4a) $y''+2y'+y=2\cos 2x+3x+e^{x}$

$(\Delta^2+2\Delta+1)y=2\cos 2x+3x+e^{x}$

$a^2+2a+1=0$ (yardımcı denklem)

$(a+1)^2=0$

$a_1=a_2=-1$

5

$u=(c_1+c_2x)e^{-x}$ (tamamlayıcı fonksiyon)

$u=c_1.e^{-x}+c_2xe^{-x}$

$2\cos 2x$ için $A\sin 2x+B\cos 2x$

$3x$ için $Cx+D$

e^x için Ee^{x}

önerilirse

$V=A\sin 2x+B\cos 2x+Cx+D+Ee^{x}$

$V'=2A\cos 2x-2B\sin 2x+C+Ee^{x}$

$V''=-4A\sin 2x-4B\cos 2x+Ee^{x}$

denklemde yerine konulursa

$\Downarrow$

$-4A\sin 2x-4B\cos 2x+Ee^{x}+4A\cos 2x-4B\sin 2x+2C+2Ee^{x}+A\sin 2x+B\cos 2x+Cx+D+Ee^{x}$

$=2\cos 2x+3x+e^{x}$

5

$-3A\sin 2x-3B\cos 2x+4A\cos 2x-4B\sin 2x+4Ee^{x}+Cx+2C+D=2\cos 2x+3x+e^{x}$

$(-3A-4B)\sin 2x+(4A-3B)\cos 2x+Cx+(2C+D)+4Ee^{x}=2\cos 2x+3x+e^{x}$

4/ $-3A-4B=0$

3/ $4A-3B=2$

$-25B=6$

$B=-\frac{6}{25}$

$A=8/25$

$C=3$

$2C+D=0$

$D=-6$

$4E=1$

$E=1/4$

R5

$V=\frac{8}{25}\sin 2x-\frac{6}{25}\cos 2x+3x-6+\frac{1}{4}e^{x}$ bulunur

genel çözüm $(y=u+v)$'den

$y=c_1.e^{-x}+c_2xe^{-x}+\frac{8}{25}\sin 2x-\frac{6}{25}\cos 2x+3x-6+\frac{1}{4}e^{x}$

$x^2 y'' - xy' + y = x\ln x$

$x = e^t$

$y' = e^{-t} Dy$

$y'' = e^{-2t} D(D-1)y$ (5)

$\cancel{e^{2t}}\cancel{e^{-2t}}(D^2 - D)y - \cancel{e^t e^{-t}} Dy + y = e^t \ln e^t$

$(D^2 - 2D + 1)y = te^t$ (5)

$r^2 - 2r + 1 = 0 \quad (r-1)^2 = 0 \to r_{1,2} = 1$ (2) (2katlı kök)

$y_h = e^t(c_1 t + c_2)$ (3)

$y_p = (at+b)e^t \cdot t^2$ (özel durum) (3)

$y_p = (at^3 + bt^2)e^t$

$y_p' = (3at^2 + 2bt)e^t + (at^3 + bt^2)e^t$

$y_p'' = (6at + 2b)e^t + 2(3at^2 + 2bt)e^t + (at^3 + bt^2)e^t$

$6at + 2b + \cancel{6at^2} + \cancel{4bt} + \cancel{at^3} + \cancel{bt^2} - \cancel{6at^2} - \cancel{4bt} - \cancel{2at^3} - \cancel{2bt^2}$
$+ \cancel{at^3} + \cancel{bt^2} = t$

$6a = 1 \to a = 1/6$

$2b = 0 \to b = 0$

$y_p = \frac{1}{6}t^3$ (5)

$y = e^t(c_1 t + c_2) + \frac{1}{6}t^3$ (1)

$t = \ln x$

$y = (c_1 \ln$

2) $x^2y'' + xy' + y = \cos(\ln x)$ diferansiyel denkleminin genel çözümünü bulunuz.

5 $\begin{cases} x = e^t \text{ dönüşümü yapılırsa} \\ x^2y'' = D(D-1)y \quad , \quad xy' = Dy \quad ; \quad D = \frac{d}{dt} \end{cases}$

Denklemde yazılırsa,

$(D(D-1) + D + 1)y = \cos(\ln e^t)$

$(D^2+1)y = \cos t \rightarrow y'' + y = \cos t$ 5

5.

$r^2 + 1 = 0 \rightarrow r = \mp i \quad ; \quad y_h = C_1 \cos t + C_2 \sin t$

$y_{ö} = (A\cos t + B\sin t)t \quad , \quad y_{ö}' = (A\cos t + B\sin t) + (-A\sin t + B\cos t)t$

10

$y_{ö}'' = 2(-A\sin t + B\cos t) + (-A\cos t - B\sin t)t$

Denklemde yazılırsa

$2(-A\sin t + B\cos t) + \cancel{(-A\cos t - B\sin t)t} + \cancel{(A\cos t + B\sin t)t} = \cos t$

$-2A = 0 \rightarrow A = 0 \;//\; ; \quad 2B = 1 \rightarrow B = \frac{1}{2} \;//$

$y_{ö} = \frac{1}{2} t \sin t \quad ; \quad y = y_h + y_{ö} \rightarrow y = C_1\cos t + C_2 \sin t + \frac{1}{2} t \sin t$

$x = e^t \rightarrow t = \ln x$ 5

$\Longrightarrow y = C_1 \cos(\ln x) + C_2 \sin(\ln x) + \frac{1}{2}\ln x . \sin(\ln x) \;//$

3) $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, b_0, b_1, b_2, b_3, b_4, b_5$ birer reel sabit olduğuna göre

$a_0y^{(7)} + a_1y^{(6)} + a_2y^{(5)} + a_3y^{(4)} + a_4y^{(3)} + a_5y'' + a_6y' + a_7y = b_0 + b_1x + b_2e^{-x} + b_3e^{-3x} + b_4\cos 2x + b_5 \sin 2x,$

diferansiyel denklemi veriliyor. Diferansiyel denklemin karakteristik kökleri $r_1 = 0$, $r_2 = 0$, $r_3 = -2$, $r_4 = -1$, $r_5 = -1$, $r_6 = 2i$, $r_7 = -2i$ olduğuna göre diferansiyel denklemin genel çözüm formunu ifade ediniz. (Hesaplama yapmayınız).

