$As-2A+Bs+B=s-1$

$$\left.\begin{aligned} As+Bs&=s \\ -2A+B&=-1 \end{aligned}\right\} \quad A=\frac{2}{3} \quad B=\frac{1}{3}$$

$$=L^{-1}\left\{\frac{2/3}{s+1}+\frac{1/3}{s-2}\right\}$$

$$y(t)=\frac{2}{3}e^{-t}+\frac{1}{3}e^{2t}$$

Örnek $y''-6y'+9y=t^2e^{3t}, \; y(0)=2, \; y'(0)=6$

$$L\{y''-6y'+9y\}=L\{t^2e^{3t}\}$$

$$s^2Y(s)-s\underbrace{y(0)}_{=2}-\underbrace{y'(0)}_{=6}-6(sY(s)-\underbrace{y(0)}_{=2})+9Y(s)=\frac{2}{(s-3)^3}$$

$$s^2Y-2s-6-6sY+12+9Y=\frac{2}{(s-3)^3}$$

$$(s^2-6s+9)Y-2(s-3)=\frac{2}{(s-3)^3}$$

$$(s-3)^2Y=\frac{2}{(s-2)^3}+2(s-3)$$

$$Y=\frac{2}{(s-3)^5}+\frac{2}{s-3}$$

$$y(t)=L^{-1}\left\{\frac{2}{(s-3)^5}+\frac{2}{s-3}\right\}=2\frac{t^4}{4!}e^{3t}+2e^{3t}$$

Çalışma soruları

Örnek $F(t)=2t^4-4t^3+2t-5 \Rightarrow L\{F(t)\}=?$

$$F(t)=2L\{t^4\}-4L\{t^3\}+2L\{t\}-5L\{1\}=2\frac{4!}{s^{4+1}}-4\frac{3!}{s^{3+1}}+2\frac{1!}{s^{1+1}}-5\frac{1}{s}$$

Örnek $F(t)=6t^2-\cos 2t$

$$F(t)=6\frac{2!}{s^{2+1}}-\frac{s}{s^2+2^2}$$

Örnek $F(t)=\sin 3t-\cos 3t$

$$L\{F(t)\}=L\{\sin 3t\}-L\{\cos 3t\}=\frac{3}{s^2+3^2}-\frac{s}{s^2+3^2}$$

Örnek $F(t) = \sin^2 3t$

$\sin^2 3t = \frac{1}{2}(1-\cos 6t)$ $\qquad$ $\cos 2x = 2\cos^2 x - 1$

$$L\left\{\frac{1}{2}\right\} - \frac{1}{2}L\left\{\cos 6t\right\} = \frac{1}{2s} - \frac{1}{2}\frac{s}{s^2+6^2}$$

Örnek $L^{-1}\left\{\frac{6s+1}{s^2+4}\right\} = ?$

$$6L^{-1}\left\{\frac{s}{s^2+2^2}\right\} + L^{-1}\left\{\frac{1}{s^2+2^2}\right\} = 6\cos 2t + \frac{1}{2}L^{-1}\left\{\frac{2}{s^2+2^2}\right\} = 6\cos 2t + \frac{1}{2}\sin 2t$$

Örnek $L^{-1}\left\{\frac{2s^2+4s-1}{s^6}\right\} = ?$

$$2L^{-1}\left\{\frac{1}{s^4}\right\} + 4L^{-1}\left\{\frac{1}{s^5}\right\} - L^{-1}\left\{\frac{1}{s^6}\right\} = \frac{2}{3!}t^3 + \frac{4}{4!}t^4 - \frac{1}{5!}t^5$$

Örnek $L^{-1}\left\{\frac{3s+1}{s^2+4s+13}\right\} = ?$

paydada kök bulunamıyor.

