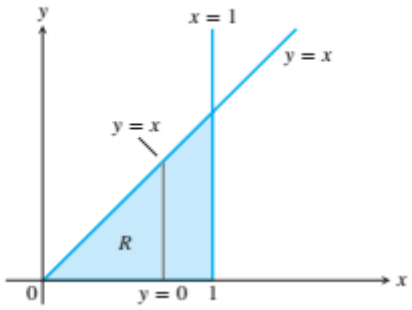
ÖRNEK:

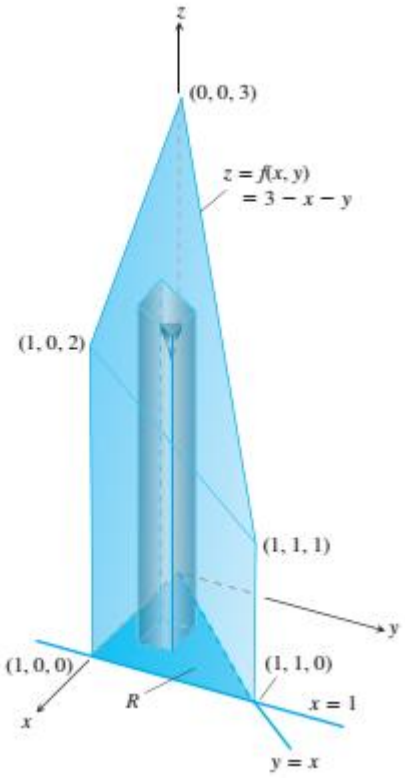
Tabanı, xy-düzleminde x-ekseni, $y = x$ ve $x = 1$ doğruları tarafından sınırlı üçgen olan ve tepesi

$$z = f(x, y) = 3 - x - y$$

düzleminde bulunan prizmanın hacmini bulun.

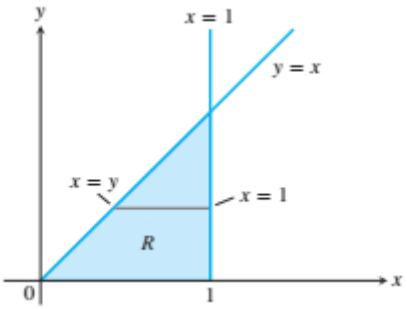
Bölge:





$$V = \int_0^1 \int_0^x (3 - x - y)\, dy\, dx = \int_0^1 \left[3y - xy - \frac{y^2}{2}\right]_{y=0}^{y=x} dx$$

$$= \int_0^1 \left(3x - \frac{3x^2}{2}\right) dx = \left[\frac{3x^2}{2} - \frac{x^3}{2}\right]_{x=0}^{x=1} = 1$$



$$V = \int_0^1 \int_y^1 (3 - x - y)\, dx\, dy = \int_0^1 \left[3x - \frac{x^2}{2} - xy\right]_{x=y}^{x=1} dy$$

$$= \int_0^1 \left(3 - \frac{1}{2} - y - 3y + \frac{y^2}{2} + y^2\right) dy$$

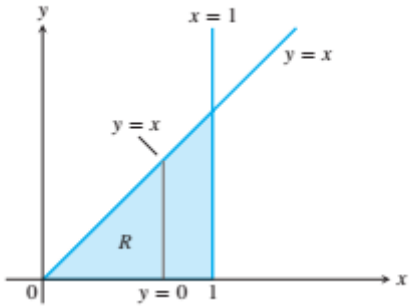
$$= \int_0^1 \left(\frac{5}{2} - 4y + \frac{3}{2}y^2\right) dy = \left[\frac{5}{2}y - 2y^2 + \frac{y^3}{2}\right]_{y=0}^{y=1} = 1$$

ÖRNEK

R, xy-düzleminde x-ekseni, $y = x$ doğrusu ve $x = 1$ doğrusuyla sınırlanan üçgen olmak üzere