$y_h = \underbrace{C_1 + C_2x + C_3 e^{-2x}}_{5} + \underbrace{(C_4 + C_5x)e^{-x}}_{5} + \underbrace{C_6\cos 2x + C_7 \sin 2x}_{5}$ 30

$y_{ö} = \underbrace{(A+Bx)x^2}_{5} + \underbrace{Cx^2e^{-x} + De^{-3x}}_{5} + \underbrace{x(E\cos 2x + F\sin 2x)}_{5}$

$y = y_h + y_{ö} \;//$

YTÜ - Fen-Edebiyat Fakültesi 2. Ara Sınav Soru ve Cevap Kağıdı		NOT TABLOSU						
		1. S	2. S	3. S				TOPLAM
Adı Soyadı								
Öğrenci Numarası	Grup No 1							
Bölümü			Sınav Tarihi		20 Ağustos 2015			
Dersin Adı	MAT2411 Diferansiyel Denklemler 2. Ara Sınavı	Sınav Süresi	45 dk	Sınav Yeri				
Dersi veren Öğretim Üyesinin Adı Soyadı			İmza					

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

1) $\sqrt{1+y^2}\,y'' = 2(y')^{3/2} y$, $y(\arctan\sqrt{8}) = \sqrt{8}$, $y'(\arctan\sqrt{8}) = 9$ başlangıç değer problemini çözünüz. (40P)

x 'in bulunmadığı denklem

$y' = p$, $y'' = p\dfrac{dp}{dy}$ 05

Denklemde yerine yazılırsa ; $\sqrt{1+y^2}\, p\dfrac{dp}{dy} = 2p^{3/2} y$

$p\left(\sqrt{1+y^2}\dfrac{dp}{dy} - 2p^{1/2}y\right) = 0$

$\boxed{\int \dfrac{dp}{\sqrt{p}} = \int \dfrac{2y}{\sqrt{1+y^2}}\,dy}$ 10 $\longrightarrow$ $\underline{2\sqrt{p} = 2\sqrt{1+y^2} + C_1}$ 10

Verilen koşulları kullanarak C_1 sabitini bulalım;

$2\sqrt{9} = 2\sqrt{1+(\sqrt{8})^2} + C_1 \Rightarrow \boxed{C_1 = 0}$ 05

$\Rightarrow \cancel{2}\sqrt{p} = \cancel{2}\sqrt{1+y^2}$ $\quad p = y'$ idi

$\dfrac{dy}{dx} = 1+y^2$

$\int dx = \int \dfrac{dy}{1+y^2}$ 05

$x = \arctan y + C_2$ koşulları kullanırsak

$\arctan\sqrt{8} = \arctan\sqrt{8} + C_2 \Rightarrow C_2 = 0$ bulunur

$\Rightarrow x = \arctan y \longrightarrow \underline{y = \tan x}$ //

05

B grubu

(B1) $(y+2)\,y''=(y')^2$ denkleminin genel çözümünü bulunuz.

$y'=p \Rightarrow y''=p\cdot\frac{dp}{dy}$

$(y+2)\,p\cdot\frac{dp}{dy}=p^2 \Rightarrow (y+2)\cdot p\cdot\frac{dp}{dy}-p^2=0$

$\Rightarrow p\left((y+2)\frac{dp}{dy}-p\right)=0$

1°) $p=0 \Rightarrow y'=0 \Rightarrow \boxed{y=c}$ tekil çözüm

2°) $(y+2)\frac{dp}{dy}-p=0 \Rightarrow \frac{dp}{p}-\frac{dy}{y+2}=0$

$\Rightarrow \ln p-\ln(y+2)=\ln c_1$

$\Rightarrow \frac{p}{y+2}=c_1 \Rightarrow p=(y+2)c_1 \Rightarrow \frac{dy}{dx}=(y+2)c_1$

$\Rightarrow \int\frac{dy}{y+2}=\int c_1\,dx \Rightarrow \ln(y+2)=c_1x+c_2$

$\Rightarrow y+2=e^{c_1x+c_2} \Rightarrow y=e^{c_1x+c_2}-2$

(B2)

$$y'' - 6y' + 9y = \frac{e^{3x}}{x^2}$$

$$r^2 - 6r + 9 = 0 \Rightarrow r_{1,2} = 3 \Rightarrow y = c_1 e^{3x} + c_2 x e^{3x}$$

$$c_1 = c_1(x), \quad c_2 = c_2(x)$$

$$y' = \underbrace{c_1' e^{3x} + c_2' x e^{3x}}_{0} + 3c_1 e^{3x} + c_2 e^{3x} + 3c_2 x e^{3x}$$

$$c_1' e^{3x} + c_2' x e^{3x} = 0 \Rightarrow \boxed{c_1' + c_2' x = 0}$$

$$y'' = \cancel{3c_1' e^{3x}} + c_2' e^{3x} + \cancel{3c_2' x e^{3x}} + 9c_1 e^{3x} + 3c_2 e^{3x} + 3c_2 e^{3x} + 9c_2 x$$

$$0 \qquad\qquad 6c_2 e^{3x}$$

$$(c_2' + 9c_1 + 6c_2 + 9c_2 x)\cancel{e^{3x}} - 6(3c_1 + c_2 + 3c_2 x)\cancel{e^{3x}} + 9(c_1 + c_2 x)\cancel{e^{3x}} = \frac{e}{x}$$

$$\boxed{c_2' = \frac{1}{x^2}}$$

$$\left.\begin{aligned} c_1' + c_2' x &= 0 \\ c_2' &= \frac{1}{x^2} \end{aligned}\right\} \Rightarrow c_2 = \int \frac{1}{x^2}\,dx = -\frac{1}{x} + K_2$$

$$c_1' + \frac{1}{x} = 0$$

$$c_1 = -\int \frac{1}{x}\,dx$$

$$c_1 = -\ln x + K_1$$

$$y = (-\ln x + K_1)e^{3x} + \left(-\frac{1}{x} + K_2\right)x e^{3x}$$

$$y = K_1 e^{3x} + K_2 x e^{3x} - (1 + \ln x)e^{3x}$$

$$3K_1 e^{3x} + K_2 e^{3x} + 3K_2 x e^{3x} -$$

(83) $x^2y'' + 2xy' - 6y = 0 \quad y(1) = 1, \; y'(1) = -6$

Euler $\Rightarrow \; x = e^t$

$\Rightarrow t = \ln x \Rightarrow \dfrac{dy}{dx} = e^{-t}\dfrac{dy}{dt}$

$$\frac{d^2y}{dx^2} = e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right)$$

$$\Rightarrow \underbrace{e^{2t} \cdot e^{-2t}}_{1}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + 2\underbrace{e^{t} \cdot e^{-t}}_{1}\frac{dy}{dt} - 6y = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} + \frac{dy}{dt} - 6y = 0 \quad \Rightarrow r^2 + r - 6 = 0$$

$$r_1 = -3 \quad r_2 = 2$$

$$\Rightarrow y = c_1 e^{-3t} + c_2 e^{2t} = c_1 \frac{1}{x^3} + c_2 x^2$$

$$y' = c_1 \frac{-3}{x^4} + 2c_2 x$$

$y(1) = 1 \Rightarrow 1 = c_1 + c_2 \Rightarrow c_2 = 1 - c_1$

$y'(1) = -6 \Rightarrow -6 = -3c_1 + 2c_2 \Rightarrow c_1 = 8/5 \quad c_2 = -3/5$

$$y = \frac{8}{5x^3} - \frac{3}{5}x^2$$

(B4) $y'' - 3y' + 2y = x^2+1 + e^x - \sin 2x$

$$\boxed{\begin{array}{l} r^2 - 3r + 2 = 0 \\ \quad -2 \\ \quad -1 \\ r_1 = 1 \ , \ r_2 = 2 \\ y_h = c_1 e^x + c_2 e^{2x} \end{array}}$$