$$L^{-1}\left\{\frac{3s+1}{\underbrace{s^2+4s+4}_{(s+2)^2}+9}\right\} = L^{-1}\left\{\frac{3s+1}{(s+2)^2+3^2}\right\} = L^{-1}\left\{\frac{3(s+2)-5}{(s+2)^2+3^2}\right\}$$

$$= L^{-1}\left\{\frac{3(s+2)}{(s+2)^2+3^2}\right\} + L^{-1}\left\{\frac{-5}{(s+2)^2+3^2}\right\}$$

$$= 3L^{-1}\left\{\frac{s+2}{(s+2)^2+3^2}\right\} - 5L^{-1}\left\{\frac{1}{(s+2)^2+3^2}\right\}$$

$$= 3L^{-1}\left\{\frac{s+2}{(s+2)^2+3^2}\right\} - \frac{5}{3}L^{-1}\left\{\frac{3}{(s+2)^2+3^2}\right\}$$

$$= 3e^{-2t}\cos 3t - \frac{5}{3}e^{-2t}\sin 3t$$

Örnek $L^{-1}\left\{\frac{5s+1}{s^2+s-2}\right\} = ?$

$$L^{-1}\left\{\frac{5s+1}{(s+2)(s-1)}\right\} = ?$$

$$\frac{5s+1}{(s+2)(s-1)} = \frac{A}{\underset{(s-1)}{s+2}} + \frac{B}{\underset{(s+2)}{s-1}} = \frac{As-A+Bs+B2}{(s+2)(s-1)}$$

$5s+1 = As - A + Bs + B2$

$$\left.\begin{aligned} As + Bs &= 5s \\ -A + B2 &= 1 \end{aligned}\right\} \quad A=3 \quad B=2$$

$$L^{-1}\left\{\frac{3}{s+2} + \frac{2}{s-1}\right\} = L^{-1}\left\{\frac{3}{s+2}\right\} + L^{-1}\left\{\frac{2}{s-1}\right\} = 3e^{-2t} + 2e^{t}$$

Örnek $L^{-1}\left\{\dfrac{5s^2+18s-36}{s^3+s^2-6s}\right\} = ?$

$$L^{-1}\left\{\frac{5s^2+18s-36}{s(s+3)(s-2)}\right\} = ?$$

$$\frac{A}{s} + \frac{B}{s+3} + \frac{C}{s-2} = \frac{5s^2+18s-36}{s(s+3)(s-2)} \qquad (As+3A)(s-2) + Bs^2 - 2Bs + Cs^2 + 3Cs = 5s^2+18s-36$$

(s^2-2) olsaydı $\dfrac{Cs+D}{s^2-2}$ olacaktı.

$(s-2)^2$ olsaydı $\dfrac{C}{s-2} + \dfrac{D}{(s-2)^2}$ olacaktı.

$As^2 - 2As + 3As - 6A + Bs^2 - 2Bs + Cs^2 + 3Cs = 5s^2 + 18s - 36$

$$\left.\begin{aligned} As^2 + Bs^2 + Cs^2 &= 5s^2 \\ As - 2Bs + 3Cs &= 18s \\ -6A &= -36 \end{aligned}\right\} \quad A=6 \quad B=-3 \quad C=2$$

$$L^{-1}\left\{\frac{6}{s} + \frac{-3}{s+3} + \frac{2}{s-2}\right\} = 12\,L^{-1}\left\{\frac{1}{s}\right\} - 3L^{-1}\left\{\frac{1}{s+3}\right\} + 2L^{-1}\left\{\frac{1}{s-2}\right\} = 6 - 3e^{-3t} + 2e^{2t}$$

Örnek $L^{-1}\left\{\dfrac{2s^2-7s+20}{s(s^2-2s+10)}\right\} = ?$

$$\frac{A}{s} + \frac{Bs+C}{s^2-2s+10} = \frac{2s^2-7s+20}{s(s^2-2s+10)}$$

$As^2 - 2As + 10A + Bs^2 + Cs = 2s^2 - 7s + 20$

$$\left.\begin{aligned} As^2 + Bs^2 &= 2s^2 \\ -2As + Cs &= -7s \\ 10A &= 20 \end{aligned}\right\} \quad A=2 \quad B=0 \quad C=-3$$

$$L^{-1}\left\{\frac{2}{s}+\frac{-3}{s^2+2s+10}\right\}=2-L^{-1}\left\{\frac{3}{(s-1)^2+3^2}\right\}=2-e^{t}\sin 3t$$

Örnek $\quad L^{-1}\left\{\frac{10s^2+4}{s(s+1)(s+2)^2}\right\}=?$

$$\frac{10s^2+4}{s(s+1)(s+2)^2}=\frac{A}{s}+\frac{B}{s+1}+\frac{C}{(s+2)^2}+\frac{D}{s+2}$$

$s=0$ için ; $A=?$ $\quad \frac{10s^2+4}{s(s+1)(s+2)^2}=\frac{A}{s}$ $\quad$ herikı tarafı s ile çarpalım.

