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Lineer Programlama Teorisi

10.11.2020

$c_j - z_j$ ler dual çözümü verir.

$c_3 - z_3 = 0 \quad c_3 = 0 \quad -z_3 = 0 \Rightarrow z_3 = 0 \Rightarrow \boxed{x_3 = 0}$

$c_4 - z_4 = -\frac{100}{7} \quad c_4 = 0 \quad z_4 = \frac{100}{7} \Rightarrow x_2 = \frac{100}{7}$

$c_5 - z_5 = -\frac{10}{7} \quad c_5 = 0 \quad z_5 = \frac{10}{7} \Rightarrow x_1 = \frac{10}{7}$

$\text{Min } z = x_1 + x_2 + x_3$

$2x_1 + x_2 + 4x_3 = 20$

$x_1 + 2x_2 + x_3 = 30$

x_1, x_2, x_3

Normal simpleks methodla çözelim.

$\text{Min } z = x_1 + x_2 + x_3 + Mx_4 + Mx_5$

$2x_1 + x_2 + 4x_3 + x_4 = 20$

$x_1 + 2x_2 + x_3 + x_5 = 30$

$x_1, \dots, x_5 \geq 0$

B		C	c_1	c_2	c_3	c_4	c_5	
			1	1	1	M	M	
i	c_i	V_0	V_1	V_2	V_3	V_4	V_5	ORAN
4	M	$x_4 = 20$	2	1	4	1	0	5
5	M	$x_5 = 30$	1	2	1	0	1	30
$c_r - z_r$		$z_0 = 50M$	$1-3M$	$1-3M$	$1-5M$	0	0	

$\leftarrow$ $\begin{pmatrix} 1/4 & 0 \\ -1/4 & 1 \end{pmatrix}^{T.G}$

$\uparrow$ TG

$(V_3, V_5) \sim \begin{pmatrix} 4 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1/4 & 0 \\ 1 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 1/4 & 0 \\ 0 & 1 & -1/4 & 1 \end{pmatrix}$

$\boxed{2}$

B		C	c_1	c_2	c_3	c_4	c_5	
			1	1	1	M	M	
j	c_j	v_0	v_1	v_2	v_3	v_4	v_5	
3	1	$x_3=5$	$1/2$	$1/4$	1	$1/4$	0	20
5	M	$x_5=25$	$1/2$	$7/4$	0	$-1/4$	1	$\frac{100}{7}$
c_r-z_r		$z_0=25M+5$	$\frac{1}{2}-\frac{1}{2}M$	$\frac{3}{4}-\frac{7}{4}M$	0	$\frac{5M}{4}-\frac{1}{4}$	0	

$\leftarrow$ TG

$\uparrow$ TG

$$\begin{pmatrix} 1 & -1/7 \\ 0 & 4/7 \end{pmatrix}$$

$$(v_3\ v_2) = \begin{pmatrix} 1 & 1/4 & 1 & 0 \\ 0 & 7/4 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1/4 & 1 & 0 \\ 0 & 1 & 0 & 4/7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & -1/7 \\ 0 & 1 & 0 & 4/7 \end{pmatrix}$$

B		C	c_1	c_2	c_3	c_4	c_5	
			1	1	1	M	M	
j	c_j	v_0	v_1	v_2	v_3	v_4	v_5	OR
3	1	$x_3=\frac{10}{7}$	$3/7$	0	1	$2/7$	$-1/7$	$10/3$
2	1	$x_2=\frac{100}{7}$	$2/7$	1	0	$-1/7$	$4/7$	$25/2$
c_r-z_r		$z_0=\frac{110}{7}$	$2/7$	0	0	$M-1/7$	$M-3/7$	

