Sol//

$$\left.\begin{aligned}\frac{dx}{dt}-\frac{dy}{dt}-y&=-e^t\\ \frac{dy}{dt}+x-y&=0\end{aligned}\right\} \qquad D=\frac{d}{dt}$$

$$\left.\begin{aligned}Dx-Dy-y&=-e^t\\ Dy+x-y&=0\end{aligned}\right\} \rightarrow \begin{aligned}Dx-(D+1)\,y&=-e^t\\ x+(D-1)\,y&=0\end{aligned}$$

$$\Delta=\begin{vmatrix} D & -(D+1)\\ 1 & D-1\end{vmatrix}=D(D-1)+D+1=D^2-\cancel{D}+\cancel{D}+1$$
$$=D^2+1$$

$$\Delta x=\begin{vmatrix} -e^t & -(D+1)\\ 0 & D-1\end{vmatrix}=(D-1)(-e^t)+(D+1).0$$
$$=-e^t+e^t=0$$

$$x=\frac{\Delta x}{\Delta}\rightarrow x=\frac{0}{D^2+1}\longrightarrow (D^2+1)x=0$$
$$\frac{d^2x}{dt^2}+x=0$$
$$r^2+1=0\rightarrow r_{1,2}=\mp i$$

$$\boxed{x=C_1\sin t+C_2\cos t}$$

$$\Delta y=\begin{vmatrix} D & -e^t\\ 1 & 0\end{vmatrix}=0-(-e^t)=e^t$$

$$y=\frac{\Delta y}{\Delta}\rightarrow y=\frac{e^t}{D^2+1}\longrightarrow (D^2+1)\,y=e^t$$
$$\frac{d^2y}{dt^2}+y=e^t$$

$$r^2+1=0\Rightarrow r_{1,2}=\mp i \qquad y_h=C_3\sin t+C_4\cos t$$

(2)

$$\left.\begin{aligned} y_p &= Ae^t \\ y_p' &= Ae^t \\ y_p'' &= Ae^t \end{aligned}\right\} \quad \begin{aligned} Ae^t + Ae^t &= e^t \\ 2A &= 1 \rightarrow A = \frac{1}{2} \\ y_p &= \frac{1}{2}e^t \end{aligned}$$

$$y = y_h + y_p \Rightarrow \boxed{y = c_3 \sin t + c_4 \cos t + \frac{e^t}{2}}$$

2 tane keyfi sabit olmalı. Bulduğumuz çözümleri en uygun denklemin birinde yerine koyalım:

2. denklemi alırsak:

$$\frac{dy}{dt} + x - y = 0$$

$$c_3 \cos t - c_4 \sin t + \cancel{\frac{e^t}{2}} + c_1 \sin t + c_2 \cos t - c_3 \sin t - c_4 \cos t - \cancel{\frac{e^t}{2}} = 0$$

$$(c_1 - c_3 - c_4)\sin t + (c_2 + c_3 - c_4)\cos t = 0$$

$$c_1 - c_3 - c_4 = 0 \rightarrow c_1 = c_3 + c_4$$
$$c_2 + c_3 - c_4 = 0 \rightarrow c_2 = c_4 - c_3$$

$$x = (c_3 + c_4)\sin t + (c_4 - c_3)\cos t$$
$$y = c_3 \sin t + c_4 \cos t + \frac{e^t}{2}$$

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$$(D-2)x+(D-4)y=e^{t}$$
$$Dx+(D-1)y=e^{4t}$$

$$\Delta=\begin{vmatrix} D-2 & D-4 \\ D & D-1 \end{vmatrix}=\cancel{D^2}-3D+2-\cancel{D^2}+4D=D+2$$

$$\Delta_1=\begin{vmatrix} e^t & D-4 \\ e^{4t} & D-1 \end{vmatrix}=(D-1)e^t-(D-4)e^{4t}=e^t-e^t-4e^{4t}+4e^{t}=0$$

$$x=\frac{\Delta_1}{\Delta} \rightarrow (D+2)x=0$$

$$\frac{dx}{dt}+2x=0 \rightarrow \int\frac{dx}{x}=\int-2dt$$

$$\ln x-\ln c_1=-2t$$

$$\frac{x}{c_1}=e^{-2t} \rightarrow \boxed{x=c_1e^{-2t}}$$

$$\Delta_2=\begin{vmatrix} D-2 & e^t \\ D & e^{4t} \end{vmatrix}=(D-2)e^{4t}-De^t=4e^{4t}-2e^{4t}-e^t=2e^{4t}-e^t$$

