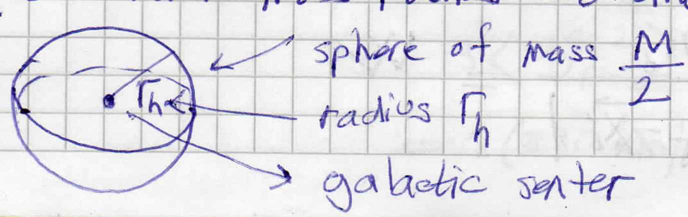
The gravitational potential energy of the system of N galaxies is

$$W = -\frac{G}{2} \sum_{\substack{i,j \\ i \neq j}} \frac{m_i m_j}{|\vec{x}_j - \vec{x}_i|},$$

which can also be written in the form

$$W = -\alpha \frac{GM^2}{r_h},$$

where $M = \sum_i m_i$ is the total mass of all the galaxies in the cluster, α is a numerical factor, and
r_h = half-mass radius of the cluster.



$\alpha \simeq 0.45$ gives a good fit to the potenti(al)

The kinetic energy of the galaxies is

$$K = \frac{1}{2} \sum_i m_i |\dot{\vec{x}}_i|^2$$

K can also be written as $K = \frac{1}{2} M \langle v^2 \rangle$,

where $\langle v^2 \rangle = \frac{1}{M} \sum_i m_i |\dot{\vec{x}}_i|^2$

Define the moment of inertia of the cluster

$$I = \sum_i m_i |\vec{x}_i|^2$$

$$\ddot{I} = \frac{d}{dt}\left(2 \sum_i m_i \, \vec{x}_i \cdot \dot{\vec{x}}_i\right) \qquad \propto K$$

$$= 2 \sum_i m_i \left(\vec{x}_i \cdot \ddot{\vec{x}}_i + \dot{\vec{x}}_i \cdot \dot{\vec{x}}_i \right)$$

$$= 2 \sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) + 4K$$

$$\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = G \sum_{\substack{i,j \\ i \neq j}} m_i m_j \frac{\vec{x}_i \cdot (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3}$$

Also $\sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j) = G \sum m_j m_i \frac{\vec{x}_j \cdot (\vec{x}_i - \vec{x}_j)}{|\vec{x}_i - \vec{x}_j|^3}$

since $\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = \sum_j m_j (\vec{x}_j \cdot \ddot{\vec{x}}_j)$

$$\Rightarrow \sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) = \frac{1}{2}\left[\sum_i m_i (\vec{x}_i \cdot \ddot{\vec{x}}_i) + \sum_j m_j \vec{x}_j \cdot \ddot{\vec{x}}_j\right]$$

$$= -\frac{1}{2} G \sum_{i,j} m_i m_j \frac{(\vec{x}_j - \vec{x}_i) \cdot (\vec{x}_j - \vec{x}_i)}{|\vec{x}_j - \vec{x}_i|^3}$$

$$= -\frac{G}{2} \sum_{i,j} \frac{m_i m_j}{|\vec{x}_j - \vec{x}_i|} = W$$

$$\Rightarrow \quad \ddot{I} = 2W + 4K$$ (this relation is known as the virial theorem)

