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ZAMAN SERILERI ANALIZI

Iki değişkenli fonksiyonlarda Maksimum ve Minimum.

1) Gerek koşullar.

$\frac{\partial f}{\partial x}=0 \quad \frac{\partial f}{\partial y}=0 \Rightarrow$ Elde edilen (x_0, y_0) kritik noktadır.

2) $\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2-\frac{\partial^2 f}{\partial x^2}\frac{\partial^2 f}{\partial y^2}=\begin{cases} <0 & \text{ise } (x_0,y_0) \text{ max yada min.} \\ >0 & \text{ise } (x_0,y_0) \text{ Semer noktası} \end{cases}$

Niçin.

Çok değişkenli fonksiyonlarda Max yada Minimum için test yapıldığında $\nabla f=0$ ve $\nabla^2 f$ hessian matrisi pozitif definit ise $\nabla f=0$ yapan nokta Mindur. Negatif definit ise $\nabla f=0$ yapan nokta Max.dur. Bu fikri yukarıdaki 2 değişkenli fonksiyonlara uygularsak.

$\nabla f=\left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right]$

$\nabla^2 f=\begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix}$ elde edili.

$\nabla f=0 \Rightarrow \frac{\partial f}{\partial x}=0 \quad \frac{\partial f}{\partial y}=0$

$\frac{\partial^2 f}{\partial x \partial y}=\frac{\partial^2 f}{\partial y \partial x}$ f sürekli ise,

$\nabla^2 f$ den $\Delta_1=\frac{\partial^2 f}{\partial x^2}>0$

$\Delta_2=\frac{\partial^2 f}{\partial x^2}\cdot\frac{\partial^2 f}{\partial y^2}-\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2>0$ $\nabla^2 f$ pozitif definit dur.

(x_0, y_0) Min noktadır.

$\boxed{2}$ yine $\nabla f = 0$ $\frac{\partial f}{\partial x} = 0$ $\frac{\partial f}{\partial y} = 0$ iken (x_0, y_0) bulunur.

$\nabla^2 f$ $\quad \Delta_1 = \frac{\partial^2 f}{\partial x^2} < 0$

$$\Delta_2 = \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0 \text{ olursa}$$

negatif definit olur. Elde edilen (x_0, y_0) noktası maksimum olur.

o halde iki kuralı birleştirirsek

$\frac{\partial f}{\partial x} = 0$ $\quad \frac{\partial f}{\partial y} = 0$ buradan (x_0, y_0) bulunur

$$\Delta_2 = \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial x \partial y^2} > 0$$

veya Max.

Min nokta. $\rightarrow \frac{\partial^2 f}{\partial x^2} > 0$

Min.

$$\frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$$

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

$\frac{\partial^2 f}{\partial x^2} < 0$ iken

f'in sürekliliğinden,

$$\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$$

özetlenirse

1) $\frac{\partial f}{\partial x} = 0$ $\quad \frac{\partial f}{\partial y} = 0$ elde edilirse ise (x_0, y_0) Max. noktadır.

(x_0, y_0) noktası elde edilir.

$\frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 > 0$ iken $\frac{\partial^2 f}{\partial x^2} > 0$ ise (x_0, y_0) Min nokta

iken $\frac{\partial^2 f}{\partial x^2} < 0$ " (x_0, y_0) Max

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Doğrusal Regrasyon denklemi

$y = ax+b \Rightarrow y_i = ax_i + b$

$\Rightarrow \hat{y}_i = ax_i + b$

verilen n tane noktaya (x_1, y_1) (x_2, y_2) ... (x_n, y_n) en yakın olan doğru denklemini belirlemeye çalışıyoruz. y_i ler veriler $\hat{y}_i$ ise verilere yakın geçen doğru denklemi olmak üzere

$$\sum_{i=1}^{n} (\hat{y}_i - y_i)^2 = \sum_{i=1}^{n} (ax_i + b - y_i)^2 = f(a,b)$$

$$\frac{\partial f}{\partial a} = 2\sum_{i=1}^{n} (ax_i + b - y_i).x_i = 0 \qquad (1)$$

$$\frac{\partial f}{\partial b} = 2\sum_{i=1}^{n} (ax_i + b - y_i)^2 . 1 = 0 \qquad (2)$$

(1) denklemden

$$\sum_{i=1}^{n} ax_i^2 + bx_i - y_i x_i = 0 \Rightarrow a\sum_{i=1}^{n} x_i^2 + b\sum_{i=1}^{n} x_i = \sum_{i=1}^{n} x_i y_i$$

