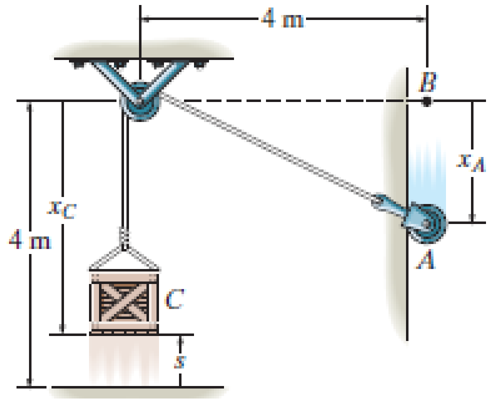
YILDIZ TEKNİK ÜNİVERSİTESİ
MAKİNE FAKÜLTESİ / MEKATRONİK MÜHENDİSLİĞİ BÖLÜMÜ

Ad-Soyad:	**No:**		**İmza:**	
MÜHENDİSLİK MEKANİĞİ-II - MKT2112	**25.4.2022 / 16:30**		**80 dakika**	
	Ara Sınav	Vize 2	Mazeret	Final	Bütünleme
1→25 2→25 3→25 4→25	**Doç. Dr. Mehmet Selçuk ARSLAN**			

1 The crate *C* is being lifted by moving the roller at *A* downward with a constant speed of $v_A = 2\ m/s$ along the guide. Determine the velocity and acceleration of the crate at the instant $s = 1\ m$. When the roller is at B, the crate rests on the ground. Neglect the size of the pulley in the calculation. *Hint:* Relate the coordinates x_C and x_A using the problem geometry, then take the first and second time derivatives.



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2 The car travels around the circular track having a radius of $r = 300\ m$ such that when it is at point *A* it has a velocity of $5\ m/s$, which is increasing at the rate of $\dot{v} = (0.06t)\ \mathrm{m/s^2}$, where t is in seconds. Determine the magnitudes of its velocity and acceleration when it has traveled one-third the way around the track.

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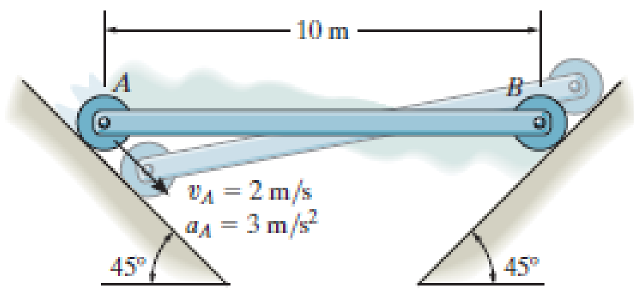
3 The rod *AB* is confined to move along the inclined planes at *A* and *B*. If point *A* has an acceleration of $3\ m/s^2$ and a velocity of $2\ m/s$, both directed down the plane at the instant the rod is horizontal, determine the angular acceleration of the rod at this instant.



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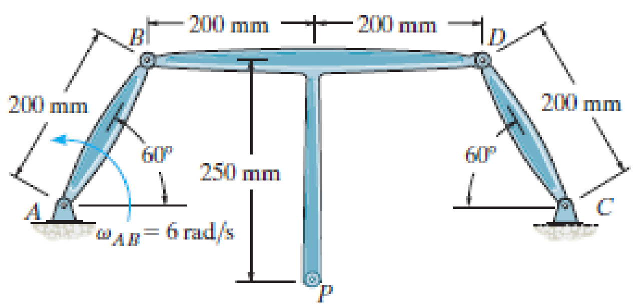
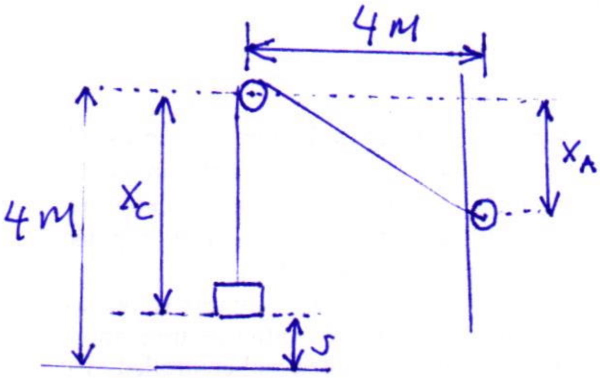
4 Member *AB* is rotating at $w_{AB} = 6\ rad/s$. Determine the velocity of point *P*, and the angular velocity of member *BPD*.



