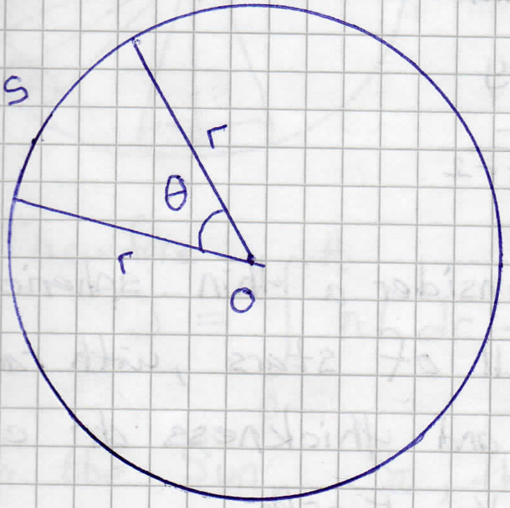
Ch. 2

The current standard model for the universe is the "Hot Big Bang" model: The universe has expanded from an initially hot and dense state to its current relatively cool and tenuous state. The expansion is still going on today.

tenuous → not thick, not dense, weak

Side Remark:

1) Angle: In 2-dimensional flat space



r = radius of a circle

s = arc length

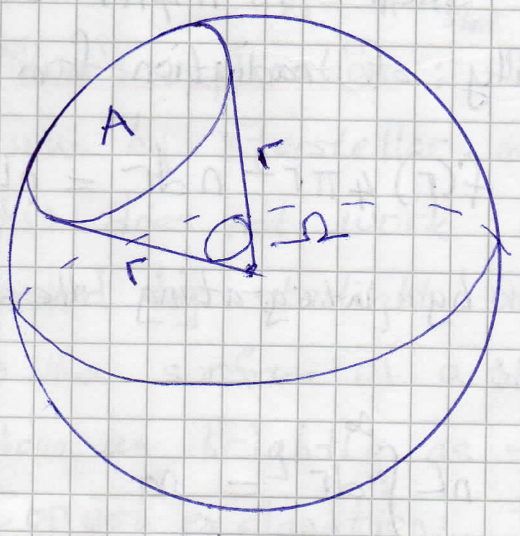
θ = angle

$$\theta \equiv \frac{s}{r} \quad \text{(in radians)}$$

when $s = r$, $\theta = 1$ rad.

when $s = 2\pi r$, $\theta = 2\pi$ rad.

2) Solid Angle:



sphere

A = area across Ω

Ω = solid angle

$$\Omega = \frac{A}{r^2} \quad \text{(in steradians)}$$

when $A = r^2$ $\Omega = 1$ sr

when $A = 4\pi r^2$ $\Omega = 4\pi$ sr

Olber's Paradox (the dark night sky paradox)

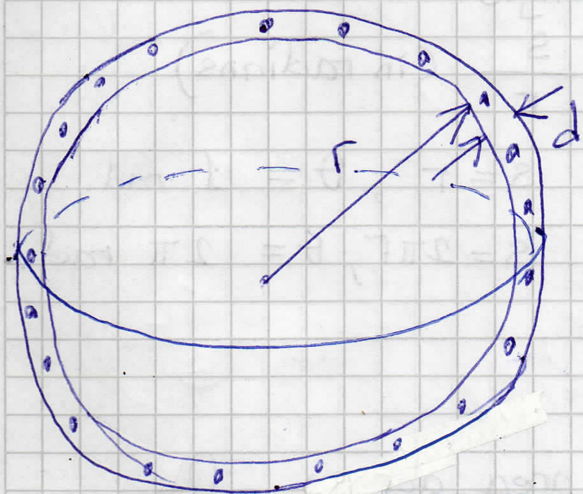
Why is the night sky dark if the Universe is infinitely large, eternal, and static? Instead, it should be bright.

How bright do we expect the night sky to be in an infinite universe.

Let be
n: the average # of ~~density of stars~~ stars per unit volume in the Universe
L: be the average stellar luminosity.

The flux received at Earth from a star of luminosity L at a distance r is given by

$$f(r) = \frac{L}{4\pi r^2}$$



Consider a thin spherical shell of stars, with radius r and thickness dr centered on the Earth

The # of stars in this thin shell is $n\, 4\pi r^2 dr$.