potential energy

When $I =$ constant

$$0 = 2W + 4K$$

or $$K = -\frac{1}{2} W$$

$$\frac{1}{2} M \langle v^2 \rangle = +\frac{1}{2} \alpha \frac{GM^2}{r_h}$$

$\Rightarrow$ the mass of a cluster of galaxies is

$$M = \frac{\langle v^2 \rangle r_h}{\alpha G}$$

Now, for the Coma cluster, the mean redshift is

$$\langle z \rangle = 0.0232$$

and the distance is

$$d_L \approx d_A \approx \frac{c}{H_0} \cdot z$$

$$\approx 4380 \times 0.0232 \text{ Mpc} = 102 \text{ Mpc}$$

The velocity dispersion of the cluster along the line of sight is

$$\sigma_r = \langle (v_r - \langle v_r \rangle)^2 \rangle^{1/2}$$

$$= 880 \text{ km/s}$$

Assuming that the velocity dispersion is isotropic

$$\langle v^2 \rangle = \langle v_r^2 + v_\theta^2 + v_\phi^2 \rangle$$

$$= 3 \langle v_r^2 \rangle = 3 \sigma_r^2$$

$$= 3 (880 \text{ km/s})^2 = 2.32 \times 10^{12} \text{ m}^2/\text{s}^2$$

Observations in the Coma cluster give

$$r_h \approx 1.5\ Mpc \approx 4.6 \times 10^{22}\ m$$

$$\Rightarrow M_{coma} = \frac{\langle v^2 \rangle r_h}{\alpha G} \approx 4 \times 10^{45} kg$$

$$\approx 2 \times 10^{15} M_\odot$$

On the other hand;

$$M_{coma\,*} \approx 2 \times 10^{13} M_\odot$$

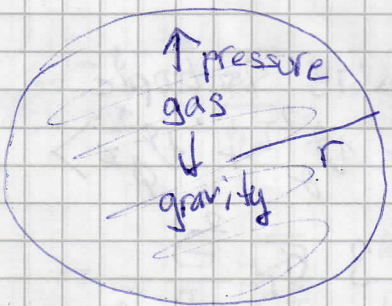
mass of stars in Coma

$$M_{coma,gas} \approx 2 \times 10^{14} M_\odot$$

$$\Rightarrow \frac{\overset{\text{calculated (virial)}}{M_{coma}}}{M_{coma,gas} + M_{coma,*}} \approx \frac{2 \times 10^{15}}{2 \times 10^{14}} = 10$$

Since the calculated mass to hold the Coma cluster together gravitationally is 10 times larger than the observed (visible) mass of the cluster, there must be invisible mass necessary to hold the cluster together. This invisible mass is the dark matter.

+ Another estimate of the Coma cluster's mass using the temperature and density of the hot gas in the cluster:



$$dP = -\rho g\, dr \ ; \ g = \frac{G M(r)}{r^2}$$

$$\frac{dP}{dr} = -\frac{G M(r) \rho_{gas}}{r^2}$$

$M(r)$ = total mass inside r including gas, stars, dark matter...

Ideal gas law

$$PV = nRT = NkT$$

$$P = \frac{N}{V} kT$$

From $\rho = \frac{N\mu}{V} \Rightarrow \frac{N}{V} = \frac{\rho}{\mu} \Rightarrow$

$$P = \frac{\rho_{gas} k T_{gas}}{\mu},$$

where μ is the mean mass per gas particle

$$\frac{dP}{dr} = \frac{k}{\mu}\left[\frac{d\rho}{dr} \cdot T + \rho \cdot \frac{dT}{dr}\right] = -\frac{GM\rho}{r^2}$$

$$\Rightarrow M = -\frac{k}{\mu G}\frac{r^2}{\rho}\left[\frac{d\rho}{dr} \cdot T + \rho \frac{dT}{d\rho}\right]$$

$\times$ by T and $\div$ by T :

$$M = -\frac{k}{\mu G} T r^2 \left[\frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{T}\frac{dT}{dr}\right]$$

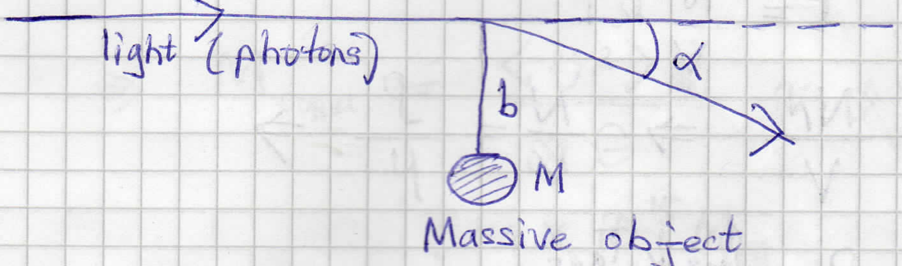
$$= -\frac{kT}{\mu G} r^2 \left[\frac{d\ln\rho}{dr} + \frac{d\ln T}{dr}\right]$$

