Örnek $y''-xy=0$ Dd'inin kuvvet serisiyle çözümünü bulunuz.

Soru bu şekilde soruluyorsa $(x-x_0)=0$ olur. (sınav sorusu)

$$\left.\begin{aligned} y&=\sum_{n=0}^{\infty} a_n x^n \\ y'&=\sum_{n=1}^{\infty} n a_n x^{n-1} \\ y''&=\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} \end{aligned}\right\}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - x\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

üsleri eşitleyelim

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

n'leri eşitleyelim

soldakinde n'ye 3 değeri verip dışarı alalım.

$$2a_2 + \sum_{n=3}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

$$2a_2 + \sum_{n=3}^{\infty} \left(n(n-1) a_n - a_{n-3}\right) x^{n-2} = 0$$

$a_2=0$ $\quad n(n-1)a_n - a_{n-3} = 0 \quad (n \geqslant 3)$ rekürans bağıntısı

$$a_n = \frac{a_{n-3}}{n(n-1)}$$

$n=3 \qquad a_3 = \frac{1}{6} a_0$

$n=4 \qquad a_4 = \frac{1}{12} a_1$

$n=5 \qquad a_5 = \frac{a_2}{5 \cdot 4} = 0$

$n=6 \qquad a_6 = \frac{a_3}{30} = \frac{1}{180} a_0$

$n=7 \qquad a_7 = \frac{a_4}{42} = \frac{1}{12 \cdot 42} a_1$

$n=8 \qquad a_8 = 0$

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$y = a_0 + a_1 x + \frac{1}{6} a_0 x^3 + \frac{1}{12} a_1 x^4 + \frac{1}{180} a_0 x^6 + \frac{1}{12 \cdot 42} a_1 x^7 + \dots$$

$$y = a_0 \left(1 + \frac{1}{6} x^3 + \frac{1}{180} x^6 + \dots\right) + a_1 \left(x + \frac{1}{12} x^4 + \frac{1}{12 \cdot 42} x^7 + \dots\right)$$

Örnek $y'' - xy' + 3y = 0$ dd'ini $x=2$ civarında kuvvet serisi yardımıyla çözünüz.

$$y'' - (x-2+2)y' + 3y = 0 \Rightarrow y'' - (x-2)y' - 2y' + 3y = 0$$

$$\sum_{n=2}^{\infty} n(n-1) a_n (x-2)^{n-2} - (x-2) \sum_{n=1}^{\infty} n a_n (x-2)^{n-1} - 2 \sum_{n=1}^{\infty} n a_n (x-2)^{n-1} + 3 \sum_{n=0}^{\infty} a_n (x-2)^n = 0$$

$$〃 \quad - \sum_{n=0}^{\infty} n a_n (x-2)^n - 2 \sum_{n=0}^{\infty} (n+1) a_{n+1} (x-2)^n + \quad 〃 \quad = 0$$

$$〃 \quad - \sum_{n=2}^{\infty} (n-2) a_{n-2} (x-2)^{n-2} - 2 \sum_{n=2}^{\infty} (n-1) a_{n-1} (x-2)^{n-2} + 3 \sum_{n=2}^{\infty} a_{n-2} (x-2)^{n-2} = 0$$

$$\sum_{n=2}^{\infty} \left(n(n-1) a_n - (n-2) a_{n-2} - 2(n-1) a_{n-1} + 3 a_{n-2} \right) (x-2)^{n-2} = 0$$

$$(x-2)^{n-2} = 0$$

$$n(n-1) a_n - (n-2) a_{n-2} - 2(n-1) a_{n-1} + 3 a_{n-2} = 0 \qquad (n \geqslant 2)$$

$$a_n = \frac{(n-2) a_{n-2} + 2(n-1) a_{n-1} - 3 a_{n-2}}{n(n-1)} = \frac{(n-5) a_{n-2} + 2(n-1) a_{n-1}}{n(n-1)}$$

$n=2 \qquad a_2 = \frac{(2-5) a_{2-2} + 2(2-1) a_{2-1}}{2(2-1)} = \frac{-3a_0 + 2a_1}{2}$

$n=3 \qquad a_3 = \frac{-2a_1 + 4a_2}{6} = \frac{-2a_1 - 6a_0 + 4a_1}{6} = \frac{-6a_0 + 2a_1}{6} = -a_0 + \frac{1}{3} a_1$

$n=4 \qquad a_4 = \frac{-a_2 + 6a_3}{12} = \frac{-\frac{-3a_0 + 2a_1}{2} - 6a_0 + 2a_1}{12} = \frac{-9a_0 + 2a_1}{24}$

$n=5 \qquad a_5 = \ldots$

$n=6 \qquad a_6 = \ldots$

$y = \ldots$

Örnek $\quad y''+y'+xy=0 \quad x=0$ noktasındaki seri çözümü elde ediniz.
(sınsa sonrası)

$$y=\sum_{n=0}^{\infty} a_n x^n, \quad y'=\sum_{n=1}^{\infty} n a_n x^{n-1}, \quad y''=\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{n=1}^{\infty} n a_n x^{n-1} + x\sum_{n=0}^{\infty} a_n x^n = 0$$

$$\text{"} \quad + \quad \text{"} \quad + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

en küçük x üssü olan $(n-2)$'yi yakalamaya çalışalım.

