# Diferansiyel Denklem Sistemleri

**Tanım:** Bir diferansiyel denklem sistemi iki yada daha fazla diferansiyel denklemden meydana gelir ve iki yada daha fazla bağımlı değişkenlerin bir tek bağımsız değişkene göre türevlerini içerirler.

örneğin,

$$\left.\begin{aligned} 8\frac{d^2x}{dt^2} &= 6x-4y \\ 2\frac{d^2y}{dt^2} &= x+y \end{aligned}\right\} \quad \text{yada} \quad \begin{aligned} x'-2x+y'-z' &= 4 \\ x'-y'+4z' &= t \\ x+y'-7z' &= t+2 \end{aligned} \quad \text{gibi}$$

**Tanım:** Diferansiyel denklem sisteminin çözümü, verilen aralıkta denklem sistemindeki herbir denklemi sağlayan ve diferansiyelleneb fonksiyonların kümesidir.

## I. Yoketme Yöntemi

Lineer denklem sistemini çözmenin en eski ve kolay yoludur.

**örn** $\left.\begin{aligned} Dy-2x &= 0 \\ Dx-3y &= 0 \end{aligned}\right\}$ dif. denk. sistemini çözünüz.

$$\begin{aligned} D/\ Dy-2x &= 0 \\ 2/\ -3y+Dx &= 0 \end{aligned}$$

$+$

$$D^2y-6y=0 \Rightarrow r^2-6=0 \Rightarrow r_{1,2}=\mp\sqrt{6}$$

$$y(t)=c_1e^{\sqrt{6}t}+c_2e^{-\sqrt{6}t}$$

$$x(t)=\frac{Dy}{2}=\frac{\sqrt{6}c_1}{2}e^{\sqrt{6}t}-\frac{\sqrt{6}}{2}c_2e^{-\sqrt{6}t}$$

**Örn** $\left.\begin{array}{l} x' = x + y + e^t \\ y' = 9x + y + \sin t \end{array}\right\}$ dif. denk. sistemini çözünüz.

$$D^{-1} / (D-1)x - y = e^t$$
$$-9x + (D-1)y = \sin t$$
$$+ \overline{\qquad\qquad\qquad\qquad}$$

$\left.\begin{array}{l} (D^2 - 2D + 1 - 9)x = \sin t \\ (D^2 - 2D - 8)x = \sin t \end{array}\right\}$ $r^2 - 2r - 8 = 0$ $\quad -4 \;\; +2$ $\quad \left.\begin{array}{l} r_1 = 4 \\ r_2 = -2 \end{array}\right\}$ $x_h = c_1 e^{4t} + c_2 e^{-2t}$

$\left.\begin{array}{l} x_ö = A\cos t + B\sin t \\ x_ö' = -A\sin t + B\cos t \\ x_ö'' = -A\cos t - B\sin t \end{array}\right\}$ $-A\cos t - B\sin t + 2A\sin t - 2B\cos t - 8A\cos t - 8B\sin t = \sin t$

$\left.\begin{array}{l} -9A - 2B = 0 \\ 2A - 9B = 1 \end{array}\right\}$ $\left.\begin{array}{l} A = \frac{2}{85} \\ B = -\frac{9}{85} \end{array}\right\}$ $x_ö = \frac{1}{85}(2\cos t - 9\sin t)$

$$x(t) = c_1 e^{4t} + c_2 e^{-2t} + \frac{1}{85}(2\cos t - 9\sin t)$$

$$y(t) = (D-1)x - e^t = 4c_1 e^{4t} - 2c_2 e^{-2t} + \frac{1}{85}(-2\sin t - 9\cos t) - c_1 e^{4t} - c_2 e^{-2t} - \frac{1}{85}(2\cos t -$$

$$y(t) = 3c_1 e^{4t} - 3c_2 e^{-2t} + \frac{1}{85}(-11\cos t + 7\sin t) - e^t$$

**Sınav sorusu** : $\left.\begin{array}{l} \frac{dx}{dt} + \frac{dy}{dt} - 2y = -e^t \\ \frac{dy}{dt} + x + y = 0 \end{array}\right\}$ dif. denk. sistemini türetme-yok etme yöntemiyle çözünüz.