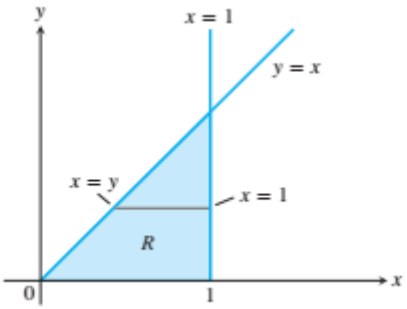
$$\iint_R \frac{\sin x}{x}\, dA$$

integralini hesaplayın.



$$\int_0^1 \left(\int_0^x \frac{\sin x}{x}\, dy\right) dx = \int_0^1 \left(y \frac{\sin x}{x}\right]_{y=0}^{y=x} dx = \int_0^1 \sin x\, dx$$

$$= -\cos(1) + 1 \approx 0.46.$$

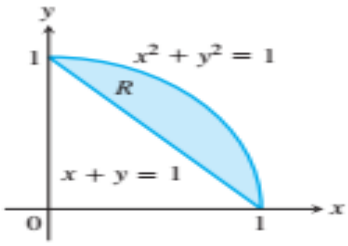


$$\int_0^1 \int_y^1 \frac{\sin x}{x}\, dx\, dy$$

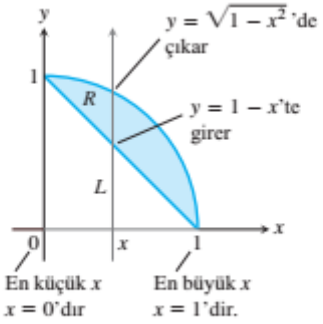
Bu durumda integrali hesaplayamayacaktık.

İNTEGRASYON SINIRLARINI BULMAK

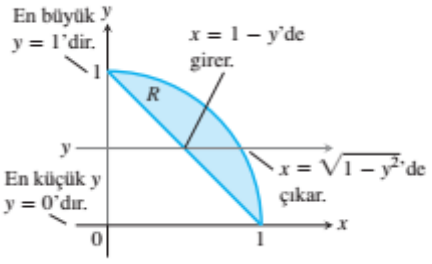
$$\iint_R f(x, y)\, dA$$



integralini şekilde verilen R bölgesinde sınırlarını belirleyelim.



$$\iint_R f(x, y)\, dA = \int_{x=0}^{x=1} \int_{y=1-x}^{y=\sqrt{1-x^2}} f(x, y)\, dy\, dx$$



$$\iint_R f(x, y)\, dA = \int_0^1 \int_{1-y}^{\sqrt{1-y^2}} f(x, y)\, dx\, dy$$

İntegrasyon sırasını değiştirmek

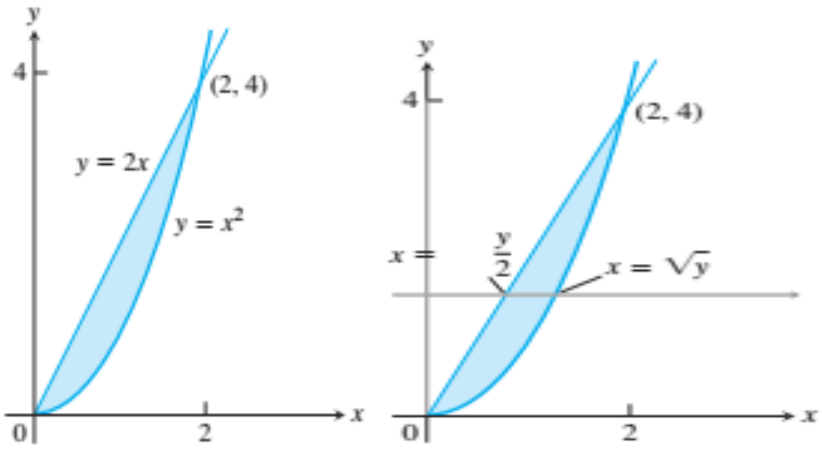
ÖRNEK

$$\int_0^2 \int_{x^2}^{2x} (4x + 2)\, dy\, dx$$

İntegralini hesaplayalım.

