OPTIMIZASYON TEKNIKLERI

$\boxed{1}$ 27./04/2020

$$\max f(x) = x_1^4 - 5x_1x_2 + 3x_2^2 - 3x_1 + 5x_2 + 2x_3$$

$$x_1 - 6x_2 = 7 \Rightarrow x_1 - 6x_2 - 7 = 0$$

$$5x_1 + 5x_3 \geq 6 \Rightarrow -5x_1 - 5x_3 + 6 \leq 0$$

$$x_1 \geq 0 \quad x_3 \geq 0$$

Sadece gerekli koşulları yazınız

$$\min f(x) = -x_1^4 + 5x_1x_2 - 3x_2^2 + 3x_1 - 5x_2 - 2x_3$$

$$L(x_1, x_2, x_3, \lambda, \mu) = -x_1^4 + 5x_1x_2 - 3x_2^2 + 3x_1 - 5x_2 - 2x_3 + \lambda(x_1 - 6x_2 - 7) + \mu(-5x_1 - 5x_3 + 6)$$

(1) $\frac{\partial L}{\partial x_1} = -4x_1^3 + 5x_2 + 3 + \lambda - 5\mu \geq 0$

(2) $\frac{\partial L}{\partial x_2} = 5x_1 - 6x_2 - 5 - 6\lambda = 0$

(3) $\frac{\partial L}{\partial x_3} = -2 - 5\mu \geq 0$

(4) $x_1 \frac{\partial L}{\partial x_1} = x_1(-4x_1^3 + 5x_2 + 3 + \lambda - 5\mu) = 0$

(5) $x_3 \frac{\partial L}{\partial x_3} = x_3(-2 - 5\mu) = 0$

(6) $\mu \geq 0$

(7) $x_1 - 6x_2 - 7 = 0$

(8) $\mu(-5x_1 - 5x_3 + 6) = 0$

(9) $x_1, x_3 \geq 0$

$\boxed{2}$ $\text{Min } f(x) = (x-4)^2 + (y-4)^2$

$$x + y = 4$$

$$x + 3y \leq 9$$

$$x \geq 0 \quad y \in \mathbb{R}$$

Kritik noktalarını bulunuz

$$L(x, y, \lambda, \mu) = (x-4)^2 + (y-4)^2 + \lambda(x+y-4) + \mu(x+3y-9)$$

(1) $\frac{\partial L}{\partial x} = 2(x-4) + \lambda + \mu \geq 0$

(2) $\frac{\partial L}{\partial y} = 2(y-4) + \lambda + 3\mu = 0$

(3) $x \frac{\partial L}{\partial x} = x\left[2(x-4) + \lambda + \mu\right] = 0$

(4) $x + y - 4 = 0$

(5) $x + 3y - 9 \leq 0$

(6) $\mu(x + 3y - 9) = 0$

(7) $\mu \geq 0 \quad x \geq 0$

Çözüm

a) $x = 0 \quad y = 0$ (4) sağlanmaz $-4 \neq 0$

b) $x = 0 \quad y \neq 0 \quad y = 4$ (4) dancü denklem

(5) sağlanmaz $(x + 3y - 9 \leq 0)$.

c) $x \neq 0 \quad y = 0 \quad 2x - 8 + \lambda + \mu = 0 \quad x = 4$ olur.

$\lambda + \mu = 0$ olur. $\lambda + 3\mu = 8 \quad \mu = -\lambda$ konursa $2\mu = 8$

$\mu = 4$ olur $(x = 4, y = 0 \quad \lambda = -4 \quad \mu = 4)$

$\boxed{3}$

d) $x \neq 0 \quad y \neq 0$

$2x - 8 + \lambda + \mu = 0$

$x + y = 4$

$2y - 8 + \lambda + 3\mu = 0$

a) $\mu \neq 0 \qquad x + 3y - 9 = 0 \qquad x + 3y = 9$

$x + y = 4$

$2y = 5$

$\boxed{y = 5/2}$

$\frac{5}{2} + \frac{3}{2} = 4$ ✓

$x + 5/2 = 4$

$x = 4 - 5/2$

$\boxed{x = 3/2}$

$\lambda + 3\mu = 3$

$\lambda + \mu = 5$

$2\mu = -2$

$\boxed{\mu = -1} \rightarrow$ (7) sağlanmaz

b) $\mu = 0 \qquad x + y - 4 = 0$

$2y - 8 + \lambda = 0$

$2x - 8 + \lambda = 0$

$2y - 2x = 0 \quad \boxed{y = x}$

$(x = 2,\ y = 2\ \lambda = 4,\ \mu = 0)$

$2x - 4 = 0 \quad x = 2 \quad y = 2$

$\lambda = 4$

$\boxed{4}$

$$\begin{bmatrix} \frac{\partial^2 L}{\partial x^2} & \frac{\partial^2 L}{\partial x \partial y} \\ \frac{\partial^2 L}{\partial y \partial x} & \frac{\partial^2 L}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\begin{bmatrix} h_1 \\ h_2 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} = \begin{bmatrix} 2h_1 & 2h_2 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

$$= 2h^2 + 2h_2^2$$

$A\ (x=4,\ y=0\ \ \lambda=-4\ \ \ell=4) \Rightarrow f(x,y)\Big|_{x=4\ y=0} = 16$

$B\ (x=2,\ y=2\ \ \lambda=4,\ \ell=0) \Rightarrow f(x,y)\Big|_{\substack{x=2 \\ y=2}} = 8$

Problem

$f(x,y) = -6x^2 + (2a+4)xy - y^2 + 4ay$ a'nın hangi değerleri için konveks yada konkav olur?

$$\nabla f = \begin{bmatrix} \frac{\partial f}{\partial x} \\ \frac{\partial f}{\partial y} \end{bmatrix} = \begin{bmatrix} -12x + (2a+4)y \\ (2a+4)x - 2y + 4a \end{bmatrix}$$

$$\nabla^2 f = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} -12 & 2a+4 \\ 2a+4 & -2 \end{bmatrix}$$

$\Delta_1 = -12 \qquad \Delta_2 = 24 - (2a+4)^2 = 0 \qquad 2a+4 = \sqrt{24}$

$2a+4 = 2\sqrt{6} \qquad 2a+4 = -2\sqrt{6}$

$a = \sqrt{6} - 2 \qquad a = -\sqrt{6} - 2$

	$-2-\sqrt{6}$		$\sqrt{6}-2$	
Δ_2	+	0	−	0	+

$a < -2-\sqrt{6}$ ve $a > \sqrt{6}-2$ konkav