Lineer Dif. Denklem

$a_0(x)y' + a_1(x)y = f(x)$ 1. mertebeden lineer dif.

$(a_0(x) \neq 0)$

$$y' + \underbrace{\frac{a_1(x)}{a_0(x)}}_{p(x)} y = \underbrace{\frac{f(x)}{a_0(x)}}_{q(x)} \rightarrow \underbrace{y' + p(x)y = q(x)}$$

$$* \boxed{1.y' + p(x)y = q(x)} *$$

$\frac{dy}{dx}$

$$\underbrace{[p(x)y - q(x)]}_{M}dx + \underbrace{1}_{N}dy = 0 \quad *$$

$$\frac{\partial M}{\partial y} = p(x) \qquad \frac{\partial N}{\partial x} = 0 \qquad \text{Tam dif. değil}$$

$$\boxed{\ln \lambda} = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} dx = \int p(x)\,dx$$

$$\boxed{\lambda = e^{\int p(x)dx}}$$ Lineer dif. denklem için int. çarpanı

✓ $$\underbrace{e^{\int p(x)dx} \cdot y' + p(x)e^{\int p(x)dx} \cdot y} = q(x)e^{\int p(x)dx}$$

$$\boxed{\left(e^{\int p(x)dx}\right)' = p(x)e^{\int p(x)dx}}$$

$$\frac{d}{dx}\left(\underbrace{e^{\int p(x)dx} \cdot y}\right) = q(x)e^{\int p(x)dx}$$

$$\int d\left(e^{\int p(x)dx} \cdot y\right) = \int q(x)e^{\int p(x)dx} \cdot dx$$

$$e^{\int p(x)dx} \cdot y = \int q(x)e^{\int p(x)dx}dx + C$$ Genel Ç.

$$y = e^{-\int p(x)dx}\left[\int q(x)e^{\int p(x)dx}dx + C\right]$$

örnek $y' - \frac{1}{x}y = x^2$ dif. denklemini çözünüz.

$p(x) = -\frac{1}{x}$ $\quad q(x) = x^2$

$$\ln\lambda = \int p(x)dx = \int -\frac{1}{x}dx = -\ln x$$

$$\boxed{\lambda = \frac{1}{x}}$$

$$\underbrace{\frac{1}{x}y' - \frac{1}{x^2}y}_{} = x$$

$$\frac{d}{dx}\left(\frac{1}{x}y\right) = x \rightarrow \int d\left(\frac{1}{x}y\right) = \int x\,dx$$

$$\frac{1}{x}y = \frac{x^2}{2} + C$$

$$\boxed{y = \frac{x^3}{2} + Cx}$$

örnek $y' = y\cot x + \sin x$

$$y' - y\cot x = \sin x$$

$$\ln\lambda = \int -\cot x\,dx = -\int \frac{\cos x}{\sin x}dx = -\ln(\sin x)$$

$$\left(\frac{1}{\sin x}\right)' = \frac{-\cos x}{\sin^2 x}$$

$$\lambda = \frac{1}{\sin x}$$

$$\underbrace{\frac{1}{\sin x}y' - \frac{1}{\sin x}\frac{\cos x}{\sin x}y}_{} = 1$$

$$\frac{d}{dx}\left(\frac{1}{\sin x}\cdot y\right) = 1 \rightarrow \int d\left(\frac{1}{\sin x}\cdot y\right) = \int 1\cdot dx$$

$$\frac{1}{\sin x}\cdot y = x + C$$

$$\boxed{y = x\sin x + C\sin x}$$

örnek $y' + y\sin x = x e^{\cos x}$

$$\ln\lambda = \int \sin x \, dx = -\cos x \Rightarrow \lambda = e^{-\cos x}$$

$$\underbrace{e^{-\cos x} y' + \sin x \, e^{-\cos x} y} = x \underbrace{e^{\cos x} e^{-\cos x}}_{1}$$

$$\frac{d}{dx}\left(e^{-\cos x} \cdot y\right) = x \qquad (e^u)' = u' e^u$$

$$\int d\left(e^{-\cos x} \cdot y\right) = \int x \, dx$$

$$\boxed{e^{-\cos x} y = \frac{x^2}{2} + C} \rightarrow \boxed{y = \frac{x^2}{2} e^{\cos x} + C e^{\cos x}}$$