$\frac{10s^2+4}{(s+1)(s+2)^2}=A$ $\quad \frac{10\cdot(0)^2+4}{(0+1)(0+2)^2}=1$ $\quad A=1$

$s=-1$ için ; $B=?$ $\quad \frac{10s^2+4}{s(s+1)(s+2)^2}=\frac{B}{(s+1)}$ $\quad$ herikı tarafı $(s+1)$ ile çarpalım.

$\frac{10s^2+4}{s(s+2)^2}=B$ $\quad \frac{10(-1)+4}{-1(-1+2)^2}=-14$ $\quad B=-14$

$s=-2$ için ; $C=?$ $\quad \frac{10s^2+4}{s(s+1)(s+2)^2}=\frac{C}{(s+2)^2}$ $\quad$ herikı tarafı $(s+2)^2$ ile çarpalım.

$\frac{10s^2+4}{s(s+1)}=C$ $\quad \frac{10(-2)^2+4}{-2(-2+1)}=22$ $\quad C=22$

$D=?$ $\quad \frac{d}{ds}\left[\frac{10s^2+4}{s(s+1)}\right]\Bigg|_{s=-2}$ $\quad \frac{20s(s^2+s)-(2s+1)(10s^2+4)}{(s^2+s)^2}\Bigg|_{s=-2}$

$\frac{-40\cdot(2)-(-3(44))}{4}=13$ $\quad D=13$

$$L^{-1}\left\{\frac{1}{s}-\frac{14}{s+1}+\frac{22}{(s+2)^2}+\frac{13}{s+2}\right\}=1-14e^{-t}+22e^{-2t}\,t+13e^{-2t}$$

Örnek $\quad y''+3y'+2y=0$, $y(0)=2$, $y'(0)=-3$ $\Rightarrow y(t)=?$

$L\{y\}=Y$

$L\{y'\}=sY-y(0)$

$L\{y''\}=s^2Y-sy(0)-y'(0)$

$L\{y''+3y'+2y\}=L\{0\}$

$L\{y''\}+3L\{y'\}+2L\{y\}=L\{0\}$

$s^2Y-sy(0)-y'(0)+3sY-y(0)+2Y=0$

$s^2Y-2s+3+3sY-6+2Y=0$

$s^2Y-2s-3+3sY+2Y=0$

Y 'yi yalnız bırakalım.

