**4.46** (*a*) Silicon at $T = 300$ K is uniformly doped with boron atoms to a concentration of $3 \times 10^{16}$ cm$^{-3}$ and with arsenic atoms to a concentration of $1.5 \times 10^{16}$ cm$^{-3}$. Is the material n type or p type? Calculate the thermal equilibrium concentrations of majority and minority carriers. (*b*) Additional impurity atoms are added such that holes are the majority carrier and the thermal equilibrium concentration is $p_0 = 5 \times 10^{16}$ cm$^{-3}$. What type and concentration of impurity atoms must be added? What is the new value of $n_0$?

(a) $N_a > N_d \Rightarrow$ p-type

Majority carriers are holes

$$p_o = N_a - N_d = 3\times10^{16} - 1.5\times10^{16} = 1.5\times10^{16}\ \text{cm}^{-3}$$

Minority carriers are electrons

$$n_o = \frac{n_i^2}{p_o} = \frac{\left(1.5\times10^{10}\right)^2}{1.5\times10^{16}} = 1.5\times10^{4}\ \text{cm}^{-3}$$

(b) Boron atoms must be added

$$p_o = N_a' + N_a - N_d \qquad 5\times10^{16} = N_a' + 3\times10^{16} - 1.5\times10^{16}$$

So $N_a' = 3.5\times10^{16}\ \text{cm}^{-3}$ $\quad n_o = \dfrac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^{3}\ \text{cm}^{-3}$

**4.55** (*a*) Silicon at $T = 300$ K is doped with donor impurity atoms at a concentration of $N_d = 6 \times 10^{15}$ cm$^{-3}$. (*i*) Determine $E_c - E_F$. (*ii*) Calculate the concentration of additional donor impurity atoms that must be added to move the Fermi energy level a distance $kT$ closer to the conduction band edge. (*b*) Repeat part (*a*) for GaAs if the original donor impurity concentration is $N_d = 1 \times 10^{15}$ cm$^{-3}$.

(a) Silicon

(i) $E_c - E_F = kT \ln\left(\dfrac{N_c}{N_d}\right) = (0.0259)\ln\left(\dfrac{2.8\times10^{19}}{6\times10^{15}}\right) = 0.2188$ eV

(ii) $E_c - E_F = 0.2188 - 0.0259 = 0.1929$ eV

$$N_d = N_c \exp\left[\frac{-(E_c - E_F)}{kT}\right] = \left(2.8\times10^{19}\right)\exp\left[\frac{-0.1929}{0.0259}\right] \quad N_d = 1.631\times10^{16}\ \text{cm}^{-3} = N_d' + 6\times10^{15}$$

$\Rightarrow N_d' = 1.031\times10^{16}\ \text{cm}^{-3}$ Additional donor atoms

(b) GaAs

(i) $E_c - E_F = (0.0259)\ln\left(\dfrac{4.7\times10^{17}}{10^{15}}\right) = 0.15936$ eV

(ii) $E_c - E_F = 0.15936 - 0.0259 = 0.13346$ eV

$$N_d = \left(4.7\times10^{17}\right)\exp\left[\frac{-0.13346}{0.0259}\right] = 2.718\times10^{15}\ \text{cm}^{-3} = N_d' + 10^{15}$$

$\Rightarrow N_d' = 1.718\times10^{15}\ \text{cm}^{-3}$ Additional donor atoms

**5.23** Consider three samples of silicon at $T = 300$ K. The n-type sample is doped with arsenic atoms to a concentration of $N_d = 5 \times 10^{16}$ cm$^{-3}$. The p-type sample is doped with boron atoms to a concentration of $N_a = 2 \times 10^{16}$ cm$^{-3}$. The compensated sample is doped with both the donors and acceptors described in the n-type and p-type samples. (*a*) Find the equilibrium electron and hole concentrations in each sample, (*b*) determine the majority carrier mobility in each sample, (*c*) calculate the conductivity of each sample, (*d*) and determine the electric field required in each sample to induce a drift current density of $J = 120$ A/cm$^2$.

n-type: $\mu_n \cong 1100\ \text{cm}^2/\text{V-s}$ p-type: $\mu_p \cong 400\ \text{cm}^2/\text{V-s}$ compensated: $\mu_n \cong 1000\ \text{cm}^2/\text{V-s}$

(a) n-type: $n_o = N_d = 5\times10^{16}\ \text{cm}^{-3}$

$$p_o = \frac{n_i^2}{n_o} = \frac{\left(1.5\times10^{10}\right)^2}{5\times10^{16}} = 4.5\times10^3\ \text{cm}^{-3}$$

p-type: $p_o = N_a = 2\times10^{16}\ \text{cm}^{-3}$ $\quad n_o = \dfrac{\left(1.5\times10^{10}\right)^2}{2\times10^{16}} = 1.125\times10^4\ \text{cm}^{-3}$

compensated: $n_o = N_d - N_a = 5\times10^{16} - 2\times10^{16} = 3\times10^{16}\ \text{cm}^{-3}$

$$p_o = \frac{\left(1.5\times10^{10}\right)^2}{3\times10^{16}} = 7.5\times10^3\ \text{cm}^{-3}$$