$$y=\frac{\Delta_2}{\Delta} \rightarrow y=\frac{2e^{4t}-e^t}{D+2} \rightarrow (D+2)y=2e^{4t}-e^t$$

$$\frac{dy}{dt}+2y=2e^{4t}-e^t$$ Lineer dif. denklem

$$\boxed{y=c_2e^{-2t}+\frac{1}{3}e^{4t}-\frac{1}{3}e^t}$$

2. denklemde yerine koyalım:

$$-2c_1e^{-2t}-2c_2e^{-2t}+\cancel{\frac{4}{3}e^{4t}}-\cancel{\frac{1}{3}e^t}-c_2e^{-2t}-\cancel{\frac{1}{3}e^{4t}}+\cancel{\frac{1}{3}e^t}=\cancel{e^{4t}}$$

$$(-2c_1-3c_2)e^{-2t}=0 \qquad -2c_1-3c_2=0$$

$$2c_1=-3c_2$$

$$c_1=-\frac{3}{2}c_2$$

$$x=-\frac{3}{2}c_2e^{-2t}$$

$$y=c_2e^{-2t}+\frac{1}{3}e^{4t}-\frac{1}{3}e^t$$

Dif. Denk. Final Sınavı / 9 Ocak 2014

S1) $x^3y'' + 2x^2y' = -1$ d.d.nin genel çözümünü bulunuz.

Ç: $\frac{1}{x}\left[x^3y'' + 2x^2y' = -1\right] \Rightarrow x^2y'' + 2xy' = -\frac{1}{x}$ } Euler d.d

$$\left.\begin{array}{l} x = e^t \\ \frac{dy}{dx} = e^{-t}\frac{dy}{dt} \\ \frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)\end{array}\right\} 5$$

d.d. yerine yazalım.

$$\left.\cancel{e^{2t}}.\cancel{e^{-2t}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + 2.\cancel{e^t}.\cancel{e^{-t}}.\frac{dy}{dt} = \frac{-1}{e^t}\right\} 3$$

$\Rightarrow y'' + y' = -e^{-t}$ s.k.l.d.d.

$$\left.\begin{array}{l} r^2 + r = r(r+1) = 0 \\ r=0 \quad r=-1 \end{array}\right\} 3 \qquad \left.y_1 = c_1 + c_2e^{-t}\right\} 2$$

$$\left.\begin{array}{l} (3)\ y_2 = kte^{-t} \\ (1)\ y_2' = k\left[e^{-t} - te^{-t}\right] = k(1-t)e^{-t} \\ (1)\ y_2'' = k\left[-1.e^{-t} + (1-t)(-)e^{-t}\right] = k(t-2)e^{-t} \end{array}\right\}$$

d.d. yerine yazalım

$$\left.k\left[\cancel{t} - 2 + 1 - \cancel{t}\right]\cancel{e^{-t}} = -\cancel{e^{-t}} \Rightarrow -k = -1 \Rightarrow \boxed{k=1}\right\} 2$$

$y_2 = te^{-t}$

$$(2)\ \left\{ y = c_1 + c_2e^{-t} + te^{-t} \Rightarrow \underbrace{\boxed{y = c_1 + \frac{c_2}{x} + \frac{\ln x}{x}}}_{3} \right. \text{ g.ç.}$$

4) $yy''+2y^2(y')^2+(y')^2=0$ diferansiyel denkleminin genel çözümünü bulunuz.

$y.y''+2y^2.(y')^2+(y')^2=0$ ⊛ x içermeyen dif. denklem

2 $y'=p$, $y''=p\frac{dp}{dy}$ 2

$y.p\frac{dp}{dy}+2y^2.p^2+p^2=0$ 2

$p\left(y\frac{dp}{dy}+2y^2p+p\right)=0$ 2

$p=0 \Rightarrow y'=0$

$y=c$ genel çözüm değildir.