(a) $y_1 = ax^2 + bx + c$

$y_1' = 2ax + b$

$y_1'' = 2a$

$2a - 3(2ax+b) + 2(ax^2+bx+c)$

$\equiv x^2+1$

$\underbrace{2a}_{1} x^2 + \underbrace{(-6a+2b)}_{0} x + \underbrace{(2a-3b+2c)}_{1}$

$$\boxed{a = \frac{1}{2} \quad b = \frac{3}{2} \quad c = 2}$$

(b) 2/ $y_2 = Axe^x \quad (r_1 = 1)$

$-3/ \ y_2' = Ae^x + Axe^x = (1+x)Ae^x$

$1/ \ y_2'' = Ae^x + Ae^x + Axe^x = (2+x)Ae^x$

$(2+x)A\cancel{e^x} - 3(1+x)A\cancel{e^x} + 2Ax\cancel{e^x} \equiv e^x$

$2A - 3A = 1 \Rightarrow \boxed{A = -1}$

(c) 2/ $y_3 = a \sin 2x + b \cos 2x$

$-3/ \ y_3' = 2a \cos 2x - 2b \sin 2x$

$1/ \ y_3'' = -4a \sin 2x - 4b \cos 2x$

$\underbrace{(-2a+6b)}_{-1} \sin 2x + \underbrace{(-6a-2b)}_{0} \cos 2x \equiv -\sin 2x$

$a = \frac{1}{20} \ , \ b = -\frac{3}{20}$

$y = c_1 e^x + c_2 e^{2x} + \left(\frac{1}{2}x^2 + \frac{3}{2}x + 2\right) - \left(xe^x\right) + \left(\frac{1}{20} \sin 2x - \frac{3}{20} \cos 2x\right)$

2.) $y''' - y'' + y' - y = e^x + 2\cos x - 4\sin x$ — Belirsiz kats. yönt

$r^3 - r^2 + r - 1 = 0 \quad r^2(r-1) + (r-1) = 0 \quad (r-1)(r^2+1) = 0$

$r_1 = 1 \quad r_{2,3} = \pm i$

$y_h = c_1 e^x + c_2 \cos x + c_3 \sin x$

$-1/ \; y_{\ddot{o}_1} = A x e^x$

$y' = A(1+x)e^x$

$-1/ \; y'' = A(2+x)e^x$

$+ \; y''' = A(3+x)e^x$

$2Ae^x = e^x \Rightarrow A = \frac{1}{2} \Rightarrow y_{\ddot{o}_1} = \frac{1}{2} x e^x$

$-1/ \; y_{\ddot{o}_2} = x(a\cos x + b\sin x)$

$y' = (a\cos x + b\sin x) + x(-a\sin x + b\cos x)$

$-1/ \; y'' = 2(-a\sin x + b\cos x) + x(-a\cos x - b\sin x)$

$+ \; y''' = 3(-a\cos x - b\sin x) + x(+a\sin x - b\cos x)$

$(-2a - 2b)\cos x + (-2b + 2a)\sin x = 2\cos x - 4\sin x$

$$\left.\begin{aligned} -\cancel{2a} - 2b &= 2 \\ -2b + \cancel{2a} &= -4 \end{aligned}\right\} \quad -4b = -2 \quad b = \frac{1}{2} \Rightarrow a = -\frac{3}{2}$$

$y_{\ddot{o}_2} = x\left(-\frac{3}{2}\cos x + \frac{1}{2}\sin x\right)$

$y_{\text{gen}} = c_1 e^x + c_2 \cos x + c_3 \sin x + \frac{1}{2} x e^x + x\left(-\frac{3}{2}\cos x + \frac{1}{2}\sin x\right)$

3.) $x^2y'' - 2xy' + 2y = 4x^3 . \ln x$ $\quad$ G.Ç. = ?

$$\left.\begin{aligned} x &= e^t \\ y' &= e^{-t} Dy \\ y'' &= e^{-2t} D(D-1)y \end{aligned}\right\}$$

$$\cancel{e^{2t}}\,\cancel{e^{-2t}}\, D(D-1)y - 2.\cancel{e^{t}}.\cancel{e^{-t}}\, Dy + 2y = 4e^{3t}.t$$

$$D^2y - Dy - 2Dy + 2y = 4te^{3t}$$

$$y'' - 3y' + 2y = 4te^{3t}$$

$$r^2 - 3r + 2 = (r-2)(r-1) = 0 \quad r_1 = 1 \quad r_2 = 2$$

$$y_h = c_1 e^t + c_2 e^{2t}$$

$$+2/ \quad y_ö = (At + B).e^{3t}$$

$$-3/ \quad y' = (\cancel{A} + 3\cancel{A}t + \cancel{3B})e^{3t}$$

$$+ \quad y'' = (\cancel{3A} + 3A + \cancel{9At} + \cancel{9B})e^{3t}$$

$$(2At + 2B + 3A)e^{3t} = 4te^{3t}$$

$$2A = 4 \quad A = 2 \;, \quad 2B + 3A = 0 \quad B = -3$$

$$y_ö = (2t - 3)e^{3t}$$

$$y_{GÇ} = c_1 e^t + c_2 e^{2t} + (2t - 3)e^{3t}$$

$$= c_1 x + c_2 x^2 + (2\ln x - 3)x^3$$

4) $(x-1)y'' + y' = (x-1)^2$ G.Ç ?

$y' = p \quad y'' = p'$

$(x-1)p' + p = (x-1)^2$

$p' + \frac{1}{x-1}p = (x-1)$ lineer d.d

$\lambda(x) = e^{\int \frac{1}{x-1}dx} = e^{\ln(x-1)} = (x-1)$

$(x-1)p' + p = (x-1)^2$

$\frac{d[(x-1)p]}{dx} = (x-1)^2$

$\int d[(x-1)p] = \int (x-1)^2 dx$

$(x-1)p = \frac{(x-1)^3}{3} + c_1$

$p = y' = \frac{(x-1)^2}{3} + \frac{c_1}{(x-1)}$

$y = \int \left[\frac{(x-1)^2}{3} + \frac{c_1}{(x-1)}\right] dx$

$= \frac{(x-1)^3}{9} + c_1 . \ln|x-1| + c_2$

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Soru 4 $\begin{array}{l} \dfrac{dx}{dt}+2x+3y=0 \\ \dfrac{dy}{dt}+2y+3x=2e^{2t} \end{array}$ denklem sistemini çözünüz.

