$$\frac{dx}{dt}+\frac{dy}{dt}-2y=-e^t \quad (1)$$

4) $$\frac{dy}{dt}+x+y=0 \quad (2)$$ diferansiyel denklem sistemini <u>türetme-yoketme yöntemiyle</u> çözünüz.

$$(2)' \Rightarrow \frac{d^2y}{dt^2}+\frac{dx}{dt}+\frac{dy}{dt}=0$$

$$(1) \Rightarrow \frac{dx}{dt}=-e^t-\frac{dy}{dt}+2y$$ yerine koy

$$\frac{d^2y}{dt^2}-e^t-\cancel{\frac{dy}{dt}}+2y+\cancel{\frac{dy}{dt}}=0$$

$$\frac{d^2y}{dt^2}+2y=e^t \qquad r^2+2=0 \qquad r_{1,2}=\mp\sqrt{2}\,i$$

$$y=c_1\cos\sqrt{2}t+c_2\sin\sqrt{2}t$$

$$y_0=Ae^t,\quad y_0'=Ae^t,\quad y_0''=Ae^t$$

$$Ae^t+2Ae^t=e^t \Rightarrow \boxed{A=\frac{1}{3}} \Rightarrow \boxed{y_0=\frac{e^t}{3}}$$

$$\Rightarrow y=c_1\cos\sqrt{2}t+c_2\sin\sqrt{2}t+\frac{e^t}{3}$$

$$(2) \Rightarrow x=-y-\frac{dy}{dt}$$

$$x=-c_1\cos\sqrt{2}t-c_2\sin\sqrt{2}t-\frac{e^t}{3}+\sqrt{2}c_1\sin\sqrt{2}t-\sqrt{2}c_2\cos\sqrt{2}t-\frac{e^t}{3}$$

$$\Rightarrow x=(-c_1-\sqrt{2}c_2)\cos\sqrt{2}t+(-c_2+\sqrt{2}c_1)\sin\sqrt{2}t-\frac{2}{3}e^t$$

Başarılar...

1) $\frac{dx}{dt} + \frac{dy}{dt} - 2y = -e^{t}$ $\quad Dx + Dy - 2y = -e^{t}$

$\frac{dy}{dt} + x + y = 0$ $\quad Dy + x + y = 0$

$Dx + (D-2)y = -e^{t}$

$x + (D+1)y = 0$

$$\Delta = \begin{vmatrix} D & D-2 \\ 1 & D+1 \end{vmatrix} = D^2 + \cancel{D} - \cancel{D} + 2 = D^{2} + 2$$

sistemin mert. verir 2 tane keyfi sbt olacak

$$\Delta_1 = \begin{vmatrix} -e^{t} & D-2 \\ 0 & D+1 \end{vmatrix}$$

$$= (D+1)(-e^{t}) - 0 = D(-e^{t}) - e^{t} = -e^{t} - e^{t} = -2e^{t}$$

$$x = \cancel{\frac{\Delta_1}{\Delta}} \; \frac{-2e^{t}}{D^2+2} \Rightarrow (D^2+2)x = -2e^{t} \qquad r^2 + 2 = 0$$

$$x'' + 2x = -2e^{t} \qquad r^2 = \mp\sqrt{2}i$$

$x_h = c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t$

$x_p = \lambda e^{t}$ (özel durumu yok $\alpha \neq r_{1,2}$)

$x_p' = \lambda e^{t}$, $x_p'' = \lambda e^{t} \Rightarrow \lambda e^{t} + 2\lambda e^{t} = -2e^{t} \Rightarrow 3\lambda = -2$
$\lambda = -2/3$

$$x = x_h + x_p = c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t - \frac{2}{3}e^{t}$$

Aynı şekilde y çözümünüde bulabiliriz

$$\Delta_2 = \begin{vmatrix} D & -e^{t} \\ 1 & 0 \end{vmatrix} = 0 + e^{t} \qquad y = \frac{\Delta_2}{\Delta} = \frac{e^{t}}{D^2+2}$$

$$(D^2+2)y = e^{t} \rightarrow y'' + 2y = e^{t}$$

$y_h = c_3 \cos\sqrt{2}t + c_4 \sin\sqrt{2}t \qquad y_p = \lambda e^{t}$

$\lambda e^{t} + 2\lambda e^{t} = e^{t} \Rightarrow 3\lambda = 1 \quad \lambda = 1/3 \Rightarrow y_p = \frac{1}{3}e^{t}$

$$y = y_p + y_h = c_3 \cos\sqrt{2}t + c_4 \sin\sqrt{2}t + \frac{1}{3}e^{t}$$

$$\frac{dy}{dx} + x + y = 0$$

$$-\sqrt{2}c_3 \sin\sqrt{2}t + \sqrt{2}c_4 \cos\sqrt{2}t + \frac{1}{3}e^t + c_1 \cos\sqrt{2}t + c_2 \sin\sqrt{2}t - \frac{2}{3}e^{2t}$$
$$+ c_3 \cos\sqrt{2}t + c_4 \sin\sqrt{2}t + \frac{1}{3}e^t = 0$$

$$(c_2 - \sqrt{2}c_3 + c_4) = 0 \qquad c_2 = \sqrt{2}c_3 - c_4$$

$$\sqrt{2}c_4 + c_1 + c_3 = 0 \qquad c_1 = -\sqrt{2}c_4 - c_3$$

$$x = (-c_3 - \sqrt{2}c_4)\cos\sqrt{2}t + (-c_4 + \sqrt{2}c_3)\sin\sqrt{2}t - \frac{2}{3}e^t$$

olarak düzenlenebilir.

$y'' - y' + xy = 0$ $x = 0$ Seri çözümü

$y = \sum_{n=0}^{\infty} a_n x^n$, $y' = \sum_{n=1}^{\infty} a_n n x^{n-1}$, $y'' = \sum_{n=2}^{\infty} a_n n(n-1) x^{n-2}$

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{\substack{n=1 \\ n-1=1}}^{\infty} a_n n x^{\overset{n-1}{n-1}} + \sum_{\substack{n=0 \\ n-3=0}}^{\infty} a_n x^{\overset{n-3}{n+1}} = 0 \quad (*)$$

x^{n-2} ye göre düzenleyelim.

$$\sum_{n=2}^{\infty} n(n-1) a_n x^{n-2} - \sum_{n=2}^{\infty} a_{n-1}(n-1) x^{n-2} + \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

$$\underbrace{2 \cdot 1 a_2}_{n=2} + \sum_{n=3}^{\infty} n(n-1) a_n x^{n-2} - \underbrace{a_1}_{n=2} - \sum_{n=3}^{\infty} a_{n-1}(n-1) x^{n-2} + \sum_{n=3}^{\infty} a_{n-3} x^{n-2} = 0$$

$$\underbrace{2a_2 - a_1}_{=0} + \sum_{n=3}^{\infty} \Big[\underbrace{n(n-1)a_n - a_{n-1}(n-1) + a_{n-3}}_{=0}\Big] x^{n-2} = 0$$

