Homojen hale Dönüştürülebilen Dif. Denklemler

$$\boxed{\frac{dy}{dx} = f\left(\frac{a_1x+b_1y+c_1}{a_2x+b_2y+c_2}\right)}$$ $a_1, a_2, b_1, b_2, c_1, c_2 \in \mathbb{R}$

$2x+5y+h$

$*$ $(a_1x+b_1y+c_1)dx + (a_2x+b_2y+c_2)dy = 0$

$a_1x+b_1y \qquad a_2x+b_2y$ $\qquad (x+y)dx+(x-y)dy=0$

1) Bu iki denklem lineer bağımsız ise;

$a_1x+b_1y \qquad a_2x+b_2y$

x_1, y_1 yeni değişkenler

h, k sabitler

$x = x_1 + h \rightarrow dx = dx_1$

$y = y_1 + k \rightarrow dy = dy_1$

$$[a_1(x_1+h)+b_1(y_1+k)+c_1]dx_1 + [a_2(x_1+h)+b_2(y_1+k)+c_2]dy_1 = 0$$

$$(a_1x_1+b_1y_1+\underbrace{a_1h+b_1k+c_1}_{0})dx_1 + (a_2x_1+b_2y_1+\underbrace{a_2h+b_2k+c_2}_{0})dy_1 = 0$$

$$\left.\begin{cases} a_1h+b_1k+c_1=0 \\ a_2h+b_2k+c_2=0 \end{cases}\right\} h, k$$

$$(a_1x_1+b_1y_1)dx_1 + (a_2x_1+b_2y_1)dy_1 = 0$$

Homojen dif. denklem

$u = \frac{y_1}{x_1} \rightarrow y_1 = ux_1$

$dy_1 = u\,dx_1 + x_1\,du$

⋮ Genel çözüm

$h, k \qquad \left.\begin{matrix} x_1 = x-h \\ y_1 = y-k \end{matrix}\right\}$ x, y cinsinden yazılacak örnek $(x+y-3)dx+(-x+y+1)dy=0$

$x+y \; ; \; -x+y \longrightarrow$ Lineer bağımsız

$$x=x_1+h \quad dx=dx_1$$
$$y=y_1+k \quad dy=dy_1$$

$$\begin{cases} h+k-3=0 & h=2 \\ -h+k+1=0 & k=1 \end{cases}$$

$$(x_1+y_1)dx_1+(-x_1+y_1)dy_1=0 \quad \text{Homojen dif.}$$

Homojen fonk

$$\frac{y_1}{x_1}=u \rightarrow y_1=ux_1$$
$$dy_1=u\,dx_1+x_1\,du$$

$$(x_1+ux_1)dx_1+(-x_1+ux_1)(u\,dx_1+x_1\,du)=0$$

$$(x_1+\cancel{ux_1}-\cancel{ux_1}+u^2x_1)dx_1+(-x_1^2+ux_1^2)du=0$$

$$x_1\cdot(1+u^2)dx_1+x_1^2\cdot(u-1)du=0 \quad \text{Değişkenlerine Ay.}$$

$$\frac{u-1}{1+u^2}=\frac{u}{1+u^2}-\frac{1}{1+u^2}$$

$$\int\frac{x_1}{x_1^2}dx_1+\int\frac{u-1}{1+u^2}du=\int 0$$

$$\ln x_1+\frac{1}{2}\ln(1+u^2)-\arctan u=C \qquad u=\frac{y_1}{x_1}$$

$$\ln x_1+\frac{1}{2}\ln\left(1+\left(\frac{y_1}{x_1}\right)^2\right)-\arctan\frac{y_1}{x_1}=C$$

$$x_1=x-2$$
$$y_1=y-1$$

$$\ln(x-2)+\frac{1}{2}\ln\left(1+\left(\frac{y-1}{x-2}\right)^2\right)-\arctan\frac{y-1}{x-2}=C$$

Genel Çözüm

2) $(\underbrace{a_1x+b_1y}_{t}+c_1)dx+(\underbrace{a_2x+b_2y}_{\lambda t}+c_2)dy=0$

$a_1x+b_1y \quad a_2x+b_2y$ Lineer bağımlı

$$a_2x+b_2y=\lambda(a_1x+b_1y)$$

$$a_1x+b_1y=t$$

$$a_1dx+b_1dy=dt \rightarrow dy=\ldots$$

Değişkenlerine ayrılabilen dif.

