$\rightarrow$ $y' = \frac{1}{x+\sin y}$ diferansiyel denklemin genel çözümünü bulunuz

1. $\frac{dy}{dx} = \frac{1}{x+\sin y}$

$\frac{dx}{dy} = x + \sin y$

$\frac{dx}{dy} - x = \sin y$ lineer

$\lambda(y) = e^{\int -dy} = e^{-y}$

$x = \frac{1}{\lambda(y)} \left[\int q(y)\lambda(y)\,dy + c \right]$

$= \frac{1}{e^{-y}} \left[\int \sin y \cdot e^{-y}\,dy + c \right]$

$= e^{y}\left[-e^{-y}\sin y + \int e^{-y}\cos y\,dy + c \right]$

$= e^{y}\left[-e^{-y}\sin y - e^{-y}\cos y - \int e^{-y}\sin y\,dy + c \right]$

$= e^{y}\left[-e^{-y}\sin y - \frac{1}{2}e^{-y}\cos y + c \right]$

$= e^{y}\left(-\frac{1}{2}e^{-y}\sin y - \frac{1}{2}e^{-y}\cos y + ce^{y} \right)$

$x = -\frac{1}{2}\sin y - \frac{1}{2}\cos y + ce^{y}$

$\sin y = u$
$\cos y\,dy = du$
$e^{-y}dy = dv$
$-e^{-y} = v$

$\cos y = u$, $-\sin y\,dy = du$
$e^{-y}dy = dv$, $-e^{-y} = v$

$I = \int e^{-y}\sin y\,dy = -e^{-y}\sin y - e^{-y}\cos y - \underbrace{\int e^{-y}\sin y\,dy}_{I}$

$I = -\frac{1}{2}(e^{-y}\sin y + e^{-y}\cos y)$

$\frac{y}{x} = u \quad y = xu \quad y' = u + xu'$

$\frac{dy}{dx} = \frac{1+e^{x/y}}{(\frac{x}{y}-1)e^{x/y}}$

$u + xu' = \frac{1+e^{1/u}}{(\frac{1}{u}-1)e^{1/u}}$

$\cancel{u} + xu' = \frac{(1+e^{\frac{1}{u}})u}{(1-u)e^{1/u}} - u$

$xu' = \frac{u + \cancel{ue^{1/u}} - \cancel{ue^{1/u}} + u^2 e^{1/u}}{(1-u)e^{1/u}}$

$1 + 2ue^{1/u} - \frac{1}{u^2}e^{1/u} \cdot u^2$

$\frac{dx}{x} = \frac{(1-u)e^{1/u}}{u + u^2 e^{1/u}}$

isinin altında, x-ek...

Diferansiyel Denklemler   25.10.2011

Uygulama

1) $xy' + 2y = 1$ diferansiyel denklemi ve $y(-1) = 2$ başlangıç koşuluyla oluşturulan başlangıç değer problemini çözünüz.

1. yol

$$x\frac{dy}{dx} + (2y-1) = 0 \quad \text{Değ. Ayrılabilir denk.}$$

$$\int\frac{dy}{2y-1} + \int\frac{dx}{x} = \int 0 \Rightarrow \frac{1}{2}\ln(2y-1) + \ln x = \ln c$$

$$x.\sqrt{2y-1} = c$$

$$\left.\begin{array}{l} x=-1 \\ y=2 \end{array}\right\} \Rightarrow -\sqrt{3} = c \Rightarrow x\sqrt{2y-1} = -\sqrt{3}$$

2. yol

$$y' + \frac{2y}{x} = \frac{1}{x} \quad \text{Lineer Dif. Denk.}$$

$$p(x) = \frac{2}{x} \qquad \lambda = e^{\int p(x)\,dx} \Rightarrow \lambda = e^{\int\frac{2dx}{x}} \Rightarrow \lambda = x^2$$

$$q(x) = \frac{1}{x} \qquad y = \frac{1}{\lambda}\left[\int q(x)\,\lambda\,dx + c\right]$$

$$y = \frac{1}{x^2}\left[\int \frac{1}{x}.x^2\,dx + c\right]$$

$$y = \frac{1}{x^2}\left[\frac{x^2}{2} + c\right] \Rightarrow y = \frac{1}{2} + \frac{c}{x^2}$$

$$\left.\begin{array}{l} x=-1 \\ y=2 \end{array}\right\} \Rightarrow 2 = \frac{1}{2} + c \Rightarrow c = \frac{3}{2} \Rightarrow y = \frac{1}{2} + \frac{3}{2x^2}$$