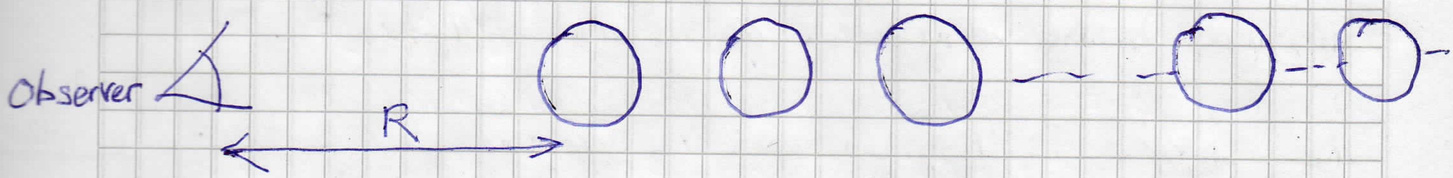
The intensity of radiation from the shell of stars is

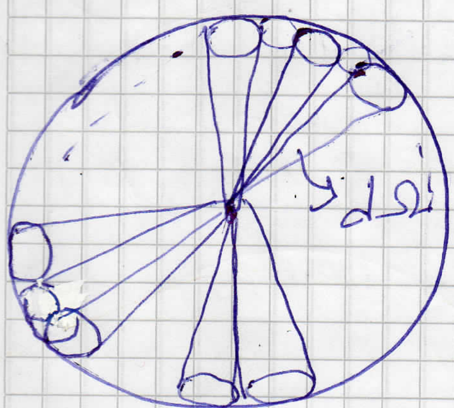
$$dJ(r) = f(r)\, 4\pi r^2 n\, dr = nL\, dr$$

The total intensity is found by integrating the spherical shells from 0 to ∞:

$$J = \int_0^\infty nL\, dr = nL \int_0^\infty dr = \infty$$

On the other hand, a finite # of visible stars each taking up a solid angle $\frac{A}{r^2}$ could block an infinite number of more distant stars, so it is not correct to integrate r to ∞.





$$\int d\Omega = 4\pi$$

$$\underbrace{\int_0^R 4\pi r^2 n \frac{A}{r^2}\, dr}_{= 4\pi n A R} = 4\pi$$

$$nAR = 1 \Rightarrow R = \frac{1}{nA}$$

Therefore

$$J = \int_0^R nL\, dr = nLR = \frac{L}{A} = \text{brightness}$$

of the Sun. So, the night sky should be as bright as the day sky. But the night sky is indeed dark! (Note: A is the cross sectional area of a star $A = \pi R_{star}^2$)

Olber's explanation : Distant stars are hidden from view by interstellar matter (dust) that absorbs starlight. This does not work because the dust would be heated by starlight until it had the same temperature as the surface of a star. After that the dust would glow as brightly as the stars themselves.

Correct explanation: The volume of the presently observable universe is not infinite, it is in fact too small to contain sufficiently many visible stars.

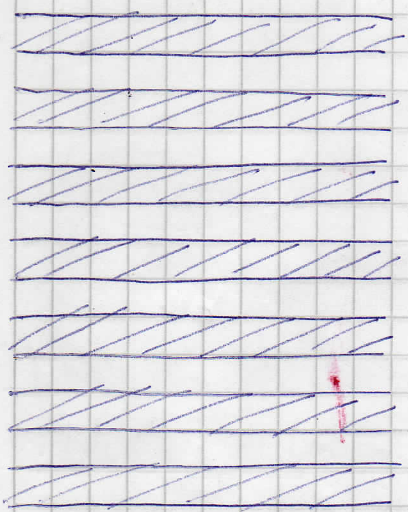
Isotropy and Homogeneouty of the Universe: on large scales:

- The universe is isotropic : There are no preferred directions in the universe on large scales.

