

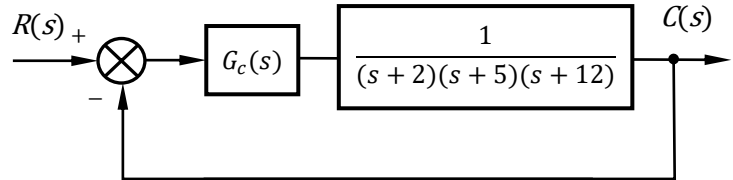
YTU FACULTY OF ELECTRICAL & ELECTRONICS ENGINEERING
DEPARTMENT OF CONTROL & AUTOMATION ENGINEERING
KOM3751 CONTROL SYSTEMS, Homework

Date: December 27, 2023

Due Date & Time: @16:00, January 4, 2024

Note: All students are required to submit a hard copy of their homework solutions to Room A-112 by 4 pm on January 4, 2024. The printed report should include all the calculations, m-files and Simulink models, plots and graphics that you produced on your own. In addition, you are expected to provide detailed analyses for each plot in your report.

Problem-1. For the control system given,



- (a) Sketch, by hand, its root locus for $G_c(s) = K$. When sketching the root locus, if necessary, make use of the asymptotes finding σ_a and θ_a that are the intersecting point and angles with the real axis, respectively using the following formula,

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\# \text{finite poles} - \# \text{finite zeros}} \quad \text{and} \quad \theta_a = \frac{(2k+1)\pi}{\# \text{finite poles} - \# \text{finite zeros}}, \text{ where } k = 0, \pm 1, \pm 2, \dots$$

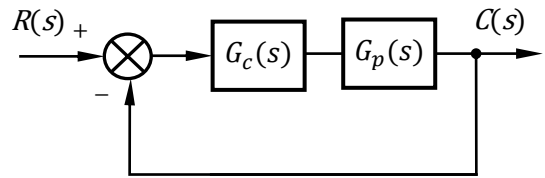
- (b) If the root locus intersects the $j\omega$ -axis, find the values of poles $s_{1,2}$ and gain K at the crossing points. Then write the range of gain K making the system stable.
- (c) Determine the type of the system (type 0, type 1, type 2, etc.).
- (d) Find the steady-state error for this P-controlled system for $G_c(s) = 162$, when a unit step and then ramp inputs are introduced.
- (e) It is determined that the root locus intersects the 10% overshoot line (which corresponds to a damping ratio of $\zeta = 0.59$) for $K = 162$, at $s = -2.62 + j3.67$. Now, justify the second order approximation by finding the third closed-loop pole at this gain, then give an estimation of the settling time using $T_s \cong 4/(\zeta\omega_n)$.
- (f) Design a PID controller to substitute for $G_c(s)$ so that the system's response becomes twice as faster with the same maximum overshoot and produces no steady-state error to a step input.
Hint: For a simple PID controller, design a PD controller to substitute for $G_c(s)$ so that the settling time is reduced 2 times while keeping the same maximum overshoot. Then add a PI controller to meet the steady-state error requirement. For simplicity, you may use the same gain you found for PD controller, also you may select the zero of PI controller at -0.1 .
- (g) Plot the responses of the P, PD and PID controlled system on the same plane for comparison.
- (h) Determine the constant of proportionality, K_p , the reset (integral) time T_i and the rate (derivative) time T_d values of the PID controller assuming that the controller has the following transfer function,

$$G_{PID}(s) = \frac{80(s + 0.4)(s + 10)}{s}$$

- (i) Realize the same PID-controller given in (h)

- i. as a block diagram where three blocks for P, I, D actions are connected in parallel. Remember to determine and write the constants of proportion, integral and derivative (K_p , K_i and K_d)
- ii. physically with a circuit having two OPAMPs. Select the first capacitor, i.e. the one at the input, as $C_1 = 10\mu\text{F}$, also select the second OPAMP's impedances as $Z_3 = Z_4 = 10k\Omega$

Problem-2. Consider a satellite position control system as presented on the right, where the plant is modelled as $G_p(s) = \frac{1}{s^2}$. A compensator will be so designed that the step response will have a settling time of 1.67 seconds and a 16.3% overshoot. Do the following,



- Find the required operating point, i.e. the coordinates of the dominant poles.
- Find the compensator pole if the compensator zero is placed at -1 . Name this compensator.
- Find the system gain.
- Find the location of the real pole for this gain (find it by hand, then you may validate your result on the root locus plot in MATLAB).
- Estimate the accuracy of your second-order approximation.
- Evaluate the steady-state error characteristics, i.e. K_p , K_v , K_a and the corresponding steady-state errors.
- Use MATLAB or any other computer program to simulate the system and evaluate the actual transient response characteristics for a step input.