$D = \dfrac{d}{dt}$

$$\begin{array}{ll} -3 / & (D+2)x+3y=0 \\ D+2 / & 3x+(D+2)y=2e^{2t} \end{array}$$

$$(D+2)^2 y - 9y = 4e^{2t} + 4e^{2t}$$

$$D^2y + 4Dy + 4y - 9y = 8e^{2t}$$

$$(D^2+4D-5)y = 8e^{2t} \quad 7/$$

$$(D^2+4D-5)x = -6e^{4}$$

5/

$$K(r) = r^2+4r-5=0$$

$-1 \quad 5$

$$r_1 = 1 \quad r_2 = -5$$

$$y_H = c_1e^{t} + c_2e^{-5t}$$

$$y_2 = 8\frac{e^{2t}}{D^2+4D-5} = 8 \cdot \frac{e^{2t}}{4+8-5} = \frac{8}{7}e^{2t} \quad 4/$$

$$\boxed{y = c_1e^{t}+c_2e^{-5t}+\frac{8}{7}e^{2t}}$$

~~10~~ $\Sigma 16$

(2. denklemde yerine yazarsak)

5/

$$3x + c_1e^{t} - 5c_2e^{-5t} + \frac{16}{7}e^{2t} + 2c_1e^{t} + 2c_2e^{-5t} + \frac{16}{7}e^{2t} = 2e^{2t}$$

5

$$3x + 3c_1e^{t} - 3c_2e^{-5t} + \frac{32}{7}e^{2t} = 2e^{2t}$$

5

$$\boxed{x = -c_1e^{t} + c_2e^{-5t} - \frac{6}{7}e^{2t}}$$

4

5

Soru 2 $2yy'' = 3yy' + (y')^2$ denkleminin genel çözümünü bulunuz ($y \geq 0$).

$y' = p \Rightarrow y'' = p\dfrac{dp}{dy}$

$2ypp' = 3yp + p^2 \Rightarrow p(2p'y - 3y - p) = 0$

1) $p = 0 \Rightarrow y' = 0 \Rightarrow y = c$

2) $2p'y - 3y - p = 0 \Rightarrow p' - \dfrac{3}{2} - \dfrac{p}{2y} = 0$

$p' - \dfrac{p}{2y} = \dfrac{3}{2}$ (Lineer d.)

$p' - \dfrac{p}{2y} = 0 \Rightarrow \dfrac{dp}{p} - \dfrac{dy}{2y} = 0$

$\ln p - \frac{1}{2}\ln y = \ln c \Rightarrow p = c\sqrt{y}$

$p' = c'\sqrt{y} + \dfrac{c}{2\sqrt{y}}$

$c'\sqrt{y} + \dfrac{c}{2\sqrt{y}} - \dfrac{c\cancel{\sqrt{y}}}{2\cancel{y}\sqrt{y}} = \dfrac{3}{2}$

$c' = \dfrac{3}{2}\dfrac{1}{\sqrt{y}} \Rightarrow \boxed{c = 3\sqrt{y} + k}$

$p = 3y + k\sqrt{y}$

$\dfrac{dy}{dx} = 3y + k\sqrt{y} \Rightarrow \displaystyle\int \frac{dy}{3y + k\sqrt{y}} = \int dx$ $\quad \left(\begin{array}{l} y = t^2 \\ dy = 2t\,dt \end{array} \right.$

$x + \ln A = \displaystyle\int \frac{2dt}{3t+k} \Rightarrow x + \ln A = \frac{2}{3}\ln|3t+k|$

$x = \ln \dfrac{|3t+k|^{2/3}}{A}$

$(3t+k)^{2/3} = Ae^x \Rightarrow 3t + k = Be^{\frac{3}{2}x}$ $\quad B = A^{2/3}$

$\boxed{3\sqrt{y} = Be^{3/2x} - k}$

S2) $y''-y=0$ d.d.nin $x=0$ noktası civarındaki kuvvet serisi çözümünü bulunuz

Ç: $y''-y=0$ d.d.nin bir çözümü

④ $y=\sum_{k=0}^{\infty} a_k x^k$ olsun.

② $y'=\sum_{k=1}^{\infty} a_k . k . x^{k-1}$; ② $y''=\sum_{k=2}^{\infty} a_k . k . (k-1) x^{k-2}$ } d.d.de yerine yazılırsa

② $\left\{ \sum_{k=2}^{\infty} a_k . k . (k-1) x^{k-2} - \sum_{k=0}^{\infty} a_k x^k = 0 \right.$

$\Rightarrow \sum_{k=0}^{\infty} a_{k+2} . (k+2)(k+1) x^k - \sum_{k=0}^{\infty} a_k x^k = 0$

③ $\Rightarrow \sum_{k=0}^{\infty} \left(a_{k+2} . (k+2)(k+1) - a_k\right) x^k = 0 \Rightarrow a_{k+2} . (k+2)(k+1) - a_k = 0$ $(k=0,1,2,\ldots)$

③ $\Rightarrow \left\{ a_{k+2} = \frac{a_k}{(k+2)(k+1)} \right.$; $k=0,1,2,\ldots$

④ $k=0 \Rightarrow a_2 = \frac{a_0}{2.1} = \frac{a_0}{2!}$; $k=1 \Rightarrow a_3 = \frac{a_1}{3.2} = \frac{a_1}{3!}$

$k=2 \Rightarrow a_4 = \frac{a_2}{4.3} = \frac{1}{4.3} . \frac{a_0}{2!} = \frac{a_0}{4!}$; $k=3 \Rightarrow a_5 = \frac{a_3}{5.4} = \frac{1}{5.4} . \frac{a_1}{3!} = \frac{a_1}{5!}$ .....

⑤ $y = \sum_{k=0}^{\infty} a_k x^k = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 + \ldots$

$\Rightarrow y = a_0 + a_1 x + \frac{a_0}{2!} . x^2 + \frac{a_1}{3!} . x^3 + \frac{a_0}{4!} . x^4 + \frac{a_1}{5!} . x^5 + \ldots$

$= a_0 \left[1 + \frac{1}{2!} . x^2 + \frac{1}{4!} . x^4 + \ldots \right] + a_1 \left[x + \frac{1}{3!} . x^3 + \frac{1}{5!} . x^5 + \ldots \right]$ veya

$\Rightarrow \boxed{y = a_0 . \sum_{k=0}^{\infty} \frac{x^{2n}}{(2n)!} + a_1 . \sum_{k=0}^{\infty} \frac{x^{2n+1}}{(2n+1)!}}$ d.d.in çözümüdür.

Dif. Denk. Final Sınavı / 9 Ocak 2014

S1) $x^3y'' + 2x^2y' = -1$ d.d.nin genel çözümünü bulunuz.