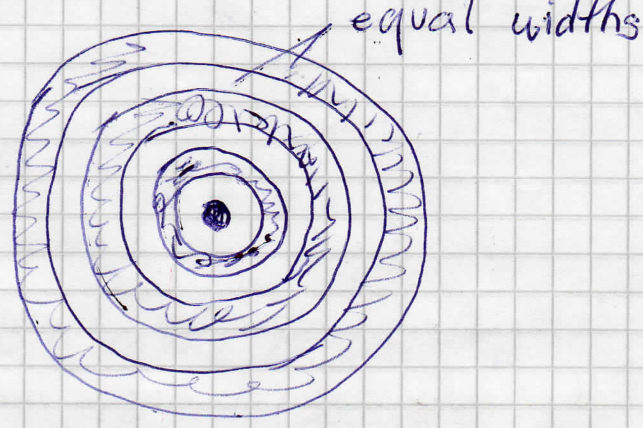
The universe is homogeneous: There are no preferred locations in the universe on large scales, it looks the same no matter you set up your telescope. The distribution of the galaxies is uniform.

Large scales means 100 Mpc or more.

Note: Homogeneouty does not imply isotropy



A pattern that is homogeneous on scales larger than the stripe width, but is anisotropic



A pattern that is isotropic about the origin, but is inhomogeneous.

The Copernican principle: There is nothing special or privileged about or location in the universe.
This principle holds only on large scales.

The cosmological principle: On large scales, the universe is homogeneous and isotropic (in three dimensional space)

Redshift and Distance:

Recall that an absorbtion spectrum occurs when light passes through a cold gas. Atoms in the gas absorb at certain frequencies and this given rise to dark lines in the spectrum.

Emission spectrum occurs when excited atomic states in a hot gas make transitions to lower levels.

Galaxy spectra at visible wavelengths contain absorbtion and emission lines.

* When a recognizable spectral line from a galaxy is observed (measured) at wavelength λ_{ob}, we know that long ago it originated (was emitted) at wavelength λ (or λ_{em}). We say that the galaxy has a redshift

$$z = \frac{\lambda_{ob} - \lambda_{em}}{\lambda_{em}}$$

when $\lambda_{ob} > \lambda_{em}$, $z > 0 \Rightarrow$ redshift

when $\lambda_{ob} < \lambda_{em}$, $z < 0 \Rightarrow$ blueshift

blue $\xrightarrow[\lambda]{}$ red

Relativistic Doppler Eff.

$$\frac{\lambda_{ob}}{\lambda_{em}} = \sqrt{\frac{1+v/c}{1-v/c}}$$

$$z = \frac{\lambda_{ob}}{\lambda_{em}} - 1 = \sqrt{\quad} - 1$$

$$\approx \frac{1+\frac{1}{2}v/c}{1-\frac{1}{2}v/c} - 1 \approx \frac{v}{c}$$

Let r be the distance from Earth to a galaxy.

Hubble's work revealed that

$$z = \frac{H_0}{c} r,$$

where H_0 = Hubble's constant. Since $z = \frac{v}{c}$

$\Rightarrow \frac{v}{c} = \frac{H_0 r}{c} \Rightarrow v = H_0 r$, where v is the velocity of the galaxy. (1929)

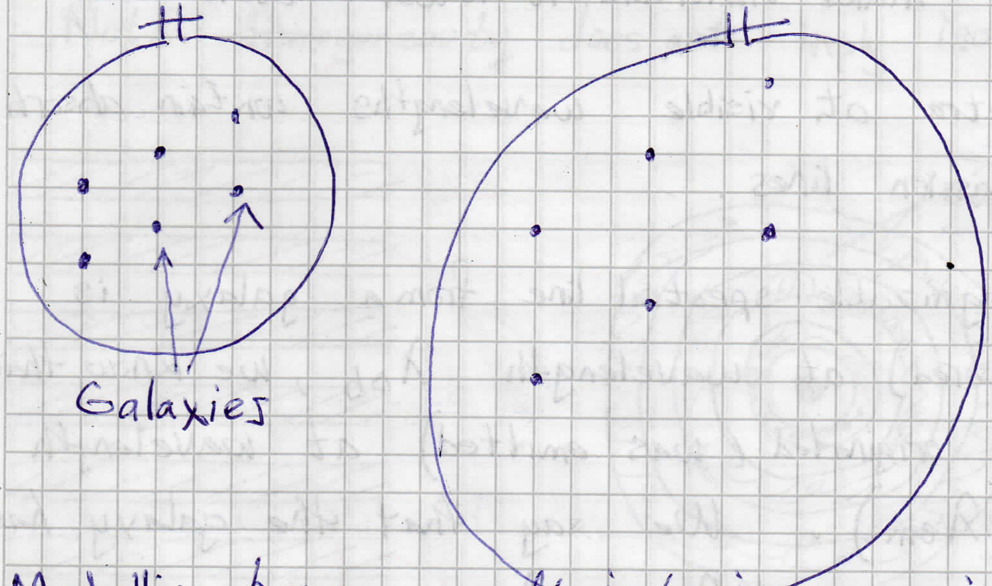
Hubble taught H_0 was $\simeq 500 \text{ km s}^{-1} \text{ Mpc}^{-1}$