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} + \sum_{\substack{n-1=1\\ n=2}}^{\infty} (n-1) a_{n-1} x^{n-2} + \sum_{\substack{n-3=0\\ n=3}}^{\infty} a_{n-3} x^{n-2} = 0$$

$n=2$'yi dışarı çıkar $\quad$ $n=2$'yi dışarı çıkar

$\downarrow \qquad \downarrow$

$2a_2 \qquad a_1$

$$2a_2+a_1+\sum_{n=3}^{\infty}\left(n(n-1)a_n+(n-1)a_{n-1}+a_{n-3}\right)x^{n-2}=0$$

$2a_2+a_1=0 \qquad a_2=\frac{-a_1}{2}$

$n(n-1)a_n+(n-1)a_{n-1}+a_{n-3}=0 \qquad n\geqslant 3$

$$a_n=-\frac{(n-1)a_{n-1}+a_{n-3}}{n(n-1)}$$

$n=3 \qquad a_3=-\frac{2a_2+a_0}{6}=\frac{a_1-a_0}{6}$

$n=4 \qquad a_4=-\frac{3a_3+a_1}{4\cdot 3}=\frac{3\,\frac{a_1-a_0}{6}+a_1}{4\cdot 3}=\frac{a_0-3a_1}{24}$

$n=5 \qquad a_5 = -\dfrac{4a_4 + a_2}{5.4} = \dfrac{4\,\frac{a_0-3a_1}{24} + a_2}{20} = \dfrac{6a_1 - a_0}{120}$

$$y = \sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$y = a_0 + a_1 x - \frac{a_1}{2} x^2 + \frac{a_1 - a_0}{6} x^3 + \frac{a_0 - 3a_1}{24} x^4 + \frac{6a_1 - a_0}{120} x^5 + \dots$$

$$y = a_0\left(1 - \frac{1}{6} x^3 + \frac{1}{24} x^4 - \frac{1}{120} x^5 + \dots\right) + a_1\left(x - \frac{1}{2} x^2 + \frac{1}{6} x^3 - \frac{3}{24} x^4 + \frac{6}{120} x^5 + \dots\right)$$

<u>Örnek</u> $\quad y'' - y = 0 \quad x=0$ civarında seri ile çözünüz. (sınav sonrası)

$$y = \sum_{n=0}^{\infty} a_n x^n, \quad y' = \sum_{n=1}^{\infty} n a_n x^{n-1}, \quad y'' = \sum_{n=2}^{\infty} n(n-1) a_n x^{n-2}$$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=0}^{\infty} a_n x^n = 0$$

$$\text{"} \qquad - \sum_{n=2}^{\infty} a_{n-2} x^{n-2} = 0$$

$$\sum_{n=2}^{\infty} \left(n(n-1) a_n - a_{n-2}\right) x^{n-2} = 0$$

$$x^{n-2} = 0$$

$$n(n-1) a_n - a_{n-2} = 0$$

$$a_n = \frac{a_{n-2}}{n(n-1)} \qquad (n \geq 2)$$

$n=2 \qquad a_2 = \dfrac{a_0}{2.1}$

$n=3 \qquad a_3 = \dfrac{a_1}{3.2}$

$n=4 \qquad a_4 = \dfrac{a_2}{4.3} = \dfrac{a_0/2}{12} = \dfrac{a_0}{24}$

$n=5 \qquad a_5 = \dfrac{a_3}{5.4} = \dfrac{a_1/6}{20} = \dfrac{a_1}{120}$

$n=6 \qquad a_6 = \dfrac{a_4}{6.5} = \dfrac{a_0/24}{30} = \dfrac{a_0}{720}$

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \dots$$

$$y = a_0 + a_1 x + \frac{a_0}{2} x^2 + \frac{a_1}{6} x^3 + \frac{a_0}{24} x^4 + \frac{a_1}{120} x^5 + \frac{a_0}{720} x^6 + \dots$$

$$y = a_0 \left(1 + \frac{x^2}{2} + \frac{x^4}{24} + \frac{x^6}{720} + \dots\right) + a_1 \left(x + \frac{x^3}{6} + \frac{x^5}{120} + \dots\right)$$