2

$y\frac{dp}{dy}+2y^2p+p=0$

$\frac{dp}{dy}+2yp+\frac{1}{y}p=0$

$\frac{dp}{dy}+p\left(2y+\frac{1}{y}\right)=0$ 3

$\int\frac{dp}{p}+\int\left(2y+\frac{1}{y}\right)dy=\int 0$

$\ln p+y^2+\ln y=\ln c_1$ 4

$\frac{p.y}{c_1}=e^{-y^2}$

$p=\frac{c_1.e^{-y^2}}{y}$ 2

$\frac{dy}{dx}=\frac{c_1.e^{-y^2}}{y}$

$\int y.e^{y^2}dy=\int c_1\,dx$ 3

$\frac{1}{2}e^{y^2}=c_1x+c_2$ 2

$\boxed{e^{y^2}=2c_1x+2c_2}$ genel çözüm

ÖR/ $\dfrac{dx}{dt} + y = 0$
$\dfrac{dy}{dt} - x = \cos t$ } diferansiyel denklem sisteminin çözümünü türetme-yok etme yöntemini kullanarak bulunuz.

$$\frac{d^2y}{dt^2} - \frac{dx}{dt} = -\sin t$$

$$\frac{d^2y}{dt^2} + y = -\sin t$$

$$r^2 + 1 = 0 \Rightarrow r_{1,2} = \mp i$$

$$y_h = c_1 \cos t + c_2 \sin t$$

$$y_ö = (A \sin t + B \cos t)t$$

$$y_ö = (A \sin t - B \sin t)$$

$$y_ö' = A \sin t + B \cos t + t(A \cos t - B \sin t + t(-A \sin t - B \cos t)$$

$$y_ö'' = A \cos t - B \sin t + A \cos t - B \sin t - A t \sin t - B t \cos t$$

$$y_ö'' = 2A \cos t - 2B \sin t - A t \sin t - B t \cos t + A t \sin t + B t \cos t$$

$$2A \cos t - 2B \sin t - A t \sin t - B t \cos t + A t \sin t + B t \cos t$$

$$2A \cos t - 2B \sin t$$

$$= -\sin t$$

$$2A = 0 \qquad -2B = -1 \Rightarrow B = 1/2$$

$$A = 0 \qquad y_ö = \frac{1}{2} t \cos t$$

$$y = y_h + y_ö = c_1 \cos t + c_2 \sin t + \frac{1}{2} t \cos t$$

$$x = \frac{dy}{dt} - \cos t$$

$$x = -c_1 \sin t + c_2 \cos t + \frac{1}{2} \cos t - \frac{t}{2} \sin t - \cos t$$

$$x = -c_1 \sin t + c_2 \cos t - \frac{1}{2} \cos t - \frac{1}{2} t \sin t$$

ÖR/ $\left.\begin{array}{l}\dfrac{dx}{dt}+y=0\\[2ex]\dfrac{dy}{dt}-x=\cos t\end{array}\right\}$ dif denk sis çöz

$$\frac{d^2y}{dt^2}-\frac{dx}{dt}=-\sin t \Rightarrow \frac{d^2y}{dx^2}+y=-\sin t$$

$r^2+1=0 \quad r_{1,2}=\mp i$

$y_h=c_1\cos t+c_2\sin t$

$y_ö=(A\sin t+B\cos t)t$

$y_{ö_1}'=A\sin t+B\cos t+t(A\cos t-B\sin t)$

$y_{ö_1}''=A\cos t-B\sin t+A\cos t-B\sin t+t(-A\sin t-B\cos t)$

$y_{ö_1}''=2A\cos t-2B\sin t-At\sin t-Bt\cos t$

$\dfrac{d^2y}{dx^2}+y=-\sin t$

$2A\cos t-2B\sin t-\cancel{At}\sin t-Bt\cancel{\cos}t+At\cancel{\sin}t+\cancel{Bt}\cos t=-\sin t$

$2A=0 \quad -2B=-1 \quad B=1/2$

$A=0$

$y_ö=\frac{1}{2}t\cos t \qquad \boxed{y=c_1\cos t+c_2\sin t+\frac{1}{2}t\cos t}$

Şimdi 1inci denklemi türetip x bulacağız.

$$\frac{d^2x}{dt^2}+\frac{dy}{dt}=0$$

$$\frac{d^2x}{dt^2}+x+\cos t=0 \qquad \frac{d^2x}{dt^2}+x=-\cos t \qquad r^2+1=0$$

$r_{1,2}=\mp i$

$x_h=c_3\cos t+c_4\sin t$

$x_ö=(A\sin t+B\cos t)t$

$x_0^{u\prime} = A\sin t + B\cos t + t(A\cos t - B\sin t)$

$x_0^{u\prime\prime} = A\cos t - B\sin t + A\cos t - B\sin t + t(-A\sin t - B\cos t)$

$\qquad\qquad - Bt\cos t$ ... $- At\sin t$

$x_0^{u\prime\prime} = 2A\cos t - 2B\sin t - At\sin t - Bt\cos t$

$\dfrac{d^2x}{dt^2} + x = -\cos t$

$2A\cos t - 2B\sin t - \cancel{At\sin t} - \cancel{Bt\cos t} + \cancel{At\sin t} + \cancel{Bt\cos t} = -\cos t$

$2A\cos t - 2B\sin t = -\cos t$

$2A = -1 \qquad A = -1/2 \qquad B = 0$

$x_0^u = -\frac{1}{2} t\sin t$

$$\boxed{x = c_3\cos t + c_4\sin t - \frac{1}{2} t\sin t}$$

$c_3, c_4 \longrightarrow c_1, c_2$ cinsinden yazılmalı

$\dfrac{dx}{dt} + y = 0$ da yazalım.