Currently : $H_0 = 68 \pm 2 \text{ km s}^{-1} \text{ Mpc}^{-1}$

$v = H_0 r$ means the universe is expanding.

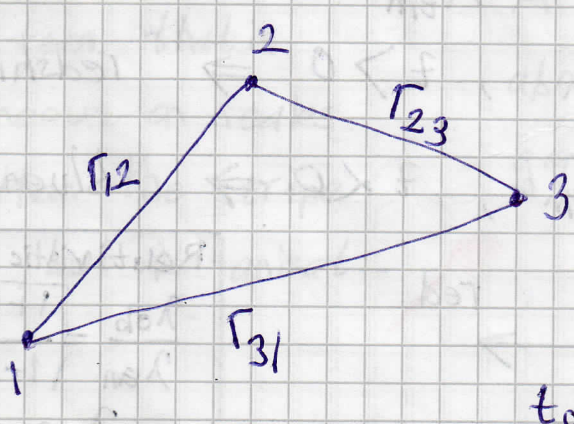
The space between galaxies increase.

The Balloon example:



Modelling homogeneous & isotropic expansion:

Consider three galaxies at positions $\vec{r_1}$, $\vec{r_2}$, and $\vec{r_3}$



$r_{12}(t) = a(t)\, r_{12}(t_0)$

$r_{23}(t) = a(t)\, r_{23}(t_0)$

$r_{31}(t) = a(t)\, r_{31}(t_0)$

t_0 = present time (today)

$a(t)$ = scale factor

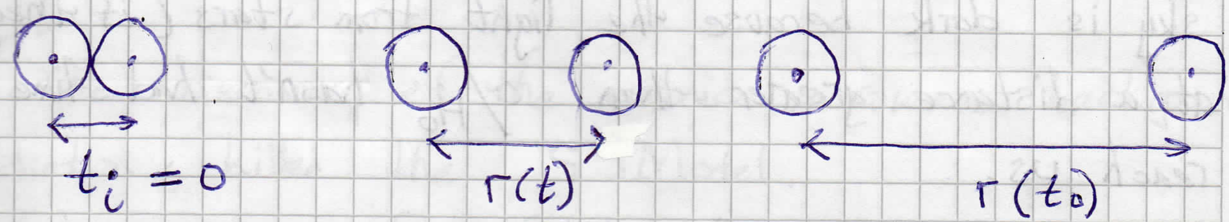
$$v_{12}(t) = \frac{dr_{12}}{dt} = \dot{a}\, r_{12}(t_0) = \frac{\dot{a}}{a}\, r_{12}(t)$$

$$v_{31}(t) = \frac{dr_{31}}{dt} = \dot{a}(t)\, r_{31}(t_0) = \frac{\dot{a}(t)}{a(t)}\, r_{31}(t)$$

$$v_{23}(t) = \frac{\dot{a}(t)}{a(t)}\, r_{23}(t)$$

Comparing with $v(t) = H(t)\, r(t) \Rightarrow H(t) = \frac{\dot{a}(t)}{a(t)}$

* That galaxies are currently moving away from each other implies they were in contact once in the past



* If there are no forces acting to accelerate or decelerate the relative motion of galaxies the expansion velocity must be constant. Then

$$r = v t \; ; \text{ or } \quad r(t_0) = v\, t_0 = H(t_0)\, r(t_0)\, t_0$$

$$\Rightarrow 1 = H(t_0)\, t_0 \Rightarrow t_0 = H(t_0)^{-1} = \text{age of univ.}$$

Today $H_0 = 68 \pm 2 \, km\, s^{-1}\, Mpc^{-1}$

$$\Rightarrow t_0 = 14.38 \pm 0.42 \text{ Gyr} \quad (\text{Actually } t_0 = 13.7 \text{ Gyr})$$

In the past the galaxies were packed much closer in a smaller volume leading to a much denser univ.