$\left.\begin{array}{l} Dx + (D-2)y = -e^t \\ \cdot D / \; x + (D+1)y = 0 \\ \hline (D - 2 - D^2 - D)y = -e^t \end{array}\right\}$

$(D^2 + 2)y = e^t$

$r^2 + 2 = 0 \Rightarrow r_{1,2} = \mp\sqrt{2}i$

$y_h = c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t$

$y_ö = Ae^t, \; y_ö' = y_ö'' = Ae^t$

$(A + 2A)e^t = e^t \Rightarrow 3A = 1 \Rightarrow A = \frac{1}{3} \qquad y_ö = \frac{1}{3}e^t$

$$y(t) = c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t + \frac{1}{3}e^t$$

$$x(t) = -(D+1)y = \sqrt{2}c_1 \sin\sqrt{2}t - \sqrt{2}c_2\cos\sqrt{2}t - \frac{1}{3}e^t - c_1\cos\sqrt{2}t - c_2 \sin\sqrt{2}t - \frac{1}{3}e^t$$

$$x(t) = (\sqrt{2}c_1 - c_2)\sin\sqrt{2}t - (\sqrt{2}c_2 + c_1)\cos\sqrt{2}t - \frac{2}{3}e^t$$

ınav sorusu: $x=x(t)$, $y=y(t)$ fonksiyonları veriliyor.

$$\left.\begin{aligned} -x+\frac{dy}{dt}+y &= e^t+\sin 3t \\ 2\frac{dx}{dt}-3x+y &= e^t+3\sin 3t-6\cos 3t+1 \end{aligned}\right\}$$

denklem sistemini "yok-etme yöntemi" kullanarak $y(t)$ nin genel çözümünü bulunuz.

$$-x+(D+1)y=e^t+\sin 3t$$

$$(2D-3)x+y \quad =e^t+3\sin 3t-6\cos 3t+1$$

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$$(2D^2-3D+2D-3+1)y=e^t+3\sin 3t-6\cos 3t+1-e^t+6\cos 3t-3\sin 3t$$

$$(2D^2-D-2)y=1$$

$$2r^2-r-2=0 \Rightarrow r_{1,2}=\frac{1\mp\sqrt{1+4.2.2}}{4}=\frac{1\mp\sqrt{17}}{4}$$

$$r_1=\frac{1+\sqrt{17}}{4} \qquad r_2=\frac{1-\sqrt{17}}{4}$$

$$y_h=c_1e^{\frac{1+\sqrt{17}}{4}t}+c_2e^{\frac{1-\sqrt{17}}{4}t}$$

$$y_ö=A \quad y_ö'=y_ö''=0 \Bigg\}\ \begin{aligned}-2A&=1\\ A&=-\tfrac{1}{2}\end{aligned} \qquad y_ö=-\frac{1}{2}$$

$$y(t)=c_1e^{\frac{1+\sqrt{17}}{4}t}+c_2e^{\frac{1-\sqrt{17}}{4}t}-\frac{1}{2}$$

ınav sorusu: $x=x(t)$, $y=y(t)$ o.ü

$$\left.\begin{aligned} \frac{dx}{dt} &= 2x+y+e^{2t}-e^{-t} \\ \frac{dy}{dt} &= -2x-y-3e^{2t} \end{aligned}\right\}$$

denklem sistemini çözerek $x(t)$ fonksiyonunu bulunuz.

$$(D-2)x-y=e^{2t}-e^{-t}$$

$$2x+(D+1)y=-3e^{2t}$$

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$$(D^2-2D+D-2+2)x=-3e^{2t}+2e^{2t}+e^{-t}+e^{2t}-e^{-t}=0$$

$$(D^2-D)x=0 \Rightarrow \left.\begin{aligned} r^2-r&=0\\ r_1&=1\\ r_2&=0\end{aligned}\right\} \quad x(t)=c_1+c_2e^t$$

