SORU $(3x-y+1)dx+(x-3y+2)dy=0$

$\underbrace{3x-y \quad x-3y}_{\text{lineer bağımsız}}$ $\quad x=x_1+h \quad y=y_1+k \quad dx=dx_1 \quad dy=dy_1$

$$[3(x_1+h)-(y_1+k)+1]dx+(x_1+h-3(y_1+k)+2)dy_1=0$$

$$[3x_1-y_1+3h-k+1]dx_1+[x_1-3y_1+h-3k+2]dy_1=0$$

$$\left.\begin{array}{l}3h-k+1=0\\ h-3k+2=0\end{array}\right\} \quad h=-\frac{1}{8} \quad k=\frac{5}{8}$$

$(3x_1-y_1)dx_1+(x_1-3y_1)dy_1=0$ Homojen dif.

$$\frac{dy_1}{dx_1}=\frac{y_1-3x_1}{x_1-3y_1} \qquad \frac{y_1}{x_1}=u \rightarrow y_1=ux_1$$

$$\frac{dy_1}{dx_1}=\frac{du}{dx_1}x_1+u$$

$$\frac{du}{dx_1}x_1+u=\frac{ux_1-3x_1}{x_1-3ux_1}$$

$$\frac{du}{dx_1}x_1+u=\frac{u-3}{1-3u}$$

$$\frac{du}{dx_1}x_1=\frac{u-3}{1-3u}-u$$

$$\frac{du}{dx_1}x_1=\frac{\cancel{u}-3-\cancel{u}+3u^2}{1-3u}$$

$$\frac{du}{dx_1}x_1=\frac{3(u^2-1)}{1-3u}$$

$$\int\frac{1-3u}{u^2-1}du=\int 3\frac{dx_1}{x_1}$$

$$\int\left(\frac{1}{u^2-1}-\frac{3u}{1-u^2}\right)du=3\int\frac{dx_1}{x_1}$$

$$\frac{1}{2}\int\left(\frac{1}{u-1}-\frac{1}{u+1}\right)du-3\int\frac{u}{1-u^2}du=3\int\frac{dx_1}{x_1}$$

$$\frac{1}{2}\left[\ln(u-1)-\ln(u+1)\right]+\frac{3}{2}\ln(1-u^2)=3\ln x_1+\ln C$$

$$\frac{1}{2}\ln\left(\frac{u-1}{u+1}\right)+\frac{3}{2}\ln(1-u^2)=\ln Cx_1^3$$

$$\ln\left[(1-u^2)^{3/2}\left(\frac{u-1}{u+1}\right)^{1/2}\right]=\ln Cx_1^3$$

$$\frac{1}{u^2-1}=\frac{A}{u-1}+\frac{B}{u+1} \quad A=\frac{1}{2},\ B=-\frac{1}{2}$$

Basit kesirlere ayırma

$$\cancel{\ln}\left[\left(1-\left(\frac{y_1}{x_1}\right)^2\right)^{3/2}\left(\frac{\frac{y_1}{x_1}-1}{\frac{y_1}{x_1}+1}\right)^{1/2}\right]=\cancel{\ln}Cx_1^3$$

$x_1=x+\frac{1}{8}$ ; $y=y-\frac{5}{8}$ $\Rightarrow$ $\boxed{\left[1-\left(\frac{y-\frac{5}{8}}{x+\frac{1}{8}}\right)^2\right]^{3/2}\left(\frac{y-\frac{5}{8}-x-\frac{1}{8}}{y-\frac{5}{8}+x+\frac{1}{8}}\right)^{1/2}=C\left(x+\frac{1}{8}\right)^3}$

SORU $y' - \frac{y}{x} = \frac{x^2}{y^2}$

$y' = \frac{y}{x} + \frac{1}{\left(\frac{y}{x}\right)^2}$ Homojen drf. $\frac{y}{x} = u \rightarrow y = ux$

$y' = u'x + u$

$u'x + \cancel{u} = \cancel{u} + \frac{1}{u^2}$

$\frac{du}{dx} = \frac{1}{u^2} \rightarrow u^2 du = dx$

$\frac{u^3}{3} = x + C$

$\boxed{\frac{1}{3}\left(\frac{y}{x}\right)^3 = x + C}$ Genel Çözüm

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SORU $xy^2 dy = (x^3 + y^3) dx$