$v = ds/dt$, $a = dv/dt$, $vdv = ads$, $v = v_o + at$, $s = s_o + v_o t + (1/2)at^2$, $v^2 = v_o^2 + 2a(s - s_o)$, $\vec{v} = v\vec{e}_t$, $\vec{a} = \dot{v}\vec{e}_t + (v^2/\rho)\vec{e}_n$, $\vec{v}_A = \vec{v}_B + \vec{v}_{A/B}$, $\rho = [1 + (y')^2]^{3/2} / y''$, $\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$, $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$, $\vec{F} = m\vec{a}$, $U_{1-2} = -w\Delta y$, $U_{1-2} = \pm\frac{1}{2}k(s_2^2 - s_1^2)$, $T_1 + U_{1-2} = T_2$, $V_1 + T_1 = V_2 + T_2$, $\Delta V_e + \Delta V_g + \Delta T = 0$, $\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$, $\vec{v}_B = \vec{v}_A + \vec{v}_{B/A}$, $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$, $w = d\theta/dt$, $\alpha = dw/dt$, $wdw = \alpha d\theta$, $w = w_o + \alpha t$, $\theta = \theta_o + w_o t + (1/2)\alpha t^2$, $w^2 = w_o^2 + 2\alpha(\theta - \theta_o)$, $v = rw$, $\vec{v} = \vec{w} x \vec{r}$, $\vec{a} = \vec{\alpha} x \vec{r} - w^2 \vec{r}$, $\vec{v} = \dot{r}\vec{e}_r + r\dot{\theta}\vec{e}_\theta$, $\vec{a} = (\ddot{r} - r\dot{\theta}^2)\vec{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\vec{e}_\theta$, $\vec{v}_B = \vec{v}_A + \vec{\omega} x\vec{r} + (\vec{v}_B)_{rel}$, $\vec{a}_B = \vec{a}_A + \vec{\alpha} x\vec{r} - w^2\vec{r} + (\vec{a}_B)_{rel} + 2\vec{\omega}x(\vec{v}_B)_{rel}$

Question	**Answers**	
1	**v =**	**a =**
2	**v =**	**a =**
3	$\boldsymbol{\alpha}$ **=**	
4	**v =**	$\boldsymbol{\omega}$ **=**

1



Total cable length:

$$x_C + \sqrt{x_A^2 + 4^2} = L$$

Take derivative wrt time twice:

$$(1) \qquad \dot{x}_C + \frac{1}{2}(x_A^2+16)^{-1/2}\, 2x_A \cdot \dot{x}_A = 0$$

$$(2) \qquad \ddot{x}_C - \frac{1}{2}(x_A^2+16)^{-3/2}\, 2x_A^2 \dot{x}_A^2 + (x_A^2+16)^{-1/2} \dot{x}_A^2 + (x_A^2+16)^{-1/2} x_A \ddot{x}_A = 0$$

when the roller is at B, $x_A = 0\,m$ and $x_C = 4\,m \Rightarrow L = 8\,m$

If $s = 1\,m \Rightarrow x_C = 3\,m$

$$\Rightarrow 3 + \sqrt{x_A^2 + 4^2} = 8 \Rightarrow x_A = 3\,m$$

and $v_A = \dot{x}_A = 2\,m/s$ is given

$\hookrightarrow$ constant $\Rightarrow \ddot{x}_A = 0$

substitute these values into (1):

$$v_C + \sqrt{3^2+16} \cdot 3 \cdot 2 = 0$$

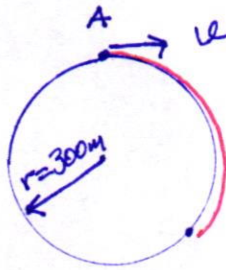
$$\boxed{v_C = -1{,}2\ m/s}$$

and into (2):

$$a_C - [3^2+16]^{-3/2}\, 3^2 2^2 + [3^2+16]^{-1/2} \cdot 2^2 + 0 = 0$$

$$\boxed{a_C = -0{,}512\ m/s^2}$$

2



Since a_t is not constant, we write that

$$dv = a_t\,dt$$

$$\int_5^v dv = \int_0^t 0{,}06t\,dt$$

$$v = 0{,}03t^2 + 5$$

At this point we need the time elapsed to travel one-third the way around the track:

