Çalışma Soruları 1

1) $y\sqrt{1-\ln^2 y}\,\sin x\,dx+(1+\cos^2 x)\,dy=0$ d.d çözünüz.

$$\underbrace{\int\frac{\sin x}{1+\cos^2 x}\,dx}+\underbrace{\int\frac{dy}{y\sqrt{1-\ln^2 y}}}=\int 0$$

$$\underbrace{\begin{aligned}\cos x&=u\\ -\sin x\,dx&=du\end{aligned}}$$

$$\int\frac{-du}{1+u^2}=\operatorname{arccot}u$$

$$=\operatorname{arccot}(\cos x)$$

$$\left.\underbrace{\begin{aligned}\ln y&=t\\ \frac{dy}{y}&=dt\end{aligned}}\right\}$$

$$\int\frac{dt}{\sqrt{1-t^2}}=\arcsin t$$

$$=\arcsin(\ln y)$$

sonuç:

$\operatorname{arccot}(\cos x)+\arcsin(\ln y)=c$

2) $2y\,dy+4x^3\sqrt{4-y^4}\,dx=0$ d.d çözünüz.

$$\underbrace{\int\frac{2y\,dy}{\sqrt{4-y^4}}}+\int 4x^3\,dx=0$$

$$\left.\begin{aligned}y^2&=t\\ 2y\,dy&=dt\end{aligned}\right\}\int\frac{dt}{\sqrt{4-t^2}}=\int\frac{\frac{1}{2}dt}{\sqrt{1-\left(\frac{t}{2}\right)^2}}=\frac{1}{2}\arcsin\left(\frac{t}{2}\right)=\frac{1}{2}\arcsin\left(\frac{y^2}{2}\right)$$

sonuç: $\frac{1}{2}\arcsin\left(\frac{y^2}{2}\right)+x^4=c$

3) $y^2y'-\sqrt{x^4-x^4y^6}=0$

$$y^2\,dy-x^2\sqrt{1-y^6}\,dx=0\Rightarrow\int\frac{y^2\,dy}{\sqrt{1-y^6}}-\int x^2\,dx=\int 0$$

$$\left.\begin{aligned}y^3&=t\\ 3y^2\,dy&=dt\end{aligned}\right\}\int\frac{dt}{3\sqrt{1-t^2}}=\frac{1}{3}\arcsin t=\frac{1}{3}\arcsin(y^3)$$

sonuç: $\frac{1}{3}\arcsin(y^3)-\frac{x^3}{3}=c$

4) $y' = \dfrac{y}{x+\sqrt{xy}}$ d.d çözünüz.

$(x+\sqrt{xy})dy - y\,dx = 0$ Homojen D.D

$$\left.\begin{array}{l} y = ux \\ dy = u\,dx + x\,du \end{array}\right\} \begin{array}{l} (x+\sqrt{ux^2})(u\,dx + x\,du) - ux\,dx = 0 \\ (\cancel{ux} + ux\sqrt{u} - \cancel{ux})dx + x(x + x\sqrt{u})du = 0 \end{array}$$

$$\frac{ux\sqrt{u}\,dx}{u\sqrt{u}\cdot x^2} + \frac{x^2(1+\sqrt{u})du}{u\sqrt{u}\cdot x^2} = 0 \quad \Big\} \int \frac{dx}{x} + \int \frac{(1+\sqrt{u})}{u\sqrt{u}}du = \int 0$$

$$\ln x + \int u^{-\frac{3}{2}}du + \int \frac{1}{u}du = c \Rightarrow \ln x - \frac{2}{\sqrt{u}} + \ln u = c$$

$$\ln x - \frac{2}{\sqrt{\frac{y}{x}}} + \ln\left(\frac{y}{x}\right) = c$$

5) $xy\,dy = (x^2+xy+y^2)dx$ diferansiyel denkleminin $x=1$ için $y=0$ koşuluna uyan özel çözümünü bulunuz.