Ç: $\frac{1}{x}\left[x^3y'' + 2x^2y' = -1\right] \Rightarrow x^2y'' + 2xy' = \frac{-1}{x}$ Euler d.d

$$\left.\begin{aligned} x &= e^t \\ \frac{dy}{dx} &= e^{-t}\frac{dy}{dt} \\ \frac{d^2y}{dx^2} &= e^{-2t}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) \end{aligned}\right\} 5 \quad \text{d.d. yerine yazalım.}$$

$$\left.\begin{aligned} &\cancel{e^{2t}}.\cancel{e^{-2t}}\left(\frac{d^2y}{dt^2} - \frac{dy}{dt}\right) + 2\cancel{e^t}.\cancel{e^{-t}}.\frac{dy}{dt} = \frac{-1}{e^t} \\ &\Rightarrow y'' + y' = -e^{-t} \quad \text{s.k.l.d.d.} \end{aligned}\right\} 3$$

$$\left.\begin{aligned} r^2 + r = r(r+1) = 0 \\ r = 0 \quad r = -1 \end{aligned}\right\} 3 \qquad \left. y_1 = c_1 + c_2e^{-t} \right\} 2$$

$$\left.\begin{aligned} &3 \quad y_2 = Kte^{-t} \\ &1 \quad y_2' = K\left[e^{-t} - te^{-t}\right] = K(1-t)e^{-t} \\ &1 \quad y_2'' = K\left[-1.e^{-t} + (1-t)(-)e^{-t}\right] = K(t-2)e^{-t} \end{aligned}\right\} \text{d.d. yerine yazalım}$$

$$\left. K\left[\cancel{t} - 2 + 1 - \cancel{t}\right]\cancel{e^{-t}} = -\cancel{e^{-t}} \Rightarrow -K = -1 \Rightarrow \boxed{K = 1} \right\} 2$$

$$y_2 = te^{-t}$$

$$2 \left\{ y = c_1 + c_2e^{-t} + te^{-t} \right. \Rightarrow \underbrace{\boxed{y = c_1 + \frac{c_2}{x} + \frac{\ln x}{x}}}_{3} \text{ g.ç.}$$

4) $y \cdot y'' + (1+y)\cdot y'^2 = 0$ d.d. nin genel çözümünü bulunuz.

Ç: x-içermez $\left.\begin{array}{l} y' = p \\ y'' = p\cdot\dfrac{dp}{dy}\end{array}\right\}$ (4) d.d. yerine yazılırsa

$$y\cdot p\cdot\frac{dp}{dy} + (1+y)\cdot p^2 = 0 \Rightarrow p\cdot\left[y\cdot\frac{dp}{dy} + (1+y)p\right] = 0$$

$$\Rightarrow y\cdot\frac{dp}{dy} + (1+y)p = 0 \Rightarrow \int\frac{dp}{p} + \int\frac{1+y}{y}\,dy = 0 \quad (3)$$

11

(2) 13

$$\Rightarrow \ln|p| + y + \ln|y| = \ln c_1 \Rightarrow \ln\frac{p\cdot y}{c_1} = -y \quad (1)$$

14

$$\Rightarrow (1)\ \frac{p\cdot y}{c_1} = e^{-y} \Rightarrow p = \frac{c_1 e^{-y}}{y}\ (1) \Rightarrow \frac{dy}{dx} = \frac{c_1 e^{-y}}{y} \quad (3)$$

16   19

(2)

$$\Rightarrow \int y e^{y}\,dy = \int c_1\,dx \Rightarrow \boxed{(y-1)e^{y} = c_1 x + c_2}$$ genel çözüm

(4)

c_2 yoksa -2 puan.

$$p\left[y\cdot\frac{dp}{dy} + (1+y)p\right] = 0$$

$$y\cdot\frac{dp}{dy} + (1+y)p = 0 \qquad \int\frac{dp}{p} + \int\frac{1+y}{y}\,dy = 0$$

$$\ln|p| + y + \ln|y| = \ln c_1$$

$$\ln\frac{py}{c_1} = -y$$

$$\frac{py}{c_1} = e^{-y}$$

$$p = \frac{c_1 e^{-y}}{y}$$

Soru 3:

$$\left.\begin{aligned} & y'' + y = t \\ & y(0)=1;\ y'(0)=0 \end{aligned}\right\}$$ başlangıç değer problemini Laplace dönüşümünü kullanarak çözünüz.

Ç:

$$y'' + y = t \Rightarrow L(y''+y) = L(t) \quad 1$$

$$\Rightarrow L(y'') + L(y) = L(t) \Rightarrow \overbrace{s^2F(s) - s\underbrace{f(0)}_{1} - \underbrace{f'(0)}_{0}}^{5} + \overbrace{F(s)}^{1} = \overbrace{\frac{1}{s^2}}^{3}$$

$$\left.\begin{aligned} & \Rightarrow (s^2+1)F(s) - s = \frac{1}{s^2} \Rightarrow (s^2+1)F(s) = s + \frac{1}{s^2} \\ & \Rightarrow (s^2+1)F(s) = \frac{1+s^3}{s^2} \Rightarrow F(s) = \frac{1+s^3}{s^2(1+s^2)} \end{aligned}\right\} 2$$

$$\frac{1+s^3}{s^2(1+s^2)} = \frac{A}{s} + \frac{B}{s^2} + \frac{Cs+D}{1+s^2} \Bigg\} 3$$

$$s(1+s^2) \qquad 1+s^2 \qquad s^2$$

$$\Rightarrow 1+s^3 = As(1+s^2) + B(1+s^2) + s^2(Ct+D)$$

$$\Rightarrow 1+s^3 = As(1+s^2) + B(1+s^2) + s^2(Ct+D)$$

$$\Rightarrow 1+s^3 = As^3 + Cs^3 \ldots$$

$$\Rightarrow 1+s^3 = (A+C)s^3 + (B+D)s^2 + As + B$$

$$\Rightarrow \left\{\begin{aligned} & \boxed{A=0} \quad \boxed{C=1} \\ & \boxed{B=1} \quad \boxed{D=-1} \end{aligned}\right\} \Rightarrow F(s) = \frac{1}{s^2} + \frac{s}{s^2+1} - \frac{1}{s^2+1} \Bigg\} 1$$

2

$$\Rightarrow L^{-1}(F(s)) = L^{-1}\left(\frac{1}{s^2} + \frac{s}{s^2+1} - \frac{1}{s^2+1}\right) 3$$

$$\Rightarrow \boxed{y(t) = t + \underbrace{\cos t}_{2} - \underbrace{\sin t}_{2}}$$ bulunur.