$$M(r) = \frac{kT\,r}{G\mu}\left[-\frac{d\ln\rho}{\frac{dr}{r}} - \frac{d\ln T}{\frac{dr}{r}}\right] = \frac{kTr}{G\mu}\left[-\frac{d\ln\rho}{d\ln r} - \frac{d\ln T}{d\ln r}\right]$$

Analysing the X-rays emitted by the intracluster gas T_{gas} and ρ_{gas} can be computed. This gives

$$M_{coma} \approx 1.3 \times 10^{15} M_\odot$$

Gravitational Lensing:

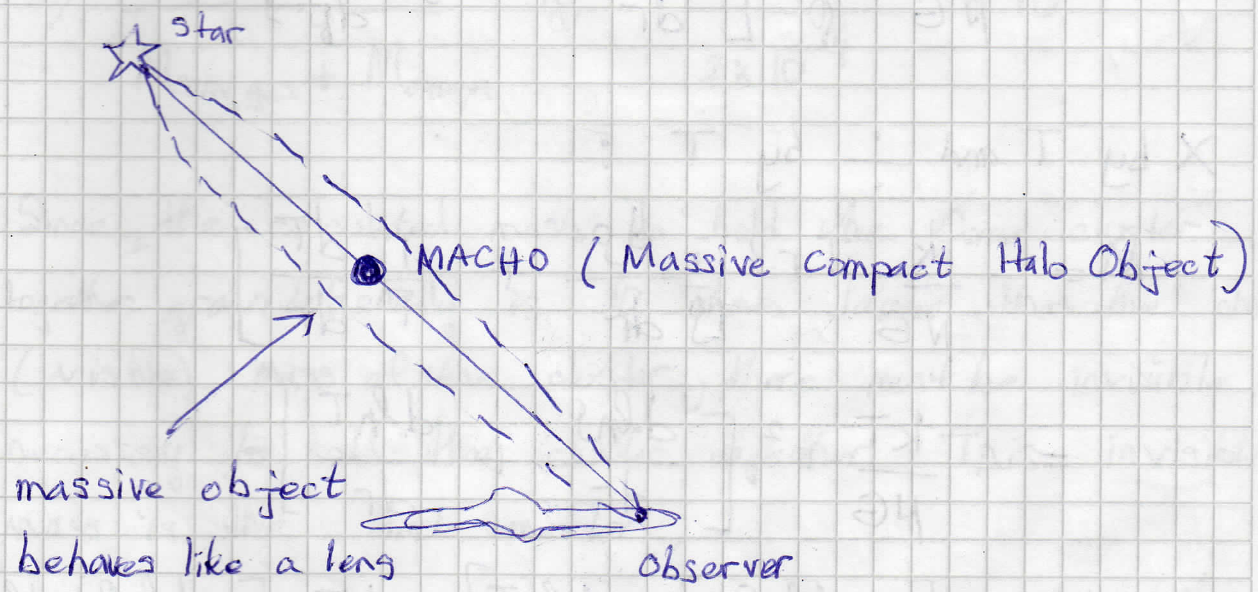


the local curvature of spacetime due to M causes light to be deflected by an angle

$$\alpha = \frac{4GM}{c^2 b}$$

For the Sun

$$\alpha_{sun} = \frac{4GM_\odot}{c^2 R_\odot} = 1.7 \text{ arcsec.}$$



Lensing events suggests that at most 8 percent of the halo mass could be in the form of MACHOs.

* The general conclusion is that most of the matter in the dark halo of our galaxy is due to nonbaryonic dark matter.