$-c_3\sin t + c_4\cos t - \frac{1}{2}\sin t - \frac{1}{2}\cancel{t\cos t} + c_1\cos t + c_2\sin t$

$+ \frac{1}{2}\cancel{t\cos t} = 0$

$(-c_3 - \frac{1}{2} + c_2)\sin t + (c_4 + c_1)\cos t = 0$

$-c_3 + c_2 - \frac{1}{2} = 0 \Rightarrow -c_3 + c_2 = \frac{1}{2} \Rightarrow \underline{c_3 = c_2 - \frac{1}{2}}$

$c_4 + c_1 = 0 \qquad \Rightarrow \underline{c_4 = -c_1}$

$x = c_3\cos t + c_4\sin t - \frac{1}{2} t\sin t$

$x = (c_2 - \frac{1}{2})\cos t - c_1\sin t - \frac{1}{2} t\sin t$

$x = -c_1\sin t + c_2\cos t - \frac{1}{2}\cos t - \frac{1}{2} t\sin t$ bulunur.

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Soru 3:

$$\left.\begin{aligned} & y''+y=t \\ & y(0)=1;\ y'(0)=0 \end{aligned}\right\}$$ başlangıç değer problemini Laplace dönüşümünü kullanarak çözünüz.

Ç:

$$y''+y=t \Rightarrow L(y''+y)=L(t) \quad 1$$

$$\Rightarrow L(y'')+L(y)=L(t) \Rightarrow \overbrace{s^2F(s)-s\underbrace{f(0)}_{1}-\underbrace{f'(0)}_{0}}^{5}+\overbrace{F(s)}^{1}=\overbrace{\frac{1}{s^2}}^{3}$$

$$\left.\begin{aligned} &\Rightarrow (s^2+1)F(s)-s=\frac{1}{s^2} \Rightarrow (s^2+1)F(s)=s+\frac{1}{s^2} \\ &\Rightarrow (s^2+1)F(s)=\frac{1+s^3}{s^2} \Rightarrow F(s)=\frac{1+s^3}{s^2(1+s^2)} \end{aligned}\right\} 2$$

$$\frac{1+s^3}{s^2(1+s^2)}=\underset{s(1+s^2)}{\frac{A}{s}}+\underset{1+s^2}{\frac{B}{s^2}}+\underset{s^2}{\frac{Cs+D}{1+s^2}} \quad \Big\} 3$$

$$\Rightarrow 1+s^3=As(1+s^2)+B(1+s^2)+s^2(Ct+D)$$

$$\Rightarrow 1+s^3\equiv(A+C)s^3+(B+D)s^2+As+B$$

$$\Rightarrow 2\left\{\begin{aligned} &\boxed{A=0} \quad \boxed{C=1} \\ &\boxed{B=1} \quad \boxed{D=-1} \end{aligned}\right\} \Rightarrow F(s)=\frac{1}{s^2}+\frac{s}{s^2+1}-\frac{1}{s^2+1} \Big\} 1$$

$$\Rightarrow L^{-1}(F(s))=L^{-1}\left(\frac{1}{s^2}+\frac{s}{s^2+1}-\frac{1}{s^2+1}\right) 3$$

$$\Rightarrow \boxed{y(t)=t+\underbrace{\cos t}_{2}-\underbrace{\sin t}_{2}} \text{ bulunur.}$$

3) $y'' + 2y' + y = t\,e^{-t}$ $\quad y(0) = 1,\ y'(0) = 2$ başlangıç değer probleminin çözümünü Laplace Dönüşümü kullanarak bulunuz.