Örnek $(x+5y+1)dx+(2x+10y+7)dy=0$

$2(x+5y)=2x+10y$ Lineer bağımlı

$$\begin{cases} t=x+5y \\ dt=dx+5dy \end{cases} \rightarrow dy=\frac{1}{5}(dt-dx)$$

$$(t+1)dx+(2t+7)\frac{1}{5}(dt-dx)=0$$

$$(5t+5)dx+(2t+7)(dt-dx)=0$$

$$(5t+5-2t-7)dx+(2t+7)dt=0$$

$$(3t-2)dx+(2t+7)dt=0 \quad \text{Değ. Ay. D. D.}$$

$$\int dx+\int\frac{2t+7}{3t-2}dt=\int 0$$

$$\begin{array}{r|l} 2t+7 & 3t-2 \\ \hline -(2t+\frac{4}{3}) & \frac{2}{3} \\ \hline \frac{25}{3} & \end{array}$$

$$\frac{2t+7}{3t-2} = \underset{\text{Bölüm}}{\frac{2}{3}} + \frac{25/3 \text{ Kalan}}{3t-2 \text{ Bölen}}$$

$$\int dx + \int\left(\frac{2}{3} + \frac{\frac{25}{3}}{3t-2}\right)dt = \int 0$$

$$x + \frac{2}{3}t + \frac{25}{9}\ln(3t-2) = C$$

$$t = x+5y$$

$$\underbrace{x + \frac{2}{3}(x+5y) + \frac{25}{9}\ln(3x+15y-2) = C}_{\text{Genel Çözüm}}$$

$$\frac{dy}{dx} = f(ax+by+c) \qquad ax+by+c = t$$

$$a + by' = t'$$

$$y' = \frac{1}{b}(t'-a)$$

$$\underbrace{\frac{1}{b}(t'-a) = f(t)}_{\text{Değiş. A.D.D}}$$

Örnek $\quad y' = \tan^2(x+y)$

$$x+y = t$$

$$1+y' = t' \rightarrow y' = t'-1$$

$$t'-1 = \tan^2 t$$

$$\frac{dt}{dx} = \underbrace{1+\tan^2 t}_{} = \frac{1}{\cos^2 t}$$

$$\int \cos^2 t\,dt = \int dx$$

$$\int \underbrace{\cos t}_{u}\,\underbrace{\cos t\,dt}_{dv} \text{ kısmi int.}$$

$$\boxed{\cos^2 kt = \frac{1+\cos 2kt}{2} \qquad \sin^2 kt = \frac{1-\cos 2kt}{2}}$$

$$\int \cos^2 t\,dt = \int \left(\frac{1+\cos 2t}{2}\right)dt = \int \left(\frac{1}{2}+\frac{1}{2}\cos 2t\right)dt$$

$$= \frac{t}{2} + \frac{1}{4}\sin 2t$$

$$\frac{t}{2} + \frac{1}{4}\sin 2t = x + C$$

$$\underbrace{\frac{x+y}{2} + \frac{1}{2}\sin\left[2(x+y)\right] = x + C}_{\text{Genel çözüm}}$$

Tam Dif. Denklemler

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

Eğer $u(x,y) = C$ ifadesinin toplam dif. yukarıdaki dif. denklem ise bu dif. denk. tam dif. tır.

$$u(x,y) = C \qquad du = 0$$

$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy$$

Tam dif.

$$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = \underbrace{M(x,y)dx + N(x,y)dy}$$

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x} = M(x,y) \quad ; \quad \frac{\partial u}{\partial y} = N(x,y)$$

$$\frac{\partial^2 u}{\partial y \partial x} = \frac{\partial M}{\partial y} \quad ; \quad \frac{\partial^2 u}{\partial x \partial y} = \frac{\partial N}{\partial x}$$

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Teorem: $M(x,y)dx + N(x,y)dy = 0$ dif. denkleminin tam dif. olması için g.y.k. $\boxed{\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}}$ dır

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$M(x,y)dx + N(x,y)dy = 0$

$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ tam dif.

$u(x,y) = C$ Genel çözüm.

$\frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy = M(x,y)dx + N(x,y)dy$

(1) $\frac{\partial u}{\partial x} = M(x,y)$ (2) $\frac{\partial u}{\partial y} = N(x,y)$ ✓

$u(x,y) = \int M(x,y)dx + h(y) \rightarrow \frac{\partial u}{\partial y} = \dots + \frac{dh}{dy} = N(x,y)$

$\frac{dh}{dy} = y$ cinsinden bir fonk. (sabit olabilir)

$\frac{\partial u}{\partial x} = 2xy$?

$\boxed{u(x,y) = x^2y + h(y)}$

$\boxed{u(x,y) = C}$

Örnek $\left(x^2+\frac{y^2}{x}\right)dx+2y\ln x\,dy=0$

$M(x,y)=x^2+\frac{y^2}{x}$ $\qquad N(x,y)=2y\ln x$

$\frac{\partial M}{\partial y}=\frac{2y}{x} = \frac{\partial N}{\partial x}=\frac{2y}{x}$

Tam dif.

$u(x,y)=C$

$\frac{\partial u}{\partial x}=x^2+\frac{y^2}{x}$

$\boxed{u(x,y)=\frac{x^3}{3}+y^2\ln x+h(y)}$

$\frac{\partial u}{\partial y}=2y\ln x+\frac{dh}{dy}$

$\frac{\partial u}{\partial y}=2y\ln x$

$\cancel{2y\ln x}+\frac{dh}{dy}=\cancel{2y\ln x}$

$\frac{dh}{dy}=0$

$h(y)=0$

$u(x,y)=C$

$\boxed{\frac{x^3}{3}+y^2\ln x=C}$ Genel çözüm

$\left(x^2+\frac{y^2}{x}\right)dx+2y\ln x\,dy=0$