örn $\left.\begin{aligned} Dx &= -y+t \\ Dy &= x-t \end{aligned}\right\}$ dif. denk. sistemini çözünüz.

$$\left.\begin{aligned} & Dx + y = t \\ D/ & -x + Dy = -t \\ + & \overline{\qquad\qquad\qquad} \\ & D^2y + y = t-1 \end{aligned}\right\}$$

$r^2+1=0 \Rightarrow r_{1,2} = \mp i$

$y_h = c_1 \cos t + c_2 \sin t$

$$\left.\begin{aligned} y_ö &= at+b \\ y_ö' &= a \\ y_ö'' &= 0 \end{aligned}\right\} \left.\begin{aligned} at+b &= t-1 \\ a &= 1 \\ b &= -1 \end{aligned}\right\} y_ö = t-1$$

$y(t) = c_1 \cos t + c_2 \sin t + t - 1$

$x(t) = Dy + t = -c_1 \sin t + c_2 \cos t + 1 + t$

örn $\left.\begin{aligned} \frac{d^2x}{dt^2} - 4y &= e^t \\ \frac{d^2y}{dt^2} - 4x &= -e^t \end{aligned}\right\}$ dif. denklem sistemini çözünüz.

$$\left.\begin{aligned} D^2/ & \; D^2x - 4y = e^t \\ 4/ & -4x + D^2y = -e^t \\ + & \overline{\qquad\qquad\qquad} \\ & D^4x - 16x = -3e^t \end{aligned}\right\}$$

$r^4 - 16 = 0 \Rightarrow (r-2)(r+2)(r^2+4) = 0$

$r_1 = 2, \; r_2 = -2, \; r_3 = 2i, \; r_4 = -2i$

$x_h = c_1 e^{2t} + c_2 e^{-2t} + c_3 \cos 2t + c_4 \sin 2t$

$$\left.\begin{aligned} & x_ö = Ae^t \\ & x_ö' = x_ö'' = x_ö''' = x_ö^{IV} = Ae^t \end{aligned}\right\} \begin{aligned} (A-16A)e^t &= -3 \\ -15A &= -3 \\ A &= \frac{1}{5} \end{aligned} \qquad x_ö = \frac{1}{5}e^t$$

$x(t) = c_1 e^{2t} + c_2 e^{-2t} + c_3 \cos 2t + c_4 \sin 2t + \frac{1}{5}e^t$

$y(t) = \frac{D^2x - e^t}{4} = \frac{4c_1e^{2t} + 4c_2e^{-2t} - 4c_3\cos 2t - 4c_4 \sin 2t + \frac{1}{5}e^t - e^t}{4}$

$y(t) = c_1 e^{2t} + c_2 e^{-2t} - c_3 \cos 2t - c_4 \sin 2t - \frac{1}{5}e^t$

ex

$$\left.\begin{aligned}\frac{dy}{dx} &= -5y - z + 1 + x^2 \\ \frac{dz}{dx} &= y - 3z + e^{2x}\end{aligned}\right\}$$ türetme-yok etme yöntemiyle çözünüz.

$$(D+5)y + z = 1 + x^2$$

$$(D+5)/ \quad -y + (D+3)z = e^{2x}$$

$$+$$

$$(D^2 + 8D + 16)z = 1 + x^2 + 2e^{2x} + 5e^{2x} = (1+x^2) + 7e^{2x}$$

$$(r+4)^2 = 0 \Rightarrow r_1 = r_2 = -4$$

$$z_h = c_1 e^{-4x} + c_2 x e^{-4x}$$

$$\left.\begin{aligned} z_{ö1} &= ax^2 + bx + c \\ z'_{ö1} &= 2ax + b \\ z''_{ö1} &= 2a \end{aligned}\right\}$$