Problem-3. Considering a similar control system as of the previous problem, where the plant is now

$$G_p(s) = \frac{1}{s(s+1)(s^2+10s+26)}$$

You are expected to

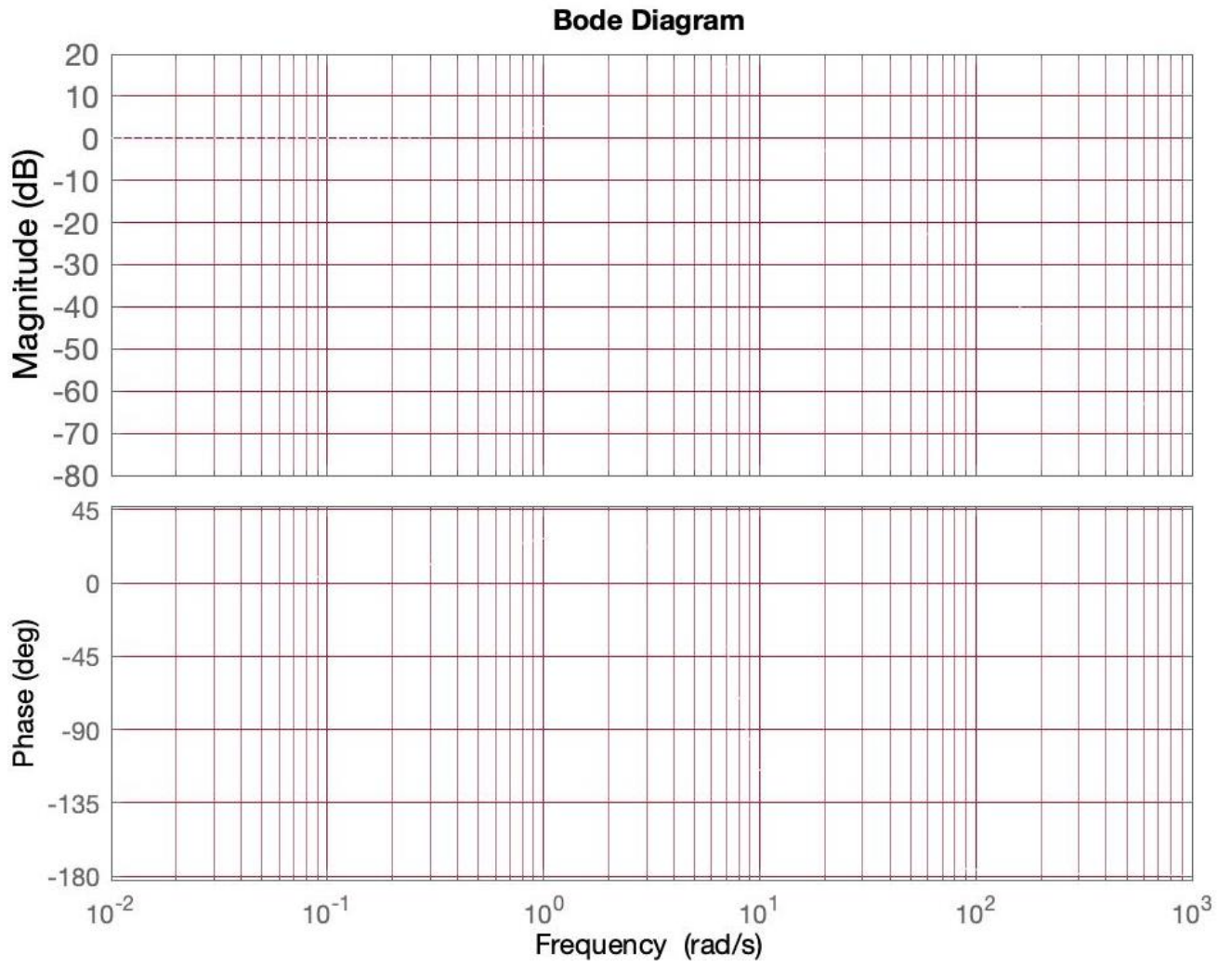
- Find the settling time for the system when it is operating with 15% overshoot.
- Find the zero of a compensator and the gain, K , so that the settling time is 7 seconds. Assume that the pole of the compensator is located at -15 .
- Use MATLAB or any other computer program to simulate the system's step response to test the compensator.
- For the same plant, design a PID controller by means of an appropriate Ziegler-Nichols method. Plot the response of the system using in MATLAB or Simulink.
- Now, propose another PID controller by halving the proportionality constant and doubling the derivative time of the previous PID controller. Plot the response of the system on the same plane as the previous one.

Problem-4. A unity feedback system has the following forward-path transfer function. Do the following.

$$G(s) = \frac{256(s+1)}{(s+4)(s^2+4s+64)}$$

- Write analytical expressions for the magnitude, $M(\omega)$, and phase, $\phi(\omega)$, responses of $G(s)$.
- Sketch by hand the asymptotic magnitude $M(\omega)$ in dB and phase $\phi(\omega)$ in degrees responses of $G(s)$ on the empty logarithmic plane presented in the next page.
- Draw the actual magnitude plot on the asymptotic magnitude sketch you provided in (b). remember the correction due to the underdamped factor of $G(s)$.
- Provide a simple table exhibiting the magnitude and phase values at low and high frequencies, i.e. at 0.01 rad/s and 1000 rad/s, respectively.
- By means of the asymptotic sketches, determine the value of the appropriate static error constant and the steady-state error per unit step.
- Obtain the actual $M(\omega)$ in dB and phase $\phi(\omega)$ in degrees responses of $G(s)$ using MATLAB.

$$T_p = \frac{\pi}{\omega_n \sqrt{1-\zeta^2}}, T_s \cong \frac{4}{\zeta \omega_n}, \%OS = 100 \cdot e^{-\zeta \pi / \sqrt{1-\zeta^2}}, \zeta = \frac{-\ln(\%OS/100)}{\sqrt{\pi^2 + \ln^2(\%OS/100)}} \quad \text{Good luck! Seref Naci Engin} \quad p. 2 \text{ of } 3$$



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