$$2y = 1 + \frac{3}{x^2}$$

$$\Rightarrow x^2(2y-1) = 3$$

$xy''' - y'' = 0$

$y'' = t \qquad y'' = t'$

$xt' - t = 0$

$$x \frac{dt}{dx} = t$$

$$\frac{dt}{t} = \frac{dx}{x}$$

$$\ln t = \ln x + \ln c_1$$

$$t = x c_1$$

$$\frac{d^2y}{dx^2} = x c_1$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = x c_1$$

$$\int d\left(\frac{dy}{dx}\right) = \int x c_1 \, dx$$

$$\frac{dy}{dx} = \frac{x^2}{2} c_1 + c_2$$

$$\underline{y = \frac{x^3}{6} c_1 + c_2 x + c_3}$$

$2y' - 3y = e^{-x}$

$2r - 3 = 0 \qquad r_1 = 3 \quad r_2 = -1$

S-2 $y^2 + xyy' = 0$ diferansiyel denklemini integral çarpanı bularak çözünüz.

$$xyy' + y^2 = 0$$

$$y' + \frac{y^2}{xy} = 0$$

$$\frac{dy}{dy} = -\frac{y}{x}$$

$$y' + \frac{y}{x} = 0$$

$$y' + p(x)y = q(x)$$ Lineer dif

$$\ln \lambda = \int p(x)\,dx$$

$$\ln \lambda = \int \frac{1}{x}\,dx$$

$$\ln \lambda = \ln x$$

$$\lambda = x$$

$$y' + \frac{y}{x} = 0$$

$$y'x + y = 0$$

$$xy' + y = 0$$

$$\frac{d}{dx}(xy) = 0$$

$$\int d(xy) = \int dx \cdot 0$$

$xy = c$ $y = \frac{c}{x}$

x (x) x

C-1 $y = Ax + B/x$ eğri ailesine ilişkin diferansiyel denklemi bulunuz.

$y = Ax + B/x \Rightarrow xy = Ax^2 + B$ (1)

(1) ifadesinin türevi alınırsa $\Rightarrow y + xy' = 2Ax$ (2) 15

(2) ifadesinin türevi alınırsa $\Rightarrow y' + y' + xy'' = 2A$ (3) 15

(2) ve (3) ifadelerinden A çekilir ve birbirlerine eşitlenirse;

$$\frac{y+xy'}{2x} = A = \frac{y'+y'+xy''}{2} \Rightarrow \frac{y+xy'}{2x} = \frac{2y'+xy''}{2}$$

$$x^2y'' + xy' - y = 0$$ 20

bulunur.

C-2 $y^2 + xyy' = 0$ diferansiyel denklemini integral çarpanı bularak çözünüz.

$$\left.\begin{aligned} & y^2dx + xydy = 0 \\ & M(x,y) = y^2 \ \ ve \ \ N(x,y) = xy \\ & \frac{\partial M}{\partial y} = 2y, \quad \frac{\partial N}{\partial x} = y, \ M_y \neq N_x \end{aligned}\right\}$$ 10 puan

birbirine eşit olmadığından tam değildir.

$$\frac{1}{M}(M_y - N_x) = \frac{2y-y}{y^2} = \frac{1}{y}$$

$$\lambda(x,y) = e^{-\int \frac{1}{y}dy} = \frac{1}{y}$$ 15

ile denklemi carptığımızda tam olur.

ydx+xdy=0 5

$$u(x,y) = \int ydx + h(y) = xy + h(y)$$ 5

$$\frac{\partial u}{\partial y} = x + h'(y) = x \ \ ise \ \ h(y) = c$$ 10

Genel çözüm

$$u(x,y) = xy + c$$ 5

$xy + c = K$

$\boxed{xy = c_1}$

diğer yolları 20 puan pay edilir

YÖK nun 2547 sayılı Kanunun **Öğrenci Disiplin Yönetmeliğinin** 9. Maddesi olan ***"Sınavlarda kopya yapmak*** ***buna teşebbüs etmek"*** fiili işleyenler **bir veya iki yarıyıl uzaklaştırma** cezası alırlar.

