OPTIMIZASYON TEKNİKLERİ 27.04.2021

1. ARA VİZE SORULARI VE ÇÖZÜMLERİ

S.1 $f = x^3 + y^3 - xy + 4$ fonksiyonu ve kritik noktalarını düşünün. Bunlara göre doğru ifade veya ifadeleri seçin. Uygun olan tüm seçenekler işaretlenmelidir.

$$\frac{\partial f}{\partial x} = 3x^2 - y = 0 \qquad\qquad \frac{\partial f}{\partial y} = 3y^2 - x = 0$$

$$\Rightarrow \quad y = 3x^2, \quad x = 3y^2 \quad \Rightarrow \quad y = 3.9y^4, \quad y = 27y^4$$

$$y - 27y^4 = 0 \qquad y(1 - 27y^3) = 0 \quad \begin{cases} y = 0 \Rightarrow x = 3y^2 \Rightarrow x = 0 \\ y = \frac{1}{3} \Rightarrow x = \frac{1}{3} \end{cases}$$

$A(0,0)$ $B(\frac{1}{3}, \frac{1}{3})$ $\Rightarrow$ iki kritik noktası vardır.

$$\nabla^2 f(x,y) = H = \begin{bmatrix} \frac{\partial^2 f}{\partial x^2} & \frac{\partial^2 f}{\partial x \partial y} \\ \frac{\partial^2 f}{\partial y \partial x} & \frac{\partial^2 f}{\partial y^2} \end{bmatrix} = \begin{bmatrix} 6x & -1 \\ -1 & 6y \end{bmatrix}$$

$$H = \nabla^2 f(A) = \nabla^2 f(0,0) = \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix} \qquad \Delta_1 = 0, \quad \Delta_2 = -1$$

(0,0) semer noktası

$$H = \nabla^2 f(B) = \nabla^2 f\left(\frac{1}{3}, \frac{1}{3}\right) = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \qquad \Delta_1 = 2 > 0, \quad \Delta_2 = 3 > 0$$

$\nabla^2 f(B)$ pozitif definit $\Rightarrow$ $(1/3, 1/3)$ yerel min nok.

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S.2 $A = \begin{bmatrix} 2 & a & 0 \\ a & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ matrisini gözönüne alınız. a'nın hangi değerleri için, A matrisi pozitif-semi-definit olacaktır.

$\Delta_1 = 2 > 0 \quad \Delta_2 = \begin{vmatrix} 2 & a \\ a & 3 \end{vmatrix} = 6 - a^2 \geq 0 \quad a^2 \leq 6 \; -\sqrt{6} \leq a \leq \sqrt{6}$

$\Delta_3 = \begin{vmatrix} 2 & a & 0 \\ a & 3 & 0 \\ 0 & 0 & 4 \end{vmatrix} = 24 + 0 + 0 - (0 + 0 + 4a^2)$

$24 - 4a^2 \geq 0 \quad 4a^2 \leq 24$

$a^2 \leq 6 \Rightarrow -\sqrt{6} \leq a \leq \sqrt{6}$

S.3 $f: \mathbb{R}^3 \to \mathbb{R}$ fonksiyonun gradyanı ∇f, Hessian matris ise H olsun ve $x^{(0)} = [0, 0, 0]^T$ noktası verilmiş olsun

$f(x_1, x_2, x_3) = x_1^2 + x_1(1 - x_2) + x_2^2 - x_2 x_3 + (x_3)^2 + x_3$

$\nabla f(x_1, x_2, x_3) = \begin{bmatrix} \frac{\partial f}{\partial x_1} \\ \frac{\partial f}{\partial x_2} \\ \frac{\partial f}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 2x_1 - x_2 + 1 \\ -x_1 + 2x_2 - x_3 \\ -x_2 + 2x_3 + 1 \end{bmatrix} \quad \nabla f(x^{(0)}) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$H = \begin{bmatrix} \frac{\partial^2 f}{\partial x_1^2} & \frac{\partial^2 f}{\partial x_1 \partial x_2} & \frac{\partial^2 f}{\partial x_1 \partial x_3} \\ \frac{\partial^2 f}{\partial x_2 \partial x_1} & \frac{\partial^2 f}{\partial x_2^2} & \frac{\partial^2 f}{\partial x_2 \partial x_3} \\ \frac{\partial^2 f}{\partial x_3 \partial x_1} & \frac{\partial^2 f}{\partial x_3 \partial x_2} & \frac{\partial^2 f}{\partial x_3^2} \end{bmatrix} = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = H(x^{(0)})$