$$\left.\begin{array}{l} y = ux \\ dy = u\,dx + x\,du \end{array}\right\} \begin{array}{l} ux^2(u\,dx + x\,du) = (x^2 + ux^2 + u^2x^2)dx \\ (\cancel{u^2x^2} - x^2 - ux^2 - \cancel{u^2x^2})dx + ux^3du = 0 \end{array}$$

$$\frac{-x^2(1+u)dx}{(1+u)x^3} + \frac{ux^3du}{(1+u)x^3} = 0 \Rightarrow -\int\frac{dx}{x} + \int\frac{u\,du}{1+u} = \int 0$$

$$-\ln x + \int\left(1 - \frac{1}{1+u}\right)du = c$$

$$-\ln x + u - \ln(u+1) = c \Rightarrow -\ln x + \frac{y}{x} - \ln\left(\frac{y}{x}+1\right) = c$$

$x=1,\ y=0$ için $-\ln 1 + 0 - \ln 1 = c \Rightarrow c=0$

Sonuç $= -\ln x + \dfrac{y}{x} - \ln\left(\dfrac{y}{x}+1\right) = 0$

7) $dy+(y.\cot x-5e^{\cos x})dx=0$ diferansiyel denklemini tam diferansiyel denklem haline getirerek, $x=\frac{\pi}{2}$ için $y=0$ koşuluna uyan özel çözümünü bulunuz.

$$\frac{\partial m}{\partial y}=\cot x \neq \frac{\partial N}{\partial x}=0$$

1°) $\mu=\mu(x)$ olsun.

$$\ln\mu=\int\frac{\frac{\partial m}{\partial y}-\frac{\partial N}{\partial x}}{N}dx=\int\frac{\cot x-0}{1}dx=\ln(\sin x)\Rightarrow\boxed{\mu=\sin x}$$

$\sin x\,dy+(y\cos x-5\sin x\,e^{\cos x})dx=0$ Tam D.D

$$\frac{\partial f}{\partial x}=y\cos x-5\sin x.e^{\cos x}$$

$$\frac{\partial f}{\partial y}=\sin x$$

$$\frac{\partial f}{\partial y}=\sin x\Rightarrow f(x,y)=y\sin x+h(x)$$

$$\frac{\partial f}{\partial x}=\cancel{y\cos x}+\frac{dh}{dx}=\cancel{y\cos x}-5\sin x\,e^{\cos x}$$

$$\int dh=\underbrace{\int-5\sin x\,e^{\cos x}\,dx}$$

$$\left.\begin{aligned}\cos x&=u\\-\sin x\,dx&=du\end{aligned}\right\}\;5\int e^{u}du=5e^{u}=5e^{\cos x}+c$$

$$h(x)=5e^{\cos x}+c$$

$$f(x,y)=y\sin x+5e^{\cos x}+c=k$$

6) $y' = \dfrac{9x+2y-7}{2x-y-3}$ d.d çözünüz.

$$(2x-y-3)dy - (9x+2y-7)dx = 0$$

$$\left.\begin{aligned} x &= x_1 + h \quad , dx = dx_1 \\ y &= y_1 + k \quad , dy = dy_1 \end{aligned}\right\}$$

$$(2x_1+2h-y_1-k-3)dy_1 - (9x_1+9h+2y_1+2k-7)dx_1 = 0$$

$$\left.\begin{aligned} 2/\ 2h-k &= 3 \\ 9h+2k &= 7 \end{aligned}\right\}$$

$$+ \quad \overline{}$$

$$13h = 13$$

$$h = 1 \Rightarrow k = -1$$

$$\left.\begin{aligned} x &= x_1 + 1 \Rightarrow \boxed{x_1 = x-1} \\ y &= y_1 - 1 \quad \boxed{y_1 = y+1} \end{aligned}\right.$$

$$(2x_1 - y_1)dy_1 - (9x_1+2y_1)dx_1 = 0$$

$$\left.\begin{aligned} y_1 &= ux_1 \\ dy_1 &= u\,dx_1 + x_1\,du \end{aligned}\right\} \begin{aligned} &(2x_1-ux_1)(u\,\underline{dx_1}+x_1\,\underline{du}) - (9x_1+2ux_1)\underline{dx_1} = 0 \\ &(\cancel{2ux_1} - u^2x_1 - 9x_1 - \cancel{2ux_1})dx_1 + x_1^2(2-u)du = 0 \end{aligned}$$