(b) n-type: $\sigma = e\mu_n n_o = \left(1.6\times10^{-19}\right)(1100)\left(5\times10^{16}\right) = 8.8\ (\Omega\text{-cm})^{-1}$

p-type: $\sigma = e\mu_p p_o = \left(1.6\times10^{-19}\right)(400)\left(2\times10^{16}\right) = 1.28\ (\Omega\text{-cm})^{-1}$

compensated: $\sigma = e\mu_n n_o = \left(1.6\times10^{-19}\right)(1000)\left(3\times10^{16}\right) = 4.8\ (\Omega\text{-cm})^{-1}$

(c) $J = \sigma \mathrm{E} \Rightarrow \mathrm{E} = \dfrac{J}{\sigma}$

n-type: $\mathrm{E} = \dfrac{120}{8.8} = 13.6$ V/cm

p-type: $\mathrm{E} = \dfrac{120}{1.28} = 93.75$ V/cm

compensated: $\mathrm{E} = \dfrac{120}{4.8} = 25$ V/cm

***5.43** In GaAs, the donor impurity concentration varies as $N_{d0}$ exp $(-x/L)$ for $0 \le x \le L$, where $L = 0.1$ $\mu$m and $N_{d0} = 5 \times 10^{16}$ cm$^{-3}$. Assume $\mu_n = 6000$ cm$^2$/V-s and $T = 300$ K. (*a*) Derive the expression for the electron diffusion current density versus distance over the given range of $x$. (*b*) Determine the induced electric field that generates a drift current density that compensates the diffusion current density.

(a) We have

$$J_{diff} = eD_n \frac{dn}{dx} = eD_n \frac{dN_d(x)}{dx} \quad = \frac{eD_n}{(-L)} \cdot N_{do} \exp\left(\frac{-x}{L}\right)$$

We have

$$D_n = \mu_n \left(\frac{kT}{e}\right) = (6000)(0.0259) \quad D_n = 155.4 \text{ cm}^2/\text{s}$$

Then

$$J_{diff} = \frac{-(1.6\times10^{-19})(155.4)(5\times10^{16})}{(0.1\times10^{-4})} \exp\left(\frac{-x}{L}\right) \qquad J_{diff} = -1.243\times10^5 \exp\left(\frac{-x}{L}\right) \text{ A/cm}^2$$

(b)

$$0 = J_{drf} + J_{diff}$$

$$J_{drf} = e\mu_n n\text{E} \ = (1.6\times10^{-19})(6000)(5\times10^{16})\left[\exp\left(\frac{-x}{L}\right)\right]\text{E}$$

$$J_{drf} = (48)\left[\exp\left(\frac{-x}{L}\right)\right]\text{E}$$

We have

$$J_{drf} = -J_{diff}$$

$$(48)\left[\exp\left(\frac{-x}{L}\right)\right]\text{E} = 1.243\times10^5 \exp\left(\frac{-x}{L}\right) \qquad \text{E} = 2.59\times10^3 \text{ V/cm}$$

**7.8** (*a*) Consider a uniformly doped silicon pn junction at $T = 300$ K. At zero bias, 25 percent of the total space charge region is in the n-region. The built-in potential barrier is $V_{bi} = 0.710$ V. Determine (*i*) $N_a$, (*ii*) $N_d$, (*iii*) $x_n$, (*iv*) $x_p$, and (*v*) $|\text{E}_{\max}|$. (*b*) Repeat part (*a*) for a GaAs pn junction with $V_{bi} = 1.180$ V.

$$x_n = 0.25W = 0.25(x_n + x_p)$$

$$0.75x_n = 0.25x_p \Rightarrow \frac{x_p}{x_n} = 3$$

$$x_n N_d = x_p N_a \Rightarrow \frac{N_d}{N_a} = \frac{x_p}{x_n} = 3 \qquad N_d = 3N_a$$

(a) $V_{bi} = (0.0259)\ln\left[\frac{N_a N_d}{(1.5\times10^{10})^2}\right]$

$$0.710 = (0.0259)\ln\left[\frac{3N_a^2}{(1.5\times10^{10})^2}\right]$$

or $3N_a^2 = (1.5\times10^{10})^2 \exp\left(\frac{0.710}{0.0259}\right)$

which yields $N_a = 7.766\times10^{15}$ cm$^{-3}$ $\quad N_d = 2.33\times10^{16}$ cm$^{-3}$

$$x_n = \left\{\frac{2\in_s V_{bi}}{e}\left(\frac{N_a}{N_d}\right)\left(\frac{1}{N_a+N_d}\right)\right\}^{1/2} = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}}\times\left(\frac{1}{3}\right)\left[\frac{1}{4(7.766\times10^{15})}\right]\right\}^{1/2}$$