$\boxed{3}$

S.3 için devamı

$$\nabla f(x^{(0)}) + H(x^{(0)})\,\nabla f(x^{(0)}) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$$

S4 Aşağıdaki matrisleri ve cinslerini eşleştirin.

$$A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{bmatrix}$$ I negatif definit

$$B = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{bmatrix}$$ II pozitif definit

$$C = \begin{bmatrix} 4 & -3 & 0 \\ -3 & 0 & 4 \\ 0 & 4 & 2 \end{bmatrix}$$ III indefinit

A matrisinde $\Delta_1 = 3 > 0$ $\quad \Delta_2 = \begin{vmatrix} 3 & 1 \\ 1 & 3 \end{vmatrix} = 9-1 = 8 > 0$ $\quad \Delta_3 = \begin{vmatrix} 3 & 1 & -1 \\ 1 & 3 & -1 \\ -1 & -1 & 5 \end{vmatrix}$

$$= 3.3.5 + (1)(-1)(-1) + 1(-1)(-1) - ((-1)3(-1) + (-1)(-1)3 + (1)(1)5)$$

$$= 45 + 1 + 1 - (3 + 3 + 5)$$

$$= 47 - 11 = 36 > 0$$

A matrisi pozitif definittir.

$\boxed{4}$

$$B = \begin{bmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{bmatrix} \qquad \Delta_1 = -1 < 0 \quad \Delta_2 = \begin{vmatrix} -1 & -1 \\ -1 & -2 \end{vmatrix} = 2-1 = 1 > 0$$

$$\Delta_3 = \begin{vmatrix} -1 & -1 & -1 \\ -1 & -2 & -2 \\ -1 & -2 & -3 \end{vmatrix} = -6-2-2-(-2-4-3)$$

$$= -10 - (-9)$$

$$= -1 < 0$$

B matrisi $\Delta_1 < 0$, $\Delta_2 > 0$, $\Delta_3 < 0$ olduğu için negatif definittir.

$$C = \begin{bmatrix} 4 & -3 & 0 \\ -3 & 0 & 4 \\ 0 & 4 & 2 \end{bmatrix} \qquad \Delta_1 = 4 \quad \Delta_2 = \begin{vmatrix} 4 & -3 \\ -3 & 0 \end{vmatrix} = -9$$

Definit değil indefinit

A - II, B - I ve C - III olur.

fonksiyonun çözümünü

S.5 $\quad f(x_1, x_2, x_3) = x_1^2 + x_1(1-x_2) + x_2^2 - x_2 x_3 + x_3^2 + x_3 \quad$ başlangıç $x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ alarak en hızlı düşüş algoritmasıyla optimal çözümünü bulunuz

$$\nabla f = \begin{bmatrix} 2x_1 + 1 - x_2 \\ -x_1 + 2x_2 - x_3 \\ -x_2 + 2x_3 + 1 \end{bmatrix} \qquad \nabla^2 f = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} = Q$$

$$g_0 = \nabla f(x_0) = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad \alpha_i = \frac{g_i' g_i}{g_i' Q g_i} \Rightarrow \alpha_0 = \frac{g_0' g_0}{g_0' Q g_0}$$

$$\|g_0\| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}$$

$$\alpha_0 = \frac{[1 \ 0 \ 1]\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}{[1, 0, 1]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} = \frac{2}{[2, -2, 2]\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}} = \frac{2}{4} = \frac{1}{2}$$

$\boxed{5}$

$$x_{i+1} = x_i + \alpha_i g_i \qquad d_0 = -g_0 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$i=0$

$$x_1 = x_0 + \alpha_0 g_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} +1 \\ 0 \\ +1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 0 \\ -1/2 \end{bmatrix}$$

$$f(x_1) = (x_1)^2 + x_1(1-x_2) + x_2^2 - x_2x_3 + x_3^2 + x_3$$

$$= \left(\frac{-1}{2}\right)^2 + \left(-\frac{1}{2}\right)(1-0) + 0^2 - 0.(-1/2) + (-1/2)^2 + (-1/2)$$

$$= \frac{1}{4} - \frac{1}{2} + \frac{1}{4} - \frac{1}{2} = \frac{1}{2} - 1 = -1/2$$

bulunan noktayı fonksiyonda yerine koyduk.