1) a) $y = c_1 x + c_2 x \ln x$ eğri ailesine ait olan diferansiyel denklemi bulunuz.

$$\left.\begin{aligned} y' &= c_1 + c_2 \ln x + c_2 \\ y'' &= \frac{c_2}{x} \end{aligned}\right\} \quad \begin{aligned} &x y' - y = x^2 y'' \\ &\boxed{x^2 y'' - x y' + y = 0} \end{aligned}$$

b) $\left(x + y\ln\left(\frac{x}{y}\right)\right)dx + x\ln\left(\frac{y}{x}\right)dy = 0$ diferansiyel denkleminin genel çözümünü bul

$$\left.\begin{aligned} x &\to tx \\ y &\to ty \end{aligned}\right\} \; t\left[x + y\ln\left(\frac{x}{y}\right)\right]dx + tx\ln\left(\frac{y}{x}\right)dy = 0 \quad \text{1. dereceden}$$

$$\left.\begin{aligned} \frac{y}{x} = u \Rightarrow y &= ux \\ dy &= udx + xdu \end{aligned}\right\} \left[x + ux\ln\left(\frac{1}{u}\right)\right]dx + x\ln u\left[udx + xdu\right]$$

$$(x - ux\ln u + ux\ln u)\,dx + x^2 \ln u\, du = 0$$

$$\int \frac{dx}{x} + \int \ln u\, du = \int 0$$

$$\ln x + u\ln u - u = \ln c$$

$$\ln x + \frac{y}{x}\ln\left(\frac{y}{x}\right) - \frac{y}{x} = \ln c$$

3-) $\frac{1}{x}y'+\frac{2}{x^2}y=-x(\operatorname{Sec}x)^2y^2$ diferansiyel denkleminin genel çözümünü bulunuz.

$y^{1-2}=y^{-1}=u$

$-y^{-2}y'=u'$ 4

$\frac{1}{x}y'y^{-2}+\frac{2}{x^2}y^{-1}=-x\sec^2 x$ 2

$-\frac{1}{x}u'+\frac{2}{x^2}u=-x\sec^2 x$ 2 Lineer Dif. Denk

$\boxed{u'-\frac{2}{x}u=x^2\sec^2 x}$ 2

$\lambda(x)=e^{\int-\frac{2}{x}dx}=e^{-2\ln x}=\frac{1}{x^2}$ 3

$\underbrace{\frac{u'}{x^2}-\frac{2}{x^3}u}=\sec^2 x$ 3

$\frac{d}{dx}\left(\frac{u}{x^2}\right)=\sec^2 x \rightarrow \frac{u}{x^2}=\int\sec^2 x\,dx=\tan x+c$ 3

$u=x^2\tan x+cx^2$ 3

$y=\frac{1}{u}=\frac{1}{x^2\tan x+cx^2}$ 3

2-) **a)** $2yy' + 4x^3\sqrt{1-y^4} = 0$ diferansiyel denkleminin $y(1) = 0$ özel çözümünü bulunuz.

$2y\,dy + 4x^3\sqrt{1-y^4}\,dx = 0$ Değişkenlere ayrılabilen dif. denk

$$\int \frac{2y\,dy}{\sqrt{1-y^4}} + \int 4x^3\,dx = \int 0$$

$$\left.\begin{array}{l} y^2 = t \\ 2y\,dy = dt \end{array}\right\} \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t = \arcsin y^2$$

$$\arcsin y^2 + x^4 = c$$

$$y(1) = 0 \Rightarrow \arcsin 0 + 1 = c \Rightarrow \boxed{c = 1}$$

b) $\frac{y}{x} + y'(2y + \ln x) = -x^2$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\underbrace{(2y + \ln x)}_{N}\,dy + \underbrace{\left(\frac{y}{x} + x^2\right)}_{m}\,dx = 0$$

$$\frac{\partial m}{\partial y} = \frac{1}{x} = \frac{\partial N}{\partial x}$$ Tam Dif. Denk.

$$\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy = m\,dx + N\,dy$$

$$\left.\begin{array}{l} \dfrac{\partial f}{\partial x} = \dfrac{y}{x} + x^2 \\ \dfrac{\partial f}{\partial y} = 2y + \ln x \end{array}\right\} \begin{array}{l} f(x,y) = y\ln x + \dfrac{x^3}{3} + h(y) \\ \dfrac{\partial f}{\partial y} = \ln x + \dfrac{dh}{dy} = 2y + \ln x \end{array}$$

$$\frac{dh}{dy} = 2y \Rightarrow h = y^2 + c$$

$$f(x,y) = y\ln x + \frac{x^3}{3} + y^2 + c = k$$

Baş

$$y' + \frac{1}{x+1} \cdot y = e^{-x}$$

$$\ln \lambda = e^{\int \frac{1}{x+1} dx} = e^{\ln|x+1|}$$

$$\boxed{\lambda = x+1}$$ 10

$$(x+1) \cdot y' + 1 \cdot y = (x+1)e^{-x}$$

$$\int [(x+1) \cdot y]' dx = \int (x+1) \cdot e^{-x} dx$$ 20

$$(x+1) \cdot y = -(x+1)e^{-x} + \int e^{-x} dx$$

$$(x+1) \cdot y = -(x+1)e^{-x} - e^{-x} + K$$

$$(x+1) \cdot y = -(x+2) \cdot e^{-x} + K$$ 10

$$y = -\frac{(x+2)}{x+1} \cdot e^{-x} + \frac{K}{x+1}$$

$f(0) = -2$

$$-2 = -2 \cdot 1 + \frac{K}{1} \Rightarrow K = 0$$

10

$$\boxed{y = -\frac{(x+2)}{x+1} \cdot e^{-x}}$$

işaret hatası → 30

4) $(y')^{3} - 6y = 0$ denk.nin genel [cut off]

