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# LINEER PROGRAMLAMA TEORISI

Saat $9^{00}$

<u>Dejenere çözüm:</u> Bilindiği gibi bir Lineer programlama problemınde satır sayısı kadar sıfırdan farklı çözüm vardı. Satır sayısından daha az sayıda sıfırdan farklı çözümün olması dejenere çözüm olur.

<u>Örnek problem</u>

$$\begin{aligned} \text{mak } z &= 3x_1 + 9x_2 \\ x_1 + 4x_2 &\le 8 \\ x_1 + 2x_2 &\le 4 \\ x_1, x_2 &\ge 0 \end{aligned} \implies \begin{aligned} \text{Mak } z &= 3x_1 + 9x_2 \\ x_1 + 4x_2 + x_3 &= 8 \\ x_1 + 2x_2 + x_4 &= 4 \\ x_1, \dots, x_4 &\ge 0 \end{aligned}$$

$\begin{pmatrix} 1/4 & 0 \\ -1/2 & 1 \end{pmatrix}$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | |
|---|---|---|---|---|---|---|---|
| | | | 3 | 9 | 0 | 0 | OR |
| | | | $V_1$ | $V_2$ | $V_3$ | $V_4$ | |
| j | $c_j$ | $V_0$ | | | | | |
| 3 | 0 | $x_3=8$ | 1 | 4 | 1 | 0 | $8/4=2$ |
| 4 | 0 | $x_4=4$ | 1 | 2 | 0 | 1 | $4/2=2$ |
| $c_j - z_j$ | | $z_0=0$ | 3 | 9 | 0 | 0 | |

oranlar aynı olduğundan tabandan $V_3$ de $V_4$'de çıkabilir. $V_3$ çıkaralım. $V_2$ de tabana girdiğine göre yeni tabanımız olur.

$$(V_2, V_4) = \begin{pmatrix} 4 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1/4 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 1/4 & 0 \\ 0 & 1 & -1/2 & 1 \end{pmatrix}$$

$(V_2, V_4)^{-1} = \begin{pmatrix} 1/4 & 0 \\ -1/2 & 1 \end{pmatrix}$ ile yukarıdaki tabloyu çarpalım.

$\boxed{2}$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | |
|---|---|---|---|---|---|---|---|
| | | | 3 | 9 | 0 | 0 | |
| j | $c_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $\theta R$ |
| 2 | 9 | $x_2=2$ | $1/4$ | 1 | $1/4$ | 0 | 8 |
| 4 | 0 | $x_4=0$ | $1/2$ | 0 | $-1/2$ | 1 | 0 |
| $c_r-z_r$ | | $z_0=9/2$ | $3/4$ | 0 | $-9/4$ | 0 | |

$$\begin{pmatrix} 1 & -1/2 \\ 0 & 2 \end{pmatrix}$$

$$(V_2, V_1) \sim \begin{pmatrix} 1 & 1/4 & 1 & 0 \\ 0 & 1/2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1/4 & 1 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & -1/2 \\ 0 & 1 & 0 & 2 \end{pmatrix}$$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ |
|---|---|---|---|---|---|---|
| | | | 3 | 9 | 0 | 0 |
| j | $c_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ |
| 2 | 9 | $x_2=2$ | 0 | 1 | $1/2$ | $-1/2$ |
| 1 | 3 | $x_1=0$ | 1 | 0 | $-1$ | 2 |
| $c_r-z_r$ | | $z_0=18$ | 0 | 0 | $-3/2$ | $-3/2$ |

$x_1=0$ $x_2=2$ $z_0=18$ iki tane sıfırdan farklı çözüm olması gerekiyordu 1 tane sıfırdan farklı çözüm var o halde dejenere çözüm.

