YILDIZ TECHNICAL UNIVERSITY, FACULTY OF CIVIL ENG. DEPT. OF GEOMATIC ENG.

ENGINEERING CALCULATIONS

LECTURE NOTES – PART 1

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Contents

1 INTRODUCTION TO GEOMATICS

Surveying is the technique, profession, and science of accurately determining the three-dimensional positions of points and the distances and angles between them. In other words, it is the art of measuring horizontal and vertical distances between objects, of measuring angles between lines, of determining the direction of lines, and of establishing points by predetermined angular and linear measurements. Distances, angles, directions, locations, elevations, areas, and volumes are thus determined from the data of the survey. These observables are usually on the surface of the Earth, and they are often used to establish land maps and boundaries for ownership or governmental purposes. The entire scope of profession is wide; it actually boils down to calculate where the land boundaries are situated. This is very important as without this service, there would not have been engineering structures, such as dams, bridges, highways, etc., planning facilities and so on.

Types of survey

As providing a general manner of survey facilities, bellows are listed as to describe some types of surveys.

Geodetic Surveys: The type of surveying that takes into account the true shape of the earth. These surveys are of high precision and extend over large areas.

Control Survey: Made to establish the horizontal and vertical positions of arbitrary points.

Boundary Survey: Made to determine the length and direction of landlines and to establish the position of these lines on the ground.

Hydrographic Survey: The survey of bodies of water made for the purpose of navigation, water supply, or sub-aqueous construction.

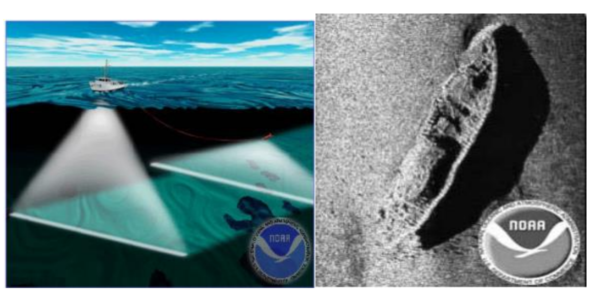


Figure 1.1 Hydrographic Survey

Mining Survey: Made to control, locate, and map underground and surface works related to mining operations.

Figure 1.2 Mine Surveying, underground and open pit

Construction Survey: Made to lay out, locate, and monitor public and private engineering works.

Figure 1.3 Construction Surveys

Route Survey: Refers to those controls, topographic, and construction surveys necessary for the location and construction of highways, railroads, canals, transmission lines, and pipelines.

Figure 1.4 Route Surveys

Photogrammetric Survey: Made to utilize the principles of aerial photogrammetry, in which measurements made on photographs are used to determine the positions of photographed objects.

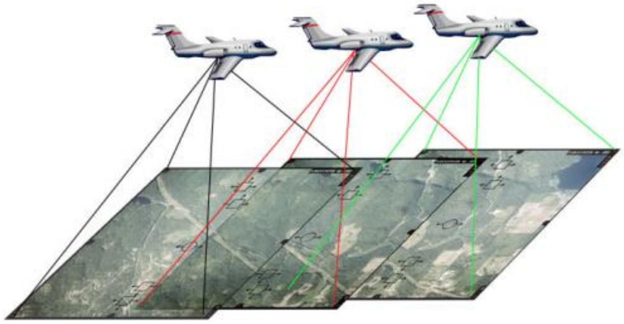


Figure 1.5 Photogrammetric Surveys

Astronomical Survey: Generally involve imaging or "mapping" of regions of the sky using telescopes.

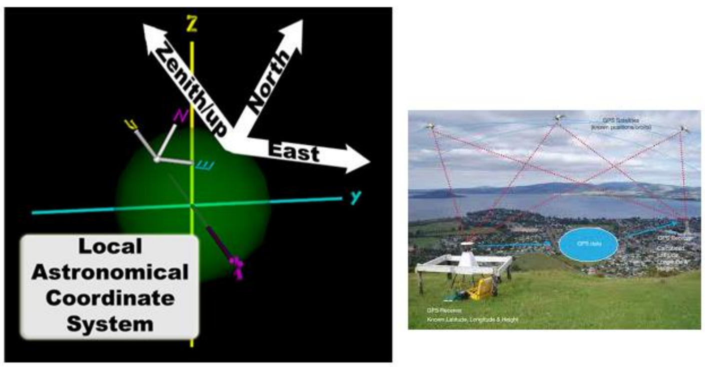


Figure 1.6 Astronomical Surveys

Land Surveys: To determine the boundaries and areas of parcels of land, also known as property survey, boundary survey or cadastral survey.

Figure 1.7 Land Surveys

Topographic Survey: Made to gather data to produce a topographic map showing the configuration of the terrain and the location of natural and manmade objects.

The purpose of topographic survey is to gather survey data about the natural and manmade features of the land as well as its elevations. Maps are then prepared from this information. The work usually consists of the following:

- Establishing horizontal and vertical control points that will serve as reference points for the survey. The most accurate method of establishing the vertical control is by levelling.
- Collecting enough horizontal and elevation of ground points to provide enough data for plotting when the map is prepared. For example, when surveying for upgrading a taxi rank, the features to be located will be, existing sidewalks, curbs, trees, island, etc. and for a road intersection features like: kerbs, islands will be located.
- The position and shape of natural and manmade features that may be required for the survey.
- Calculating distances, angles, and elevations.