4) *Laplace Dönüşümünü* kullanarak $y'' + 9y = 10e^{-x}$ diferansiyel denkleminin $y(0) = 0$, $y'(0) = 0$ koşullarına uyan çözümünü bulunuz.

$$s^2 Y(s) + s\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{0} + 9Y(s) = \frac{10}{s+1} \quad 4$$

$$(s^2+9)Y(s) = \frac{10}{s+1} \Rightarrow Y(s) = \frac{10}{(s+1)(s^2+9)} \quad 2$$

2 2

$$\frac{A}{s+1} + \frac{Bs+C}{s^2+9} = \frac{10}{(s+1)(s^2+9)}$$

$$(A+B)s^2 + (B+C)s + (9A+C) = 10$$

$$\left.\begin{array}{l} A+B=0 \\ B+C=0 \\ 9A+C=10 \end{array}\right\} \begin{array}{l} A=1 \quad 1 \\ B=-1 \quad 1 \\ C=1 \quad 1 \end{array}$$

$$y(t) = L^{-1}\left\{\frac{1}{s+1}\right\} - L^{-1}\left\{\frac{s}{s^2+9}\right\} + L^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$\boxed{y(t) = e^{-t} - \cos 3t + \frac{1}{3}\sin 3t}$$

4 4 4

Başarılar...

3) $\left.\begin{array}{l} \dfrac{dx}{dt} = y + 1 \\ \dfrac{dy}{dt} = -x + \dfrac{1}{Sint} \end{array}\right\}$ Diferansiyel denklem sistemini türetme-yok etme yöntemiyle çözünüz.

$$y = \frac{dx}{dt} - 1 \Rightarrow \frac{dy}{dt} = \frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2} + x = \frac{1}{\sin t}} \quad 6$$

$$r^2 + 1 = 0 \Rightarrow r_{1,2} = \mp i \quad 1 \qquad x_h = c_1 \cos t + c_2 \sin t \quad \underline{\underline{2}}$$

$$\left.\begin{array}{l} \sin t / \; c_1' \cos t + c_2' \sin t = 0 \\ \cos t / \; -c_1' \sin t + c_2' \cos t = \dfrac{1}{\sin t} \end{array}\right\} \quad 4$$

$+$

$$c_2' = \cot t \rightarrow \boxed{c_2 = \ln(\sin t) + k_2} \quad 3$$

$$c_1' = -c_2' \tan t = -(\cot t).(\tan t) = -1 \Rightarrow \boxed{c_1 = -t + k_1} \quad 3$$

$$\boxed{x = k_1 \cos t + k_2 \sin t - t \cos t + \sin t \ln(\sin t)} \quad 3$$

$$y = \frac{dx}{dt} - 1 = -k_1 \sin t + k_2 \cos t - \cos t + t \sin t + \cos t \ln(\sin t) + \sin t \cdot \frac{\cos t}{\sin t} - 1 \quad 3$$

Başarılar ...

2) $y = xy' + \sqrt{(y')^2 + 1}$ diferansiyel denklemini çözünüz.

2 $y' = p \rightarrow y = xp + \sqrt{p^2+1}$ Clairaut D.D

5 $\underbrace{y'}_{p} = p + xp' + \dfrac{2pp'}{2\sqrt{p^2+1}}$

$xp' + \dfrac{pp'}{\sqrt{p^2+1}} = 0 \rightarrow p'\left[x + \dfrac{p}{\sqrt{p^2+1}}\right] = 0$ 3

$y = xc_1 + \sqrt{c_1^2+1}$

1) $p' = 0 \rightarrow p = c_1 \Rightarrow \boxed{y = c_1 x + c_2}$ Genel çözüm 5

2) $x = \dfrac{-p}{\sqrt{p^2+1}}$

$y = \dfrac{-p^2}{\sqrt{p^2+1}} + \sqrt{p^2+1} = \dfrac{1}{\sqrt{p^2+1}}$

$\boxed{x^2 + y^2 = 1}$ Tekil çözüm 10

Soru 3 Laplace dönüşümünü kullanarak $y''+4y'+5y=10e^{t}$ dif. denkleminin $y(0)=0$, $y'(0)=0$ koşuluna uyan çözümünü bulunuz.

$$L\{y''\}+4L\{y'\}+5L\{y\}=L\{10e^{t}\}$$

$$\underbrace{s^2Y(s)}_{1}-s\underbrace{y(0)}_{0}-\underbrace{y'(0)}_{0\ (1)}+\underbrace{4sY(s)}_{1}-4\underbrace{y(0)}_{0}+\underbrace{5Y(s)=\frac{10}{s-1}}_{1}$$

4

$$\underbrace{[s^2+4s+5]Y(s)=\frac{10}{s-1}}_{1} \Rightarrow \underbrace{Y(s)=\frac{10}{(s-1)(s^2+4s+5)}}_{1}$$ 2

$$y(t)=L^{-1}\left\{\frac{10}{(s-1)(s^2+4s+5)}\right\}$$

$$\underset{1}{\frac{A}{s-1}}+\underset{1}{\frac{Bs+C}{s^2+4s+5}}=\frac{10}{(s-1)(s^2+4s+5)}$$

$$\left.\begin{aligned}A+B&=0\\4A-B+C&=0\\5A-C&=10\end{aligned}\right\}\quad \begin{aligned}A&=1 \quad 1\\B&=-1 \quad 1\\C&=-5 \quad 1\end{aligned}$$

$$\left.\begin{aligned}A&=\frac{1}{10}\\B&=-\frac{1}{10}\\C&=-\frac{5}{10}=-\frac{1}{2}\end{aligned}\right\}$$

$$y(t)=L^{-1}\left\{\frac{1}{s-1}\right\}+L^{-1}\left\{\frac{-s-5}{s^2+4s+5}\right\}$$ 1

$$y(t)=\overset{2}{e^{t}}-\overset{3}{L^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\}}-\overset{3}{L^{-1}\left\{\frac{3}{(s+2)^2+1}\right\}}$$

$$\boxed{y(t)=e^{t}-e^{-2t}\cos t-3e^{-2t}\sin t}$$

5

$$\underbrace{\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}+\mathcal{L}^{-1}\left\{\frac{-s}{}\right\}-\mathcal{L}^{-1}\left\{\frac{5}{}\right\}}_{21}$$

A, B, C yanlış?