$\boxed{3}$

$\max z = 5x_1 + 4x_2$

$x_1 + 2x_2 \geq 6$

$4x_1 + 2x_2 \leq 8$

$x_1, x_2 \geq 0$

$\max z = 5x_1 + 4x_2 - Mx_5$

$x_1 + 2x_2 - x_3 + x_5 = 6$

$4x_1 + 2x_2 + x_4 = 8$

$x_1, \dots x_5 \geq 0$

$$\begin{pmatrix} 1/2 & 0 \\ -1 & 1 \end{pmatrix}$$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | |
|---|---|---|---|---|---|---|---|---|
| | | | 5 | 4 | 0 | 0 | $-M$ | |
| j | $c_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | ØR |
| 5 | $-M$ | $x_5 = 6$ | 1 | 2 | $-1$ | 0 | 1 | 3 |
| 4 | 0 | $x_4 = 8$ | 4 | 2 | 0 | 1 | 0 | 4 |
| $c_r - z_r$ | | $z_0 = -6M$ | $5+M$ | $4+2M$ | $-M$ | 0 | 0 | |

$$(v_2, v_4)\ N \begin{pmatrix} 2 & 0 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix} N \begin{pmatrix} 1 & 0 & 1/2 & 0 \\ 2 & 1 & 0 & 1 \end{pmatrix}$$

$$N \begin{pmatrix} 1 & 0 & 1/2 & 0 \\ 0 & 1 & -1 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & -1/6 \\ 0 & 1/3 \end{pmatrix}$$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | |
|---|---|---|---|---|---|---|---|---|
| | | | 5 | 4 | 0 | 0 | $-M$ | |
| j | $c_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | ØR |
| 2 | 4 | $x_2 = 3$ | 1/2 | 1 | $-1/2$ | 0 | 1/2 | 6 |
| 4 | 0 | $x_4 = 2$ | 3 | 0 | 1 | 1 | $-1$ | 2/3 |
| $c_r - z_r$ | | $z_0 = 12$ | 3 | 0 | 2 | 0 | $-M-2$ | |

$$(v_2, v_1)\ N \begin{pmatrix} 1 & 1/2 & 1 & 0 \\ 0 & 3 & 0 & 1 \end{pmatrix} N \begin{pmatrix} 1 & 1/2 & 1 & 0 \\ 0 & 1 & 0 & 1/3 \end{pmatrix}$$

$$N \begin{pmatrix} 1 & 0 & 1 & -1/6 \\ 0 & 1 & 0 & 1/3 \end{pmatrix}$$

$\boxed{4}$

$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | |
|---|---|---|---|---|---|---|---|---|
| | | | 5 | 4 | 0 | 0 | $-M$ | OR |
| J | $C_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | |
| 2 | 4 | $x_2 = 8/3$ | 0 | $\cancel{1/2}$ | $-2/3$ | $-1/6$ | $2/3$ | X |
| 1 | 5 | $x_1 = 2/3$ | 1 | 0 | $1/3$ | $1/3$ | $-1/3$ | 2 |
| $c_r - z_r$ | | $z_0 = 37/3$ | 0 | 0 | $+1$ | $1/3$ | $-M-1$ | |

$\uparrow$

$$(v_2, v_3) \sim \begin{pmatrix} 1 & -2/3 & 1 & 0 \\ 0 & 1/3 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2/3 & 1 & 0 \\ 0 & 1 & 0 & 3 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 3 \end{pmatrix}$$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ |
|---|---|---|---|---|---|---|---|
| | | | 5 | 4 | 0 | 0 | $-M$ |
| J | $C_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ |
| 2 | 4 | $x_2 = 4$ | 2 | 1 | 0 | $1/2$ | 0 |
| 3 | 0 | $x_3 = 2$ | 3 | 0 | 1 | 1 | $-1$ |
| $c_r - z_r$ | | $z_0 = 16$ | $-3$ | 0 | 0 | $-2$ | $-M$ |

$x_2^* = 4 \quad x_3^* = 2 \quad z_0^* = 16$

5 sınırlı değişkenler

$x_j \leq s_j \Rightarrow s_j - x_j = x_j'$ diyelim. $x_j = s_j - x_j'$

değerini amaç fonksiyonunda ve kısıtlarda yerine koyalım

Örnek:

$$\text{Mak } z = x_1 + x_2$$

$$-x_1 + 2x_2 \leq 8$$

$$x_1 + 2x_2 \leq 10$$

$$x_1 \leq 8 \quad x_2 \geq 0$$

$x_1 \leq 8 \Rightarrow \underbrace{8 - x_1}_{x_1'} \geq 0 \quad x_1' = 8 - x_1 \quad x_1 = 8 - x_1'$ olur.