$$\frac{-x_1(\cancel{u^2+9})dx_1}{(\cancel{u^2+9})x_1^2} + \frac{\cancel{x_1^2}(2-u)du}{(u^2+9)\cancel{x_1^2}} = 0$$

$$-\int \frac{dx_1}{x_1} + \int \frac{2du}{u^2+9} - \int \frac{u\,du}{u^2+9} = \int 0$$

$$-\ln x_1 + \frac{2}{3}\arctan\frac{u}{3} - \frac{1}{2}\ln(u^2+9) = c$$

$$-\ln x_1 + \frac{2}{3}\arctan\left(\frac{y_1}{3x_1}\right) - \frac{1}{2}\ln\left(\left(\frac{y_1}{x_1}\right)^2+9\right) = c$$

$$-\ln(x-1) + \frac{2}{3}\arctan\left(\frac{y+1}{3(x-1)}\right) - \frac{1}{2}\ln\left(\left(\frac{y+1}{x-1}\right)^2+9\right) = c$$

8) $y' = \frac{y}{x} + e^{\frac{y}{x}}$ diferansiyel denklemini çözünüz.

$$\left.\begin{aligned} &\frac{y}{x} = u \Rightarrow y = ux \\ &dy = u\,dx + x\,du \end{aligned}\right\} \qquad dy = \left(\frac{y}{x} + e^{\frac{y}{x}}\right)dx$$

$$u\,dx + x\,du = (u + e^u)\,dx \Rightarrow x\,du = (\cancel{u} + e^u - \cancel{u})\,dx$$

$$\frac{x\,du}{e^u . x} = \frac{e^u\,dx}{e^u . x} \Rightarrow e^{-u}\,du = \frac{dx}{x}$$

$$-e^{-u} = \ln x + c \Rightarrow -e^{-\frac{y}{x}} = \ln x + c$$

9) $\cos y\,dx + (x\sin y - 2)\,dy = 0$ diferansiyel denklemini çözünüz.

$$\frac{\partial m}{\partial y} = -\sin y \neq \frac{\partial N}{\partial x} = \sin y$$

1°) $\mu = \mu(x)$ olsun.

$$\ln \mu = \int \frac{m_y - N_x}{N}\,dx = \int \frac{-\sin y - \sin y}{(x\sin y - 2)}\,dx \neq \mu(x)$$

2°) $\mu = \mu(y)$ olsun.

$$\ln \mu = \int \frac{N_x - m_y}{m}\,dy = \int \frac{\sin y + \sin y}{\cos y}\,dy = 2\int \frac{\sin y}{\cos y}\,dy = -2\ln(\cos y)$$

$$\Rightarrow \mu = \frac{1}{\cos^2 y}$$

$$\frac{1}{\cos y}\,dx + \left(\frac{x\sin y}{\cos^2 y} - \frac{2}{\cos^2 y}\right)dy = 0 \qquad \text{Tam D.D}$$

$$\frac{\partial f}{\partial x} = \frac{1}{\cos y} \quad ; \quad \frac{\partial f}{\partial y} = \frac{x\sin y}{\cos^2 y} - \frac{2}{\cos^2 y}$$

$$f(x,y)=\frac{x}{\cos y}+h(y)$$

$$\frac{\partial f}{\partial y}=\cancel{\frac{+x\sin y}{\cos^2 y}}+\frac{dh}{dy}=\cancel{\frac{x\sin y}{\cos^2 y}}-\frac{2}{\cos^2 y}$$

$$\Rightarrow dh=-2\int\frac{dy}{\cos^2 y}=-2\int\sec^2 y\,dy=-2\tan y+c$$

$$f(x,y)=\frac{x}{\cos y}-2\tan y+c=k$$

10) $y^2dx+x(x-y)dy=0$ diferansiyel denkleminin $x=-1$ için $y=1$ koşuluna uyan özel çözümünü bulunuz.