İntegrasyon bölgesi:

$$x^2 \leq y \leq 2x \text{ ve } 0 \leq x \leq 2$$



$$\int_0^4 \int_{y/2}^{\sqrt{y}} (4x + 2)\, dx\, dy$$

Değeri=8

İki Katlı İntegrallerin Özellikleri

$f(x, y)$ ve $g(x, y)$ fonksiyonları sürekli ise

1. *Sabitle Çarpım:* $\iint_R cf(x, y)\, dA = c \iint_R f(x, y)\, dA$ (herhangi bir c sayısı)

2. *Toplam ve Fark:*

$$\iint_R (f(x, y) \pm g(x, y))\, dA = \iint_R f(x, y)\, dA \pm \iint_R g(x, y)\, dA$$

3. *Baskınlık:*

(a) R üzerinde $f(x, y) \geq 0$ ise $\iint_R f(x, y)\, dA \geq 0$

(b) R üzerinde $f(x, y) \geq g(x, y)$ ise

$$\iint_R f(x, y)\, dA \geq \iint_R g(x, y)\, dA$$

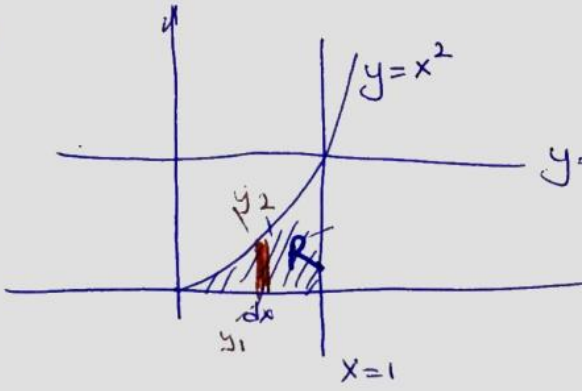
4. R, üst üste binmeyen R_1 ve R_2 gibi iki bölgenin bileşimi ise (Şekil 15.7)*:*

$$\iint_R f(x, y)\, dA = \iint_{R_1} f(x, y)\, dA + \iint_{R_2} f(x, y)\, dA$$

Int. Sınırlarına göre integrasyon sınırlarının değişm

$$I = \int_{0}^{1} \int_{x=\sqrt{y}}^{1} e^{y/x}\, dx\,dy = ?$$

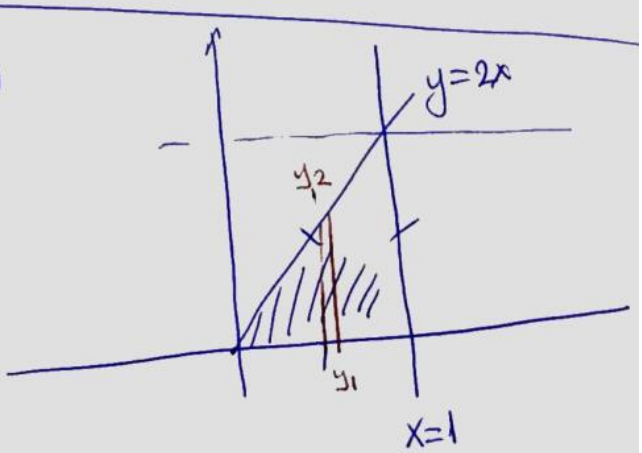
R: $\begin{pmatrix} x=\sqrt{y} & x=1 \\ y=0 & y=1 \end{pmatrix}$