Amaç fonksiyonunda ve kısıtlarda $x_1' = 8 - x_1 \Rightarrow x_1 = 8 - x_1'$ koyalım.

$$\text{Mak } z = 8 - x_1' + x_2$$

$$-(8 - x_1') + 2x_2 \leq 8$$

$$8 - x_1' + 2x_2 \leq 10$$

$$8 - x_1' \leq 8 \quad x_2 \geq 0$$

$$x_1', x_2 \geq 0$$

$$\text{Mak } z = -x_1' + x_2 + 8$$

$$x_1' + 2x_2 \leq 16$$

$$-x_1' + 2x_2 \leq 2$$

$$x_1' \geq 0, \quad x_2 \geq 0$$

$$x_1', x_2 \geq 0$$

$$\text{Mak } z = -x_1' + x_2 + 8$$

$$x_1' + 2x_2 + x_3 = 16$$

$$-x_1' + 2x_2 + x_4 = 2$$

$$x_1', x_2 \ldots x_4 \geq 0$$

6

| B | | C | $c_1'$ | $c_2$ | $c_3$ | $c_4$ | |
|---|---|---|---|---|---|---|---|
| | | | $-1$ | $1$ | $0$ | $0$ | |
| $j$ | $c_j$ | $v_0$ | $v_1'$ | $v_2$ | $v_3$ | $v_4$ | OR |
| 3 | 0 | $x_3=16$ | 1 | 2 | 1 | 0 | 8 |
| 4 | 0 | $x_4=2$ | $-1$ | 2 | 0 | 1 | 1 |
| $c_r-z_r$ | | $z_0=0$ | $-1$ | 1 | 0 | 0 | |

$\begin{pmatrix} 1 & -1 \\ 0 & 1/2 \end{pmatrix} \leftarrow$

$v_2$, girer ; $v_4$ çıkar.

$$(V_3, V_2) = \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix} N \begin{pmatrix} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 1/2 \end{pmatrix}$$

$$N \begin{pmatrix} 1 & 0 & 1 & -1 \\ 0 & 1 & 0 & 1/2 \end{pmatrix}$$

| B | | C | $c_1'$ | $c_2$ | $c_3$ | $c_4$ | |
|---|---|---|---|---|---|---|---|
| | | | $-1$ | $1$ | $0$ | $0$ | |
| $j$ | $c_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | |
| 3 | 0 | $x_3=14$ | 2 | 0 | 1 | $-1$ | |
| 2 | 1 | $x_2=1$ | $-1/2$ | 1 | 0 | $1/2$ | |
| $c_r-z_r$ | | $z_0=1$ | $-1/2$ | 0 | 0 | $-1/2$ | |

$x_2=1 \quad x_3=14 \quad x_1'=0$. olduğundan

$x_1=8-x_1'$ $\boxed{x_1=8}$ elde edilir.

$x_1^*=8 \quad x_2^*=1 \quad x_3^*=14 \quad x_4^*=0$

$z^*=9$

**7** $0 \le x_j \le s_j$ olması durumu

Örnek

$\text{Mak } z = -x_1 + 2x_2$

$-3x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 16$

$0 \le x_1 \le 9 \quad x_2 \ge 0$ $x_1$'in pozitif olduğu kesin olduğu için $x_1 \le 9$'u kısıtlara ilave etmemiz yeterli olur.