$$\left.\begin{aligned} y&=ux \\ dy&=udx+xdu \end{aligned}\right\}\quad u^2x^2dx+x(x-ux)(udx+xdu)=0$$

$$(\cancel{u^2x^2}+ux^2-\cancel{u^2x^2})dx+x^3(1-u)du=0$$

$$\frac{ux^2dx}{u.x^3}+\frac{x^3(1-u)du}{u.x^3}=0$$

$$\int\frac{dx}{x}+\int\left(\frac{1-u}{u}\right)du=\int 0$$

$$\ln x+\ln u-u=\ln c\Rightarrow\frac{ux}{c}=e^u\Rightarrow\frac{y}{x}\cdot\frac{x}{c}=e^{y/x}$$

$$y=ce^{y/x}$$

$x=-1,\ y=1$ için $\quad 1=c.e^{-1}\Rightarrow c=e$

$$\boxed{y=e^{\frac{y}{x}+1}}$$

Çalışma Soruları 2

1) $x^2y+(y-x^3)y'=0$ diferansiyel denklemini $\mu=\mu(y)$ şeklinde y'ye bağlı bir integrasyon çarpanı bularak çözünüz.

$\mu=\mu(y)$ olsun.

$$\underbrace{(x^2y)}_{m}dx+\underbrace{(y-x^3)}_{N}dy=0$$

$$\frac{\partial m}{\partial y}=x^2, \quad \frac{\partial N}{\partial x}=-3x^2$$

$$\ln\mu=\int\frac{N_x-m_y}{m}dy=\int\frac{-3x^2-x^2}{x^2y}dy=\int\frac{-4dy}{y} \Rightarrow \mu=\frac{1}{y^4}$$

$$\frac{x^2}{y^3}dx+\left(\frac{1}{y^3}-\frac{x^3}{y^4}\right)dy=0$$

$$\left.\begin{aligned}\frac{\partial f}{\partial x}&=\frac{x^2}{y^3}\\ \frac{\partial f}{\partial y}&=\boxed{\frac{1}{y^3}-\frac{x^3}{y^4}}\end{aligned}\right\} \quad \begin{aligned}&f(x,y)=\frac{x^3}{3y^3}+h(y)\\ &\frac{\partial f}{\partial y}=-\cancel{\frac{x^3}{y^4}}+\frac{dh}{dy}=\frac{1}{y^3}-\cancel{\frac{x^3}{y^4}}\end{aligned}$$

$$\frac{dh}{dy}=\frac{1}{y^3} \Rightarrow \int dh=\int\frac{dy}{y^3} \Rightarrow h=-\frac{1}{y^2}+c$$

$$f(x,y)=\frac{x^3}{3y^3}-\frac{1}{y^2}+c=K$$

2) $y'\sec x + y^2 = y$ diferansiyel denklemini çözünüz.

$y'\sec x - y = -y^2$ Bernoulli D.D

$$\left.\begin{aligned} y^{1-2} &= y^{-1} = u \\ -y^{-2}y' &= u' \end{aligned}\right\} \quad \begin{aligned} &\underbrace{y'y^{-2}}\sec x - y^{-1} = -1 \\ &-u'\sec x - u = -1 \end{aligned}$$

$$u' + \frac{u}{\sec x} = \frac{1}{\sec x} \Rightarrow \boxed{u' + u\cos x = \cos x}$$

Lineer D.D

$$\lambda(x) = e^{\int \cos x\, dx} = e^{\sin x}$$

$$u(x) = \frac{1}{e^{\sin x}}\Big[\underbrace{\int e^{\sin x}\cdot \cos x\, dx}\, + c\Big]$$

$$\left.\begin{aligned} \sin x &= t \\ \cos x\, dx &= dt \end{aligned}\right\} \int e^t dt = e^t = e^{\sin x}$$

$$u(x) = e^{-\sin x}\left[e^{\sin x} + c\right] = 1 + ce^{-\sin x}$$

$$y(x) = \frac{1}{u} = \frac{1}{1 + ce^{-\sin x}}$$

3) Özel bir çözümü $y_1 = x$ olan $y' = \frac{y^2}{x^2} - \frac{y}{x} + 1$ diferansiyel denklemini çözünüz.

$$\left.\begin{aligned} y &= y_1 + \frac{1}{u} = x + \frac{1}{u} \\ y' &= 1 - \frac{u'}{u^2} \end{aligned}\right\} \quad \begin{aligned} &\cancel{1} - \frac{u'}{u^2} = \left(\cancel{1} + \frac{2}{ux} + \frac{1}{u^2x^2}\right) - \left(\cancel{1} + \frac{1}{ux}\right) + \cancel{1} \\ &-\frac{u'}{u^2} = \frac{1}{ux} + \frac{1}{u^2x^2} \Rightarrow u' = -\frac{u}{x} - \frac{1}{x^2} \end{aligned}$$