$2a_2 - a_1 = 0$ $n(n-1)a_n - a_{n-1}(n-1) + a_{n-3} = 0$

$a_2 = \frac{c_1}{2}$ $a_n = \frac{(n-1)a_{n-1} - a_{n-3}}{n(n-1)}$ $n \geq 3$

$n=3$ $a_3 = \frac{2c_2 - c_0}{3 \cdot 2} = \frac{2\left(\frac{c_1}{2}\right) - c_0}{3 \cdot 2} = \frac{c_1 - c_0}{6}$

$n=4$ $c_4 = \frac{3 \cdot c_3 - c_1}{4 \cdot 3} = \frac{3\left(\frac{c_1 - c_0}{6}\right) - c_1}{4 \cdot 3} = \frac{\frac{c_1}{2} - \frac{c_0}{2} - c_1}{12} = -\frac{c_1 - c_0}{24}$

Eğer (*) ifadesini

x^{n-1} göre düzenlersek

toplamda $n \to n+1$

$$\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} - \sum_{n=1}^{\infty} n a_n x^{n-1} + \sum_{n=0}^{\infty} a_n x^{n+1} = 0$$

$$\underbrace{\sum_{n=1}^{\infty}(n+1)(n)a_{n+1}x^{n-1}}_{n=2 \text{ kaydıralım}} - \underbrace{\sum_{n=1}^{\infty} n a_n x^{n-1}}_{n=2\text{'den başlatalım}} + \sum_{n=2}^{\infty} a_{n-2} x^{n-1} = 0$$

$$2.1a_2 + \sum_{n=2}^{\infty}(n+1)n a_{n+1} - a_1 - \sum_{n=2}^{\infty} n a_n x^{n-1} + \sum_{n=2}^{\infty} a_{n-2} x^{n-1} = 0$$

$$\underbrace{2a_2 - a_1}_{=0} + \sum_{n=2}^{\infty}\underbrace{\left[(n+1)n a_{n+1} - n a_n + a_{n-2}\right]}_{=0} x^{n-1}$$

$$2a_2 = a_1$$
$$a_2 = \frac{a_1}{2}$$

$$(n+1)n a_{n+1} - n a_n + a_{n-2} = 0$$

$$a_{n+1} = \frac{n a_n - a_{n-2}}{(n+1)n} \qquad n \geq 2$$

Bir önceki rekurans (yineleme) bağıntısı ile
Bu bağıntıyı karşılaştıralım.

$$a_n = \frac{(n-1)a_{n-1} - a_{n-3}}{n(n-1)} \quad n \geq 3 \qquad \Bigg| \qquad a_{n+1} = \frac{n a_n - a_{n-2}}{(n+1)n} \quad n \geq 2$$

$n \to n+1$

$$a_{n+1} = \frac{(n+1-1)a_{n+1-1} - a_{n+1-3}}{(n+1)(n+1-1)}$$

aynı bağıntı elde edilir.

3) $y'' - y' + xy = 0$ denkleminin x=0 noktası civarında seri çözümünü elde ediniz.

$$y = \sum_{n=0}^{\infty} c_n x^n \quad , \quad y' = \sum_{n=1}^{\infty} n c_n \quad , \quad y'' = \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2}$$

$$\Rightarrow \sum_{n=2}^{\infty} n(n-1) c_n x^{n-2} - \sum_{n=1}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$$2c_2 + \sum_{n=3}^{\infty} n(n-1) c_n x^{n-2} - c_1 - \sum_{n=2}^{\infty} n c_n x^{n-1} + \sum_{n=0}^{\infty} c_n x^{n+1} = 0$$

$$(2c_2 - c_1) + \sum_{n=3}^{\infty} \{ n(n-1) C_n - (n-1) C_{n-1} + C_{n-3} \} x^{n-2} = 0$$

$$2c_2 - c_1 = 0 \quad , \quad n(n-1) C_n - (n-1) C_{n-1} + C_{n-3} = 0 \quad n \geq 3$$

$$c_2 = \frac{c_1}{2} \quad , \quad c_n = \frac{(n-1) C_{n-1} - C_{n-3}}{n(n-1)} \quad n \geq 3$$

$$c_2 = \frac{c_1}{2} \qquad\qquad c_4 = \frac{3 \cdot c_3 - c_1}{4 \cdot 3} = -\frac{c_1 - c_0}{24}$$

$$c_3 = \frac{2c_2 - c_0}{3 \cdot 2} = \frac{c_1 - c_0}{6} \qquad c_5 = \frac{4 \cdot c_4 - c_2}{5 \cdot 4} = \frac{-4c_1 - c_0}{120} \ \ldots$$

$$y = c_0 + c_1 x + c_2 x^2 + \ldots$$

$$y = c_0 + c_1 x + \frac{c_1}{2} x^2 + \frac{c_1 - c_0}{6} x^3 + \frac{(-c_1 - c_0)}{24} x^4 + \frac{(-4c_1 - c_0)}{120} x^5 + \ldots$$

$$y = c_0 \left(1 - \frac{x^3}{6} - \frac{x^4}{24} - \frac{x^5}{120} - \ldots \right) + c_1 \left(x + \frac{x^2}{2} + \frac{x^3}{6} - \frac{x^4}{24} - \frac{4x^5}{120} - \ldots \right)$$

genel çöz.

2) $y''+2y'+y=8e^{-t}$ $\quad y(0)=2,\ y'(0)=-1$ Başlangıç değer problemini Laplace dönüşümü kullanarak çözünüz.

$$\mathcal{L}\{y''+2y'+y\}=\mathcal{L}\{8e^{-t}\}$$

$$s^2Y(s)-s\,y(0)-y'(0)+2(sY(s)-y(0))+Y(s)=\frac{8}{s+1}$$

$$s^2Y(s)+2sY(s)+Y(s)-2s-3=\frac{8}{s+1}$$

$$(s^2+2s+1)Y(s)-2s-3=\frac{8}{s+1}$$

$$Y(s)=\frac{8}{s+1}+\frac{2s+3}{(s+1)^2}$$

$$\frac{2s+3}{(s+1)^2}=\frac{2(s+1)+1}{(s+1)^2}=\frac{2}{s+1}+\frac{1}{(s+1)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\}=\mathcal{L}^{-1}\left\{\frac{8}{(s+1)^3}+\frac{2s+3}{(s+1)^2}\right\}$$

$$y(t)=\mathcal{L}^{-1}\{Y(s)\}$$

$$y(t)=8e^{-t}\frac{t^2}{2!}+2e^{-t}.1+e^{-t}.t$$

$$y(t)=(4t^2+t+2)e^{-t}$$

2) $y'' + 2y' + y = 4e^{-t}$ denkleminin $y(0) = 2,\ y'(0) = -1$ başlangıç değer problemini Laplace dönüşümü kullanarak çözünüz.