$$I = \int_{x=0}^{x=1} dx \int_{y=0}^{y=x^2} e^{y/x}\, dy = \frac{1}{2} //$$

$$\int_{0}^{2} \left(\int_{x=y/2}^{1} \sin x^2\, dx \right) dy$$

R: $\begin{pmatrix} x=y/2 & y=0 \\ x=1 & y=2 \end{pmatrix}$



$$I = \int_{0}^{1} dx \sin x^2 \int_{y=0}^{y=2x} dy = 1 - \cos 1 //$$

) a) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx = ?$

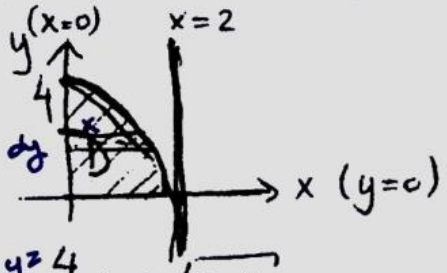
b) R bölgesi $y = x, y = 2x$ ve $x + y = 2$ doğrularıyla sınırlı bölge olmak üzere

$$\iint_R xy\,dA$$

integralini hesaplayınız.

a) $\int_0^2 \int_0^{4-x^2} \frac{xe^{2y}}{4-y}\,dy\,dx$

D: $x=0$ $x=2$ $y=0$ $y=4-x^2$

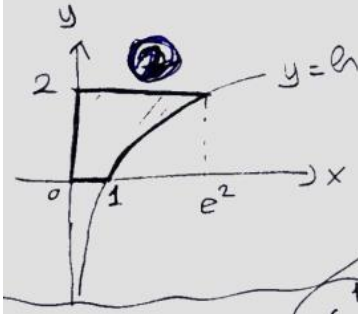


$$I = \int_{y=0}^{y=4} \int_0^{\sqrt{4-y}} \frac{xe^{2y}}{4-y}\,dx\,dy = \int_0^4 \frac{x^2}{2}\left(\frac{e^{2y}}{4-y}\right)\Big|_0^{\sqrt{4-y}}\,dy$$

$$= \frac{1}{2}\int_0^4 \frac{(4-y)e^{2y}}{4-y}\,dy = \frac{1}{2}\cdot\frac{1}{2}e^{2y}\Big|_0^4$$

$$= \frac{e^8-1}{4}$$

S. 4-a) $y = \ln x$ eğrisi, $y = 0$, $y = 2$ doğruları ve y-ekseni ile sınırlanmış bölgenin **alanını,** *iki katlı integral* ile hesaplayınız. (13 P)



1. YOL : $A = \int_0^2 \int_0^{e^y} dx\,dy$ → Σ8

$A = \int_0^2 e^y dy = (e^2 - 1)$ br² // → Σ3

2. YOL : $A = \int_0^1 \int_0^2 dy\,dx + \int_1^{e^2} \int_{\ln x}^2 dy\,dx = 2 + \int_1^{e^2} (2 - \ln x)\,dx$

$= 2 + \left[x(2 - \ln x) + x\right]\Big|_1^{e^2}$

$= 2 + \left[e^2(2-1) + e^2 - 2 - 1\right] = e^2 - 1$ //

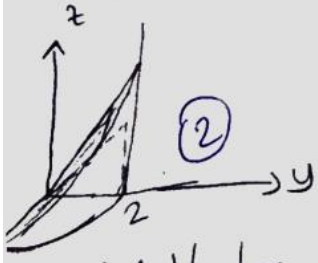
$\int (2 - \ln x)\,dx = x(2 - \ln x) + \int dx$

$= x(2 - \ln x) + x + C$

$2 - \ln x = u$; $dv = dx$

$-\frac{1}{x}dx = du$ $v = x$

S. 4-b) $z = 0$ ve $z = y$ düzlemleri ve $x^2 + y^2 = 4$ silindiri ile oluşturulmuş cismin, xy-düzleminin üstünde kalan kısmının **hacmini** *iki katlı integral* ile bulunuz. (12 P)



$V = \iint_D y\,dA = \int_{-2}^{2} \int_0^{\sqrt{4-x^2}} y\,dy\,dx = \int_{-2}^{2} \frac{y^2}{2}\Big|_0^{\sqrt{4-x^2}} dx$

$V = \frac{1}{2}\int_{-2}^{2} (4 - x^2)\,dx = \frac{1}{2}\left(4x - \frac{x^3}{3}\right)\Big|_{-2}^{2} = \frac{16}{3}$ br³ //