$y' = p \Rightarrow \quad y = \frac{1}{6}p^3 \Rightarrow \quad p = \frac{1}{2}p^2\frac{dp}{dx} \Rightarrow p\left(1 - \frac{1}{2}p\frac{dp}{dx}\right)$

$\Rightarrow \boxed{p=0}$ (1) yada $\boxed{p$ [cut off]

$\Rightarrow \boxed{y=0}$ (1), $\boxed{p^2 = 4x + 2c}$ (2) $\Rightarrow \boxed{36y^2 = (4x+2c)^3}$ // genel

5) $y = x(1+y') + y'^2$ $\rightarrow x(1+p)+p^2$ dif. denk.ini çözünüz. (parametrik [cut off]

$\boxed{y' = p \Rightarrow \quad y'' = p'}$ (3) $\cancel{y'} = 1 + \cancel{y'} + xy'' + 2y'y'' \Rightarrow (x+2p)$ [cut off]

$\Rightarrow \boxed{\frac{dx}{dp} + x + 2p = 0}$ lin.dif.denk. (2) $\quad \frac{dx}{dp} + x = 0 \Rightarrow \frac{dx}{x} = -$ [cut off]

$\Rightarrow \ln x = -p + \ln c \Rightarrow \boxed{x$ [cut off]

$\Rightarrow \frac{dx}{dp} = \frac{dc}{dp}\cdot e^{-p} - e^{-p}c(p) \Rightarrow \frac{dc}{dp}e^{-p} - \cancel{e^{-p}c(p)} + \cancel{c(p)e^{-p}} + 2$ [cut off]

$\Rightarrow \frac{dc}{dp} = -2pe^{p} \Rightarrow c = -2\int pe^{p}dp = -2\left(pe^{p} - \int e^{p}dp\right) = \boxed{-$ [cut off]

$\boxed{\begin{array}{l} p = u \Rightarrow dp = du \\ e^{p}dp = dv \Rightarrow v = e^{p} \end{array}}$ $\Rightarrow \boxed{\begin{array}{l} x = -2(p-1) + ke^{-p} \\ y = 2 - p^2 + ke^{-p}(1+p) \end{array}}$ (5)

$y = x(1+p) + p^2$

$\quad -(2(1-p) + ke^{-p})(1+p) + p^2$

$$\Rightarrow \frac{y}{x}\, y - \left(\frac{y}{x}\right) \quad \ln\left(\frac{y}{x}\right)$$

$$\Rightarrow \left(\frac{y}{x}\,dy - \left(\frac{y}{x}\right)^2 dx\right) \ln\left(\frac{y}{x}\right) = dx \quad \text{Homoje}$$

$$\boxed{\frac{y}{x} = u} \Rightarrow y = ux \Rightarrow \boxed{dy = u\,dx + x\,du}$$

$$\Rightarrow (u(u\,dx + x\,du) - u^2 dx) \ln u = dx$$

$$\Rightarrow (u^2 \cancel{dx} + ux\,du - \cancel{u^2 dx}) \ln u = dx$$

$$\Rightarrow u \ln u\, du - \frac{dx}{x} = 0 \quad , \quad \boxed{t = \ln u} \Rightarrow dt =$$

$$\boxed{u\,du = ds} \Rightarrow \boxed{\frac{u^2}{2}}$$

$$\Rightarrow \int u \ln u\, du - \ln x = c$$

$$\Rightarrow \frac{u^2}{2} \ln u - \int \frac{u}{2}\, du - \ln x = c \Rightarrow \frac{u^2 \ln u}{2} - \frac{u^2}{4}$$

$$\Rightarrow 2u^2 \ln u - u^2 - 4 \ln x = 4c \Rightarrow \boxed{\left(\frac{y}{x}\right)^2 \left(2 \ln\left(\frac{y}{x}\right) - \right.}$$

2) $\underbrace{(a^2x^2 + y\cos x)}_{P(x,y)} dx + \underbrace{(\sin x + y^2)}_{Q(x,y)} dy = 0$ dif. d