$$\mathcal{L}\{y''+2y'+y\} = \mathcal{L}\{4e^{-t}\}$$

$$s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{-1} + 2[sY(s) - \underbrace{y(0)}_{2}] + Y(s) = \frac{4}{s+1}$$

$$s^2Y(s) + 2sY(s) + Y(s) - 2s - 3 = \frac{4}{s+1}$$

$$\underbrace{(s^2+2s+1)}_{(s+1)^2} Y(s) - 2s - 3 = \frac{4}{s+1}$$

$$Y(s) = \frac{4}{(s+1)^3} + \frac{2s+3}{(s+1)^2}$$

$$\frac{2s+3}{(s+1)^2} = \frac{2(s+1)+1}{(s+1)^2} = \frac{2}{s+1} + \frac{1}{(s+1)^2}$$

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{4}{(s+1)^3} + \frac{2}{s+1} + \frac{1}{(s+1)^2}\right\}$$

$$y(t) = \mathcal{L}^{-1}\{Y(s)\}$$

$$y(t) = 4 \cdot e^{-t} \cdot \frac{t^2}{2!} + 2e^{-t} \cdot 1 + e^{-t} \cdot t$$

$$y(t) = (2t^2 + t + 2)e^{-t}$$

$$\frac{dx}{dt}-\frac{dy}{dt}-y=-e^t \quad (1)$$

4) $$\frac{dy}{dt}+x-y=0 \quad (2)$$ denklem sistemini türetme – yok etme yöntemiyle çözünüz.

$$(2)' \Rightarrow \frac{d^2y}{dt^2}+\frac{dx}{dt}-\frac{dy}{dt}=0$$

$$(1) \Rightarrow \frac{dx}{dt}=-e^t+\frac{dy}{dt}+y$$ yerine koy

$$\frac{d^2y}{dt^2}-\frac{dy}{dt}-e^t+\frac{dy}{dt}+y=0$$

$$\frac{d^2y}{dt^2}+y=e^t \qquad r^2+1=0 \qquad r_{1,2}=\mp i$$

$$y=c_1\cos t+c_2\sin t$$

$$y_0=Ae^t \qquad y_0'=Ae^t \qquad y_0''=Ae^t$$

$$Ae^t+Ae^t=e^t \rightarrow 2Ae^t=e^t \Rightarrow \boxed{A=\frac{1}{2}} \Rightarrow \boxed{y_0=\frac{e^t}{2}}$$

$$y=c_1\cos t+c_2\sin t+\frac{e^t}{2}$$

$$(2) \Rightarrow x=y-\frac{dy}{dt} \Rightarrow x=c_1\cos t+c_2\sin t+\frac{e^t}{2}+c_1\sin t-c_2\cos t-\frac{e^t}{2}$$

$$x=(c_1-c_2)\cos t+(c_1-c_2)\sin t$$

S-4) $y''+2y'+y=\sin t+\cos t$, $y(0)=0$ $y'(0)=0$ diferansiyel denklemini Laplace ve invers laplace yöntemi kullanarak çözünüz.

C-4) $\mathcal{L}[y''+2y'+y]=\mathcal{L}[\sin t]+\mathcal{L}[\cos t]$ (2), $\mathcal{L}[y]=Y(s)$

$\mathcal{L}[y'']=s^2\cdot Y(s)-s\cdot y(0)-y'(0)$ (2), $\mathcal{L}[y']=sY(s)-y(0)$ (2)

$$s^2Y(s)-s\cdot\underbrace{y(0)}_{0}-\underbrace{y'(0)}_{0}+2sY(s)-2\underbrace{y(0)}_{0}+Y(s)=\frac{1}{s^2+1}+\frac{s}{s^2+1}$$ (2)

$$(s^2+2s+1)\cdot Y(s)=\frac{s+1}{s^2+1}, \quad (s+1)^2\cdot Y(s)=\frac{s+1}{s^2+1}$$

$$Y(s)=\frac{1}{(s^2+1)(s+1)}$$ (2) $$\frac{1}{(s^2+1)(s+1)}=\frac{As+B}{\underset{s+1}{s^2+1}}+\frac{C}{\underset{s^2+1}{s+1}}$$ (2)

$$1\equiv As^2+As+Bs+B+Cs^2+C$$

$$\left.\begin{aligned}A+C&=0\\A+B&=0\\B+C&=1\end{aligned}\right\}\quad C=1/2 \quad B=1/2 \quad A=-1/2.$$ (3)

$$Y(s)=\frac{1}{2}\left(\frac{-s}{s^2+1}\right)+\frac{1}{2}\left(\frac{1}{s^2+1}\right)+\frac{1}{2(s+1)}.$$

$$\mathcal{L}^{-1}[Y(s)]=\mathcal{L}^{-1}\left[-\frac{1}{2}\cdot\frac{s}{s^2+1}\right]+\mathcal{L}^{-1}\left[\frac{1}{2}\cdot\frac{1}{s^2+1}\right]+\frac{1}{2}\mathcal{L}^{-1}\left[\frac{1}{s+1}\right]$$ (3)

$$y(t)=-\frac{1}{2}\cos t+\frac{1}{2}\sin t+\frac{1}{2}e^{-t}$$ (5)

S-3) $y''-y'-y=0$ diferansiyel denkleminin $x=0$ noktası civarındaki çözümünü kuvvet serisini kullanarak, serinin ~~a_{k+2}, a_{k+1} ve~~ a_k katsayıları arasında bir ~~bağıntı bulunuz~~ rekürans (indirgeme) bağıntısını bulunuz.