18

Soru 4 $\left.\begin{aligned} &\frac{dx}{dt} + 2y - x = 0 \\ &\frac{dy}{dt} + y - x = -\sin t \end{aligned}\right\}$ denklem sistemini çözünüz.

iki denklem taraf tarafa çıkarılırsa

$$\boxed{\frac{dx}{dt} - \frac{dy}{dt} + y = \sin t} \quad 2$$

2. denklem türetilirse

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} - \frac{dx}{dt} = -\cos t \rightarrow \boxed{\frac{dx}{dt} = \frac{d^2y}{dt^2} + \frac{dy}{dt} + \cos t} \quad 2$$

Bu iki denklemden

$$\frac{d^2y}{dt^2} + y = \sin t - \cos t \quad 2$$

$$r^2 + 1 = 0 \rightarrow r_{1,2} = \mp i \;(1) \rightarrow y_h = c_1\cos t + c_2 \sin t \quad 1$$

$$y_ö = (A\cos t + B\sin t)t \quad 2$$

$$y_ö' = (-A\sin t + B\cos t)t + (A\cos t + B\sin t) \quad 1$$

$$y_ö'' = (-A\cos t - B\sin t)t + 2(-A\sin t + B\cos t) \quad 1$$

$$(-A\cos t - B\sin t)t + 2(-A\sin t + B\cos t) + (A\cos t + B\sin t)t = \sin t - \cos t \quad 2$$

$$-2A = 1 \rightarrow \boxed{A = -\frac{1}{2}} \;1 \qquad 2B = -1 \rightarrow \boxed{B = -\frac{1}{2}} \;1$$

$$y_ö = -\frac{1}{2}t\cos t - \frac{1}{2}t\sin t \quad 2$$

$$\boxed{y = y_h + y_ö = c_1\cos t + c_2\sin t - \frac{1}{2}t\cos t - \frac{1}{2}t\sin t} \quad 2$$

$$x = \frac{dy}{dt} + y + \sin t = -c_1\sin t + c_2\cos t - \frac{1}{2}\cos t + \frac{t}{2}\sin t - \frac{1}{2}\sin t - \frac{1}{2}t\cos t$$
$$+ c_1\cos t + c_2\sin t - \frac{1}{2}t\cos t - \frac{1}{2}t\sin t + \sin t \quad 5$$

$$x = c_1(\cos t - \sin t) + c_2(\cos t + \sin t) - \underline{\cos t} + \underline{\sin t} - t\cos t$$

3) $y'' + 2y' + y = \left(\frac{1}{e^x}\right)\ln x$ diferansiyel denkleminin çözümünü sabitin değişimi yöntemini kullanarak bulunuz.

$r^2 + 2r + 1 = 0 \qquad r_{1,2} = -1$

$y = c_1 e^{-x} + c_2 x e^{-x}$

$c_1' e^{-x} + c_2' x e^{-x} = 0$

$-c_1' e^{-x} + c_2' (e^{-x} - x e^{-x}) = \frac{1}{e^x} \ln x$

$c_2'(x) = \ln x$

$c_2 = x \ln x - x + k_2$

$c_1'(x) = -x \ln x$

$c_1 = \frac{x^2}{4} - \frac{x^2}{2} \ln x + k_1$

$y = \left(\frac{x^2}{4} - \frac{x^2}{2} \ln x + k_1\right) e^{-x} + (x \ln x - x + k_2) x e^{-x}$

$y=(D-1)x = x'-x$

$=2e^{2t}(at+b + a$

6) $\dfrac{dx}{dt}=x+y$, $\dfrac{dy}{dt}=-x+3y$ diferansiyel denklem sistemini çözünüz.

$y=e^{2t}(at+a+b$

$$\frac{d^2x}{dt^2}=\frac{dx}{dt}+\frac{dy}{dt} \quad 2$$

$$\frac{d^2x}{dt^2}=\frac{dx}{dt}-x+3y \quad 2 \qquad y=\frac{dx}{dt}-x$$

$$\frac{d^2x}{dt^2}=\frac{dx}{dt}+3\left(\frac{dx}{dt}-x\right)-x=0 \quad 2$$

$$\frac{d^2x}{dt^2}-4\frac{dx}{dt}+4x=0 \quad 3$$

$$r^2-4r+4=0 \qquad (r-2)^2=0 \quad r_{1,2}=2 \quad 4$$

$$x=(c_1t+c_2)e^{2t}=c_1te^{2t}+c_2e^{2t} \quad 4$$

$$y=\frac{dx}{dt}-x$$

$$y=c_1e^{2t}+c_1te^{2t}+c_2e^{2t} \quad 8$$

$(D-2)^2y=0$

D^2-4D

$(D-1)x-y=0$ $\qquad (D-2)^2=0$

$x+(D-3)y=0$ $\qquad (D-2)^2x=0$

$x=e^{2t}(at+b)$

$\cancel{24}-\cancel{25}-\cancel{26}-\cancel{27}-\cancel{28}-\cancel{32}$

0252311-2802311 DİFERANSİYEL DENKLEMLER
FİNAL SINAVI (II. SEANS)

13.01.2009

No :
Ad Soyad :
Grup :
Sınav Süresi 75 Dakikadır.

1	2	3	4	Toplam

1) $x^2y''+xy'+y=Cos(\ln x)$ diferansiyel denklemini çözünüz.

$x=e^t \qquad y'=e^{-t}\dfrac{dy}{dt} \qquad y''=e^{-2t}\left(\dfrac{d^2y}{dt^2}-\dfrac{dy}{dt}\right)$

$$e^{2t}\cdot e^{-2t}\left(\frac{d^2y}{dt^2}-\frac{dy}{dt}\right)+e^t\cdot e^{-t}\frac{dy}{dt}+y=\cos t$$

$$\frac{d^2y}{dt^2}+y=\cos t$$

$$r^2+1=0 \qquad r=\mp i$$

$$\underline{y_G=c_1\cos t+c_2\sin t}$$

$$y_ö=(A\cos t+B\sin t)t=At\cos t+Bt\sin t$$

$$y_ö'=A\cos t-At\sin t+B\sin t+Bt\cos t$$

$$y_ö''=-2A\sin t-At\cos t+2B\cos t-Bt\sin t$$

$$-2A\sin t-At\cos t+2B\cos t-Bt\sin t+At\cos t+Bt\sin t=\cos$$

$$A=0 \qquad B=\frac{1}{2} \qquad \underline{y_ö=\frac{1}{2}t.\sin t}$$

$$y=y_G+y_ö=c_1\cos t+c_2\sin t+\frac{1}{2}t.\sin t$$

$$y=c_1\cos(\ln x)+c_2\sin(\ln x)+\frac{1}{2}\ln x.\sin(\ln x)$$

4) $y^{IV} + 2y''' - 3y'' = x + 3e^{2x}$ d.d. belirsiz kat. çözünüz

Ç: İkinci tarafsız denk.in karakteristik denklemi

$r^4 + 2r^3 - 3r^2 = 0$ ② $\Rightarrow r^2(r^2 + 2r - 3) = 0$

$r_{1,2} = 0$ $\quad r_3 = -3$ $\quad r_4 = 1$ ③

$$\boxed{y_1 = c_1 + c_2 x + c_3 e^{-3x} + c_4 e^{x}}$$ ⑤ +1

Top. 10

$y_2 = x^2 \cdot (ax + b) = ax^3 + bx^2$ ④

$y_2' = 3ax^2 + 2bx$

$y_2'' = 6ax + 2b$

$y_2''' = 6a$

$y_2^{IV} = 0$

d.d. yerine yazılır $\Rightarrow$

$2 \cdot 6a - 3 \cdot (6ax + 2b) = x$

$-18ax + 12a - 6b = x$

$\boxed{a = -\frac{1}{18}}$ $\quad \boxed{b = -\frac{1}{9}}$ ③ +1

$$\boxed{y_2 = -\frac{1}{18}x^3 - \frac{1}{9}x^2}$$

④

$y_3 = ke^{2x}$

$y_3' = 2ke^{2x}$

$y_3'' = 4ke^{2x}$

$y_3''' = 8ke^{2x}$

$y_3^{IV} = 16ke^{2x}$

d.d. yerine yazılır $\Rightarrow$

$(16k + 2 \cdot 8k - 3 \cdot 4k)\cancel{e^{2x}} = 3\cancel{e^{2x}}$

$20k = 3 \Rightarrow \boxed{k = 3/20}$ $\quad \boxed{y_3 = \frac{3}{20}e^{2x}}$

③ +

$$\boxed{y = c_1 + c_2 x + c_3 e^{-3x} + c_4 e^{x} - \frac{1}{18}x^3 - \frac{1}{9}x^2 + \frac{3}{20}e^{2x}}$$ +2