$\leftarrow$ TG

$\uparrow$ TG

$$\cancel{\begin{pmatrix} 7/3 & 0 \\ -2/3 & 1 \end{pmatrix}}$$

$$(v_1, v_2) \sim \begin{pmatrix} 3/7 & 0 & 1 & 0 \\ 2/7 & 1 & 0 & 1 \end{pmatrix} \sim \cancel{\begin{pmatrix} 1 & 0 & 7/3 & 0 \\ 2/7 & 1 & 0 & 1 \end{pmatrix}}$$

$$\sim \cancel{\begin{pmatrix} 1 & 0 & 7/3 & 0 \\ 0 & 1 & -2/3 & 1 \end{pmatrix}}$$

$z_4 \leftarrow x_4 \rightarrow y_1$

$z_5 \leftarrow x_5 \rightarrow y_2$

$c_4 - z_4 = M - 1/7 \Rightarrow z_4 = 1/7 \rightarrow y_1 = 1/7$

$c_5 - z_5 = M - 3/7 \Rightarrow z_5 = 3/7 \rightarrow y_2 = 3/7$

[3] Primal, Dualini bulalım

$3x_1+2x_2 \geq 4$

$2x_1+x_2 \geq 3$

$-x_1+3x_2 \geq 5$

$-2x_1+x_2 \geq 2$

$4x_1+2x_2 \geq 5$

$x_1, x_2 \geq 0$

$\text{Min } z = x_1+x_2$

$\Rightarrow$

$3y_1+2y_2-y_3-2y_4+4y_5 \leq 1$

$2y_1+y_2+3y_3+y_4+2y_5 \leq 1$

$y_1, y_2, y_3, y_4, y_5 \geq 0$

$\text{Mak } W = 4y_1+3y_2+5y_3+2y_4+5y_5$

$x_1+x_2 \leq 50$

$2x_1+x_2 \leq 110$

$x_1+2x_2 \leq 80$

$x_1+5x_2 \leq 185$

$5x_1+6x_2 \leq 300$

$x_1, x_2 \geq 0$

$\text{Mak } z = 10x_1+15x_2$

$y_1+2y_2+y_3+y_4+5y_5 \geq 10$

$y_1+y_2+2y_3+5y_4+6y_5 \geq 15$

$y_1, \dots, y_5 \geq 0$

$\text{Min } W = 50y_1+110y_2+80y_3+185y_4+300y_5$

$x_1+x_2+x_3 \geq 4$

$2x_1-4x_2-x_3 \leq 10$

$x_1+2x_2+5x_3 = 8$

$x_1, x_2, x_3 \geq 0$

$\text{Mak } z = 3x_1+4x_2-x_3$

1) yol makz olduğuna göre bütün kısıtları küçük yapalım

$-x_1-x_2-x_3 \leq -4$

$2x_1-4x_2-x_3 \leq 10$

$x_1+2x_2+5x_3 = 8$

$\text{Mak } z = 3x_1+4x_2-x_3$

$\leftarrow$ Duali

$-y_1+2y_2+y_3 \geq 3$

$-y_1-4y_2+2y_3 \geq 4$

$-y_1-y_2+5y_3 \geq 8$

$\text{Min } w = -4y_1+10y_2+8y_3$

$y_1, y_2 \geq 0$ y_3 işaretten bağımsız.

[4] DUAL SIMPLEKS ALGORITMASI

$x_1 + x_2 + x_3 \geq 6$

$x_1 - 5x_2 - x_3 \geq 4$

$x_1 + 5x_2 + x_3 \geq 24$

$x_1, x_2, x_3 \geq 0$

$\text{Min } Z = 3x_1 + 6x_2 + x_3$ Dual simpleks algoritmasıyla çözünüz.

$x_1 + x_2 + x_3 - x_4 = 6$

$x_1 - 5x_2 - x_3 - x_5 = 4$ ⇒ Her iki tarafı (−) ile çarpalım.