C-3) $y''-y'-y=0$ Diferansiyel denkleminin bir çözümü $y=\sum_{k=0}^{\infty} a_k \cdot x^k$ olsun (4p)

$$y'=\sum_{k=1}^{\infty} k\, a_k \cdot x^{k-1} \quad (4p), \quad y''=\sum_{k=2}^{\infty} k(k-1) \cdot a_k \cdot x^{k-2} \quad (4p)$$

y, y' ve y'' ifadelerini D.D de yerlerine yazalım

$$\sum_{k=2}^{\infty} k(k-1)\, a_k \cdot x^{k-2} - \sum_{k=1}^{\infty} k \cdot a_k \cdot x^{k-1} - \sum_{k=0}^{\infty} a_k \cdot x^k = 0 \quad (3p)$$

$z=k-2$ alalım $\qquad t=k-1$ alalım.

$$\sum_{z=0}^{\infty} (z+2)(z+1)\, a_{z+2} \cdot x^z - \sum_{t=0}^{\infty} (t+1)\, a_{t+1}\, x^t - \sum_{k=0}^{\infty} a_k \cdot x^k = 0$$

$z=k$ yazılabilir $\qquad t=k$ yazılabilir

$$\sum_{k=0}^{\infty} (k+2)(k+1)\, a_{k+2}\, x^k - \sum_{k=0}^{\infty} (k+1)\, a_{k+1}\, x^k - \sum_{k=0}^{\infty} a_k x^k = 0$$

(10p)

$$\sum_{k=0}^{\infty} \left[(k+2)(k+1) \cdot a_{k+2} - (k+1)\, a_{k+1} - a_k\right] x^k = 0$$

$$(k+2)(k+1) \cdot a_{k+2} - (k+1)\, a_{k+1} - a_k = 0$$

$$\boxed{a_{k+2} = \frac{a_{k+1}}{k+1} + \frac{a_k}{(k+1)(k+2)}}$$

(4) $xy' - y = e^{y'}$ $\quad y = xy' + \varphi(y')$

1)

$$\frac{dx}{dt}+\frac{dy}{dt}-x+3y=6e^{-t}$$
$$\frac{dx}{dt}+\frac{dy}{dt}+2x=0$$

$$D=\frac{d}{dt}$$

$$Dx+Dy-x+3y=6e^{-t} \qquad (D-1)x+(D+3)y=6e^{-t}$$
$$Dx+Dy+2x=0 \qquad (D+2)x+Dy=0$$

$\Rightarrow \begin{vmatrix} D-1 & D+3 \\ D+2 & D \end{vmatrix} = -5D-6 \Rightarrow D$ ye göre 1.dereceden, sadece 1 tane int. sabiti içerecektir.

$$[(D+2)(D+3)-D(D-1)]y=(D+2)6e^{-t}$$
$$(6D+6)y=D(6e^{-t})+12e^{-t}$$
$$6(D+1)y=-6e^{-t}+12e^{-t}$$
$$(D+1)y=e^{-t} \Rightarrow D+1=0 \Rightarrow D=-1 \Rightarrow y_p=Ce^{-t}$$
$$y_0=Ate^{-t} \Rightarrow (D+1)Ate^{-t}=e^{-t}$$
$$A(e^{-t}-te^{-t}+te^{-t})=e^{-t} \Rightarrow A=1$$
$$y_0=te^{-t}$$
$$y=Ce^{-t}+te^{-t}$$

Verilen denklem ler taraf tarafa çıkarılırsa;

$$-x+3y-3x=6e^{-t} \Rightarrow -3x+3y=6e^{-t}$$

$$x=y-2e^{-t}=Ce^{-t}+te^{-t}-2e^{-t}$$

$$x=(C-2)e^{-t}+te^{-t}$$

2) $\dfrac{dx}{dt}+\dfrac{dy}{dt}-x-3y=0$

$\dfrac{dx}{dt}+\dfrac{dy}{dt}+x=e^{3t}$ denklem sisteminin genel çözümünü bulunuz.

5 $\left[\begin{array}{l}(D-1)x+(D-3)y=0\\(D+1)x+Dy=e^{3t}\end{array}\right.$ 3

$\Delta=\begin{vmatrix}D-1 & D-3\\ D+1 & D\end{vmatrix}=D+3$ 2

6 $x=\dfrac{\begin{vmatrix}0 & D-3\\ e^{3t} & D\end{vmatrix}}{\Delta}=\dfrac{3(-e^{3t}+e^{3t})}{\Delta}=0$ 2

$(D+3)x=0$ 2

$r+3=0$

$r=-3$

$x=c_1e^{-3t}$ 2

9 $y=\dfrac{\begin{vmatrix}D-1 & 0\\ D+1 & e^{3t}\end{vmatrix}}{\Delta}=\dfrac{3e^{3t}-e^{3t}}{\Delta}$ 2

$(D+3)y=2e^{3t}$ 2 $r=-3$

$y_h=c_2e^{-3t}$ 2

$y_ö=Ke^{3t}$ $y_ö'=3Ke^{3t}$ $3Ke^{3t}+3Ke^{3t}=2e^{3t}$

$K=\frac{1}{3}$ $y_ö=\frac{1}{3}e^{3t}$ 2

$y=c_2e^{-3t}+\frac{1}{3}e^{3t}$ 1

5 $(D+1)x+Dy=e^{3t}$ denk. yer. yaz

$-3c_1e^{-3t}+c_1e^{-3t}-3c_2e^{-3t}+\cancel{e^{3t}}=\cancel{e^{3t}}$ 2

$-2c_1e^{-3t}=3c_2e^{-3t}$

$\boxed{c_2=-\frac{2}{3}c_1}$ 1

2 $\begin{cases}x=c_1e^{-3t}\\ y=-\frac{2}{3}c_1e^{-3t}+\frac{1}{3}e^{3t}\end{cases}$

1) $y''+2y'+5y=0$, $y(0)=2$, $y'(0)=-1$ başlangıç değer probleminin çözümünü Laplace dönüşümünü kullanarak bulunuz.

$$\mathcal{L}\{y''+2y'+5y\} = \mathcal{L}\{0\}$$

$$\boxed{[s^2Y(s) - s\underbrace{y(0)}_{2} - \underbrace{y'(0)}_{-1}] + 2[sY(s) - \underbrace{y(0)}_{2}] + 5Y(s) = 0}$$

3 2 1

$$s^2Y(s) - 2s + 1 + 2sY(s) - 4 + 5Y(s) = 0$$ 1

$$(s^2+2s+5)Y(s) = 2s+3$$

$$\boxed{Y(s) = \frac{2s+3}{s^2+2s+5}}$$ 3

$$\boxed{\frac{2s+3}{(s+1)^2+4} = \frac{2s}{(s+1)^2+4} + \frac{3}{(s+1)^2+4}}$$

1 3 3

$$\mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{2s+2}{(s+1)^2+4} + \frac{1}{(s+1)^2+4}\right\}$$

$$\boxed{y(t) = 2e^{-t}\cos 2t + \frac{1}{2}e^{-t}\sin 2t}$$

4 4

... yerine yazıldığında

$$\sum_{n=2}^{\infty} n(n-1)c_n t^{n-2} - \sum_{n=1}^{\infty} n c_n t^n - \sum_{n=0}^{\infty} c_n t^n = 0$$

bulunur. Birinci ve üçüncü serilerin ilk terimleri dışarı alınırsa

$$2c_2 + \sum_{n=3}^{\infty} n(n-1)c_n t^{n-2} - \sum_{n=1}^{\infty} n c_n t^n - c_0 - \sum_{n=1}^{\infty} c_n t^n = 0$$