$$g_1 = \begin{bmatrix} 2x_1+1-x_2 \\ -x_1+2x_2-x_3 \\ -x_2+2x_3+1 \end{bmatrix} = \begin{bmatrix} 2(-1/2)+1-0 \\ -(-1/2)+2.0+1/2 \\ -0+2.(-1/2)+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \qquad \|g_1\| = \sqrt{0^2+1^2+0^2} = 1$$

$$\alpha_1 = \frac{g_1' g_1}{g_1' Q g_1} = \frac{[0,1,0]\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{[0,1,0]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} = \frac{1}{[-1,2-1]\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}} = \frac{1}{2}$$

$$x_2 = x_1 - \alpha_1 g_1 = \begin{bmatrix} -1/2 \\ 0 \\ -1/2 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1/2 \\ -1/2 \end{bmatrix}$$

$$f(x_2) = x_1^2 + x_1(1-x_2) + x_2^2 - x_2x_3 + x_3^2 + x_3$$

$$= \frac{1}{4} + (-1/2)(1-(-1/2)) + (-1/2)^2 - (-1/2)(-1/2) + (-1/2)^2 - 1/2$$

$$= \cancel{\frac{1}{4}} - 1/2\ \frac{3}{2} + \cancel{\frac{1}{4}} - \cancel{1/4} + \cancel{1/4} - \cancel{1/2}$$

$$= \cancel{1/4} - \frac{3}{4} + \cancel{\frac{1}{4}} \cancel{-\frac{1}{4}} + \cancel{\frac{1}{4}} - \cancel{1/2}$$

$$= -3/4$$

$$g_2 = \begin{bmatrix} 2x_1+1-x_2 \\ -x_1+2x_2-x_3 \\ -x_2+2x_3+1 \end{bmatrix} = \begin{bmatrix} 2(-1/2)+1-(-1/2) \\ -(-1/2)+2(-1/2)-(-1/2) \\ -(-1/2)+2(-1/2)+1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}$$

$$\alpha_2 = \frac{g_2' g_2}{g_2' Q g_2} = \frac{[1/2, 0\ 1/2]\begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}}{[1/2, 0\ 1/2]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}} = \frac{1/2}{[1, -1, 1]\begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix}} = \frac{1/2}{1}$$

$$x_3 = x_2 - \alpha_2 g_2 = \begin{bmatrix} -1/2 \\ -1/2 \\ -1/2 \end{bmatrix} - 1/2 \begin{bmatrix} 1/2 \\ 0 \\ 1/2 \end{bmatrix} = \begin{bmatrix} -1/2 - \frac{1}{4} \\ -1/2 - 0 \\ -\frac{1}{2} - \frac{1}{4} \end{bmatrix} = \begin{bmatrix} -3/4 \\ 0 \\ -3/4 \end{bmatrix}$$

Konjugate gradyant Methodu.

$$g_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad d_0 = -g_0 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} \quad \|g_0\| = \sqrt{2}$$

$$\alpha_k = \frac{-d_k' g_k}{d_k' Q d_k} \Rightarrow \alpha_0 = \frac{-d_0' g_0}{d_0' Q d_0} = -\frac{[-1, 0\ -1]\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}{[-1, 0, -1]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}}$$

$$\alpha_0 = \frac{2}{[-2, 2, -2]\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}} = \frac{2}{4} = 1/2$$

$$x_1 = x_0 + \alpha_0 d_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 1/2 \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 0 \\ -1/2 \end{bmatrix}$$

$$f(x_1) = x_1^2 + x_1(1-x_2) + x_2^2 - x_2 x_3 + x_3^2 + x_3$$

$$= 1/4 - 1/2\ (1-0) + 0^2 - 0 + 1/4 - 1/2$$

$$= \cancel{1/4} - \cancel{1/2} + \cancel{1/4} - 1/2 = -1/2$$

$\boxed{7}$

$$g_1 = \begin{bmatrix} 2x_1+1-x_2 \\ -x_1+2x_2-x_3 \\ -x_2+2x_3+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad \|g_1\| = 1$$

$$\alpha_1 = \frac{-d_1' g_1}{d_1' Q d_1} = \qquad \beta_0 = \frac{g_1' \cdot Q\, d_0}{d_0' Q\, d_0} = \frac{[0,1,0]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}}{[-1,0,-1]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}}$$

$$\beta_k = \frac{g_{k+1}' Q d_k}{d_k' Q d_k} \qquad d_{k+1} = -g_{k+1} + \beta_k d_k$$

$$\alpha_1 = \frac{-[-1/2, -1, -1/2]\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{[-1/2, -1, -1/2]\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}\begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix}} \qquad \beta_0 = \frac{[-1 \;\; 2 \;\; -1]\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}}{[-2, 2, -2]\begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}} = \frac{2}{4} = 1/2$$

$$d_1 = -g_1 + \beta_0 d_0 = -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1/2 \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix}$$