$x_1 + 5x_2 + x_3 - x_6 = 24$

$\text{Min } Z = 3x_1 + 6x_2 + x_3$

$-x_1 - x_2 - x_3 + x_4 = -6$

$-x_1 + 5x_2 + x_3 + x_5 = -4$

$-x_1 - 5x_2 - x_3 + x_6 = -24$

$\begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & 1 \\ 0 & 0 & -1 \end{pmatrix}$

B		C	c_1	c_2	c_3	c_4	c_5	c_6
			3	6	1	0	0	0
j	c_j	V_0	V_1	V_2	V_3	V_4	V_5	V_6
4	0	$x_4 = -6$	−1	−1	−1	1	0	0
5	0	$x_5 = -4$	−1	5	1	0	1	0
6	0	$x_6 = -24$	−1	−5	−1	0	0	1
$c_r - z_r$			3	6	1	0	0	0

← T.Ç

$\text{Min}\left\{ \left|\frac{3}{-1}\right|, \left|\frac{6}{-5}\right|, \left|\frac{1}{-1}\right| \right\} \Rightarrow V_3$ Tabana girer

sonra

önce V_6 Tabandan çıkar.

$(4,5,3) \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$

$\begin{pmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 1/2 & 1 \end{pmatrix}$

Çarp. 3. Satır (−1) ile çarp

$\sim \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & -1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{pmatrix}$

V_1 tabana girer,

oran yapıldığından V_5 satırında sadece V_1'in katsayısı (−2)

T.Ç ←

V_5 tabandan çıkar.

B		C	c_1	c_2	c_3	c_4	c_5	c_6	
			3	6	1	0	0	0	
j	c_j	V_0	V_1	V_2	V_3	V_4	V_5	V_6	
4	0	$x_4 = 18$	0	4	0	1	0	−1	
5	0	$x_5 = -28$	−2	0	0	0	1	1	
3	1	$x_3 = 24$	1	5	1	0	0	−1	
$c_r - z_r$			2	1	0	0	0	1	

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yeni Taban $(V_4\ V_1,\ V_3) = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & -2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$

ikinci satır (−2) ile böl

$$N \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$N \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1/2 & 0 \\ 0 & 0 & 1 & 0 & 1/2 & 1 \end{pmatrix}$$

B		C	C_1	C_2	C_3	C_4	C_5	C_6
			3	6	1	0	0	0
j	C_j	v_0	v_1	v_2	v_3	v_4	v_5	v_6
4	0	$x_4=18$	0	4	0	1	0	−1
1	3	$x_1=14$	1	0	0	0	−1/2	−1/2
3	1	$x_3=10$	0	5	1	0	1/2	−1/2
C_r-Z_r		$Z_0=52$	0	1	0	0	1	2

$C_r - Z_r \geq 0$ ve $x_1, x_3, x_4 \geq 0$ olduğundan problem bitmiştir.

$x_1^* = 14 \quad x_2^* = 0 \quad x_3^* = 10 \quad x_4^* = 18 \quad x_5^* = x_6^* = 0 \quad Z^* = 52$

__Örnek__

$\text{Mak } z = 2x_1 + x_2$

$x_1 + x_2 \leq 4$

$2x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

a) problemi simpleks methodla çözünüz

b) Dualini alınız

c) optimal duali çözümü, çözmeden bulunuz.

$\text{Mak } z = 2x_1 + x_2$

$x_1 + x_2 + x_3 = 4$

$2x_1 - x_2 + x_4 = 2$

$x_1, \ldots, x_4 \geq 0$

$\begin{pmatrix} 1 & -1/2 \\ 0 & 1/2 \end{pmatrix}$

B		C	C_1	C_2	C_3	C_4	
			2	1	0	0	
j	C_j	v_0	v_1	v_2	v_3	v_4	OR
3	0	$x_3=4$	1	1	1	0	4
4	0	$x_4=2$	2	−1	0	1	1
C_r-Z_r		$Z_0=0$	2	1	0	0	

T.G.

v_4 çıkar v_1 Tabana girer.