$\frac{dy}{dx} = \frac{x^3 + y^3}{xy^2}$ Homojen drf. $\frac{y}{x} = u \rightarrow y = ux$

$y' = u'x + u$

$u'x + u = \frac{x^3 + u^3 x^3}{u^2 x^3} \Rightarrow u'x + u = \frac{1 + u^3}{u^2}$

$\frac{du}{dx} x = \frac{1 + u^3}{u^2} - u$

$\frac{du}{dx} x = \frac{1 + \cancel{u^3} - \cancel{u^3}}{u^2} \Rightarrow u^2 du = \frac{dx}{x}$

$\frac{u^3}{3} = \ln x + C$

$\boxed{\frac{1}{3}\left(\frac{y}{x}\right)^3 = \ln x + C}$

Genel Çözüm

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SORU $y' = \frac{1 - x - y}{1 + x + y}$, $1 - (x+y)$

$x + y = t$

$1 + y' = t' \Rightarrow y' = t' - 1$

$t' - 1 = \frac{1 - t}{1 + t}$

$\frac{dt}{dx} = \frac{1 - t}{1 + t} + 1$

$\frac{dt}{dx} = \frac{1 - \cancel{t} + 1 + \cancel{t}}{1 + t} \Rightarrow$

$\frac{dt}{dx} = \frac{2}{1 + t}$

$\int (1 + t) dt = \int 2 dx$

$t + \frac{t^2}{2} = 2x + C$

$\boxed{(x + y) + \frac{(x + y)^2}{2} = 2x + C}$ Genel Çözüm

SORU

$$y' = \frac{x+y-4}{x-y-6}$$

$x+y \quad x-y$ : lineer bağımsız

$x = x_1 + h \quad dx = dx_1$

$y = y_1 + k \quad dy = dy_1$

$$\frac{dy_1}{dx_1} = \frac{x_1 + y_1 + \overbrace{h+k-4}^{=0}}{x_1 - y_1 + \underbrace{h-k-6}_{=0}}$$

$$\left.\begin{array}{l} h+k-4=0 \\ h-k-6=0 \end{array}\right\} \begin{array}{l} h=5 \\ k=-1 \end{array}$$

$$\frac{dy_1}{dx_1} = \frac{x_1+y_1}{x_1-y_1}$$

$$\frac{y_1}{x_1} = u \rightarrow y_1 = u x_1$$

$$\frac{dy_1}{dx_1} = \frac{du}{dx_1} x_1 + u$$

$$\frac{du}{dx_1} x_1 + u = \frac{x_1 + u x_1}{x_1 - u x_1}$$

$$\frac{du}{dx_1} x_1 = \frac{1+u}{1-u} - u \longrightarrow \frac{du}{dx_1} x_1 = \frac{1+\cancel{u}-\cancel{u}+u^2}{1-u}$$

$$\frac{du}{dx_1} x_1 = \frac{1+u^2}{1-u}$$

$$\int \frac{1-u}{1+u^2} du = \int \frac{dx_1}{x_1}$$

$$\int \left( \frac{1}{1+u^2} - \frac{u}{1+u^2} \right) du = \int \frac{dx_1}{x_1}$$

$$\arctan u - \frac{1}{2} \ln(1+u^2) = \ln x_1 + C$$

$$\arctan\left(\frac{y_1}{x_1}\right) - \frac{1}{2} \ln\left(1 + \left(\frac{y_1}{x_1}\right)^2\right) = \ln x_1 + C$$

$$\left.\begin{array}{l} x_1 = x-5 \\ y_1 = y+1 \end{array}\right\}$$

$$\boxed{\arctan\left(\frac{y+1}{x-5}\right) - \frac{1}{2} \ln\left(1 + \left(\frac{y+1}{x-5}\right)^2\right) = \ln x_1 + C}$$

Genel Çözüm.

