Örnek

$$y' + 2\frac{y}{x} = 12\sqrt{y}\,x^4 \; ; \; y(0) = 4$$

$$\frac{y'}{\sqrt{y}} + \frac{2}{x}\sqrt{y} = 12x^4$$

$$z = \sqrt{y}$$

$$n = \frac{1}{2}$$

$$z = \frac{1}{y^{\frac{1}{2}-1}} = \frac{1}{y^{-1/2}} = y^{1/2}$$

$$z = \sqrt{y}$$

$$z' = \frac{y'}{2\sqrt{y}} \Rightarrow 2z' = \frac{y'}{\sqrt{y}}$$

$$2z' + \frac{2}{x}z = 12x^4 \quad \text{Lineer dif.}$$

$$z' + \frac{1}{x}z = 6x^4 \qquad \ln\lambda = \int \frac{1}{x}\,dx$$

$$\ln\lambda = \ln x \Rightarrow \boxed{\lambda = x}$$

$$\underbrace{xz' + z}_{\frac{d}{dx}(x\cdot z)} = 6x^5$$

$$\frac{d}{dx}(x \cdot z) = 6x^5$$

$\int d(x.z) = \int 6x^5 dx$

$x.z = x^6 + C$ Genel çözüm

$z = x^5 + \frac{C}{x} \longrightarrow \sqrt{y} = x^5 + \frac{C}{x}$

$y(0) = 4 \Leftrightarrow x = 0 \quad y = 4 \Big| \; x\sqrt{y} = x^6 + C$

$0\sqrt{4} = 0^6 + C \Rightarrow C = 0$

$x\sqrt{y} = x^6 \Rightarrow \sqrt{y} = x^5$

$\boxed{y = x^{10}}$ özel çözüm

Örnek

$y' - y = y^2 e^{-x} \sin x \qquad \boxed{y' + p(x)y = q(x)y^n}$

$\uparrow n = 2 \qquad z = \frac{1}{y^{2-1}} = \frac{1}{y}$

$\frac{y'}{y^2} - \frac{1}{y} = e^{-x} \sin x$

$z = \frac{1}{y} \Rightarrow z' = \frac{-y'}{y^2}$

$-z' - z = e^{-x} \sin x$

$z' + 1z = -e^{-x} \sin x$ lineer dif. denk.

$\ln \lambda = \int 1\, dx = \int dx = x$

$\lambda = e^x$

$e^x z' + e^x z = e^x \cdot (-e^{-x} \sin x)$

$$\underbrace{e^x z' + e^x z}_{\frac{d}{dx}(e^x z)} = -\sin x$$

$$\frac{d}{dx}(e^x z) = -\sin x \Rightarrow \int d(e^x z) = \int -\sin x\, dx$$

$$e^x z = \cos x + C$$

$$z = e^{-x}\cos x + Ce^{-x}$$

$$\frac{1}{y} = \underbrace{e^{-x}\cos x + Ce^{-x}}_{e^{-x}(\cos x + C)}$$

$$\boxed{y = \frac{e^x}{\cos x + C}}$$

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b) Riccati Dif. Denklemi:

$y_1(x)$ bir özel çözüm olmak üzere,

$$y' + p(x)y + q(x)y^2 + r(x) = 0$$

Riccati dif. denklemi

$u = u(x)$ olmak üzere

1°) $y = y_1 + u$ dönüşümü $\longrightarrow$ Bernoulli dif.

2°) $y = y_1 + \frac{1}{u}$ dönüşümü $\longrightarrow$ $\boxed{\text{Lineer dif.}}$

Örnek $y' = -y^2 + 2x^2 y + 2x - x^4$ dif. denklemine ait bir özel çözüm $y_1 = x^2$ old. göre dif. denklemin genel çözümünü bulunuz.

$y = x^2 + \frac{1}{u} \rightarrow y' = 2x - \frac{u'}{u^2}$ $\boxed{u' + p(x)u = q(x)}$

$\cancel{2x} - \frac{u'}{u^2} = -\left(x^2 + \frac{1}{u}\right)^2 + 2x^2\left(x^2 + \frac{1}{u}\right) + \cancel{2x} - x^4$

$-\frac{u'}{u^2} = \cancel{-x^4} - \cancel{\frac{2x^2}{u}} - \frac{1}{u^2} + \cancel{2x^4} + \cancel{\frac{2x^2}{u}} - \cancel{x^4}$

$-u^2 / \left(-\frac{u'}{u^2} = -\frac{1}{u^2}\right)$

$u' = 1 \rightarrow$ Lineer dif.