genel çözüm

7 + 7 + 7

3) $y'' + y' + xy = 0$ denkleminin $x = 0$ noktası civarında seri çözümünü elde ediniz.

$$y = \sum_{n=0}^{\infty} c_n x^n \quad , \quad y' = \sum_{n=1}^{\infty} n c_n x^{n-1} \qquad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} + \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$$2c_2 + \sum_{n=3}^{\infty} n(n-1) c_n x^{n-2} + c_1 + \sum_{n=2}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$$(2c_2 + c_1) + \sum_{n=3}^{\infty} \{ n(n-1) c_n + (n-1) c_{n-1} + c_{n-3} \} x^{n-2} = 0$$

$$2c_2 + c_1 = 0 \quad , \quad n(n-1) c_n + (n-1) c_{n-1} + c_{n-3} = 0 \quad (n \geq 3)$$

$$c_2 = -\frac{c_1}{2} \quad , \quad c_n = -\frac{(n-1) c_{n-1} + c_{n-3}}{n(n-1)} \quad (n \geq 3).$$

$$c_2 = -\frac{c_1}{2}$$

$$c_3 = \frac{-2c_2 + c_0}{2 \cdot 3} = \frac{c_1 - c_0}{6}$$

$$c_4 = -\frac{3c_3 + c_1}{4 \cdot 3} = \frac{c_0 - 3c_1}{24}$$

$$c_5 = -\frac{4c_4 + c_1}{5 \cdot 4} = \frac{6c_1 - c_0}{120}$$

$$\vdots$$

$$y = c_0 + c_1 x - \frac{c_1}{2} x^2 + \frac{c_1 - c_0}{6} x^3 + \frac{c_0 - 3c_1}{24} x^4 + \frac{6c_1 - c_0}{120} x^5 + \cdots$$

$$y = c_0\left(1 - \frac{x^3}{6} + \frac{x^4}{24} - \frac{x^5}{120} + \frac{5x^6}{720} - \cdots\right) + c_1\left(x - \frac{x^2}{2} + \frac{x^3}{6} - \frac{3x^4}{24} + \frac{6x^5}{120} - \cdots\right)$$

gen. çöz.

4) $\frac{dx}{dt}-\frac{dy}{dt}-y=-e^t$ (1)

$\frac{dy}{dt}+x-y=0$ (2) denklem sistemini türetme – yok etme yöntemiyle çözünüz.

$(2)' \Rightarrow \frac{d^2y}{dt^2}+\frac{dx}{dt}-\frac{dy}{dt}=0$

yerine koy

$(1) \Rightarrow \frac{dx}{dt}=-e^t+\frac{dy}{dt}+y$

$\frac{d^2y}{dt^2}-\frac{dy}{dt}-e^t+\frac{dy}{dt}+y=0$

$\frac{d^2y}{dt^2}+y=e^t \qquad r^2+1=0 \qquad r_{1,2}=\mp i$

$y=c_1\cos t+c_2\sin t$

$y_0=Ae^t \qquad y_0'=Ae^t \qquad y_0''=Ae^t$

$Ae^t+Ae^t=e^t \rightarrow 2Ae^t=e^t \Rightarrow \boxed{A=\frac{1}{2}} \Rightarrow \boxed{y_0=\frac{e^t}{2}}$

$y=c_1\cos t+c_2\sin t+\frac{e^t}{2}$

$(2) \Rightarrow x=y-\frac{dy}{dt} \Rightarrow x=c_1\cos t+c_2\sin t+\cancel{\frac{e^t}{2}}+c_1\sin t-c_2\cos t-\cancel{\frac{e^t}{2}}$

$x=(c_1-c_2)\cos t+(c_1-c_2)\sin t$

Başarılar...

S.1) $\frac{d^2y}{dx^2} + \frac{dz}{dx} = y - 2z - \sin x + \cos x$

$\frac{dy}{dx} + \frac{d^2z}{dx^2} = y - 2z + \cos x$

diferansiyel denklem sistemini "yok etme yöntemiyle" kullanarak $z = z(x)$'i bulunuz.

$(D^2-1)y + (D+2)z = -\sin x + \cos x$

10

$-D-1)\ (D-1)y + (D^2+2)z = \cancel{}\ (-D-1)\cos x$

$\cancel{(D^2-1)y} + (D+2)z = -\sin x + \cos x$

$\cancel{(-D^2+1)y} + (-D^3-2D-D^2-2)z = \sin x - \cos x$

$(-D^3 - D^2 - D)z = 0$ 5

$(D^3 + D^2 + D)z = 0$

$D(D^2+D+1) = 0 \quad D_1 = 0 \quad D_{2,3} = \frac{-1 \mp \sqrt{3}i}{2}$

$z = z(x) = e^{-\frac{1}{2}x}\left(C_1 \cos\frac{\sqrt{3}}{2}x + C_2 \sin\frac{\sqrt{3}}{2}x\right)$

10

4) $y'' + k^2 y = a \cos kx$ dif denk çözünüz

$r^2 + k^2 = 0$ 2 $\quad r = \mp ik$ 2

$y_G = c_1 \cos kx + c_2 \sin kx$ 6

$y_ö = (A \cos kx + B \sin kx)x$ 5

$y_ö' = x(-Ak \sin kx + Bk \cos kx) + (A \cos kx + B \sin kx)$

$y_ö'' = x(-Ak^2 \cos kx - Bk^2 \sin kx) - (2Ak \sin kx - 2Bk \cos kx$

$-Ak^2 \cos kx - Bk^2 \sin kx) - 2Ak \sin kx + 2Bk \cos kx$

$k^2 Ax \cos kx + k^2 Bx \sin kx \equiv a \cos kx$

$-2Ak \sin kx + 2Bk \cos kx \equiv a \cos kx$ 5

$A = 0 \qquad B = \dfrac{a}{2k}$ 3

$y_ö = \dfrac{ax}{2k} \sin kx$

$y = c_1 \cos kx + c_2 \sin kx + \dfrac{ax}{2k} \sin kx$ 2