(2) denkleminden

$$a\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} b - \sum_{i=1}^{n} y_i = 0 \qquad a\sum_{i=1}^{n} x_i + b.n = \sum_{i=1}^{n} y_i$$

$$\Delta = \begin{vmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} x_i & n \end{vmatrix}$$

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$$\Delta a = \begin{vmatrix} \sum_{i=1}^{n} x_i y_i & \sum_{i=1}^{n} x_i \\ \sum_{i=1}^{n} y_i & n \end{vmatrix} \qquad a = \frac{\Delta a}{\Delta}$$

$$\Delta_b = \begin{vmatrix} \sum_{i=1}^{n} x_i^2 & \sum_{i=1}^{n} x_i y_i \\ \sum_{i=1}^{n} x_i & \sum_{i=1}^{n} y_i \end{vmatrix} \qquad b = \frac{\Delta b}{\Delta}$$

$$\frac{\partial^2 f}{\partial a^2} = \sum_{i=1}^{n} 2 x_i^2 \qquad \frac{\partial^2 f}{\partial a \partial b} = \sum_{i=1}^{n} 2 x_i$$

$$\frac{\partial^2 f}{\partial b \partial a} = \sum_{i=1}^{n} 2 x_i \qquad \frac{\partial^2 f}{\partial b^2} = 2\sum_{i=1}^{n} 1$$

$$\begin{vmatrix} 2\sum_{i=1}^{n} x_i^2 & 2\sum_{i=1}^{n} x_i \\ 2\sum_{i=1}^{n} x_i & \underbrace{2\sum_{i=1}^{n} 1}_{2n} \end{vmatrix} = 4n\sum_{i=1}^{n} x_i^2 - 4\left(\sum_{i=1}^{n} x_i\right)^2 > 0$$

$$n\sum_{i=1}^{n} x_i^2 > \left(\sum_{i=1}^{n} x_i\right)^2$$

$$3(1^2+2^2+3^2) \overset{?}{>} (1+2+3)^2$$

$$\underbrace{3(1+4+9)}_{42} > 36$$

$\frac{\partial^2 f}{\partial a^2} = 2\sum_{i=1}^{n} x_i^2 > 0$ ve $\frac{\partial^2 f}{\partial a^2} \cdot \frac{\partial^2 f}{\partial b^2} - \left(\frac{\partial^2 f}{\partial a \partial b}\right)^2 > 0$ olması noktalara en yakın doğrunun geçtiğini ortaya çıkarır.

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x_i	y_i	$x_i \cdot y_i$	x_i^2
0	1	0	0
1	3	3	1
2	2	4	4
3	4	12	9
4	5	20	16
$\sum_{i=1}^{5} x_i = 10$	$\sum_{i=1}^{5} y_i = 15$	$\sum x_i y_i = 39$	$\sum x_i^2 = 30$

$$30a + b10 = 39$$

$$10a + 5b = 15$$

$$\Delta = \begin{vmatrix} 30 & 10 \\ 10 & 5 \end{vmatrix} = 50$$

$$\Delta_a = \begin{vmatrix} 39 & 10 \\ 15 & 5 \end{vmatrix} = 195 - 150 = 45$$

$$\Delta_b = \begin{vmatrix} 30 & 39 \\ 10 & 15 \end{vmatrix} = 450 - 390 = 60$$

$$a = \frac{\Delta a}{a} = \frac{45}{50} = \frac{9}{10} = 0,9 \qquad b = \frac{\Delta b}{b} = \frac{60}{50} = 1,2$$

$$\boxed{y = 0.9x + 1.2}$$

Seri yarısı ortalaması

$$\frac{3}{2}\begin{pmatrix} 1 & 3 \\ 2 & 5 \end{pmatrix} 4 \qquad (3/2, 4) \qquad M = \frac{3}{\frac{9}{2} - \frac{3}{2}} = \frac{3}{3} = 1$$

$$\underbrace{}_{3} \quad \cancel{4}$$

$$\frac{9}{2}\begin{pmatrix} 4 & 6 \\ 5 & 8 \end{pmatrix} 7 \qquad (9/2, 7) \qquad y - 4 = 1(x - 3/2)$$

$$y = 4 + x - 3/2 \Rightarrow y = x + 5/2$$

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x	y
1	2
2	3
3	5
4	8
5	6
6	4

$$2\begin{pmatrix}1 & 2\\2 & 3\\3 & 5\end{pmatrix}10/3 \qquad 5\begin{pmatrix}4 & 8\\5 & 6\\6 & 4\end{pmatrix}6$$