$Y(s^2+3s+2)=2s+3$

$$Y=\frac{2s+3}{s^2+3s+2}=\frac{2s+3}{(s+2)(s+1)}$$

$$Y=\frac{A}{s+2}+\frac{B}{s+1}$$

$A(s+1)+B(s+2)=2s+3$

$As+A+Bs+2s=2s+3$

$A+B=2$

$A+2B=3$

$A=1 \quad B=1$

$$Y=\frac{1}{s+2}+\frac{1}{s+1}$$

$L^{-1}\{Y\}=y(t)=?$

$L^{-1}\{Y\}=e^{-2t}+e^{-t}=y(t)$

Örnek $\quad y''+y=-15\sin 4t \quad , y(0)=2 \quad , y'(0)=5$

$L\{y\}=Y$

$L\{y'\}=sY-y(0)$

$L\{y''\}=s^2Y-sy(0)-y'(0)$

$L\{y''+y\}=-15L\{\sin 4t\}$

$$s^2Y-sy(0)-y'(0)+Y=-15\frac{4}{s^2+4^2}$$

$$s^2Y-2s-5+Y=-15\frac{4}{s^2+4^2}$$

$$Y(s^2+1)=-\frac{60}{s^2+4^2}+2s+5$$

$$Y=-\frac{60}{(s^2+1)(s^2+16)}+\frac{2s}{s^2+1}+\frac{5}{s^2+1}$$

$$\frac{-60}{(s^2+1)(s^2+16)}=\frac{As+B}{s^2+1}+\frac{Cs+D}{s^2+16}$$

$As^3+s^2B+16As+16B+Cs^3+s^2D+Cs+D=-60$

$s^3(A+C)=0$

$s^2(B+D)=0$

$s(16A+C)=0$

$16B+D=-60$

$A=0 \quad B=-4 \quad C=0 \quad D=4$

$$\frac{-4}{s^2+1}+\frac{4}{s^2+16}$$

$$Y=-\frac{4}{s^2+1}+\frac{4}{s^2+16}+\frac{2s}{s^2+1}+\frac{5}{s^2+1}$$

$$Y=\left\{-4\frac{1}{s^2+1}+\frac{4}{s^2+4^2}+2\frac{s}{s^2+1^2}+5\frac{1}{s^2+1}\right\}$$

$y(t)=-4\sin t+\sin 4t+2\cos t+5\sin t$

Örnek $y''+2y'+y=4e^{-t}$, $y(0)=2$, $y'(0)=-1$ başlangıç değer problemini Laplace dönüşümünü kullanarak çözünüz. (sınav sorusu)

$$s^2Y-sy(0)-y'(0)+2sY-2y(0)+Y=4\frac{1}{s+1}$$

$$s^2Y-2s+1+2sY-4+Y=\frac{4}{s+1}$$

$$Y(s^2+2s+1)=\frac{4}{s+1}+2s+3$$

$$Y=\frac{4}{(s+1)^3}+\frac{2s+3}{(s+1)^2}$$

$$\frac{2s+3}{(s+1)^2}=\frac{A}{s+1}+\frac{B}{(s+1)^2}$$

$$2s+3=A(s+1)+B$$

$$2s+3=As+A+B$$

$A=2$ $B=1$

$$Y=\frac{4}{(s+1)^3}+\frac{2}{s+1}+\frac{1}{(s+1)^2}$$

$$Y=4\frac{1}{(s+1)^3}+2\frac{1}{s+1}+\frac{1}{(s+1)^2}$$

$$y(t)=4e^{-t}\frac{t^2}{2!}+2e^{-t}+e^{-t}\frac{t}{1!}$$

Örnek $y''+2y'+5y=0$, $y(0)=2$, $y'(0)=-1$

$$L\{y''\}+2L\{y'\}+5\{y\}=0$$

$$L\{y(t)\}=Y(s)$$

$$L^{-1}\{Y(s)\}=y(t)$$

$$s^2Y-s\underbrace{y(0)}_{=2}-\underbrace{y'(0)}_{=-1}+2(sY-\underbrace{y(0)}_{=2})+5Y=0$$

s^2+2s+5 şeklinde çıkmalı, aksi halde hata yapmışız demektir.

$$s^2Y-2s+1+2sY-4+5Y=0$$

$$Y(s^2+2s+5)-2s+1-4=0$$

$$Y=\frac{2s+3}{s^2+2s+5}$$

s^2+2s+5 $y''+2y'+5y$ 1 2 5 O.K.

$$y(t)=L^{-1}\{Y\}=L^{-1}\left\{\frac{2s+3}{s^2+2s+5}\right\}=L^{-1}\left\{\frac{2s}{(s+1)^2+2^2}\right\}+L^{-1}\left\{\frac{3}{(s+1)^2+2^2}\right\}$$

$$=2L^{-1}\left\{\frac{s+1-1}{(s+1)^2+2^2}\right\}+3L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}=2L^{-1}\left\{\frac{s+1}{(s+1)^2+2^2}\right\}-2L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}+3L^{-1}\left\{\frac{1}{(s+1)^2+2^2}\right\}$$

$$=2e^{-t}\cos 2t-2e^{-t}\frac{\sin 2t}{2}+3e^{-t}\frac{\sin 2t}{2}=2e^{-t}\cos 2t+\frac{1}{2}e^{-t}\sin 2t$$

Örnek $y''+y'+y=e^{-t}$, $y(0)=0$, $y'(0)=0$

$$L\{y''\}+L\{y'\}+L\{y\}=L\{e^{-t}\}=\frac{1}{s+1}$$

$$s^2Y-s\underbrace{y(0)}_{=0}-\underbrace{y'(0)}_{=0}-sY-\underbrace{y'(0)}_{=0}+Y=\frac{1}{s+1}$$

$$Y(s^2+s+1)=\frac{1}{s+1}$$

s^2+s+1 $\quad y''+y'+y$ $\quad 1\ 1\ 1$ $\quad$ O.K.