$\Rightarrow x_n = 9.93\times10^{-6}$ cm

or $x_n = 0.0993\,\mu$m

$$x_p = \left\{\frac{2(11.7)(8.85\times10^{-14})(0.710)}{1.6\times10^{-19}}\times\left(\frac{3}{1}\right)\left[\frac{1}{4(7.766\times10^{15})}\right]\right\}^{1/2} = 2.979\times10^{-5}\text{ cm}$$

or $x_p = 0.2979\,\mu$m

$$|\mathrm{E}_{\max}| = \frac{eN_d x_n}{\in_s} = \frac{(1.6\times10^{-19})(2.33\times10^{16})(0.0993\times10^{-4})}{(11.7)(8.85\times10^{-14})} = 3.58\times10^4\text{ V/cm}$$

(b) From part (a), we can write

$$3N_a^2 = (1.8\times10^6)^2 \exp\left(\frac{1.180}{0.0259}\right)$$

which yields $N_a = 8.127\times10^{15}$ cm$^{-3}$

$N_d = 2.438\times10^{16}$ cm$^{-3}$

$$x_n = \left\{\frac{2(13.1)(8.85\times10^{-14})(1.180)}{1.6\times10^{-19}}\times\left(\frac{1}{3}\right)\left[\frac{1}{4(8.127\times10^{15})}\right]\right\}^{1/2}$$

$= 1.324\times10^{-5}$ cm

or $x_n = 0.1324\,\mu$m

$$x_p = \left\{\frac{2(13.1)(8.85\times10^{-14})(1.180)}{1.6\times10^{-19}}\times\left(\frac{3}{1}\right)\left[\frac{1}{4(8.127\times10^{15})}\right]\right\}^{1/2} = 3.973\times10^{-5}\text{ cm}$$

or $x_p = 0.3973\,\mu$ m

$$|E_{max}| = \frac{eN_d x_n}{\in_s} = \frac{(1.6\times10^{-19})(2.438\times10^{16})(0.1324\times10^{-4})}{(13.1)(8.85\times10^{-14})} = 4.45\times10^4 \text{ V/cm}$$

**7.19** A silicon $n^+p$ junction is biased at $V_R = 5$ V. (*a*) Determine the change in built-in potential barrier if the doping concentration in the p region increases by a factor of 3. (*b*) Determine the ratio of junction capacitance when the acceptor doping is $3N_a$ compared to that when the acceptor doping is $N_a$. (*c*) Why does the junction capacitance increase when the doping concentration increases?

(a) $V_{bi}(3N_a) - V_{bi}(N_a)$

$$= V_t \ln\left[\frac{N_d(3N_a)}{n_i^2}\right] - V_t \ln\left[\frac{N_d N_a}{n_i^2}\right] = V_t\left\{\ln(3) + \ln\left[\frac{N_d N_a}{n_i^2}\right]\right\} - V_t \ln\left[\frac{N_d N_a}{n_i^2}\right] = V_t \ln(3) = (0.0259)\ln(3)$$

$$= 0.02845 \text{ V}$$

(b) $C' \cong \left\{\frac{e \in_s N_a}{2(V_{bi} + V_R)}\right\}^{1/2}$

So $\frac{C'(3N_a)}{C'(N_a)} = \left\{\frac{3N_a}{N_a}\right\}^{1/2} = \sqrt{3} = 1.732$

(c) For a larger doping, the space charge width narrows which results in a larger capacitance.

**7.22** Consider a uniformly doped GaAs pn junction at $T = 300$ K. The junction capacitance at zero bias is $C_j(0)$ and the junction capacitance with a 10-V reverse-biased voltage is $C_j(10)$. The ratio of the capacitances is

$$\frac{C_j(0)}{C_j(10)} = 3.13$$

Also under reverse bias, the space charge width into the p region is 0.2 of the total space charge width. Determine (*a*) $V_{bi}$ and (*b*) $N_a$, $N_d$.