$u' + \frac{u}{x} = -\frac{1}{x^2}$ Lineer D.D

$$\lambda(x) = e^{\int \frac{dx}{x}} = e^{\ln x} = x \quad \left\{\begin{aligned} u(x) &= \frac{1}{x}\left[\int x\cdot\left(-\frac{1}{x^2}\right)dx + c\right] \\ u(x) &= \frac{1}{x}\left[-\ln x + c\right] \end{aligned}\right.$$

$$y(x) = x + \frac{x}{(c - \ln x)}$$

4) $y = xy' + y' - (y')^2$ d.d çözünüz.

$y' = p \Rightarrow y = xp + p - p^2$

$$\underbrace{y'}_{\cancel{p}} = \cancel{p} + xp' + p' - 2pp'$$

$p'\,[x + 1 - 2p] = 0$

a. $p' = 0 \Rightarrow p = c \Rightarrow y' = c \Rightarrow y = cx + g(c)$ Genel çözüm

b. $\boxed{x = 2p - 1}$

$$y = xp + p - p^2 = (2p-1).p + p - p^2 = 2p^2 - p + p - p^2 = \boxed{p^2}$$

$y = \left(\frac{x+1}{2}\right)^2$ Tekil çözüm

5) $y + xy' = (y')^4$ d.d çözünüz.

$y' = p \Rightarrow y + xp = p^4$

$$\underbrace{y'}_{p} + p + xp' = 4p^3p'$$

$2p = p'(4p^3 - x) \Rightarrow \frac{dx}{dp}.2p = \cancel{\frac{dp}{dx}}(4p^3 - x).\cancel{\frac{dx}{dp}}$

$2p\frac{dx}{dp} = 4p^3 - x \Rightarrow 2p\frac{dx}{dp} + x = 4p^3 \Rightarrow \frac{dx}{dp} + \frac{x}{2p} = 2p^2$ Lineer d.d

$\lambda(p) = e^{\int \frac{dp}{2p}} = e^{\frac{1}{2}\ln p} \Rightarrow \lambda = \sqrt{p}$

$x = \frac{1}{\sqrt{p}}\left[\int \sqrt{p}\,.\,2p^2 dp + c\right] = \frac{1}{\sqrt{p}}\left[\frac{4}{5}p^2\sqrt{p} + c\right]$ Çözümün parametrik denklemleri

$y + xp = p^4 \Rightarrow y = p^4 - p\left(\frac{1}{\sqrt{p}}\left[\frac{4}{5}p^2\sqrt{p} + c\right]\right) = p^4 - \frac{4}{5}p^3 - c\sqrt{p}$

6. $\dfrac{y'}{x} + \dfrac{2}{x^2} y = -x(\sec^2 x)y^2$ d.d çözünüz.

$$\left.\begin{aligned} y^{1-2} &= y^{-1} = u \\ -y^{-2}y' &= u' \end{aligned}\right\} \quad \frac{y'y^{-2}}{x} + \frac{2y^{-1}}{x^2} = -x(\sec^2 x)$$

$$-\frac{u'}{x} + \frac{2u}{x^2} = -x(\sec^2 x) \quad \text{Lineer D.D}$$

$$u' - \frac{2u}{x} = x^2 \sec^2 x$$

$$\lambda = e^{\int -\frac{2dx}{x}} = e^{-2\ln x} = \frac{1}{x^2}$$

$$u = x^2 \left[\int \frac{1}{\cancel{x^2}} \cancel{x^2} \sec^2 x \, dx + c \right] = x^2 (\tan x + c)$$

$$y = \frac{1}{u} = \frac{1}{x^2(\tan x + c)}$$

7. $\dfrac{y}{x} + y'(2y + \ln x) = -x^2$ d.d çözünüz.