$$Y=\frac{1}{(s+1)(s^2+s+1)}$$

$$L^{-1}\{Y\}=L^{-1}\left\{\frac{1}{(s+1)\underbrace{(s^2+s+1)}_{\text{reel kök yok}}}\right\}$$

$$\frac{A}{s+1}+\frac{Bs+C}{s^2+s+1}=\frac{1}{(s+1)(s^2+s+1)}$$

$$As^2+As+A+Bs^2+Bs+Cs+C=1$$

$$\left.\begin{aligned}A+C&=1\\A+B&=0\\A+B+C&=0\end{aligned}\right\}\quad A=1\quad B=-1\quad C=0$$

$$L^{-1}\left\{\frac{1}{s+1}-\frac{s}{s^2+s+1}\right\}$$

$$L^{-1}\left\{\frac{1}{s+1}\right\}-L^{-1}\left\{\frac{s}{s^2+s+1}\right\}$$

$$e^{-t}-L^{-1}\left\{\frac{s}{(s+\frac{1}{2})^2+\frac{3}{4}}\right\}$$

$$e^{-t}-L^{-1}\left\{\frac{s+\frac{1}{2}-\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right\}$$

$$e^{-t}-L^{-1}\left\{\frac{s+\frac{1}{2}}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right\}+\frac{1}{2}L^{-1}\left\{\frac{1}{(s+\frac{1}{2})^2+(\frac{\sqrt{3}}{2})^2}\right\}$$

$$e^{-t}-e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t+\frac{1}{2}e^{-\frac{1}{2}t}\frac{\sin\frac{\sqrt{3}}{2}t}{\frac{\sqrt{3}}{2}}$$

$$e^{-t}-e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t+\sqrt{3}e^{-\frac{1}{2}t}\sin\frac{\sqrt{3}}{2}t$$

Örnek $y''-y=4\cos t-4\sin t$, $y(0)=0$, $y'(0)=0$

$$L\{y''\}-L\{y\}=4L\{\cos t\}-4L\{\sin t\}$$

$$s^2Y-sy(0)-y'(0)-Y=4\frac{s}{s^2+1}-4\frac{1}{s^2+1}$$

$$Y(s^2-1)=4\frac{s}{s^2+1}-4\frac{1}{s^2+1}=4\frac{s-1}{s^2+1}$$

s^2-1 $\quad y''-y$ $\quad 1\ -1$ $\quad$ O.K.

$$Y=\frac{4(s-1)}{\underbrace{(s^2-1)}_{(s-1)(s+1)}(s^2+1)}=\frac{4}{(s+1)(s^2+1)}$$

$$L^{-1}\{Y\}=L^{-1}\left\{\frac{4}{(s+1)(s^2+1)}\right\}=4L^{-1}\left\{\frac{1}{(s+1)(s^2+1)}\right\}$$

$$Y = \frac{A}{s+1} + \frac{Bs+C}{s^2+1}$$

$$As^2 + A + Bs^2 + Bs + Cs + C = 1$$

$$\left.\begin{array}{l} A+B=0 \\ A+C=1 \\ B+C=0 \end{array}\right\} \quad A=\frac{1}{2} \quad B=-\frac{1}{2} \quad C=\frac{1}{2}$$

$$4\,\mathcal{L}^{-1}\left\{\frac{1/2}{s+1}\right\} + 4\,\mathcal{L}^{-1}\left\{\frac{-1/2\, s + 1/2}{s^2+1}\right\}$$

$$4\,\frac{1}{2}\,\mathcal{L}^{-1}\left\{\frac{1}{s+1}\right\} + 4\left(-\frac{1}{2}\right)\mathcal{L}^{-1}\left\{\frac{s-1}{s^2+1}\right\}$$

$$2e^{-t} - 2\,\mathcal{L}^{-1}\left\{\frac{s}{s^2+1}\right\} - 2\,\mathcal{L}^{-1}\left\{\frac{-1}{s^2+1}\right\}$$

$$2e^{-t} - 2\cos t + 2\sin t$$