$(a \in \mathbb{R})$

$$\Rightarrow \frac{\partial P}{\partial y} = \cos x = \frac{\partial Q}{\partial x} = \cos x \quad \text{Tam Dif.}$$

$$\Rightarrow (a^2x^2 + y\cos x)dx + (\sin x + y^2)\,dy = du(x,y) = \frac{\partial u}{\partial x}$$

$$\Rightarrow \frac{\partial u}{\partial x} = a^2x^2 + y\cos x \Rightarrow \int \partial u = \int (a^2x^2 + y\cos x)\,dx$$

$$\Rightarrow \boxed{u(x,y) = a^2 \frac{x^3}{3} + y\sin x}$$

$$\Rightarrow \frac{\partial u(x,y)}{\partial y} = \sin x + R'(y) = \sin x + y^2 \Rightarrow R'(y) = y^2 =$$

$$\quad a^2 \; 3 \qquad u^3 \qquad \boxed{a^2 3}$$

1. $y = ce^{\left(\frac{x}{y}\right)}$ eğri ailesine ait diferansiyel denklemi bulunuz.

$$y' = \left(\frac{y - xy'}{y^2}\right)ce^{\left(\frac{x}{y}\right)} \Rightarrow \frac{dy}{dx} = \left(\frac{y - x\dfrac{dy}{dx}}{y^2}\right).y \Rightarrow y.\frac{dx}{dy} = x + y$$

2) $(2x + y - 3)dx + (x + 2y - 3)dy = 0$ diferansiyel denkleminin genel çözümünü bulunuz.

$x = x_1 + h; \quad y = y_1 + k \Rightarrow dx = dx_1 \ dy = dy_1$

$[2(x_1 + h) + (y_1 + k) - 3]dx_1 + [(x_1 + h) + 2(y_1 + k) - 3]dy_1 = 0$

$[(2x_1 + y_1) + (2h + k - 3)]dx_1 + [(x_1 + 2y_2) + (h + 2k - 3)]dy_1 = 0$

$(2h + k - 3) = 0$

$h + 2k - 3) = 0 \Rightarrow h = 1; k = 1$

Alındığında homojen denklem

$(2x_1 + y_1)dx_1 + (x_1 + 2y_2)dy_1 = 0$ olur.

$$\frac{dy_1}{dx_1} = -\frac{(2x_1 + y_1)}{(x_1 + 2y_2)}$$

$$\frac{dy_1}{dx_1} = -\frac{x_1(2 + \dfrac{y_1}{x_1})}{x_1(1 + 2\dfrac{y_1}{x_1})}$$

$$\frac{y_1}{x_1} = t \Rightarrow y_1 = x_1 t \Rightarrow \frac{dy_1}{dx_1} = t + \frac{dt}{dx_1}x_1$$

$$t+\frac{dt}{dx_1}x_1=-\frac{(2+t)}{(1+2t)}$$

$$\frac{dt}{dx_1}x_1=-\frac{(2+t)}{(1+2t)}-t$$

$$\frac{dt}{dx_1}x_1=\frac{-2(t^2+t+1)}{(1+2t)}$$

$$\frac{dx_1}{x_1}=\frac{(1+2t)}{-2(t^2+t+1)}$$

$$\int\frac{dx_1}{x_1}=\int\frac{(1+2t)}{-2(t^2+t+1)}$$

$$\ln x_1-\ln c=-\frac{1}{2}\ln(t^2+t+1)$$

$$\frac{x_1}{c}=(t^2+t+1)^{-\frac{1}{2}}$$

$$x_1=\frac{c}{\sqrt{(t^2+t+1)}}$$

$$x_1=\frac{c}{\sqrt{(\frac{y_1}{x_1})^2+\frac{y_1}{x_1}+1)}}$$

$$x-1=\frac{c}{\sqrt{(\frac{y-1}{x-1})^2+\frac{y-1}{x-1}+1)}}$$

Not: Tam diferansiyel denklem olarak da çözebilir.

3) $y'^2 + (e^y - e^x)y' - e^{x+y} = 0$ dif. Denkleminin çözümünü bulunuz.
Bu denklem $(y' + e^y)(y' - e^x) = 0$ şeklinde yazılırsa birinci mertebeden ve birinci dereceden iki dif. Denklemin çarpımı olarak yazılmış olur. O halde

$$y' - e^x = 0$$
$$y' = e^x$$
$$dy = e^x dx$$
$$y = e^x + c$$
$$y - e^x - c = 0$$

$$y' - e^y = 0$$
$$y' = -e^y$$
$$dy = -e^y dx$$
$$e^{-y} dy = -dx$$
$$-e^{-y} = -x - c$$
$$e^{-y} = x + c$$
$$e^{-y} - x - c = 0$$

Olup genel çözüm

$$\boxed{(y - e^x - c)(e^{-y} - x - c) = 0}$$

olur

4) $2xydx + (y - x^2)dy = 0$ denklemini $y(0) = -e$ koşulu altında çözünüz.