The observation of galactic redshifts led naturally to The Big Bang model: The universe expands from an initially highly dense state to its current low-density state.

t = Approximate age of the universe = the Hubble time is a natural time scale for our expanding univ.

The Hubble distance $\frac{c}{H_0} = 4380 \pm 130 \, Mpc$ is a natural distance scale.

Radius of the observed universe $R \approx \frac{c}{H_0}$

$$R \approx 4500 \text{ Mpc} = 4500 \times 10^6 \times 3 \times 10^{16} \text{ m}$$
$$= 135 \times 10^{24} \text{ m} = 1.4 \times 10^{26} \text{ m}$$

The finite size of the universe (not the finite age) provides the resolution for Olber's paradox: the night sky is dark because the light from stars (if they exist) at a distance greater than c/H_0 hasn't had time to reach us.

* Hubble's law $v(t) = H(t)\, r(t)$ occurs naturally in a Big Bang model for the universe
* the homogeneous & isotropic expansion causes the density of the universe to decrease steadily from its initial high value.

The Steady State Model (1940s, Hermann Bondi, Thomas Gold, Fred Hoyle):

Based on the perfect cosmological principle, which states that not only are there no privileged locations in space, there are no privileged moments in time.

* mean density ρ and H remain constant with time

$$v(t) = \frac{dr(t)}{dt} = H\, r(t)$$

$$\int \frac{dr}{r} = H \int dt$$

$$\ln r = Ht + \text{const.}$$

$r = C e^{Ht}$, C const.

$r \to 0$ as $t \to -\infty$; the steady state model is infinitely old

Volume of a spherical region $V = \frac{4\pi}{3} r^3 \propto e^{3Ht}$

the density of this sphere $\rho_0 = \frac{M}{V}$ must remain constant.

$$M = \rho_0 V \quad ; \quad \frac{dM}{dt} = \rho_0 \frac{dV}{dt} = \rho_0 \, 3HV$$

* matter must be continously created at the rate $\frac{dM}{dt}$

* The discovery of the cosmic microwave background radiation killed the SSModel

Elementary Particles:

$\underbrace{\text{Proton , Neutron}}_{\text{Baryons}}$, Electron , Neutrino , Photon , - -

Baryons are made of three quarks

$p : (uud)$, $n : (udd)$

1st generation	2nd Generation	3rd Generation
ν_e, e^-, u, d		
Doublets: $\begin{pmatrix} u \\ d \end{pmatrix}_L \begin{matrix} \leftarrow q = \frac{2}{3}e \\ \leftarrow q = -\frac{1}{3}e \end{matrix}$	$\begin{pmatrix} c \\ s \end{pmatrix}_L$	$\begin{pmatrix} t \\ b \end{pmatrix}_L$
$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$
u_R, d_R, e^-_R	c_R, s_R, μ^-_R	t_R, b_R, τ^-_R

$\underbrace{\qquad\qquad\qquad\qquad}_{SU(2)_L \times U(1)_Y \text{ Model}}$

Neutrinos are not massless, but have very small masses.

$$\left(m(\nu_e) + m(\nu_\mu) + m(\nu_\tau) \right) c^2 \leq 0.3 \text{ eV}$$

Photons : $E_\gamma = hf$; $c = \lambda f$

The energy density of photons in the frequency range f and $f + df$ is given by the blackbody function

$$\varepsilon(f)\, df = \left(\frac{8\pi h}{c^3} \frac{f^3}{e^{hf/k_B T} - 1} \right) df$$

Integrating this over all frequencies yields the total energy density for blackbody radiation

$$\varepsilon_\gamma = \alpha T^4 \quad ; \quad \alpha = \frac{\pi^2 k_B^4}{15 \hbar^3 c^3}$$

Since the energy of a photon is $E_\gamma = hf$, the number density of photons in the range f and $f + df$ is

$$n(f)\, df = \frac{\varepsilon(f)\, df}{hf} = \left(\frac{8\pi}{c^3} \frac{f^2}{e^{hf/k_B T} - 1} \right) df$$

Integrating this over all frequencies yields the number density of photons in blackbody radiation.

$$n_\gamma = \beta T^3 \quad ; \quad \beta = \frac{2.4041}{\pi^2} \frac{k_B^3}{\hbar^3 c^3}$$

According to observational astronomers:

Dark Matter: Any massive component of the universe that is too dim to be detected using current technology

According to theoretical astrophysicists dark matter is any massive component of the universe which does not emit, absorb, or scatter light at all.