$$2a + 16ax + 8b + 16ax^2 + 16bx + 16c = 1 + x^2$$

$$a = \frac{1}{16}, \quad 16a + 16b = 0 \quad b = -\frac{1}{16} \qquad 2a + 8b + 16c = 1$$

$$\frac{2}{16} - \frac{8}{16} + 16c = 1 \Rightarrow c = \frac{11}{128}$$

$$z_{ö1} = \frac{1}{16}x^2 - \frac{1}{16}x + \frac{11}{128}$$

$$\left.\begin{aligned} z_{ö2} &= ke^{2x} \\ z'_{ö2} &= 2ke^{2x} \\ z''_{ö2} &= 4ke^{2x} \end{aligned}\right\}$$

$$4k + 16k + 16k = 7$$

$$36k = 7$$

$$k = \frac{7}{36}$$

$$z_{ö2} = \frac{7}{26}e^{2x}$$

$$z = c_1 e^{-4x} + c_2 x e^{-4x} + \frac{x^2}{16} - \frac{x}{16} + \frac{11}{128} + \frac{7}{26}e^{2x}$$

$$y = (D+3)z - e^{2x} = -4c_1 e^{-4x} + c_2 e^{-4x} - 4c_2 x e^{-4x} + \frac{x}{8} - \frac{1}{16} + \frac{7}{13}e^{2x}$$

$$+ 3c_1 e^{-4x} + 3c_2 x e^{-4x} + \frac{3x^2}{16} - \frac{3x}{16} + \frac{33}{128} + \frac{21}{26}e^{2x} - e^{2x}$$

$$= -c_1 e^{-4x} + c_2 e^{-4x} - c_2 x e^{-4x} + \frac{3x^2}{16} - \frac{x}{16} + \frac{25}{128} + \frac{9}{26}e^{2x}$$

## II. Determinant Yöntemi

$L_1, L_2, L_3$ ve $L_4$ sabit katsayılı lineer diferansiyel operatörler olm
üzere, x ve y'ye bağlı bir lineer denklem sistemi

$$L_1 x + L_2 y = g_1(t)$$
$$L_3 x + L_4 y = g_2(t)$$

olarak yazılabilir. Determinant yöntemi kullanılarak

$$\begin{vmatrix} L_1 & L_2 \\ L_3 & L_4 \end{vmatrix} x(t) = \begin{vmatrix} g_1(t) & L_2 \\ g_2(t) & L_4 \end{vmatrix}$$

$$\begin{vmatrix} L_1 & L_2 \\ L_3 & L_4 \end{vmatrix} y(t) = \begin{vmatrix} L_1 & g_1(t) \\ L_3 & g_2(t) \end{vmatrix}$$

bulunur.

$$\begin{vmatrix} L_1 & L_2 \\ L_3 & L_4 \end{vmatrix} \neq 0 \text{ ise,}$$

(+) Herbir diferansiyel denklemin karakteristik denklemleri ve homo
kısımları aynıdır.

(+)

Eğer

$$\begin{vmatrix} L_1 & L_2 \\ L_3 & L_4 \end{vmatrix} = 0 \text{ ise, sistemin}$$

**Ör.** $\left.\begin{aligned} Dx &= -y + t \\ Dy &= x - t \end{aligned}\right\}$ d.s. çözünüz.

$$\left.\begin{aligned} Dx + y &= t \\ -x + Dy &= -t \end{aligned}\right\} \quad \begin{vmatrix} D & 1 \\ -1 & D \end{vmatrix} x = \begin{vmatrix} t & 1 \\ -t & D \end{vmatrix} \quad \text{ve} \quad \begin{vmatrix} D & 1 \\ -1 & D \end{vmatrix} y = \begin{vmatrix} D & t \\ -1 & -t \end{vmatrix}$$