örnek $x\,dy + y\,dx = e^x dx \; ; \; y(0) = 1$ 2. yol

$$\boxed{(y - e^x)\,dx + x\,dy = 0}$$

$$x\frac{dy}{dx} + y = e^x$$

Lineer dif.

$$y' + p(x) y = q(x)$$

$$xy' + y = e^x$$

$$* \quad y' + \frac{y}{x} = \frac{e^x}{x}$$

$$\ln\lambda = \int \frac{1}{x} dx = \ln x \rightarrow \lambda = x$$

$$\underbrace{xy' + y} = e^x$$

$$\frac{d}{dx}(xy) = e^x \rightarrow \int d(xy) = \int e^x dx$$

$$\checkmark \quad xy = e^x + C$$

$$* \quad y = \cancel{\frac{e^x}{x}} + \frac{C}{x}$$

$\boxed{y(0)=1}$ $\quad x=0 \quad y=1$

$$0\cdot 1 = e^0 + C \rightarrow \underbrace{0\cdot 1}_{0} = 1 + C \rightarrow C=-1$$

$$xy = e^x - 1 \rightarrow y = \frac{e^x-1}{x} \quad \text{özel çözüm}$$

$$\left(\lim_{x\to 0} \frac{e^x-1}{x} \overset{\frac{0}{0}}{=} \lim_{x\to 0} \frac{e^x}{1} = 1 \right)$$

Lineer Dif. Denkleme Dönüştürülebilen Dif. Denklemler

1) Bernoulli Dif. Denk: $p(x), q(x)$ tanımlı ve sürekli

$n\neq 0$ ve $n\neq 1$ olmak üzere

$n\in\mathbb{R}$ $\qquad y' + p(x)y = q(x)y^n$ $\qquad \left(\begin{array}{ll} y'+p(x)y = q(x) & n=0 \\ y'+p(x)y = q(x)y & n=1 \end{array} \right)$

$$y' + p(x)y = q(x)y^n$$

$$\left(\frac{y'}{y^n}\right) + p(x)\overbrace{\frac{1}{y^{n-1}}}^{z} = q(x) \qquad z = \frac{1}{y^{n-1}}$$

Değişken değiştirme uygulanacak

$$y' + p(x)y = q(x)y^n \rightarrow z = \frac{1}{y^{n-1}} \rightarrow z' + p_1(x)z = q_1(x)$$

Lineer dif.

Örnek: $y' + y = xy^3$ dif. denk. çözünüz.

$n=3$ $\qquad z = \frac{1}{y^{n-1}} = \frac{1}{y^2}$

$$\frac{y'}{y^3} + \frac{y}{y^3} = x \rightarrow \boxed{\frac{y'}{y^3} + 1\cdot\frac{1}{y^2} = x}$$

$$z = \frac{1}{y^2} \rightarrow z' = -2\frac{y'}{y^3} \rightarrow \frac{y'}{y^3} = -\frac{z'}{2}$$

y^{-2} $\quad -2y^{-3}y'$ $\qquad -\frac{z'}{2} + z = x$

$z' - 2z = -2x$ Lineerdif.

$$\ln\lambda = \int -2\,dx = -2x \rightarrow \lambda = e^{-2x}$$

$$\underbrace{e^{-2x}z' - 2e^{-2x}z}_{} = -2xe^{-2x}$$

$$\frac{d}{dx}\left(e^{-2x}\cdot z\right) = -2xe^{-2x} \rightarrow \int d\left(e^{-2x}\cdot z\right) = \int -2xe^{-2x}\,dx$$

$uv - \int v\,du$

$$\int xe^{-2x}\,dx = -\frac{x}{2}e^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$x = u \quad e^{-2x}dx = dv$

$dx = du \quad -\frac{1}{2}e^{-2x} = v$

$$= -\frac{x}{2}e^{-2x} - \frac{1}{4}e^{-2x}$$

$$e^{-2x}\cdot z = xe^{-2x} + \frac{1}{2}e^{-2x} + C$$

$$z = \boxed{x + \frac{1}{2} + Ce^{2x} = \frac{1}{y^2}}$$

SORU $\quad y' + 2\frac{y}{x} = 12\sqrt{y}\,x^4 \; ; \; y(0) = 4$

Cevap: $\quad y = x^{10}$