SORU

$$4xy\,dx + (x^2+y^2+4)\,dy = 0$$

$$\frac{\partial M}{\partial y} = 4x \qquad \frac{\partial N}{\partial x} = 2x \quad \text{Tam dif. değil}$$

$$\lambda = \lambda(y)$$

$$\ln\lambda = \int \frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M}\,dy$$

$$\ln\lambda = \int \frac{2x-4x}{4xy}\,dy = \int \frac{-\cancel{2x}}{\cancel{4}_2\cancel{x}y}\,dy = -\frac{1}{2}\ln y$$

$$\lambda = y^{-1/2} = \frac{1}{y^{1/2}} = \frac{1}{\sqrt{y}}$$

$$\frac{1}{\sqrt{y}}4xy\,dx + \frac{1}{\sqrt{y}}(x^2+y^2+4)\,dy = 0$$

$$4x\sqrt{y}\,dx + \left(\frac{x^2}{\sqrt{y}} + y\sqrt{y} + \frac{4}{\sqrt{y}}\right)dy = 0$$

$$\frac{\partial M_1}{\partial y} = {}^2\cancel{4}x\frac{1}{\cancel{2}\sqrt{y}} = \frac{\partial N_1}{\partial x} \quad \text{Tam dif.}$$

$$u(x,y) = C$$

$$\frac{\partial u}{\partial x} = 4x\sqrt{y} \qquad \frac{\partial u}{\partial y} = \frac{x^2}{\sqrt{y}} + y\sqrt{y} + \frac{4}{\sqrt{y}}$$

$$u(x,y) = \int 4x\sqrt{y}\,dx$$

$$u(x,y) = 2x^2\sqrt{y} + h(y)$$

$$\downarrow$$

$$\frac{\partial u}{\partial y} = \cancel{2}x^2\frac{1}{\cancel{2}\sqrt{y}} + \frac{dh}{dy}$$

$$\cancel{\frac{x^2}{\sqrt{y}}} + \frac{dh}{dy} = \cancel{\frac{x^2}{\sqrt{y}}} + y^{3/2} + 4y^{1/2}$$

$$\int dh = \int (y^{3/2} + 4y^{1/2})\,dy$$

$$h(y) = 2\frac{y^{5/2}}{5} + 4.\frac{2y^{3/2}}{3}$$

$$u(x,y) = C$$

$$\boxed{2x^2\sqrt{y} + \frac{2}{5}\sqrt{y^5} + \frac{8}{3}\sqrt{y^3} = C} \quad \text{Genel Çözüm}$$

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SORU $(3x^2-y^2)dy - 2xy\,dx = 0$

1. Yol Homojen dif.

$$\frac{dy}{dx} = \frac{2xy}{3x^2-y^2}$$

$$\frac{y}{x} = u \rightarrow \begin{array}{l} y = ux \\ y' = u'x + u \end{array}$$

$$u'x + u = \frac{2ux^2}{3x^2-u^2x^2}$$

$$\frac{du}{dx}x = \frac{2u}{3-u^2} - u$$

$$\frac{du}{dx}x = \frac{\overset{-u}{\cancel{2u-3u}}+u^3}{3-u^2}$$

$$\int \frac{3-u^2}{u^3-u}\,du = \int \frac{dx}{x}$$

$$\int \frac{2+1-u^2}{u(u^2-1)}\,du$$

$$\int \left(\frac{2}{u(u^2-1)} - \frac{1}{u}\right)du$$

$$\frac{2}{u(u-1)(u+1)} = \frac{A}{u} + \frac{B}{u-1} + \frac{C}{u+1}$$

$$A = -2 \quad B = 1 \quad C = 1$$

$$\rightarrow \int \left(\frac{-2}{u} + \frac{1}{u+1} + \frac{1}{u-1}\right)du = \int \frac{dx}{x}$$

$$-2\ln u + \ln(u+1) + \ln(u-1) = \ln x + \ln c$$

$$\ln\frac{(u+1)(u-1)}{u^2} = \ln Cx$$

$$\frac{u^2-1}{u^2} = Cx \rightarrow 1 - \frac{1}{u^2} = Cx$$

$$u = \frac{y}{x} \Rightarrow \boxed{1 - \frac{x^2}{y^2} = Cx} \quad \text{Genel Ç.}$$