$$L\{y''\} + 2L\{y'\} + L\{y\} = L\{t.e^{-t}\} \quad ①$$

$$\underbrace{s^2 y(s) - s\,y(0) - y'(0)}_{③} + 2\,[\underbrace{s\,y(s) - y(0)}_{②}] + \underbrace{y(s)}_{①} = \underbrace{\frac{1}{(s+1)^2}}_{②}$$

$$(s^2 + 2s + 1)\,y(s) = s + 4 + \frac{1}{(s+1)^2}$$

$$y(s) = \frac{s+4}{(s+1)^2} + \frac{1}{(s+1)^4} \quad ②$$

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$$y(s) = \frac{s+1}{(s+1)^2} + \frac{3}{(s+1)^2} + \frac{1}{(s+1)^4}$$

$$y(s) = \frac{1}{(s+1)} + \frac{3}{(s+1)^2} + \frac{1}{(s+1)^4} \quad ④$$

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$$L^{-1}\{y(s)\} = L^{-1}\left\{\frac{1}{s+1}\right\} + 3L^{-1}\left\{\frac{1}{(s+1)^2}\right\} + L^{-1}\left\{\frac{1}{(s+1)^4}\right\} \quad ②$$

$$y(t) = \underbrace{e^{-t}}_{②} + \underbrace{3\,t\,e^{-t}}_{③} + \underbrace{\frac{t^3 e^{-t}}{3!}}_{③}$$

3) $xy''+2y=0$ diferansiyel denkleminin $x_0=1$ civarındaki seri çözümünü bulunuz.

2 $\cdot\ y=\sum_{n=0}^{\infty} a_n(x-1)^n$, 2 $\cdot\ y'=\sum_{n=1}^{\infty} a_n . n(x-1)^{n-1}$, 2 $\cdot\ y''=\sum_{n=2}^{\infty} a_n n(n-1)(x-1)^{n-2}$

2 $x.\sum_{n=2}^{\infty} a_n n(n-1)(x-1)^{n-2}+2\sum_{n=0}^{\infty} a_n(x-1)^n=0$ $\qquad * \ x=(x-1)+1$

2 $\sum_{n=2}^{\infty} a_n n(n-1)(x-1)^{n-1}+\sum_{n=2}^{\infty} a_n n(n-1)(x-1)^{n-2}+\sum_{n=0}^{\infty} 2a_n(x-1)^n=0$

$\sum_{n=1}^{\infty} a_{n+1}(n+1)n(x-1)^n+\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)(x-1)^n+\sum_{n=0} 2a_n(x-1)^n=0$

$\sum_{n=0}^{\infty}\left[a_{n+1}(n+1)n+a_{n+2}(n+2)(n+1)+2a_n\right](x-1)^n=0$,

$a_{n+1}(n+1)n+a_{n+2}(n+2)(n+1)+2a_n=0 \qquad (n\geq 0)$

6 $\cdot\ a_{n+2}=\dfrac{-2a_n-n(n+1)a_{n+1}}{(n+2)(n+1)}$

$a_{n+2}=-\dfrac{2a_n}{(n+2)(n+1)}-\dfrac{n}{n+2}a_{n+1} \quad , \quad n\geq 0$

1 $\cdot$ $n=0$ için $\quad a_2=-\dfrac{2a_0}{2}=-a_0$

1 $\cdot$ $n=1$ için $\quad a_3=-\dfrac{\cancel{2}a_1}{3.\cancel{2}}-\dfrac{1}{3}a_2=-\dfrac{a_1}{3}+\dfrac{a_0}{3}$

1 $\cdot$ $n=2$ için $\quad a_4=\dfrac{a_1}{6}$

1 $\cdot$ $n=3$ için $\quad a_5=-\dfrac{a_0}{30}-\dfrac{a_1}{15}$

$\vdots \qquad \vdots$

$y=\sum_{n=0}^{\infty} a_n(x-1)^n=a_0+a_1(x-1)+a_2(x-1)^2+\dots$

3 $\cdot\ y=a_0+a_1(x-1)+(-a_0)(x-1)^2+\left(-\dfrac{a_1}{3}+\dfrac{a_0}{3}\right)(x-1)^3+\dfrac{a_1}{6}(x-1)^4+$
$+\left(-\dfrac{a_0}{30}-\dfrac{a_1}{15}\right)(x-1)^5+\dots$

2 $\cdot\ y=a_0\left[1-(x-1)^2-\dfrac{1}{3}(x-1)^3-\dfrac{1}{30}(x-1)^5+\dots\right]+a_1\left[(x-1)-\dfrac{1}{3}(x-1)^3+\right.$
$\left.\dfrac{1}{6}(x-1)^4-\dfrac{1}{15}(x-1)^5+\dots\right]$

Başarılar dilerim.