$\boxed{6}$

$$(v_3, v_1) \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 2 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & \frac{1}{2} \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 1 & -\frac{1}{2} \\ 0 & 1 & 0 & \frac{1}{2} \end{pmatrix}$$

$\begin{pmatrix} 2/3 & 0 \\ 1/3 & 1 \end{pmatrix}$ T.G ←

B		C	c_1 2	c_2 1	c_3 0	c_4 0	
j	c_j	v_0	v_1	v_2	v_3	v_4	θR
3	0	$x_3=3$	0	$3/2$	1	$-1/2$	2
1	2	$x_1=1$	1	$-1/2$	0	$1/2$	X
c_r-z_r		$z_0=2$	0	2	0	-1	

↑ T.G

$$(v_2, v_1) \sim \begin{pmatrix} 3/2 & 0 & 1 & 0 \\ -1/2 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 2/3 & 0 \\ -1/2 & 1 & 0 & 1 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & 0 & 2/3 & 0 \\ 0 & 1 & 1/3 & 1 \end{pmatrix}$$

B		C	c_1 2	c_2 1	c_3 0	c_4 0	
j	c_j	v_0	v_1	v_2	v_3	v_4	θR
2	1	$x_2=2$	0	1	$2/3$	$-1/3$	
1	2	$x_1=2$	1	0	$1/3$	$1/3$	
c_r-z_r		$z=5$	0	0	$-4/3$	$-1/3$	

$x_3 \to y_1$ $\quad c_3 - z_3 = -4/3 \Rightarrow z_3 = 4/3 \to y_1$

$x_4 \to y_2$ $\quad c_4 - z_4 = -1/3 \Rightarrow z_4 = 1/3 \to y_2$

$\text{Max } z = 2x_1 + x_2$

$x_1 + x_2 \leq 4$

$2x_1 - x_2 \leq 2$

$x_1, x_2 \geq 0$

Duale $\Rightarrow$

$y_1 + 2y_2 \geq 2$

$y_1 - y_2 \geq 1$

$y_1, y_2 \geq 0$

$\text{Min } w = 4y_1 + 2y_2$

$\boxed{7}$

$\text{maks } z = -4x_1 - 2x_2$

$3x_1 + x_2 \geq 27$
$x_1 + x_2 \geq 21$
$x_1 + 2x_2 \geq 30$
$x_1, x_2 \geq 0$

} Lineer Programlama Problemini dual simpleks algoritmasıyla çözünüz

$\text{Maks } z = -4x_1 - 2x_2$ | $\text{Maks } z = -4x_1 - 2x_2$
$3x_1 + x_2 - x_3 = 27$ | $-3x_1 - x_2 + x_3 = -27$
$x_1 + x_2 - x_4 = 21 \Rightarrow$ | $-x_1 - x_2 + x_4 = -21$
$x_1 + 2x_2 - x_5 = 30$ | $-x_1 - 2x_2 + x_5 = -30$

$x_1, \ldots, x_5 \geq 0$

B		C	c_1	c_2	c_3	c_4	c_5
			-4	-2	0	0	0
j	c_j	v_0	v_1	v_2	v_3	v_4	v_5
3	0	$x_3 = -27$	-3	-1	1	0	0
4	0	$x_4 = -21$	-1	-1	0	1	0
5	0	$x_5 = -30$	-1	-2	0	0	1
$c_r - z_r$		$z_0 = 0$	-4	-2	0	0	0

$\begin{pmatrix} 1 & 0 & -1/2 \\ 0 & 1 & -1/2 \\ 0 & 0 & -1/2 \end{pmatrix}$

TÇ ←

$\min\left\{\frac{-4}{-1}, \frac{-2}{-2}, \right\} = 1 \quad v_2 \rightarrow$ girer.