$(2, 10/3)$ ve $(5, 6)$

$$M = \frac{6-10/3}{5-2} = \frac{8/3}{3} = 8/9$$

$$y - 10/3 = 8/9\,(x-2)$$

$$y = \frac{10}{3} + 8/9\,x - \frac{16}{9}$$

$$y = 8/9\,x + 14/9$$

Eğer eğrimiz $y = a.b^x$ ise. $\log y = \log a + \log b^x$

$\Rightarrow \log y = \log a + x \log b \,(1)$ bu denklemde en küçük karelere uygulanabilir.

$x \log y = x \log a + x^2 \log b \,(2)$ elde edilir.

(1) ve (2) denklemlerinin toplamları alınarak

$$\sum \log y = \sum \log a + \log b \sum x = n \log a + \log b \sum x$$

$$\sum x \log y = \log a \sum x + \log b \sum x^2 = \log a \sum x + \log b \sum x^2$$

Ü/negrimiz x	y	log y	x log y	x^2
1	2	log2=0.301	0.301	1
2	3	log3=0.477	0.954	4
3	5	log5=0.698	2.094	9
4	6	log6=0.778	3.112	16
+ 5	+ 4	+ log4=0.602	+ 3.01	+ 25
15		2.856	9.471	55

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$2.856 = 5\log a + 15\log b \quad /3$

$9.471 = 15\log a + 55\log b$

$8.568 = 15\cancel{\log a} + 45\log b$

$9.471 = 15\cancel{\log a} + 55\log b$

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$-0.903 = -10\log b$

$\log b = \frac{0.903}{10}$

$\log b = 0.0903$

$b = 1.23$

$2.856 = 5\log a + 15 \cdot 0.0903$

$2.856 - 1.3545 = 5\log a$

$1.501 = 5\log a$

$0.5005 = \log a$

$3.16 = a$

$y = a \cdot b^x$

$$\boxed{y = 3.16(1.23)^x}$$

Hareketli ortalama: Hareketli ortalama mevsimle ilgilidir. Hareketli ortalamanın kaçlı olduğunun bilinmesi gerekir. Yani bir yılın kaça bölündüğünü bilmek gerekir.

Tekli hareketli ortalama

$$x_t^* = \frac{1}{2m+1}\sum_{j=-m}^{m} x_{t+j} \qquad t = m+1, m+2, \ldots, n-m$$

n ↓ Veri sayısı

$2M+1 = 3 \quad M = 1 \quad n = 7$ olsun

$$x_t^* = \frac{1}{3}\sum_{j=-1}^{1} x_{t+j}$$

$x_t^* = \frac{1}{3}(x_{t-1} + x_t + x_{t+1}) \quad t-1 = 1 \Rightarrow t = 2$

$x_2^* = \frac{1}{3}(x_1+x_2+x_3)$

$x_3^* = \frac{1}{3}(x_2+x_3+x_4)$

$x_4^* = \frac{1}{3}(x_3+x_4+x_5)$

$x_5^* = \frac{1}{3}(x_4+x_5+x_6)$

$x_6^* = \frac{1}{3}(x_5+x_6+x_7)$

örnek

x	y
1	3
2	4
3	9
4	11
5	13
6	16
7	15

$2m+1=5$ olsun. $\boxed{M=2}$

$$x_t^* = \frac{1}{5}\sum_{j=-2}^{2} x_{t+j} \Rightarrow y_t^* = \frac{1}{5}(y_{t-2}+y_{t-1}+y_t+y_{t+1}+y_{t+2})$$

$t-2=1$ $\boxed{t=3}$

$$y_3^* = \frac{1}{5}(y_1+y_2+y_3+y_4+y_5) = \frac{1}{5}(3+4+9+11+13) = \frac{40}{5} = 8$$

$$y_4^* = \frac{1}{5}(y_2+y_3+y_4+y_5+y_6) = \frac{1}{5}(4+9+11+13+16) = \frac{53}{5} = 10.6$$

$$y_5^* = \frac{1}{5}(y_3+y_4+y_5+y_6+y_7) = \frac{1}{5}(9+11+13+16+17) = \frac{66}{5} = 13.2$$