$$\alpha_1 = \frac{1}{[\underbrace{-1+1}_{0}; \underbrace{1/2 - 2 + 1/2}_{-1}, \underbrace{0}_{0}]\begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix}} = \frac{1}{+1} = 1$$

$$x_2 = x_1 + \alpha_1 d_1 = \begin{bmatrix} -\frac{1}{2} \\ 0 \\ -1/2 \end{bmatrix} + 1 \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

$$g_2 = \nabla f(x_2) = \begin{bmatrix} 2x_1+1-x_2 \\ -x_1+2x_2-x_3 \\ -x_2+2x_3+1 \end{bmatrix} = \begin{bmatrix} -2+1+1 \\ 1-2+1 \\ 1-2+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = -1$

$x_2 = -1$

$x_3 = -1$

Problem biter.

$\boxed{8}$ Newton Methodu

$$H = \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \Rightarrow H^{-1} = \left[\begin{array}{ccc|ccc} 2 & -1 & 0 & 1 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{array}\right]$$

$$\sim \begin{bmatrix} 1 & -1/2 & 0 & 1/2 & 0 & 0 \\ -1 & 2 & -1 & 0 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 3/2 & -1 & 1/2 & 1 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -1/2 & 0 & 1/2 & 0 & 0 \\ 0 & 1 & -2/3 & 1/3 & 2/3 & 0 \\ 0 & -1 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & -1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -2/3 & 1/3 & 2/3 & 0 \\ 0 & 0 & 4/3 & 1/3 & 2/3 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & -1/3 & 2/3 & 1/3 & 0 \\ 0 & 1 & -2/3 & 1/3 & 2/3 & 0 \\ 0 & 0 & 1 & 1/4 & 1/2 & 3/4 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 3/4 & \cancel{1/2} & +1/4 \\ 0 & 1 & 0 & 1/2 & 1 & 1/2 \\ 0 & 0 & 1 & 1/4 & 1/2 & 3/4 \end{bmatrix}$$

$$H^{-1} = \begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix} \qquad x_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_{k+1} = x_k - \left(\nabla^2 f(x_k)\right)^{-1} \nabla f(x_k)$$

$$x_1 = x_0 - H^{-1} \nabla f(x_0) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 3/4 & 1/2 & 1/4 \\ 1/2 & 1 & 1/2 \\ 1/4 & 1/2 & 3/4 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix}$$

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fletcher reeves ile çözülsün

$$x_{k+1} = x_k + \alpha_k d_k$$

$$\alpha_k = \frac{-d_k' g_k}{d_k' Q d_k}$$

$$d_{k+1} = -g_{k+1} + \beta_k d_k$$

$$\beta_k = \frac{g'_{k+1} \cdot g_{k+1}}{g'_k \cdot g_k}$$

$$g_0 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \qquad d_0 = -g_0 = \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}$$

$$\alpha_0 = \frac{-d_0' g_0}{d_0' Q d_0} = \frac{-[-1, 0, -1] \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}}{[-1, 0, -1] \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}} = \frac{2}{[-2\ 2\ -2] \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}}$$

$$\alpha_0 = \frac{2}{4} = 1/2 \qquad \nabla f(x_1) = g_1 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = g_1$$

$$x_1 = x_0 + \alpha_0 d_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} + 1/2 \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ 0 \\ -1/2 \end{bmatrix}$$

$$\beta_0 = \frac{g_1' g_1}{g_0' g_0} = \frac{[0, 1, 0] \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{[-1, 0, -1] \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix}} = \frac{1}{2}$$

$\boxed{10}$

$d_1 = -g_1 + \beta_0 d_0$

$d_1 = -\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 1/2 \begin{bmatrix} -1 \\ 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix}$

$\alpha_1 = \dfrac{-d_1' g_1}{d_1' \varphi d_1} = \dfrac{-\begin{bmatrix} -1/2 & -1, & -1/2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}}{\begin{bmatrix} -1/2, & -1, & -1/2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix} \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix}}$

$= \dfrac{1}{\begin{bmatrix} \underbrace{-1+1}_{0}, & \underbrace{-1/2 - 1 + 1/2}_{-1}, & \underbrace{1-1}_{0} \end{bmatrix} \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix}}$

$= \dfrac{1}{1} = 1$

$x_2 = x_1 + \alpha_1 d_1$

$x_2 = \begin{bmatrix} -1/2 \\ 0 \\ -1/2 \end{bmatrix} + 1 \begin{bmatrix} -1/2 \\ -1 \\ -1/2 \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \\ -1 \end{bmatrix} \quad \nabla f(x_2) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$