ur. Birinci seride n yerine $n+2$ yazılırsa

$$2c_2 - c_0 + \sum_{n=1}^{\infty} \{(n+2)(n+1)c_{n+2} - (n+1)c_n\} t^n = 0 \quad \text{olur.}$$

$$2c_2 - c_0 = 0 \quad \text{ve} \quad (n+2)c_{n+2} - c_n = 0 \quad (n \geq 1)$$

$$\Rightarrow c_2 = 2c_0 \quad \text{ve} \quad c_{n+2} = \frac{1}{n+2} c_n \quad (n \geq 1)$$

$$c_3 = \frac{1}{3}c_1, \quad c_4 = \frac{1}{4}c_2 = \frac{1}{4.2}c_0, \quad c_5 = \frac{1}{5}c_3 = \frac{1}{5\,3}c_1$$

$$\vdots$$

$$\ldots \quad c_{2n} = \frac{1}{2.4.6\ldots(2n)}c_0, \quad c_{2n+1} = \frac{1}{3.5.7\ldots(2n+1)}c_1, \quad (n \geq 1)$$

Böylece

$$y(t) = c_0 + c_1 t + c_2 t^2 + \ldots$$

$$= c_0\left\{1 + \frac{t^2}{2} + \frac{t^4}{4.2} + \ldots\right\} + c_1\left\{t + \frac{t^3}{3} + \frac{t^5}{3.5} + \ldots\right\}$$

$$= c_0 \sum_{n=0}^{\infty} \frac{t^{2n}}{2.4.6\ldots(2n)} + c_0 \sum_{n=0}^{\infty} \frac{t^{2n+1}}{3.5.7\ldots(2n+1)}$$

$a_{n+2}(n+2)(n+1)$

$$a_{n+2} = \frac{-a_n - 2a_{n+1}}{(n+2)} \quad n \geq 1$$

$Dy - (D+1)z = e^t$
$y + (D-1)z = e^{2t}$

$\Delta = D^2 + 1$

$\Delta_1 = 3e^{2t}$ $\qquad y = c_1 \sin t + c_2 \cos t + \frac{3}{5} e^{2t}$

$\Delta_2 = 2e^{2t} + e^t$

$z = \frac{2e^{2t} + e^t}{D^2 + 1}$ $\qquad z = c_3 \sin t + c_4 \cos t + \frac{2}{5} e^{2t} + \frac{1}{2} e^t$

2. dalı yerine yaz?

$\underbrace{(c_1 - c_3 - c_4)}_{0} \sin t + \underbrace{(c_2 + c_3 - c_4)}_{0} \cos t = 0$

$(c_1 + c_2) - \cancel{2} c_4 = 0$

$c_4 = \frac{c_1 + c_2}{2}$ $\qquad c_3 = \frac{c_1 - c_2}{2}$

... Kağıdı

Adı Soyadı		1. S	2. S	3.S	4. S	Toplam
Numarası						
Bölümü	Grup NO					
Dersin Adı			100	Tarih	14.11.2015	
Öğretim Üyesi	MAT2411 DİFERANSİYEL DENKLEMLER	Süre	90 dk	Sınıf		
	CEVAP ANAHTARI	İmza				

YÖK nun 2547 sayılı Kanunun *Öğrenci Disiplin Yönetmeliğinin* 9. Maddesi olan *"Sınavlarda kopya yapmak ve yaptırmak veya buna teşebbüs etmek"* fiili işleyenler bir veya iki yarıyıl uzaklaştırma cezası alırlar.

1) a) $y = c_1x + c_2x\ln x$ eğri ailesine ait olan diferansiyel denklemi bulunuz.

$$\left.\begin{aligned} y' &= c_1 + c_2\ln x + c_2 \\ y'' &= \frac{c_2}{x} \end{aligned}\right\} \quad xy' - y = x^2y''$$

$$\boxed{x^2y'' - xy' + y = 0}$$

b) $\left(x + y\ln\left(\frac{x}{y}\right)\right)dx + x\ln\left(\frac{y}{x}\right)dy = 0$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\left.\begin{aligned} x &\to tx \\ y &\to ty \end{aligned}\right\} \quad t\left[x + y\ln\left(\frac{x}{y}\right)\right]dx + tx\ln\left(\frac{y}{x}\right)dy = 0$$ 1. dereceden homojen dif. denk.

$$\left.\begin{aligned} \frac{y}{x} = u &\Rightarrow y = ux \\ dy &= u\,dx + x\,du \end{aligned}\right\} \quad \left[x + ux\ln\left(\frac{1}{u}\right)\right]dx + x\ln u\left[u\,dx + x\,du\right] = 0$$

$$(x - ux\ln u + ux\ln u)\,dx + x^2\ln u\,du = 0$$

$$\int\frac{dx}{x} + \int \ln u\,du = \int 0$$

$$\ln x + u\ln u - u = \ln c$$

$$\ln x + \frac{y}{x}\ln\left(\frac{y}{x}\right) - \frac{y}{x} = \ln c$$

Başarılar...

$$\frac{1}{y^2} = 6x^2 - 6 + Ke \Rightarrow \boxed{y = \sqrt{6x^2 + 6 + Ke^x}}$$

Bo

3-) $\frac{1}{x}y' + \frac{2}{x^2}y = -x(\text{Sec}\, x)^2\, y^2$ diferansiyel denkleminin genel çözümünü bulunuz.

$y^{1-2} = y^{-1} = u$

$-y^{-2}y' = u'$ 4

$\frac{1}{x}y'y^{-2} + \frac{2}{x^2}y^{-1} = -x\sec^2 x$ 6

$-\frac{1}{x}u' + \frac{2}{x^2}u = -x\sec^2 x$ 2 Lineer Dif Denk

$\boxed{u' - \frac{2}{x}u = x^2\sec^2 x}$ 2

$\lambda(x) = e^{\int -\frac{2}{x}dx} = e^{-2\ln x} = \frac{1}{x^2}$ 3

$\underbrace{\frac{u'}{x^2} - \frac{2}{x^3}u}_{} = \sec^2 x$ 3

$\frac{d}{dx}\left(\frac{u}{x^2}\right) = \sec^2 x \rightarrow \frac{u}{x^2} = \int \sec^2 x\, dx = \tan x + c$ 3

$u = x^2\tan x + cx^2$ 3

$y = \frac{1}{u} = \frac{1}{x^2\tan x + cx^2}$ 3

3) a) $2yy' + 4x^3\sqrt{1-y^4} = 0$ diferansiyel denkleminin $y(1)=0$ özel çözümünü bulunuz.