$M = 2xy, N = y - x^2 \;\; M_y = 2x, \;\; N_x = -2x$

$$\frac{M_y - N_x}{-M} = \frac{N_x - M_y}{M} = \frac{-2x - 2x}{2xy} = -\frac{2}{y}$$ (sadece y'ye bağlı)

$$\Rightarrow \frac{t'(y)}{t(y)} = -\frac{2}{y} \Rightarrow \ln t = -2\ln y \Rightarrow t = y^{-2} = \frac{1}{y^2}$$ integral çarpanı

Yeni denklem

$\frac{2x}{y}dx+(\frac{1}{y}-\frac{x^2}{y^2})dy=0, \Rightarrow M_y=-\frac{2x}{y^2}$ $N_x=-\frac{2x}{y^2}$ Tam diferansiye

denklemdir. (1) $f_x=M=\frac{2x}{y}$ (2) $f_y=N=\frac{1}{y}-\frac{x^2}{y^2}$ olacak şekilde f

vardır. $f=\int\frac{2x}{y}dx+A(y)=\frac{x^2}{y}+A(y)$

(2) $f_y=N:-\frac{x^2}{y^2}+A'(y)=\frac{1}{y}-\frac{x^2}{y^2}\Rightarrow A'(y)=\frac{1}{y}$

$A(y)=\ln|y|+c_1\Rightarrow f=\frac{x^2}{y}+\ln|y|+c_1$

Genel Çözüm:

$\frac{x^2}{y}+\ln|y|+c_1=c_2\Rightarrow\frac{x^2}{y}+\ln|y|=c$

$koşul: x=0$

$y=-e \quad 0+\ln|-e|=c\Rightarrow c=\ln e=1$

$Çözüm: \frac{x^2}{y}+\ln|y|=1$

5) $y'''+4y''+4y'=0$ diferansiyel denkleminin genel çözümü

Karakteristik denklemi:

$r^3+4r^2+4r=0$

$r_1=0,\ r_{2,3}=-2$

$y=c_1+(c_2+c_3x)e^{-2x}$

2) $y' = \cot g^2(x+y)$ diferansiyel denkleminin genel çöz
bulunuz.

$$\frac{dy}{dx} = \operatorname{cotg}^2(x+y) \Rightarrow \operatorname{cotg}^2(x+y)dx - dy = 0$$

$$x + y = u \Rightarrow dx + dy = du$$

$$\operatorname{cotg}^2 u dx - (du - dx) = 0$$

$$\int (\operatorname{cotg}^2 u + 1)dx - \int du = 0$$

$$\int dx - \int \frac{du}{\operatorname{cotg}^2 u + 1} = 0$$

$$\int dx - \int \sin^2 u du = 0$$

$$x - \int \frac{1 - \cos 2u}{2} du = 0$$

$$x - \frac{1}{2}u + \frac{1}{2}\frac{1}{2}\sin 2u = c$$

$$x - \frac{1}{2}(x+y) + \frac{1}{2}\frac{1}{2}\sin 2(x+y) = c$$

$\frac{}{y} + \ln|y| = 1$

5) $y''' + 4y'' + 4y' + 3y = 0$ diferansiyel denkleminin

Karakteristik denklemi:

$$r^3 + 4r^2 + 4r + 3 = 0$$

$$r_1 = -3,\ r_{2,3} = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$$

$$y = c_1 e^{-3x} + e^{-\frac{1}{2}}\left(c_2 \cos \frac{\sqrt{3}}{2} x + c_3 \sin \frac{\sqrt{3}}{2} x\right)$$

1- Yonca
2- Ayten
3- Selmahan
4- M.E kitar

Ad:
Soyad:
Numara:

23.07.2009

1	2	3	4	toplam

0252311 KODLU DİFERANSİYEL DENKLEMLER 1.VİZE SORULARI

1) $\frac{y}{x}-\arcsin(\frac{\ln C}{x})=0$ verilen eğri ailesinin diferansiyel denklemini bulunuz.

2) $\frac{y}{2y'}-\frac{y^2y'^2}{2}=x$ diferansiyel denkleminin genel çözümünü bulunuz.

3) $\frac{y+\ln x}{x^2}dx=\frac{1}{x}dy$ diferansiyel denkleminin genel çözümünü bulunuz.

4) $y'''-5y''+8y'-4y=0$ diferansiyel denkleminin genel çözümünü bulunuz.