$\text{mak } z = -x_1 + 2x_2$

$-3x_1 + 4x_2 \le 24$

$x_1 + 2x_2 \le 16$

$x_1 \le 9$

$x_1, x_2 \ge 0$

$\Rightarrow$

$\text{Mak } z = -x_1 + 2x_2$

$-3x_1 + 4x_2 + x_3 = 24$

$x_1 + 2x_2 + x_4 = 16$

$x_1 + x_5 = 9$

$x_1, \ldots, x_5 \ge 0$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | |
|---|---|---|---|---|---|---|---|---|
| | | | -1 | 2 | 0 | 0 | 0 | |
| j | $c_j$ | $v_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | OR |
| 3 | 0 | $x_3 = 24$ | -3 | 4 | 1 | 0 | 0 | $24/4 = 6$ |
| 4 | 0 | $x_4 = 16$ | 1 | 2 | 0 | 1 | 0 | $16/2 = 8$ |
| 5 | 0 | $x_5 = 9$ | 1 | 0 | 0 | 0 | 1 | $\times$ |
| $c_j - z_j$ | | $z_0 = 0$ | -1 | 2 | 0 | 0 | 0 | |

$\begin{pmatrix} 1/4 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \leftarrow$ T.G

$\uparrow$ T.G

$(v_2, v_4, v_5) \sim \begin{pmatrix} 4 & 0 & 0 & 1 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 2 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 0 & 0 & 1/4 & 0 & 0 \\ 0 & 1 & 0 & -1/2 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 \end{pmatrix} \quad (v_2, v_4, v_5)^{-1} = \begin{pmatrix} 1/4 & 0 & 0 \\ -1/2 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\boxed{8}$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | |
|---|---|---|---|---|---|---|---|---|
| | | | $-1$ | $2$ | $0$ | $0$ | $0$ | |
| $j$ | $c_j$ | $u_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | OR |
| 2 | 2 | $x_2=6$ | $-3/4$ | 1 | $1/4$ | 0 | 0 | X |
| 4 | 0 | $x_4=4$ | $5/2$ | 0 | $-1/2$ | 1 | 0 | $8/5$ |
| 5 | 0 | $x_5=9$ | 1 | 0 | 0 | 0 | 1 | 9 |
| $c_r-z_r$ | | $z_0=12$ | $1/2$ | 0 | $-1/2$ | 0 | 0 | |

$\begin{pmatrix} 1 & 3/10 & 0 \\ 0 & 2/5 & 0 \\ 0 & -2/5 & 1 \end{pmatrix}$ TQ $\leftarrow$

$\uparrow$ TQ

$$(v_2, v_1\ v_5) = \begin{pmatrix} 1 & -3/4 & 0 & 1 & 0 & 0 \\ 0 & 5/2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$N \begin{pmatrix} 1 & -3/4 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 2/5 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix} N \begin{pmatrix} 1 & 0 & 0 & 1 & 3/10 & 0 \\ 0 & 1 & 0 & 0 & 2/5 & 0 \\ 0 & 0 & 1 & 0 & -2/5 & 1 \end{pmatrix}$$

| B | | C | $c_1$ | $c_2$ | $c_3$ | $c_4$ | $c_5$ | |
|---|---|---|---|---|---|---|---|---|
| | | | $-1$ | $2$ | $0$ | $0$ | $0$ | |
| $j$ | $c_j$ | $u_0$ | $v_1$ | $v_2$ | $v_3$ | $v_4$ | $v_5$ | OR |
| 2 | 2 | $x_2=11/5$ | 0 | 1 | $1/10$ | $3/10$ | 0 | |
| 1 | $-1$ | $x_1=8/5$ | 1 | 0 | $-1/5$ | $2/5$ | 0 | |
| 5 | 0 | $x_5=37/5$ | 0 | 0 | $1/5$ | $-2/5$ | 1 | |
| $c_r-z_r$ | | $z_0=14/5$ | 0 | 0 | $-2/5$ | $-1/5$ | 0 | |

$x_1^* = 8/5 \quad x_2^* = 11/5 \quad x_3^* = x_4^* = 0 \quad x_5^* = 37/5$

$z_0^* = 14/5$