$2y\,dy + 4x^3\sqrt{1-y^4}\,dx = 0$ değişkenlere ayrılabilen dif. denk

$$\int \frac{2y\,dy}{\sqrt{1-y^4}} + \int 4x^3\,dx = \int 0$$

$$\left.\begin{aligned} y^2 &= t \\ 2y\,dy &= dt \end{aligned}\right\} \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t = \arcsin y^2$$

$\arcsin y^2 + x^4 = c$

$y(1)=0 \rightarrow \arcsin 0 + 1 = c \rightarrow \boxed{c=1}$

b) $\frac{y}{x} + y'(2y + \ln x) = -x^2$ diferansiyel denkleminin genel çözümünü bulunuz.

$$\underbrace{(2y+\ln x)}_{N}\,dy + \underbrace{\left(\frac{y}{x}+x^2\right)}_{m}\,dx = 0$$

$$\frac{\partial m}{\partial y} = \frac{1}{x} = \frac{\partial N}{\partial x}$$ Tam Dif. Denk

$$\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy = m\,dx + N\,dy$$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= \frac{y}{x} + x^2 \\ \frac{\partial f}{\partial y} &= 2y + \ln x \end{aligned}\right\} \begin{aligned} & f(x,y) = y\ln x + \frac{x^3}{3} + h(y) \\ & \frac{\partial f}{\partial y} = \ln x + \frac{dh}{dy} = 2y + \ln x \end{aligned}$$

$$\frac{dh}{dy} = 2y \rightarrow h = y^2 + c$$

$$f(x,y) = y\ln x + \frac{x^3}{3} + y^2 + c = k$$

Ba

4) $y = -x(y')^2 + \ln y'$ diferansiyel denkleminin genel çözümünü bulunuz.

$y' = p$ (1)

$y = -xp^2 + \ln p$ (2)

$\underbrace{y'}_{p} = -p^2 - 2xpp' + \dfrac{p'}{p}$ (2)

$\Rightarrow p'\left[\dfrac{1}{p} - 2xp\right] = p^2 + p$ (2) Lagrange dif. denk

$$\frac{dx}{dp} = \frac{\left[\frac{1}{p} - 2xp\right]}{p^2+p} \rightarrow \frac{dx}{dp} + \frac{2x}{p+1} = \frac{1}{p^2(p+1)}$$ (5) Linear dif. denk

$$\lambda(p) = e^{\int \frac{2dp}{p+1}} = e^{2\ln(p+1)} = (p+1)^2$$ (2)

$$\underbrace{(p+1)^2\frac{dx}{dp} + 2(p+1)x}_{} = \frac{p+1}{p^2}$$

$$\frac{d}{dp}\left((p+1).x\right) = \left(\frac{1}{p} + \frac{1}{p^2}\right)$$ (2)

4 → ~~35~~, ~~39~~ 32, ~~42~~ 23, ~~44~~ 35, ~~45~~, 46 34

~~19~~ 78, ~~30~~, ~~21~~ 25,

3 → ~~19~~ 78, ~~21~~ 25, 59, ~~42~~ 23,

$(p+1)x = -p^{-1} + \ln p + C$ (2)

$$\boxed{x = \frac{-p^{-1} + \ln p + C}{p+1}}$$ (3)

$$\boxed{y = -\frac{-p^{-1} + \ln p + C}{p+1} \cdot p^2 + \ln p}$$ (3)

Başarılar

3) $y'' + y' + xy = 0$ denkleminin $x = 0$ noktası civarında seri çözümünü elde ediniz.

A/ $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} = -2e^{3x} - 6xe^{3x}$

$r^2 - 3r = 0 \quad r(r-3) = 0 \quad r = 0 \; r = 3 \quad \boxed{y = c_1 + c_2 e^{3x}}$

$\boxed{y_{ö1} = kxe^{3x}} \quad y'_{ö1} = ke^{3x} + 3kxe^{3x} \quad y''_{ö1} = 3ke^{3x} + 3ke^{3x} + 9kxe^{3x}$

$\cancel{3ke^{3x}} + 3k\cancel{e^{3x}} + 3ke^{3x} + \cancel{9kxe^{3x}} - \cancel{3kxe^{3x}} = -2e^{3x} \quad 3k = -2 \quad k = -\frac{2}{3} \quad y_{ö1} = -\frac{2}{3}xe^{3x}$

$\boxed{y_{ö2} = z(x)e^{3x}} \quad y'_{ö2} = z'e^{3x} + 3ze^{3x} \quad y''_{ö2} = z''e^{3x} + 6z'e^{3x} + 9ze^{3x}$

$z'' + 6z' + \cancel{9z} - 3z' - \cancel{9z} = -6x \quad z'' + 3z' = -6x \quad r^2 + 3r = 0 \quad r(r+3) = 0 \quad r = 0$

$z_ö = (ax+b)x = ax^2 + bx \quad z' = 2ax + b \quad z'' = 2a$

$2a + 6ax + 3b = -6x \quad a = -1 \quad 2a + 3b = 0 \quad -2 + 3b = 0 \quad 3b = 2 \quad b = \frac{2}{3} \quad z_ö = -x^2 + \frac{2}{3}x$

$y_{ö2} = \left(-x^2 + \frac{2}{3}x\right)e^{3x}$

$y = c_1 + c_2 e^{3x} - \cancel{\frac{2}{3}xe^{3x}} + \left(-x^2 + \cancel{\frac{2}{3}x}\right)e^{3x}$

$y = c_1 + c_2 e^{3x} - x^2 e^{3x}$

/ $y' - 3y = -2xe^{3x}$

$\ln\lambda = -\int 3\,dx \quad \ln\lambda = -3x + C$

$\lambda = e^{-3x+C}$

$e^{-3x+C}y' - 3e^{-3x+C}y = -2xe^{C}$

$e^{-3x+C}y = \int -2xe^{C}\,dx$

$e^{-3x+C}y = -x^2e^{C}$

$e^{-3x}y = -x^2 + K$

$-e^{-3x}y - x^2 = K$

$-e^{-3x}y = K + x^2$

$y = \dfrac{x^2 + K}{-e^{-3x}}$

$y = \dfrac{-x^2 - K}{e^{-3x}}$

Başarılar...