Süre:80 dak.dır

$$\frac{y'.x-1.y}{x^2}-\frac{-\frac{\ln C}{x^2}}{\sqrt{1-\left(\frac{\ln C}{x}\right)^2}}=0$$

$$\frac{y'.x-y}{x^2}-\frac{-\frac{1}{x}\sin\frac{y}{x}}{\sqrt{1-\sin^2\frac{y}{x}}}=0$$

$$y'\frac{1}{x}\cos\frac{y}{x}-\left(\cos\frac{y}{x}\right).\frac{y}{x^2}+\frac{1}{x}\sin\frac{y}{x}=0$$

$$\boxed{x.y'.\cos\frac{y}{x}-y.\cos\frac{y}{x}+x.\sin\frac{y}{x}=0}$$

②

$$x = \frac{y}{2y'} - \frac{y^2 y'^2}{2} \qquad y' = p \qquad x = \frac{y}{2p} - \frac{y^2 \cdot p^2}{2}$$

$$x = f(y, p) \qquad \frac{dx}{dy} = \frac{\partial f}{\partial y} + \frac{\partial f}{\partial p} \cdot \frac{\partial p}{\partial y}$$

$$\frac{dx}{dy} = \frac{1}{p} = \left(\frac{1}{2p} - \frac{\cancel{2} y p^2}{\cancel{2}}\right) + \left(-\frac{y}{2p^2} - \frac{\cancel{2} y^2 p}{\cancel{2}}\right)\frac{dp}{dy}$$

$$\left(\underbrace{\frac{1}{p} - \frac{1}{2p}} + y p^2\right) + \left(\frac{y}{2p^2} + y^2 p\right)\frac{dp}{dy} = 0$$

$$\left(\frac{1}{2p} + y p^2\right) + \frac{y}{p}\left(\frac{1}{2p} + \frac{y}{p}\right)\frac{dp}{dy} = 0$$

$$\left(\frac{1}{2p} + y p^2\right)\left(1 + \frac{y}{p} \cdot \frac{dp}{dy}\right) = 0$$

$$1 + \frac{y}{p}\frac{dp}{dy} = 0 \Rightarrow \frac{dy}{y} + \frac{dp}{p} = 0$$

$$\ln y + \ln p = \ln c$$

$$\boxed{y = \frac{c}{p}}$$

$$x = \frac{\frac{c}{p}}{2p} - \frac{\frac{c^2}{\cancel{p}} \cdot p^{\cancel{2}}}{2}$$

$$\boxed{x = \frac{c}{2p^2} - \frac{c^2 p}{2}}$$

(3) $\frac{y+\ln x}{x^2}dx = \frac{1}{x}dy$

$\frac{y+\ln x}{x^2}dx - \frac{1}{x}dy = 0$

$\frac{\partial P}{\partial y} = \frac{1}{x^2}$ $\quad \frac{\partial Q}{\partial x} = \frac{1}{x^2}$ $\quad$ T. D D

$\frac{\partial f}{\partial x} = \frac{y+\ln x}{x^2}$

$f(x,y) = \int \Big(\frac{y}{x^2} + \overbrace{\frac{\ln x}{x^2}}^{\text{Kısmi int.}}\Big)dx$

$= -\frac{y}{x} - \frac{1}{x}(\ln x + 1) + K(y)$

$\frac{\partial f}{\partial y} = -\cancel{\frac{1}{x}} + K'(y) = -\cancel{\frac{1}{x}}$

$K'(y) = 0 \quad K(y) = C_1$

$f(x,y) = -\frac{y}{x} - \frac{1}{x}(\ln x + 1) + C_1$

$-\frac{y}{x} - \frac{1}{x}(\ln x + 1) + C_1 = C$

veya $\quad -\frac{y}{x} - \frac{1}{x}(\ln x + 1) = C$

$$y''' - 5y'' + 8y' - 4y = 0$$

$$r^3 - 5r^2 + 8r - 4 = 0$$

$$r_1 = 1 \Rightarrow r_{2,3} = 2$$

$$y = c_1 e^x + c_2 e^{2x} + c_3 x e^{2x} \quad //$$

...renci Disiplin Yönetmeliğinin 9. Maddesi olan "Sınavlarda ...
...teşebbüs etmek" fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

1-) Find the general solution of $x^2y' = y - xy$.

$$x^2 \frac{dy}{dx} = y(1-x)$$

$$\frac{dy}{dx} = y\frac{(1-x)}{x^2} \quad 2$$

$$\int \frac{dy}{y} = \int \frac{(1-x)}{x^2} dx \quad 5$$

$$\ln y = \int \left(\frac{1}{x^2} - \frac{1}{x}\right) dx + c$$

5 5 5

$$\ln y = -\frac{1}{x} - \ln x + c$$

$$\ln y + \frac{1}{x} + \ln x = c \quad 3$$

1) $\left(x^2+xy\right)y'+3xy+y^2=0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\left(1+\frac{y}{x}\right)dy+\left(\frac{3y}{x}+\frac{y^2}{x^2}\right)dx=0$$ Homojen D.D.