$\frac{du}{dx} = 1 \rightarrow \int du = \int dx$

$u(x) = x + C$

$\boxed{y = x^2 + \frac{1}{x+C}}$ Genel Çözüm

Örnek $(1+x^3)y' - x^2y + y^2 + 2x = 0$ dif. denkleminin bir özel çözümü $y_1 = -x$ old. göre dif denklemin genel çözümü?

$y = -x^2 + \frac{1}{u} \rightarrow y' = -2x - \frac{u'}{u^2}$

$(1+x^3)\left(-2x - \frac{u'}{u^2}\right) - x^2\left(-x^2 + \frac{1}{u}\right) + \left(-x^2 + \frac{1}{u}\right)^2 + 2x = 0$

$-\cancel{2x} - \frac{u'}{u^2} - \cancel{2x^4} - \frac{u'}{u^2}x^3 + \cancel{x^4} - \frac{x^2}{u} + \cancel{x^4} - \frac{2x^2}{u} + \frac{1}{u^2} + \cancel{2x} = 0$

$-u^2 / -\frac{u'}{u^2}(1+x^3) - \frac{3x^2}{u} + \frac{1}{u^2} = 0$

$(1+x^3)\underbrace{u'} + 3x^2\underbrace{u} - 1 = 0$ lineer drf.

$* \quad u' + \frac{3x^2}{1+x^3} u = \frac{1}{1+x^3}$ lineer drf.

$\ln \lambda = \int \frac{3x^2}{\underbrace{1+x^3}} dx = \ln(1+x^3)$

$\boxed{\lambda = 1+x^3}$

$\underbrace{(1+x^3)u' + 3x^2 u} = 1$

$\frac{d}{dx}\left[(1+x^3)u\right] = 1 \rightarrow \int d\left[(1+x^3)u\right] = \int 1\,dx$

$(1+x^3)u = x + C$

$u = \frac{x+C}{1+x^3}$

$y = -x^2 + \frac{1}{u}$

$\boxed{y = -x^2 + \frac{1+x^3}{x+C}}$ Genel Çözüm

örnek $y' - \frac{y}{x} + 9x^2 = y^2 \qquad y_1 = 3x$

$y = 3x + \frac{1}{u} \qquad y' = 3 - \frac{u'}{u^2}$

$$3-\frac{u'}{u^2}-\frac{1}{x}\cdot\left(3x+\frac{1}{u}\right)+9x^2=\left(3x+\frac{1}{u}\right)^2$$

$$\cancel{3}-\frac{u'}{u^2}-\cancel{3}-\frac{1}{ux}+\cancel{9x^2}=\cancel{9x^2}+\frac{6x}{u}+\frac{1}{u^2}$$

$$-u^2/-\frac{u'}{u^2}-\frac{1}{ux}=\frac{6x}{u}+\frac{1}{u^2}$$

$$u'+\frac{u}{x}=-6xu-1 \qquad u'+p(x)u=q(x)$$

$$u'+\underbrace{\left(6x+\frac{1}{x}\right)}u=-1 \quad \text{Lineer dif.}$$

$$\ln\lambda=\int\left(6x+\frac{1}{x}\right)dx$$

$$\ln\lambda=3x^2+\ln x \rightarrow \ln\lambda-\ln x=3x^2$$

$$\lambda=e^{3x^2+\ln x} \qquad \ln\frac{\lambda}{x}=3x^2$$

$$\lambda=e^{3x^2}e^{\ln x} \qquad \frac{\lambda}{x}=e^{3x^2}\rightarrow\lambda=xe^{3x^2}$$

$$\lambda=xe^{3x^2}$$

$$\underbrace{xe^{3x^2}u'+xe^{3x^2}\left(6x+\frac{1}{x}\right)u}=-xe^{3x^2}$$

$$\frac{d}{dx}\left(xe^{3x^2}\cdot u\right)=-xe^{3x^2}$$

⋮ ?

SORU $y' + y^2 + \frac{1}{x}y - \frac{4}{x^2} = 0 \quad y_1 = \frac{2}{x}$

Genel Çözüm

Cevap $y = \frac{2}{x} + \frac{1}{cx^5 - \frac{x}{4}}$

$y' + p(x)y = q(x)y^n \quad n \neq 0,\ n \neq 1$

Bernoulli Dif.

$y' + p(x)y + q(x)y^2 + r(x) = 0 \quad y_1 *$

Riccati dif.