$$\underbrace{\left(\frac{y}{x} + x^2\right)}_{m} dx + \underbrace{(2y + \ln x)}_{N} dy = 0$$

$$\frac{\partial m}{\partial y} = \frac{1}{x} = \frac{\partial N}{\partial x} = \frac{1}{x} \quad \text{Tam D.D}$$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \frac{y}{x} + x^2 \\ \frac{\partial f}{\partial y} &= 2y + \ln x \end{aligned}\right\} \quad \begin{aligned} & f(x,y) = y\ln x + \frac{x^3}{3} + h(y) \\ & \frac{\partial f}{\partial y} = \ln x + \frac{dh}{dy} = 2y + \ln x \end{aligned}$$

$$\frac{dh}{dy} = 2y \Rightarrow h = y^2 + c$$

$$f(x,y) = y\ln x + \frac{x^3}{3} + y^2 + c = k$$

8) $\left(x+y\ln\left(\frac{x}{y}\right)\right)dx+x\ln\left(\frac{y}{x}\right)dy=0$ d.d çözünüz.

$$\left.\begin{array}{l} y=ux \\ dy=udx+xdu \end{array}\right\} \left(x+ux\ln\left(\frac{1}{u}\right)\right)dx+x\ln u\left(udx+xdu\right)=0$$

$$\left(x+ux\underbrace{\ln\left(\frac{1}{u}\right)}_{\ln u^{-1}}+ux\ln u\right)dx+x^2\ln u\,du=0$$

$$\left(x-\cancel{ux\ln u}+\cancel{ux\ln u}\right)dx+x^2\ln u\,du=0$$

$$\frac{x\,dx}{x^2}+\frac{x^2\ln u\,du}{x^2}=\frac{0}{x^2} \Rightarrow \int\frac{dx}{x}+\underbrace{\int\ln u\,du}_{I}=\int 0$$

$I: \int \ln u\,du$

$$\left.\begin{array}{ll} \ln u=\alpha & du=d\beta \\ \frac{du}{u}=d\alpha & u=\beta \end{array}\right\} I=\alpha\beta-\int\beta\,d\alpha=u\ln u-\int u\frac{du}{u}=u\ln u-u$$

$$\ln x+u\ln u-u=\ln c \Rightarrow \ln x+\frac{y}{x}\ln\left(\frac{y}{x}\right)-\frac{y}{x}=c$$

9. $y=c_1x+c_2x\ln x$ eğri ailesine ait olan d.d bulunuz.

$$\left.\begin{array}{l} y=c_1x+c_2x\ln x \\ y'=c_1+c_2\ln x+c_2 \\ y''=\frac{c_2}{x} \end{array}\right\} \begin{array}{l} \boxed{c_2=xy''} \\ xy'-y=c_2x \Rightarrow \boxed{c_2=y'-\frac{y}{x}} \end{array}$$

$$xy''=y'-\frac{y}{x} \Rightarrow x^2y''-xy'+y=0$$

10. $y=-x(y')^2+\ln y'$ d.d çözünüz.

$y'=p \Rightarrow y=-xp^2+\ln p$

$$\underbrace{y'}_{p}=-p^2-2xpp'+\frac{p'}{p}$$

$$p^2+p=p'\left(\frac{1}{p}-2xp\right) \Rightarrow \frac{dx}{dp}(p^2+p)=\cancel{\frac{dp}{dx}}\left(\frac{1}{p}-2xp\right)\cdot\cancel{\frac{dx}{dp}}$$

$$(p^2+p)\frac{dx}{dp}+2xp=\frac{1}{p} \quad \text{Lineer d.d}$$

$$\frac{dx}{dp}+\frac{2xp}{p(p+1)}=\frac{1}{p^2(1+p)}$$

$$\lambda(p)=e^{\int\frac{2p\,dp}{p(p+1)}}=e^{\int\frac{2dp}{p+1}}=e^{2\ln(p+1)}=(p+1)^2$$

$$x(p)=\frac{1}{(p+1)^2}\left[\int \cancel{(p+1)^2}\cdot\frac{1}{p^2\cancel{(1+p)}}dp+c\right]=\frac{1}{(p+1)^2}\left[\ln p-\frac{1}{p}+c\right]$$

$$y(p)=-xp^2+\ln p=\frac{-p^2}{(p+1)^2}\left[\ln p-\frac{1}{p}+c\right]+\ln p$$