$$\left.\begin{aligned} (D^2+1)x &= 1+t \\ (D^2+1)y &= -1+t \end{aligned}\right\} \quad \begin{aligned} & r^2+1=0 \Rightarrow r_{1,2} = \mp i \\ & x_h = c_1 \cos t + c_2 \sin t \end{aligned}$$

$$\left.\begin{aligned} x_ö &= at+b \\ x_ö' &= a \\ x_ö'' &= 0 \end{aligned}\right\} \left.\begin{aligned} at+b &= 1+t \\ a &= 1 \\ b &= 1 \end{aligned}\right\} x_ö = t+1 \qquad x(t) = c_1 \cos t + c_2 \sin t + t + 1$$

$$y_h = c_3 \cos t + c_4 \sin t$$

$$\left.\begin{aligned} y_ö &= at+b \\ y_ö' &= a \\ y_ö'' &= 0 \end{aligned}\right\} \left.\begin{aligned} at+b &= -1+t \\ a &= 1 \\ b &= -1 \end{aligned}\right\} y_ö = t-1 \qquad y(t) = c_3 \cos t + c_4 \sin t + t - 1$$

$$-c_1 \sin t + c_2 \cos t + \cancel{1} = -c_3 \cos t - c_4 \sin t - \cancel{t} + \cancel{1} + \cancel{t}$$

$$c_1 = c_4$$

$$c_2 = -c_3$$

$$x(t) = c_4 \cos t - c_3 \sin t + t + 1$$

$$y(t) = c_3 \cos t + c_4 \sin t + t - 1$$

Sınav sorusu: $(D+1)x + 2Dy = 6\cos 2t$

$$Dx + (D-1)y = t$$

dif. denk. sistemi veriliyor.

a) Çözümden gelen keyfi sabit sayısını belirleyiniz.

b) Sistemin genel çözümünü bulunuz.

a) $1+1=2$ sabit olmalı.

b) $\begin{vmatrix} D+1 & 2D \\ D & D-1 \end{vmatrix} x = \begin{vmatrix} 6\cos 2t & 2D \\ t & D-1 \end{vmatrix}$ ; $\begin{vmatrix} D+1 & 2D \\ D & D-1 \end{vmatrix} y = \begin{vmatrix} D+1 & 6\cos 2t \\ D & t \end{vmatrix}$

$$(D^2+1)x = 2 + 6\cos 2t + 12\sin 2t$$

$$(D^2+1)y = -(t+1+12\sin 2t)$$

$r^2+1=0 \Rightarrow r_{1,2} = \mp i$ $\qquad x_h = c_1\cos t + c_2\sin t$

$x_{ö1} = a \quad x_{ö1}' = x_{ö1}'' = 0 \Rightarrow a = 2 \quad x_{ö1} = 2$

$$\left.\begin{aligned} x_{ö2} &= A\cos 2t + B\sin 2t \\ x_{ö2}' &= -2A\sin 2t + 2B\cos 2t \\ x_{ö2}'' &= -4A\cos 2t - 4B\sin 2t \end{aligned}\right\} \begin{aligned} &-3A\cos 2t - 3B\sin 2t = 6\cos 2t + 12\sin 2t \\ &\left.\begin{aligned} A &= -2 \\ B &= -4 \end{aligned}\right\} x_{ö2} = -2\cos 2t - 4\sin 2t \end{aligned}$$

$$x(t) = c_1\cos t + c_2\sin t + 2 - 2\cos 2t - 4\sin 2t$$

$y_h = c_3\cos t + c_4\sin t$

$y_{ö1} = at + b \quad y_{ö1}' = a \ , \ y_{ö1}'' = 0 \ \left\} \begin{aligned} &at+b = -t-1 \\ &a=-1 \ , \ b=-1 \end{aligned}\right. \qquad y_{ö1} = -t-1$