$\boxed{\frac{y}{x}=u}$ 3 $y=ux$ $\boxed{dy=u\,dx+x\,du}$ 3

$(1+u)(u\,dx+x\,du)+(3u+u^2)\,dx=0$ 2

$x(1+u)\,du+(2u^2+4u)\,dx=0$ 2

$\boxed{\int\frac{1+u}{2u^2+4u}\,du=-\int\frac{dx}{x}}$ 3 $\quad u^2+2u=t$

$(2u+2)\,du=dt$ 2

$\frac{1}{4}\int\frac{dt}{t}=-\int\frac{dx}{x}$

3 2 1

$\ln|t|=-4\ln|x|+\ln c$

$\ln|u^2+2u|=-4\ln|x|+\ln c$ 1

$\left(\frac{y^2}{x^2}+\frac{2y}{x}\right)x^4=c$ 2

$\boxed{y^2x^2+2yx^3=c}$

$\underbrace{(3xy+y^2)}_{P}dx+\underbrace{(x^2+xy)}_{Q}dy=0$

$\frac{\partial P}{\partial y}=3x+2y\neq\frac{\partial Q}{\partial x}=2x+y$ 3

$\ln\lambda=\int\frac{\frac{\partial P}{\partial y}-\frac{\partial Q}{\partial x}}{Q}\,dx=\int\frac{1}{x}\,dx\Rightarrow\ln\lambda=\ln x$ 2

$\boxed{\lambda=x}$ 5

T.D.D.

$\boxed{(3x^2y+y^2x)\,dx+(x^3+x^2y)\,dy=0}$ 3

$\frac{\partial u}{\partial x}=3x^2y+y^2x\quad\frac{\partial u}{\partial y}=x^3+x^2y$ 2

$u(x,y)=\int(3x^2y+y^2x)\,dx=x^3y+\frac{x^2y^2}{2}+R(y)$ 3

$\frac{\partial u}{\partial y}=x^3+x^2y+R'(y)=x^3+x^2y$ 2

$R'(y)=0\quad\boxed{R(y)=K}$ 2

$u(x,y)=x^3y+\frac{x^2y^2}{2}+K=c_1$ 2 $2(c_1-K)=c$

$\boxed{x^2y^2+2yx^3=c}$ 1

b) $y''+2y'+5y=0,\ y(0)=2,\ y'(0)=-1$ başlangıç değer probleminin çözümünü ~~Laplace dönüşümünü~~ kullanarak bulunuz.

$r^2+2r+5=0 \qquad b^2-4ac=4-4.1.5=-16<0$

$r_{1,2}=\frac{-2\mp4i}{2}=-1\mp2i \qquad \alpha=-1 \quad \beta=2$

$y=e^{-x}\left[c_1\cos x+c_2\sin x\right]$

$2=e^{0}\left[c_1\cos 0+c_2\sin 0\right]\Rightarrow c_1=2$ //

$y'=-e^{-x}\left[c_1\cos x+c_2\sin x\right]+e^{-x}\left[-c_1\sin x+c_2 C\right.$

$y'(0)=-(c_1)+c_2=-1$

$-c_1+c_2=-1 \Rightarrow c_1=3$

özel çözüm

$y=e^{-x}(3\cos x+2\sin x)$ //

b) $y''+2y'+5y=0$, $y(0)=2$, $y'(0)=-1$ başlangıç değer probleminin çözümünü Laplace dönüşümünü kullanarak bulunuz.

$r^2+2r+5=0 \qquad b^2-4ac=4-4.1.5=-16<0$

$r_{1,2}=\dfrac{-2\mp 4i}{2}=-1\mp 2i \qquad \alpha=-1 \quad \beta=2$

$y=e^{-x}\left[c_1\cos x+c_2\sin x\right]$

$2=e^{0}\left[c_1\cos 0+c_2\sin 0\right]\Rightarrow \quad c_1=2$

$y'=-e^{-x}\left[c_1\cos x+c_2\sin x\right]+e^{-x}\left[-c_1\sin x+c_2\cos x\right]$

$y'(0)=-(c_1)+c_2=-1$

$-c_1+c_2=-1 \Rightarrow c_1=3$

özel çözüm

$y=e^{-x}(3\cos x+2\sin x)$