$$ds = v\,dt$$

$$\int_0^s ds = \int_0^t (0{,}03t^2+5)\,dt$$

$$s = 0{,}01t^3 + 5t = \frac{1}{3}\,2\pi\cdot 300$$

$$0{,}01t^3 + 5t - 628{,}318 = 0$$

$$t = 35{,}58\,s$$

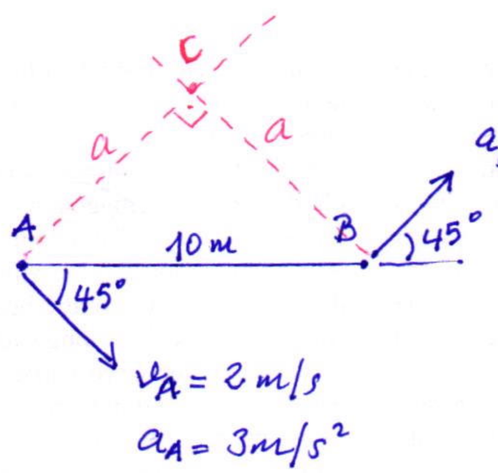
$$v = 0{,}03t^2+5 = 0{,}03\cdot 35{,}58^2+5$$

$$\cong \boxed{43\ m/s}\ (42{,}978\ m/s)$$

$$a_n = v^2/\rho = 43^2/300 = 6{,}163\ m/s^2$$

$$a_t = 0{,}06\cdot 35{,}58 = 2{,}135\ m/s^2 \quad a = \sqrt{a_t^2+a_n^2} = \boxed{6{,}52\ m/s^2}$$

3



$a^2 + a^2 = 100$

$a = \sqrt{50}$

The relative accel. eqn.:

$$\underline{a}_B = \underline{a}_A + \underbrace{\underline{a}_{B/A}}_{(\underline{a}_{B/A})_t + (\underline{a}_{B/A})_n}$$

$$(\underline{a}_{B/A})_t = \underline{\alpha} \times \underline{r}_{B/A} \qquad (\underline{a}_{B/A})_n = -\omega^2 \underline{r}_{B/A}$$

$$a_B \cos 45\, \underline{i} + a_B \sin 45\, \underline{j} = 3\cos 45\, \underline{i} - 3\sin 45\, \underline{j} + \alpha \underline{k} \times 10\, \underline{i} - \omega^2 10\, \underline{i}$$

using IC of zero vel., we can find ω

$$v_A = \omega \cdot a$$

$$2 = \omega \cdot \sqrt{50} \Rightarrow \omega = 0{,}283\ rad/s$$

Equating $\underline{i}$ and $\underline{j}$ components:

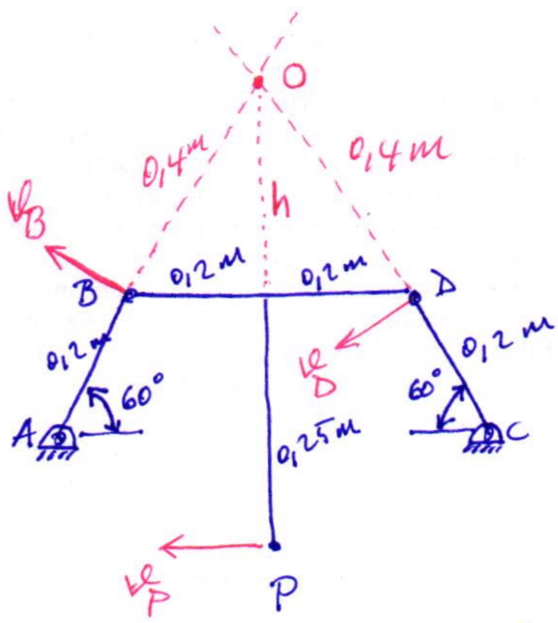
$$a_B \cos 45 = 3\cos 45 - 0{,}283^2 \cdot 10$$

$$a_B \sin 45 = -3\sin 45 + \alpha \cdot 10$$

$$a_B = 1{,}87\ m/s^2$$

$$\boxed{\alpha = 0{,}344\ rad/s^2}$$

4



$h = 0{,}2 \cdot \tan 60$

$\overline{PO} = 0{,}25 + 0{,}2 \cdot \tan 60$
$= 0{,}596 \text{ m}$

$$\omega_{BPD} = \frac{v_B}{\overline{BO}} = \frac{\omega_{AB} \cdot \overline{AB}}{\overline{BO}} = \frac{6 \cdot 0{,}2}{0{,}4} = \boxed{3 \text{ rad/s}}$$

$$v_P = \omega_{BPD} \cdot \overline{PO} = 3 \cdot 0{,}596 = \boxed{1{,}789 \text{ m/s}}$$