4) $xy''' - y'' = 0$

$y'' = t \quad y''' = t'$

$xt' - t = 0 \qquad x\dfrac{dt}{dx} = t$

$$\frac{dt}{t} = \frac{dx}{x}$$

$$\ln t = \ln x + \ln c_1$$

$$t = xc_1$$

$$\frac{d^2y}{dx^2} = xc_1$$

$$\frac{d}{dx}\left(\frac{dy}{dx}\right) = xc_1$$

$$\int d\left(\frac{dy}{dx}\right) = \int xc_1\,dx$$

$$\frac{dy}{dx} = \frac{x^2}{2}c_1 + c_2$$

$$\underline{y = \frac{x^3}{6}c_1 + c_2x + c_3}$$

5) $y'' - 2y' - 3y = e^{-x}$

$r^2 - 2r - 3 = 0 \qquad r_1 = 3 \quad r_2 = -1$

$3 \quad -1$

$$y = c_1e^{3x} + c_2e^{-x}$$

$$c_1'e^{3x} + c_2'e^{-x} = 0$$

$$\underline{3c_1'e^{3x} - \cancel{c_2'}e^{-x} = e^{-x}}$$

$$4c_1'e^{3x} = e^{-x}$$

$$c_1' = \frac{e^{-4x}}{4}$$

$$\frac{dc_1}{dx} = \frac{1}{4}e^{-4x}$$

$$c_1 = \frac{-1}{16}e^{-4x} + k_1$$

$$c_1'e^{3x} + c_2'e^{-x} = 0$$

$$\frac{e^{-4x}}{4}e^{3x} = -c_2'e^{-x}$$

$$\frac{e^{-x}}{4} = -c_2'e^{-x}$$

$$c_2' = \frac{-1}{4}$$

$$dc_2 = \frac{-1}{4}dx$$

$$c_2 = \frac{-1}{4}x + k_2$$

$$y = \left(\frac{-1}{16}e^{-4x} + k_1\right)e^{3x} + \left(\frac{-1}{4}x + k_2\right)e^{-x}$$

Soru 4

$$\left.\begin{aligned} &\frac{dx}{dt}+2y-x=0 \\ &\frac{dy}{dt}+y-x=-\sin t \end{aligned}\right\}$$ denklem sistemini çözünüz.

I. denklem taraf tarafa çıkarılırsa

$$\frac{dx}{dt}-\frac{dy}{dt}+y=\sin t \qquad (2)$$

2. denklem türetilirse

$$\frac{d^2y}{dt^2}+\frac{dy}{dt}-\frac{dx}{dt}=-\cos t \rightarrow \boxed{\frac{dx}{dt}=\frac{d^2y}{dt^2}+\frac{dy}{dt}+\cos t} \qquad (2)$$

Bu iki denklemden

$$\frac{d^2y}{dt^2}+y=\sin t-\cos t \qquad (2)$$

$$r^2+1=0 \rightarrow r_{1,2}=\mp i \ (1) \rightarrow y_h=c_1\cos t+c_2\sin t \ (1)$$

$$y_{ö}=(A\cos t+B\sin t)\,t \qquad (2)$$

$$y_{ö}'=(-A\sin t+B\cos t)\,t+(A\cos t+B\sin t) \qquad (1)$$

$$y_{ö}''=(-A\cos t-B\sin t)\,t+2(-A\sin t+B\cos t) \qquad (1)$$

$$(A\cos t-B\sin t)\,t+2(-A\sin t+B\cos t)+(A\cos t+B\sin t)\,t=\sin t-\cos t \qquad (2)$$

$$-2A=1 \rightarrow \boxed{A=-\tfrac{1}{2}} \ (1) \qquad 2B=-1 \rightarrow \boxed{B=-\tfrac{1}{2}} \ (1)$$

$$y_{ö}=-\frac{1}{2}t\cos t-\frac{1}{2}t\sin t \qquad (2)$$

$$\boxed{y=y_h+y_{ö}=c_1\cos t+c_2\sin t-\frac{1}{2}t\cos t-\frac{1}{2}t\sin t} \qquad (2)$$

$$x=\frac{dy}{dt}+y+\sin t=-c_1\sin t+c_2\cos t-\frac{1}{2}\cos t+\frac{t}{2}\sin t-\frac{1}{2}\sin t-\frac{1}{2}t\cos t$$
$$+c_1\cos t+c_2\sin t-\frac{1}{2}t\cos t-\frac{1}{2}t\sin t+\sin t \qquad (5)$$

$$x=c_1(\cos t-\sin t)+c_2(\cos t+\sin t)-\frac{\cos t}{2}+\frac{\sin t}{2}-t\cos t$$

Soru 3 Laplace dönüşümünü kullanarak $y''+4y'+5y=10e^{t}$ dif. denkleminin $y(0)=0$, $y'(0)=0$ koşuluna uyan çözümünü bulunuz.

$$L\{y''\}+4L\{y'\}+5L\{y\}=L\{10e^{t}\}$$

$$\cancel{s^2Y(s)-s\,y(0)}-\underbrace{y'(0)}_{0}+\underbrace{4sY(s)-4y(0)}_{1}+\underbrace{5Y(s)=\frac{10}{s-1}}_{1}$$

4 1 1 1 1

$$[s^2+4s+5]\,Y(s)=\frac{10}{s-1} \Rightarrow Y(s)=\frac{10}{(s-1)(s^2+4s+5)}$$ 2

1 1

$$y(t)=L^{-1}\left\{\frac{10}{(s-1)(s^2+4s+5)}\right\}$$

$$\frac{A}{s-1}+\frac{Bs+C}{s^2+4s+5}=\frac{10}{(s-1)(s^2+4s+5)}$$

1 1

$$\left.\begin{aligned} A+B&=0\\ 4A-B+C&=0\\ 5A-C&=10 \end{aligned}\right\} \quad A=1 \;\; 1,\quad B=-1 \;\; 1,\quad C=-5 \;\; 1$$

$A=\frac{1}{10}$, $B=-\frac{1}{10}$, $C=-\frac{5}{10}=-\frac{1}{2}$

$$y(t)=L^{-1}\left\{\frac{1}{s-1}\right\}+L^{-1}\left\{\frac{-s-5}{s^2+4s+5}\right\}$$ 1

2 3 3

$$y(t)=e^{t}-L^{-1}\left\{\frac{s+2}{(s+2)^2+1}\right\}-L^{-1}\left\{\frac{3}{(s+2)^2+1}\right\}$$

$$\boxed{y(t)=e^{t}-e^{-2t}\cos t-3e^{-2t}\sin t}$$

5

$$\mathcal{L}^{-1}\left\{\frac{1}{s-1}\right\}+\mathcal{L}^{-1}\left\{\frac{-s}{}\right\}-\mathcal{L}^{-1}\left\{\frac{5}{}\right\}$$

21

A, B, C yanlış

18

Cevap Anahtarı

Soru 1 $x^2y' + x^2y^2 - 3xy + 3 = 0$ denkleminin bir özel çözümü $y_1 = \frac{1}{x}$ olduğuna göre genel çözümünü bulunuz.

