OPTİMİZASYON TEKNİKLERİ

**Problem** 11.05.2020

**[1]** $f(x_1, x_2) = 4x_1^2 + x_2^2 - 2x_1x_2$ $[1, 1]^T$ başlangıç çözümünü kullanarak Newton metoduyla çözünüz

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \end{bmatrix} = \begin{bmatrix} 8x_1 - 2x_2 \\ 2x_2 - 2x_1 \end{bmatrix} \Rightarrow \begin{bmatrix} 8 & -2 \\ -2 & 2 \end{bmatrix} = \nabla^2 f$$

$$(\nabla^2 f)^{-1} = \left[\begin{array}{cc|cc} 8 & -2 & 1 & 0 \\ -2 & 2 & 0 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -1/4 & 1/8 & 0 \\ -2 & 2 & 0 & 1 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & -1/4 & 1/8 & 0 \\ 0 & \frac{3}{2} & 1/4 & 1 \end{array}\right] \sim \left[\begin{array}{cc|cc} 1 & -1/4 & 1/8 & 0 \\ 0 & 1 & 1/6 & 2/3 \end{array}\right]$$

$$\sim \left[\begin{array}{cc|cc} 1 & 0 & 1/6 & 1/6 \\ 0 & 1 & 1/6 & 2/3 \end{array}\right] \Rightarrow (\nabla^2 f)^{-1} = \begin{bmatrix} 1/6 & 1/6 \\ 1/6 & 2/3 \end{bmatrix}$$

$$x_1 = x_0 - (\nabla^2 f)^{-1} \nabla f(x_0)$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/6 & 1/6 \\ 1/6 & 2/3 \end{bmatrix} \begin{bmatrix} 8x_1 - 2x_2 \\ 2x_2 - 2x_1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1/6 & 1/6 \\ 1/6 & 2/3 \end{bmatrix} \begin{bmatrix} 6 \\ 0 \end{bmatrix}$$

$$x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$x_0 \begin{cases} x_1 = 1 \\ x_2 = 1 \end{cases}$ olabilir $x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\nabla f(x_1) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ olur. optimal çözümdür.

$\boxed{2}$ $\min f(x) = -x_1 x_2 x_3$

$$x_1 x_2 + x_2 x_3 + x_3 x_1 = \frac{A}{2}$$

$$L(x_1, x_2, x_3) = -x_1 x_2 x_3 + \lambda \left( x_1 x_2 + x_2 x_3 + x_3 x_1 - \frac{A}{2} \right)$$

$$\frac{\partial L}{\partial x_1} = -x_2 x_3 + \lambda x_2 + \lambda x_3 = 0 \quad (1)$$

$$\frac{\partial L}{\partial x_2} = -x_1 x_3 + \lambda (x_1 + x_3) = 0 \quad (2)$$

$$\frac{\partial L}{\partial x_3} = -x_1 x_2 + \lambda (x_2 + x_1) = 0 \quad (3)$$

$$x_1 x_2 + x_2 x_3 + x_3 x_1 - \frac{A}{2} = 0 \quad (4)$$

a) $x_1 = 0 \Rightarrow x_2 x_3 = \frac{A}{2}$ (2)den $\lambda x_3 = 0$

(3)den. $\lambda x_2 = 0$ Bu denklemler (1) de yerine yazılırsa $-x_2 x_3 = 0$ olur. Bu durum (4) koşulunu sağlamaz ($-\frac{A}{2} = 0$ olması) Aynı şekilde ($x_2 = 0$ ve $x_3 = 0$ olamazlar)

b) $\lambda = 0$ ise $x_1 x_2 = x_2 x_3 = x_3 x_1 = 0$ olur.

Buda (4)'ü sağlamaz $-\frac{A}{2} = 0$ olur.

öyle ise $x_1 \neq 0$ $x_2 \neq 0$ $x_3 \neq 0$ $\lambda \neq 0$ dır.

$\boxed{3}$ öyleyse $\lambda, x_1, x_2, x_3 \neq 0$ olur.

(1) ile $x_1$, (2)'yi $x_2$ ile (3)'ü $x_3$ le çarparsak

ve çarpılmış halde (1) den (2) yi çıkarırsak

$$-x_1x_2x_3 + \lambda x_1(x_2+x_3) = 0$$

$$-x_1x_2x_3 + x\, x_2(x_1+x_3) = 0$$

$$\lambda x_1(x_2+x_3) - \lambda x_2(x_1+x_3) = 0$$

$$\cancel{\lambda x_1x_2} + \lambda x_1x_3 \cancel{-\lambda x_1x_2} - \lambda x_2x_3 = 0$$

$$\lambda x_3(x_1-x_2) = 0 \quad \lambda \neq 0 \quad x_3 \neq 0$$

$$x_1 = x_2$$

Benzer yolla

(1) den (3)'ü çıkarırsak

$$-x_1x_2x_3 + \lambda x_1(x_2+x_3) = 0$$

$$-x_1x_2x_3 + \lambda x_3(x_2+x_1) = 0$$

$$\lambda x_1(x_2+x_3) - \lambda x_3(x_2+x_1) = 0$$

$$\lambda x_1x_2 + \cancel{\lambda x_1x_3} - \lambda x_3x_2 \cancel{-\lambda x_3x_1} = 0$$