$(3,4,2) \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & -2 & 0 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & -1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1/2 \end{pmatrix}$

$\sim \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & -1/2 \\ 0 & 1 & 0 & 0 & 1 & -1/2 \\ 0 & 0 & 1 & 0 & 0 & -1/2 \end{pmatrix}$

$\boxed{8}$

$T.C_1 \leftarrow$ $\begin{pmatrix} -2/5 & 0 & 0 \\ -1/5 & 1 & 0 \\ 1/5 & 0 & 1 \end{pmatrix}$

B		C	c_1	c_2	c_3	c_4	c_5	
			-4	-2	0	0	0	
j	c_j	V_0	V_1	V_2	V_3	V_4	V_5	
3	0	$x_3=-12$	$-5/2$	0	1	0	$-1/2$	
4	0	$x_4=-6$	$-1/2$	0	0	1	$-1/2$	
2	-2	$x_2=15$	$1/2$	1	0	0	$-1/2$	
c_r-z_r		$z_0=-15$	-3	0	0	0	-1	

$$\min\left\{\frac{-3}{-5/2}\ \ \frac{-1}{-\frac{1}{2}}\right\}=\left\{6/5,\ 2\right\} \quad v_1 \text{ girer.}$$

$$(v_1,v_4,v_2)\sim\begin{pmatrix} -5/2 & 0 & 0 & 1 & 0 & 0 \\ -1/2 & 1 & 0 & 0 & 1 & 0 \\ 1/2 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\sim\begin{pmatrix} 1 & 0 & 0 & -2/5 & 0 & 0 \\ -1/2 & 1 & 0 & 0 & 1 & 0 \\ 1/2 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}\sim\begin{pmatrix} 1 & 0 & 0 & -2/5 & 0 & 0 \\ 0 & 1 & 0 & -1/5 & 1 & 0 \\ 0 & 0 & 1 & 1/5 & 0 & 1 \end{pmatrix}$$

$\begin{pmatrix} 1 & 1/2 & 0 \\ 0 & -5/2 & 0 \\ 0 & -3/2 & 1 \end{pmatrix}$ $T.C_1 \leftarrow$

B		C	c_1	c_2	c_3	c_4	c_5	
			-4	-2	0	0	0	
j	c_j	V_0	V_1	V_2	V_3	V_4	V_5	
1	-4	$x_1=24/5$	1	0	$-2/5$	0	$1/5$	
4	0	$x_4=-18/5$	0	0	$1/5$	1	$-2/5$	
2	-2	$x_2=63/5$	0	1	$1/5$	0	$-3/5$	
c_r-z_r		$z_0=-222/5$	0	0	$-6/5$	0	$-2/5$	

v_4 çıkar v_5 girer

$$(v_1,v_5,v_2)\sim\begin{pmatrix} 1 & 1/5 & 0 & 1 & 0 & 0 \\ 0 & -2/5 & 0 & 0 & 1 & 0 \\ 0 & -3/5 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$$\sim\begin{pmatrix} 1 & 1/5 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & -5/2 & 0 \\ 0 & -3/5 & 1 & 0 & 0 & 1 \end{pmatrix}$$

$\boxed{9}$.

$$\begin{pmatrix} 1 & 0 & 0 & 1 & 1/2 & 0 \\ 0 & 1 & 0 & 0 & -5/2 & 0 \\ 0 & 0 & 1 & 0 & -3/2 & 1 \end{pmatrix}$$

B		C	c_1	c_2	c_3	c_4	c_5
			-4	-2	0	0	0
j	c_j	v_0	v_1	v_2	v_3	v_4	v_5
1	-4	$x_1 = 3$	1	0	$-3/10$	$1/2$	0
5	0	$x_5 =$ [illegible]	0	0	$-1/2$	$-5/2$	1
2	-2	$x_2 = 18$	0	1	$-1/10$	$-3/2$	0
$c_r - z_r$		$z_0 = -48$	0	0	$-7/5$	-1	0