$y_{ö2} = A\cos 2t + B\sin 2t \ \left\} \begin{aligned} &-3A\cos 2t - 3B\sin 2t = -12\sin 2t \\ &A=0 \ , \ B=4 \qquad y_{ö2} = 4\sin 2t \end{aligned}\right.$

$$y(t) = c_3\cos t + c_4\sin t - t - 1 + 4\sin 2t$$

$-c_1\sin t + c_2\cos t + \cancel{4\sin 2t} - \cancel{8\cos 2t} - c_3\sin t + c_4\cos t - \cancel{1} + \cancel{8\cos 2t}$
$\quad - c_3\cos t - c_4\sin t + \cancel{t} + \cancel{1} - \cancel{4\sin 2t} = \cancel{t}$

$-(c_1+c_3+c_4)\sin t + (c_2+c_4-c_3)\cos t = 0$

$\quad = -c_3 - c_4$

$c_2 = c_3 - c_4$

$$\left.\begin{aligned} x(t) &= (-c_3 - c_4)\cos t + (c_3 - c_4)\sin t + 2 - 2\cos 2t - 4\sin 2 \\ y(t) &= c_3 \cos t + c_4 \sin t - t - 1 + 4\sin 2t \end{aligned}\right.$$

Sınav sorusu = $\left.\begin{aligned} \frac{dx}{dt} + 2x + 3y &= 0 \\ \frac{dy}{dt} + 2y + 3x &= 2e^{2t} \end{aligned}\right\}$ d.s çözünüz.

$$\left.\begin{aligned} (D+2)x + 3y &= 0 \\ 3x + (D+2)y &= 2e^{2t} \end{aligned}\right\} \begin{vmatrix} D+2 & 3 \\ 3 & D+2 \end{vmatrix} x = \begin{vmatrix} 0 & 3 \\ 2e^{2t} & D+2 \end{vmatrix}$$

$$\begin{vmatrix} D+2 & 3 \\ 3 & D+2 \end{vmatrix} y = \begin{vmatrix} D+2 & 0 \\ 3 & 2e^{2t} \end{vmatrix}$$

$$\left.\begin{aligned} (D^2+4D-5)x &= -6e^{2t} \\ (D^2+4D-5)y &= 8e^{2t} \end{aligned}\right\} \quad r^2 + 4r - 5 = 0 \quad \left.\begin{aligned} r_1 &= 1 \\ r_2 &= -5 \end{aligned}\right\} x_h = c_1 e^{t} + c_2 e^{-5t}$$

$+5 \quad -1$

$$\left.\begin{aligned} x_ö &= Ae^{2t} \\ x_ö' &= 2Ae^{2t} \\ x_ö'' &= 4Ae^{2t} \end{aligned}\right\} \quad \begin{aligned} 4A + 8A - 5A &= -6 \\ 7A &= -6 \\ A &= -\frac{6}{7} \end{aligned} \qquad x_ö = -\frac{6}{7}e^{2t}$$

$$x(t) = c_1 e^{t} + c_2 e^{-5t} - \frac{6}{7}e^{2t}$$

$$y_h = c_3 e^{t} + c_4 e^{-5t}$$

$$\left.\begin{aligned} y_ö &= Ae^{2t} \\ y_ö' &= 2Ae^{2t} \\ y_ö'' &= 4Ae^{2t} \end{aligned}\right\} \quad \begin{aligned} 4A + 8A - 5A &= 8 \\ 7A &= 8 \\ A &= \frac{8}{7} \end{aligned} \qquad y_ö = \frac{8}{7}e^{2t}$$

$$y(t) = c_3 e^{t} + c_4 e^{-5t} + \frac{8}{7}e^{2t}$$

$$e^{t} - 5c_2 e^{-5t} - \cancel{\frac{12}{7}e^{2t}} + 2c_1 e^{t} + 2c_2 e^{-5t} - \cancel{\frac{12}{7}e^{2t}} + 3c_3 e^{t} + 3c_4 e^{-5t} + \cancel{\frac{24}{7}e^{2t}} = 0$$

$$(3c_1 + 3c_3)e^{t} + (-3c_2 + 3c_4)e^{-5t} = 0$$

$$\left.\begin{aligned} c_1 &= -c_3 \\ c_2 &= c_4 \end{aligned}\right\} \begin{aligned} x(t) &= -c_3 e^{t} + c_4 e^{-5t} - \frac{6}{7}e^{2t} \\ y(t) &= c_3 e^{t} + c_4 e^{-5t} + \frac{8}{7}e^{2t} \end{aligned}$$