SORU

2. Yol

$$\overbrace{(3x^2-y^2)}^{N}dy\ \overbrace{-2xy}^{M}dx=0$$

$$\frac{\partial M}{\partial y}=-2x \neq \frac{\partial N}{\partial x}=6x$$

$$\lambda=\lambda(y)$$

$$\ln\lambda=\int\frac{\frac{\partial N}{\partial x}-\frac{\partial M}{\partial y}}{M}dy=\int\frac{6x-(-2x)}{-2xy}dy$$

$$\ln\lambda=\int\frac{\cancel{8}^{4}\cancel{x}}{-\cancel{2}\cancel{x}y}dy=-4\int\frac{dy}{y}$$

$$\ln\lambda=-4\ln y \Rightarrow \lambda=\frac{1}{y^4}$$

$$\frac{1}{y^4}(3x^2-y^2)dy-2xy\frac{1}{y^4}dx=0$$

$$-2\frac{x}{y^3}dx+\left(\frac{3x^2}{y^4}-\frac{1}{y^2}\right)dy=0$$

$$\frac{\partial M_1}{\partial y}=6\frac{x}{y^4}=\frac{\partial N_1}{\partial x}\quad \text{Tam dif.}$$

$$u(x,y)=C$$

$$\frac{\partial u}{\partial x}=-\frac{2x}{y^3}$$

$$u(x,y)=\int-\frac{2x}{y^3}dx$$

$$u(x,y)=-\frac{x^2}{y^3}+h(y)$$

$$\downarrow$$

$$\frac{\partial u}{\partial y}=+\frac{3x^2}{y^4}+\frac{dh}{dy}$$

$$\frac{\partial u}{\partial y}=\frac{3x^2}{y^4}-\frac{1}{y^2}$$

$$\cancel{\frac{3x^2}{y^4}}+\frac{dh}{dy}=\cancel{\frac{3x^2}{y^4}}-\frac{1}{y^2}$$

$$\int dh=\int\frac{1}{y^2}dy$$

$$h(y)=-\frac{1}{y}$$

$$u(x,y)=C$$

$$\boxed{-\frac{x^2}{y^3}-\frac{1}{y}=C}\quad \text{Genel Çözüm}$$

SORU

$$(y+x^3e^x)dx-xdy=0$$

$$\frac{\partial M}{\partial y}=1\neq\frac{\partial N}{\partial x}=-1$$

$$\lambda=\lambda(x)$$

$$\ln\lambda=\int\frac{\frac{\partial M}{\partial y}-\frac{\partial N}{\partial x}}{N}dx=\int\frac{\overbrace{1-(-1)}^{2}}{-x}dx$$

$$\ln\lambda=-2\ln x\rightarrow\lambda=\frac{1}{x^2}$$

$$\frac{1}{x^2}(y+x^3e^x)dx-\frac{1}{x^2}xdy=0$$

$$\underbrace{\left(\frac{y}{x^2}+xe^x\right)}dx\ \underbrace{-\frac{1}{x}}dy=0$$

$$\frac{\partial M_1}{\partial y}=\frac{1}{x^2}=\frac{\partial N_1}{\partial x}$$

$$\frac{\partial u}{\partial x}=\frac{y}{x^2}+xe^x$$

$$\cancel{\frac{y}{x^2}}+\frac{dk}{dx}=\cancel{\frac{y}{x^2}}+xe^x$$

$$\int dk=\int xe^x dx$$

$$k(x)=e^x(x-1)$$

$$\frac{\partial u}{\partial y}=-\frac{1}{x}$$

$$u(x,y)=\int-\frac{1}{x}dy$$

$$u(x,y)=-\frac{y}{x}+k(x)$$

$$\downarrow$$

$$\frac{\partial u}{\partial x}=\frac{y}{x^2}+\frac{dk}{dx}$$

$$u(x,y)=C$$

$$\boxed{\frac{y}{x^2}+e^x(x-1)=C}\quad \text{Genel Çözüm}$$

SORU $(x^2+y^2+2x)dx + 2y\,dy = 0$

$$\frac{\partial M}{\partial y} = 2y \neq \frac{\partial N}{\partial x} = 0$$

$\lambda = \lambda(x)$

$$\ln\lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}dx = \int \frac{2y-0}{2y}dx$$