Massive Neutrinos may be dark matter or weakly interacting massive particles (WIMPs) may be dark matter.

Cosmic Microwave Background (CMB):

* discovered in 1965 by Arno Penzias and Robert Wilson
* has a blackbody spectrum with

$$T_0 = 2.7255 \pm 0.0006 \text{ K}$$

The energy density of the CMB is from

$$\varepsilon_\gamma = \alpha T_0^4 = 0.2606 \text{ MeV m}^{-3}$$

The # density of CMB is from

$$n_\gamma = \beta T_0^3 = 4.107 \times 10^8 \text{ m}^{-3}$$

$$= 410.7 \text{ cm}^{-3} \approx 411 \text{ cm}^{-3}.$$

The mean energy of a CMB photon is

$$E_{mean} = \frac{\varepsilon_\gamma}{n_\gamma} = 6.344 \times 10^{-4} \text{ eV}$$

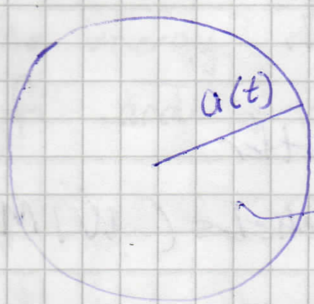
* The CMB can be explained by the Big Bang model
* The early universe was very dense and opaque (not letting light to pass). The universe became transparent

when $T = 2970\,K$. $(t \ll t_0)$

$\uparrow$ today

The drop in temperature is due to the expansion of the universe.

$$V \propto a(t)^3$$



Observable universe

photon gas with energy density $\varepsilon_\gamma = \alpha T^4$

Photons have momentum and the photon gas have a pressure

$$P_\gamma = \frac{\varepsilon_\gamma}{3}$$

The photon gas obeys the 1st law of thermodynamics

$$dQ = dE + PdV$$

$dQ \downarrow$ amount of heat flow into or out of the volume V

dE: change in internal energy

In a homogeneous universe $dQ = 0 \Rightarrow$

$$\frac{dE}{dt} = -P(t)\frac{dV}{dt}$$

$$E = \varepsilon_\gamma V = \alpha T^4 V$$

$$P = P_\gamma = \frac{1}{3}\alpha T^4$$

$$\frac{dE}{dt} = \alpha\left(4T^3\frac{dT}{dt}V + T^4\frac{dV}{dt}\right) = -\underbrace{\frac{1}{3}\alpha T^4}_{P(t)}\frac{dV}{dt}$$

dividing by VT^4: $\frac{4}{T}\frac{dT}{dt} + \frac{1}{V}\frac{dV}{dt} = -\frac{1}{3}\frac{1}{V}\frac{dV}{dt}$

$$4 \frac{1}{T}\frac{dT}{dt} = -\frac{4}{3}\frac{1}{V}\frac{dV}{dt}$$

$$\frac{1}{T}\frac{dT}{dt} = -\frac{1}{3V}\frac{dV}{dt}$$

since $V \propto a(t)^3$, $\frac{1}{V}\frac{dV}{dt} = \frac{3}{a}\frac{da}{dt}$

$$\Rightarrow \frac{1}{T}\frac{dT}{dt} = -\frac{1}{a}\frac{da}{dt}$$

$$\frac{1}{dt}\int \frac{dT}{T} = -\frac{1}{dt}\int \frac{da}{a}$$

$$\frac{d(\ln T)}{dt} = -\frac{d(\ln a)}{dt}$$

$$\Rightarrow T(t) \propto a(t)^{-1} = \frac{1}{a(t)}$$

$$\frac{T(t_{transparent})}{T_0} = \frac{a_0(t_0)}{a(t_{transparent})} = 1090$$

today's value

$$\Rightarrow T(t_{transparent}) = 1090\, T_0(t_0)$$

HW 1

Ryden 1st Ed.

2.2, 2.5