Örn $\left.\begin{aligned} x' &= 3x - y - 1 \\ y' &= x + y + 4e^t \end{aligned}\right\}$ sistemini çözünüz.

$$\left.\begin{aligned} (D-3)x + y &= -1 \\ -x + (D-1)y &= 4e^t \end{aligned}\right\} \quad \begin{vmatrix} D-3 & 1 \\ -1 & D-1 \end{vmatrix} = (D^2-4D+3)+1 = (D-2)^2$$

$$(D-2)^2 x = \begin{vmatrix} -1 & 1 \\ 4e^t & D-1 \end{vmatrix} = 1-4e^t \Rightarrow (D-2)^2 x = 1-4e^t$$

$$(r-2)^2 = 0 \Rightarrow r_1 = r_2 = 2 \qquad x_h = c_1 e^{2t} + c_2 t e^{2t}$$

$$x_{ö1} = A, \quad x_ö' = x_ö'' = 0 \quad \left.\begin{aligned} 4A &= 1 \\ A &= \tfrac{1}{4} \end{aligned}\right\} \quad x_{ö1} = \frac{1}{4}$$

$$x_{ö2} = Ae^t, \quad x_ö' = x_ö'' = Ae^t \quad \left.\begin{aligned} (A-4A+4A)e^t &= -4e^t \\ A &= -4 \end{aligned}\right\} \quad x_{ö2} = -4e^t$$

$$x(t) = c_1 e^{2t} + c_2 t e^{2t} + \frac{1}{4} - 4e^t$$

$$(D-2)^2 y = \begin{vmatrix} D-3 & -1 \\ -1 & 4e^t \end{vmatrix} = 4e^t - 12e^t + 1 = -1 - 8e^t$$

$$y_h = c_3 e^{2t} + c_4 t e^{2t}$$

$$y_{ö1} = A, \quad y_{ö1}' = y_{ö1}'' = 0 \quad \Big\} \ 4A = -1 \rightarrow A = \frac{-1}{4} \qquad y_{ö1} = \frac{-1}{4}$$

$$y_{ö2} = Ae^t \quad y_{ö2}' = y_{ö2}'' = Ae^t \quad \left.\begin{aligned} (A-4A+4A)e^t &= -8e^t \\ A &= -8 \end{aligned}\right\} \quad y_{ö2} = -8e^t$$

$$y(t) = c_3 e^{2t} + c_4 t e^{2t} - \frac{1}{4} - 8e^t$$

$$2c_1 e^{2t} + c_2 e^{2t} + 2c_2 t e^{2t} - \cancel{4e^t} - 3c_1 e^{2t} - 3c_2 t e^{2t} - \cancel{\tfrac{3}{4}} + \cancel{12e^t}$$

$$+ c_3 e^{2t} + c_4 t e^{2t} - \cancel{\tfrac{1}{4}} - \cancel{8e^t} = \cancel{-1}$$

$$\left.\begin{aligned} &(-c_1 + c_2 + c_3)e^{2t} + (-c_2 + c_4)te^{2t} = 0 \\ &c_1 = c_2 + c_3 = c_3 + c_4 \end{aligned}\right\} \quad \begin{aligned} x(t) &= (c_3 + c_4)e^{2t} + c_4 t e^{2t} + \frac{1}{4} - 4e^t \\ y(t) &= c_3 e^{2t} + c_4 t e^{2t} - \frac{1}{4} - 8e^t \end{aligned}$$