(a) We have

$$\frac{C_j'(0)}{C_j'(10)} = \frac{\left[\frac{\in_s N_a N_d}{2(V_{bi})(N_a + N_d)}\right]^{1/2}}{\left[\frac{\in_s N_a N_d}{2(V_{bi} + V_R)(N_a + N_d)}\right]^{1/2}} \qquad \frac{C_j'(0)}{C_j'(10)} = 3.13 = \left(\frac{V_{bi} + V_R}{V_{bi}}\right)^{1/2}$$

For $V_R = 10$ V, we find

$$(3.13)^2 V_{bi} = V_{bi} + 10 \qquad V_{bi} = 1.137 \text{ V}$$

(b) $x_p = 0.2W = 0.2(x_p + x_n)$

$$\frac{x_p}{x_n} = 0.25 = \frac{N_d}{N_a}$$

$$V_{bi} = V_t \ln\left(\frac{N_a N_d}{n_i^2}\right)$$

$$1.137 = (0.0259)\ln\left[\frac{0.25 N_a^2}{(1.8\times10^6)^2}\right]$$

$$N_a = \frac{1.8\times10^6}{\sqrt{0.25}}\exp\left[\frac{1.137}{2(0.0259)}\right]$$

$$N_a = 1.23\times10^{16}\ \text{cm}^{-3}$$

$$N_d = 3.07\times10^{15}\ \text{cm}^{-3}$$

**8.5** Consider a GaAs pn junction with doping concentrations $N_a = 5 \times 10^{16}$ cm$^{-3}$ and $N_d = 10^{16}$ cm$^{-3}$. The junction cross-sectional area is $A = 10^{-3}$ cm$^2$ and the applied forward-bias voltage is $V_a = 1.10$ V. Calculate the (*a*) minority electron diffusion current at the edge of the space charge region, (*b*) minority hole diffusion current at the edge of the space charge region, and (*c*) total current in the pn junction diode.

(a) $$J_n(-x_p) = \frac{eD_n n_{po}}{L_n}\exp\left(\frac{V_a}{V_t}\right) = \frac{en_i^2}{N_a}\sqrt{\frac{D_n}{\tau_{no}}}\cdot\exp\left(\frac{V_a}{V_t}\right) = \frac{(1.6\times10^{-19})(1.8\times10^6)^2}{5\times10^{16}}\sqrt{\frac{205}{5\times10^{-8}}}\times\exp\left(\frac{1.10}{0.0259}\right)$$

$$= 1.849\ \text{A/cm}^2$$

$$I_n = AJ_n(-x_p) = (10^{-3})(1.849)\ \text{A}$$

or $I_n = 1.85$ mA

(b) $$J_p(x_n) = \frac{eD_p p_{no}}{L_p}\exp\left(\frac{V_a}{V_t}\right) = \frac{en_i^2}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\cdot\exp\left(\frac{V_a}{V_t}\right) = \frac{(1.6\times10^{-19})(1.8\times10^6)^2}{10^{16}}\sqrt{\frac{9.80}{10^{-8}}}\times\exp\left(\frac{1.10}{0.0259}\right)$$

$$= 4.521\ \text{A/cm}^2$$

$$I_p = AJ_p(x_n) = (10^{-3})(4.521)\ \text{A}$$

or $I_p = 4.52$ mA

(c) $I = I_n + I_p = 1.85 + 4.52 = 6.37$ mA

**8.8** A one-sided p$^+$n silicon diode has doping concentrations of $N_a = 5 \times 10^{17}$ cm$^{-3}$ and $N_d = 8 \times 10^{15}$ cm$^{-3}$. The minority carrier lifetimes are $\tau_{n0} = 10^{-7}$ s and $\tau_{p0} = 8 \times 10^{-8}$ s. The cross-sectional area is $A = 2 \times 10^{-4}$ cm$^2$. Calculate the (*a*) reverse-biased saturation current, and (*b*) the forward-bias current at (*i*) $V_a = 0.45$ V, (*ii*) $V_a = 0.55$ V, and (*iii*) $V_a = 0.65$ V.

(a) $J_s = en_i^2\left[\frac{1}{N_a}\sqrt{\frac{D_n}{\tau_{no}}} + \frac{1}{N_d}\sqrt{\frac{D_p}{\tau_{p0}}}\right] = \left(1.6\times10^{-19}\right)\left(1.5\times10^{10}\right)^2 \times \left[\frac{1}{5\times10^{17}}\sqrt{\frac{25}{10^{-7}}} + \frac{1}{8\times10^{15}}\sqrt{\frac{10}{8\times10^{-8}}}\right]$

$J_s = 5.145\times10^{-11}$ A/cm $^2$

$I_s = AJ_s = \left(2\times10^{-4}\right)\left(5.145\times10^{-11}\right) = 1.029\times10^{-14}$ A

(b) $I = I_s \exp\left(\frac{V_a}{V_t}\right)$

(i) $I = \left(1.029\times10^{-14}\right)\exp\left(\frac{0.45}{0.0259}\right) = 3.61\times10^{-7}$ A

(ii) $I = \left(1.029\times10^{-14}\right)\exp\left(\frac{0.55}{0.0259}\right) = 1.72\times10^{-5}$ A

(iii) $I = \left(1.029\times10^{-14}\right)\exp\left(\frac{0.65}{0.0259}\right) = 8.16\times10^{-4}$ A