Çözüm

(4) $y = \frac{1}{x} + \frac{1}{u}$, $y' = -\frac{1}{x^2} - \frac{u'}{u^2}$

$$x^2\left(-\frac{1}{x^2} - \frac{u'}{u^2}\right) + x^2\left(\frac{1}{x^2} + \frac{2}{ux} + \frac{1}{u^2}\right) - 3x\left(\frac{1}{x} + \frac{1}{u}\right) + 3 = 0$$

$$-1 - \frac{u'x^2}{u^2} + 1 + \frac{2x}{u} + \frac{x^2}{u^2} - 3 - \frac{3x}{u} + 3 = 0$$ (4)

(5) $\boxed{u' + \frac{u}{x} = 1}$ lineer dif. denk.

$$\lambda(x) = e^{\int \frac{dx}{x}} = e^{\ln x} = x$$ (3)

$$u'x + u = x \Rightarrow \frac{d}{dx}(u.x) = x$$

$$ux = \frac{d}{dx}(x) = \frac{x^2}{2} + c$$ (3)

$\boxed{u = \frac{x}{2} + \frac{c}{x}}$ (4)

$\boxed{y = \frac{1}{x} + \frac{2x}{x^2 + 2c}}$ (2)

Soru 2 $xy'' - y' - \frac{3y}{x} = 4x^2 \ln x$ diferansiyel denklemini çözünüz.

$x^2y'' - xy' - 3y = 4x^3 \ln x$ 1

$x = e^t$

3 $y' = e^{-t}Dy$

$y'' = e^{-2t}D(D-1)y$

$e^{2t} \cdot e^{-2t}D(D-1)y - e^{-t}Dy - 3y = 4e^{3t} \cdot t$

$(D^2 - 2D - 3)y = 4te^{3t}$ 3

$r^2 - 2r - 3 = 0$

$-3 \quad +1$

$r_1 = 3, \quad r_2 = -1$ 1

$\boxed{y_h = c_1 e^{3t} + c_2 e^{-t}}$ 2

2 $y_ö = (at+b)te^{3t} = (at^2+bt)e^{3t}$

2 $y_ö' = (2at+b)e^{3t} + 3(at^2+bt)e^{3t}$

2 $y_ö'' = 2ae^{3t} + 6(2at+b)e^{3t} + 9(at^2+bt)e^{3t}$

3

$\left[2a + 6(2at+b) + 9(at^2+bt) - 2(2at+b) - 6(at^2+bt) - 3(at^2+bt)\right]e^{3t} = 4te^{3t}$

$2a + 8at + 4b = 4t$

$2a + 4b = 0 \Rightarrow \boxed{b = -\frac{1}{4}}$ 1

$\boxed{a = \frac{1}{2}}$ 1

$y_ö = \left(\frac{1}{2}t^2 - \frac{1}{4}t\right)e^{3t}$ 2

$y = y_h + y_ö = c_1 x^3 + \frac{c_2}{x} + \left(\frac{1}{2}(\ln x)^2 - \frac{1}{4}(\ln x)\right)x^3$ 2

3) $\left.\begin{aligned} \frac{dx}{dt} &= y+1 \\ \frac{dy}{dt} &= -x+\frac{1}{Sint} \end{aligned}\right\}$ Diferansiyel denklem sistemini türetme-yok etme yöntemiyle çözünüz.

$$y=\frac{dx}{dt}-1 \Rightarrow \frac{dy}{dt}=\frac{d^2x}{dt^2}$$

$$\boxed{\frac{d^2x}{dt^2}+x=\frac{1}{\sin t}}$$

$$r^2+1=0 \Rightarrow r_{1,2}=\mp i \qquad x_h=C_1\cos t+C_2\sin t$$

$$\begin{aligned} \sin t / \quad & c_1'\cos t+c_2'\sin t=0 \\ \cos t / \quad & -c_1'\sin t+c_2'\cos t=\frac{1}{\sin t} \end{aligned}$$

$+$

$$c_2'=\cot t \Rightarrow \boxed{c_2=\ln(\sin t)+k_2}$$

$$c_1'=-c_2'\tan t=-(\cot t).(\tan t)=-1 \Rightarrow \boxed{c_1=-t+k_1}$$

$$\boxed{x=k_1\cos t+k_2\sin t-t\cos t+\sin t\ln(\sin t)}$$

$$y=\frac{dx}{dt}-1=-k_1\sin t+k_2\cos t-\cos t+t\sin t+\cos t\ln(\sin t)+\sin t.\frac{\cos t}{\sin t}-1$$

2) $y = xy' + \sqrt{(y')^2 + 1}$ diferansiyel denklemini çözünüz.

$y' = p \rightarrow y = xp + \sqrt{p^2+1}$ Clairaut D.D

$$\underbrace{y'}_{p} = p + xp' + \frac{2pp'}{2\sqrt{p^2+1}}$$

$$xp' + \frac{pp'}{\sqrt{p^2+1}} = 0 \Rightarrow p'\left[x + \frac{p}{\sqrt{p^2+1}}\right] = 0$$

$$y = xc_1 + \sqrt{c_1^2+1}$$

1) $p' = 0 \rightarrow p = c_1 \Rightarrow \boxed{y = c_1 x + c_2}$ Genel çözüm

2) $x = \dfrac{-p}{\sqrt{p^2+1}}$

$y = \dfrac{-p^2}{\sqrt{p^2+1}} + \sqrt{p^2+1} = \dfrac{1}{\sqrt{p^2+1}}$

$\Bigg\}\ \boxed{x^2 + y^2 = 1}$ Tekil çözüm

4) *Laplace Dönüşümünü* kullanarak $y'' + 9y = 10e^{-x}$ diferansiyel denkleminin $y(0) = 0$, $y'(0) = 0$

koşullarına uyan çözümünü bulunuz.

$$s^2 Y(s) + s\underbrace{y(0)}_{0} - \underbrace{y'(0)}_{0} + 9Y(s) = \frac{10}{s+1}$$

$$(s^2+9)Y(s) = \frac{10}{s+1} \Rightarrow Y(s) = \frac{10}{(s+1)(s^2+9)}$$

$$\frac{A}{s+1} + \frac{Bs+C}{s^2+9} = \frac{10}{(s+1)(s^2+9)}$$

$$(A+B)s^2 + (B+C)s + (9A+C) = 10$$

$$\left.\begin{array}{l} A+B=0 \\ B+C=0 \\ 9A+C=10 \end{array}\right\} \begin{array}{l} A=1 \\ B=-1 \\ C=1 \end{array}$$

$$y(t) = L^{-1}\left\{\frac{1}{s+1}\right\} - L^{-1}\left\{\frac{s}{s^2+9}\right\} + L^{-1}\left\{\frac{1}{s^2+9}\right\}$$

$$\boxed{y(t) = e^{-t} - \cos 3t + \frac{1}{3}\sin 3t}$$