$$\ln\lambda = \int dx = x \rightarrow \lambda = e^x$$

$$\underbrace{e^x(x^2+y^2+2x)dx}_{x^2e^x+y^2e^x+2xe^x} + 2ye^x dy = 0$$

$$\frac{\partial M_1}{\partial y} = 2ye^x = \frac{\partial N_1}{\partial x} \quad \text{Tam dif}$$

$$U(x,y) = C$$

$$\frac{\partial u}{\partial x} = x^2e^x + y^2e^x + 2xe^x$$

$$\cancel{y^2e^x} + \frac{dk}{dx} = x^2e^x + \cancel{y^2e^x} + 2xe^x$$

$$dk = (x^2e^x + 2xe^x)dx$$

$$\int dk = \int (x^2+2x)e^x dx$$

2 kere kısmi integrasyon yapılacak

$$k(x) = x^2e^x$$

$$\frac{\partial u}{\partial y} = 2ye^x$$

$$u(x,y) = \int 2ye^x dy$$

$$u(x,y) = y^2e^x + k(x)$$

$$\downarrow$$

$$\frac{\partial u}{\partial x} = y^2e^x + \frac{dk}{dx}$$

$$u(x,y) = C$$

$$\boxed{y^2e^x + x^2e^x = C} \quad \text{Genel çözüm}$$

SORU $(xy+x^2+1)dx + x^2dy = 0$

$$\frac{\partial M}{\partial y} = x \neq \frac{\partial N}{\partial x} = 2x$$

$\lambda = \lambda(x)$

$$\ln\lambda = \int \frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N}dx = \int \frac{x-2x}{x^2}dx = \int -\frac{1}{x}dx$$

$$\ln\lambda = -\ln x \rightarrow \lambda = \frac{1}{x}$$

$$\frac{1}{x}(xy+x^2+1)dx + \frac{1}{x}x^2dy = 0$$

$$(y+x+\tfrac{1}{x})dx + x\,dy = 0$$

$$\frac{\partial M_1}{\partial y} = 1 = \frac{\partial N_1}{\partial x} \quad \text{Tam dif.}$$

$$U(x,y) = C$$

$$\frac{\partial u}{\partial x} = y + x + \frac{1}{x}$$

$$\cancel{y} + \frac{dk}{dx} = \cancel{y} + x + \frac{1}{x}$$

$$\int dk = \int (x + \tfrac{1}{x})dx$$

$$k(x) = \frac{x^2}{2} + \ln x$$

$$\frac{\partial u}{\partial y} = x$$

$$u(x,y) = \int x\,dy$$

$$u(x,y) = xy + k(x)$$

$$\downarrow$$

$$\frac{\partial u}{\partial x} = y + \frac{dk}{dx}$$

$$U(x,y) = C$$

$$\boxed{xy + \frac{x^2}{2} + \ln x = C}$$

Örnek $\left(x\cos^2\left(\frac{y}{x}\right)-y\right)dx+x\,dy=0$ , $y(1)=\frac{\pi}{4}$ şartını sağlayan özel çözümü bulunuz.

Homojen dd. Önce genel çözüm bulunur.

Her iki tarafı x'e bölelim.

$$\left(\cos^2\left(\frac{y}{x}\right)-\frac{y}{x}\right)dx+dy=0$$

$\frac{y}{x}=u \quad y=xu \quad y'=x'u+x \quad dy=u\,dx+x\,du$

$$(\cos^2u-u)dx+u\,dx+x\,du=0$$

$$\cos^2u\,dx-\cancel{u\,dx}+\cancel{u\,dx}+x\,du=0$$

$$\cos^2u\,dx=-x\,du$$

$$\frac{dx}{x}=-\frac{du}{\cos^2u}$$

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$$\ln x=-\tan u+c$$

$\ln x=-\tan\left(\frac{y}{x}\right)+c$ Genel çözüm

$x=1$ , $y=\frac{\pi}{4}$ ise;

$$\ln 1=-\tan\left(\frac{\pi}{4}\right)+c$$

$$0=-1+c$$

$$c=1$$

$\ln x=-\tan\left(\frac{y}{x}\right)+1$ Özel çözüm