$$\lambda x_2(x_1-x_3) = 0 \quad \lambda \neq 0 \quad x_2 \neq 0$$

$$x_1 = x_3$$

$$x_1 = x_2 = x_3$$

$\boxed{4}$

$$x_1 x_2 + x_2 x_3 + x_3 x_1 - \frac{A}{2} = 0$$

$$3 x_1^2 = \frac{A}{2}$$

$$x_1^2 = \frac{A}{6} \Rightarrow x_1 = \sqrt{\frac{A}{6}}$$

(1) denklemlerinde yerine konursa

$$-x_2 x_3 + \lambda x_2 + \lambda x_3 = 0$$

$$-\frac{A}{6} + \lambda\left(\sqrt{\frac{A}{6}} + \frac{\sqrt{A}}{6}\right) = 0$$

$$\lambda = \frac{\frac{A}{6}}{2\sqrt{\frac{A}{6}}} = \frac{1}{2}\sqrt{\frac{A}{6}}$$

$$\nabla^2 L = \begin{bmatrix} \frac{\partial^2 L}{\partial x_1^2} & \frac{\partial^2 L}{\partial x_1 \partial x_2} & \frac{\partial^2 L}{\partial x_1 \partial x_3} \\ \frac{\partial^2 L}{\partial x_2 \partial x_1} & \frac{\partial^2 L}{\partial x_2^2} & \frac{\partial^2 L}{\partial x_2 \partial x_3} \\ \frac{\partial^2 L}{\partial x_3 \partial x_1} & \frac{\partial^2 L}{\partial x_3 \partial x_2} & \frac{\partial^2 L}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} 0 & -x_3+\lambda & -x_2+\lambda \\ -x_3+\lambda & 0 & -x_1+\lambda \\ -x_2+\lambda & -x_1+\lambda & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\sqrt{\frac{A}{6}} + \frac{1}{2}\sqrt{\frac{A}{6}} & -\sqrt{\frac{A}{6}} + \frac{1}{2}\sqrt{\frac{A}{6}} \\ -\frac{\sqrt{A}}{6} + \frac{1}{2}\sqrt{\frac{A}{6}} & 0 & -\sqrt{\frac{A}{6}} + \frac{1}{2}\sqrt{\frac{A}{6}} \\ -\frac{\sqrt{A}}{6} + \frac{1}{2}\sqrt{\frac{A}{6}} & -\frac{\sqrt{A}}{6} + \frac{1}{2}\sqrt{\frac{A}{6}} & 0 \end{bmatrix}$$

$\boxed{5}$

$$\nabla^2 L = \begin{bmatrix} 0 & -\frac{1}{2}\sqrt{\frac{A}{6}} & -\frac{1}{2}\sqrt{\frac{A}{6}} \\ -\frac{1}{2}\sqrt{\frac{A}{6}} & 0 & -\frac{1}{2}\sqrt{\frac{A}{6}} \\ -\frac{1}{2}\sqrt{\frac{A}{6}} & -\frac{1}{2}\sqrt{\frac{A}{6}} & 0 \end{bmatrix}$$

$$\nabla^2 L = -\frac{1}{2}\sqrt{\frac{A}{6}} \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} h_1 & h_2 & h_3 \end{bmatrix} \cdot \left(-\frac{1}{2}\sqrt{\frac{A}{6}}\right) \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

$$-\frac{1}{2}\sqrt{\frac{A}{6}} \begin{bmatrix} h_2+h_3, & h_1+h_3, & h_1+h_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

$$-\frac{1}{2}\sqrt{\frac{A}{6}} \left[ \underbrace{h_1h_2 + h_1h_3 + h_1h_2 + h_3h_2 + h_1h_3 + h_2h_3} \right]$$

? Belirsiz ilerlenebilir yönlere ihtiyacımız var.

$\nabla g \cdot h = 0 \qquad g: x_1x_2 + x_1x_3 + x_2x_3 - \frac{A}{2} = 0$

$$\nabla g = \begin{bmatrix} \underbrace{x_2+x_3}, & x_1+x_3 & x_1+x_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

$$\begin{bmatrix} 2\sqrt{\frac{A}{6}}, & 2\sqrt{\frac{A}{6}}, & 2\sqrt{\frac{A}{6}} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

$\boxed{6}$ $2\sqrt{\frac{A}{6}}\begin{bmatrix}1, & 1, & 1\end{bmatrix}\begin{bmatrix}h_1\\h_2\\h_3\end{bmatrix}=0$

$2\sqrt{\frac{A}{6}}\neq 0 \qquad h_1+h_2+h_3=0$

$h_3=-(h_1+h_2)$

$$\nabla^2 L=-\frac{1}{2}\sqrt{\frac{A}{6}}\Big[h_1h_2+h_1(-(h_1+h_2))+h_1h_2+h_2(-(h_1+h_2))$$
$$+h_1(-(h_1+h_2))+h_2(-(h_1+h_2))\Big]$$

$$=-\frac{1}{2}\sqrt{\frac{A}{6}}\Big[\cancel{h_1h_2}-h_1^2-\cancel{h_1h_2}+\cancel{h_1h_2}-\cancel{h_2h_1}-h_2^2$$
$$-h_1^2-h_1h_2-h_2h_1-h_2^2\Big]$$

$$=-\frac{1}{2}\sqrt{\frac{A}{6}}\Big[-2h_1^2-2h_1h_2-2h_2^2\Big]$$

$$=\underbrace{-\frac{1}{2}\sqrt{\frac{A}{6}}}_{<0}\Big[\underbrace{-h_1^2-h_2^2}_{<0}-\underbrace{(h_1+h_2)^2}_{<0}\Big]$$

$=\cdot>0$ olur.

$A\left(x_1=\sqrt{\frac{A}{6}},\ x_2=\sqrt{\frac{A}{6}},\ x_3=\sqrt{\frac{A}{6}}\right)$

noktası Min nokta olur. $f(x_1,x_2,x_3)=x_1x_2x_3$ içinde

fakat $f(x_1,x_2,x_3)=-x_1x_2x_3$ için max