Topography is a field of geoscience and planetary science comprising the study of surface shape and features of the Earth and other observable astronomical objects including planets, moons, and asteroids. It is also the description of such surface shapes and features (especially their depiction in maps). The topography of an area could also mean the surface shape and features them.

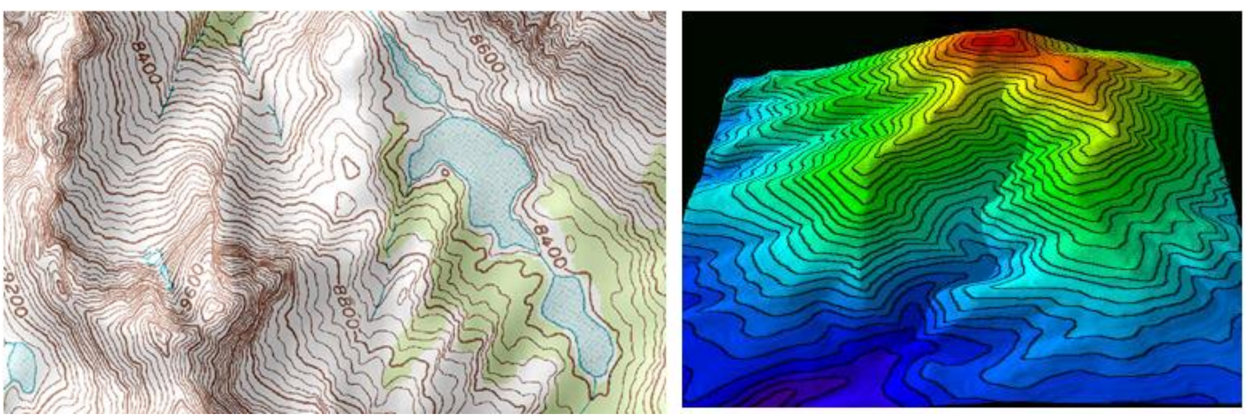


Figure 1.8 Example of topography

Standards and Specifications

A standard attempts to define the quality of the work in a way that is ideally independent of the equipment or technology in use. A specification describes how to achieve a certain standard with a given set of tools, equipment, or technology.

The followings are typical definitions of a standard:

1. An exact value, or concept thereof, established by authority, custom, or common consent, to serve as a rule or basis of comparison in measuring quantity, content, extent, value, quality, and capacity.
2. The type, model, or example commonly or generally accepted or adhered to; criterion set for the establishment of a practice or procedure.
3. The minimum accuracies deemed necessary to meet specific objectives; a reasonably accepted error; a level of precision of closure; a numerical limit on the uncertainty of coordinates.

Specifications are the field operations or procedures required to meet a particular standard; the specified precision and allowable tolerances for data collection and/or application, the limitations of the geometric form of acceptable network figures, monumentation, and description of points.

2 MEASURING SYSTEM and UNITS

The International System of Units (SI, abbreviated from the French Système international (d'unités)) is the modern form of the metric system, and is the most widely used system of measurement. It comprises a coherent system of units of measurement built on **seven base units**, which are the ampere (electric current), kelvin (temperature), second (time), meter (length), kilogram (mass), candela (brightness), mole (quantity), and a set of twenty prefixes to the unit names and unit symbols that may be used when specifying multiples and fractions of the units.

A unit of measurement is a definite magnitude of a physical quantity, defined and adopted by convention or by law that is used as a standard for measurement of the same physical quantity. The modern metric system consists of four electromechanical base units representing seven fundamental dimensions of measure: length, mass, time, electromagnetism, thermodynamic temperature, luminous intensity, and quantity of substance. The units are:

- For length: meter = m
- For mass: kilogram = kg
- For time: second = s
- For electric current: Ampere = A
- For temperature: Kelvin = K
- For light intensity: Candela = cd
- For quantity of substance: Mole = mol

The utilization of prefixes leads to decimal multiples and fractions of these units.

These are denoted as:

	term	*prefix*		*term*	*prefix*
10^1	deca	da	10^{-1}	deci	d
10^2	Hector	h	10^{-2}	centi	c
10^3	kilo	k	10^{-3}	milli	m
10^6	mega	M	10^{-6}	micro	μ
10^9	giga	G	10^{-9}	nano	n
10^{12}	tera	T	10^{-12}	pico	p

Basic SI Units

The Units for Length, Area and Volume

These are based on the form of 1875 metric convention, and have since been expanded only upwards and downwards by several powers of ten.

a) The SI unit for length is the base unit meter (m).

$1\ decametre = 10^1\ m = 1\ dam$	$1\ decimetre = 10^{-1}\ m = 1\ dm$
$1\ hectometre = 10^2\ m = 1\ hm$	$1\ centimetre = 10^{-2}\ m = 1\ cm$
$1\ kilometre = 10^3\ m = 1\ km$	$1\ millimetre = 10^{-3}\ m = 1\ mm$
$1\ megametre = 10^6\ m = 1\ Mm$	$1\ micrometre = 10^{-6}\ m = 1\ \mu m$
$1\ gigametre = 10^9\ m = 1\ Gm$	$1\ nanometre = 10^{-9}\ m = 1\ nm$
$1\ terametre = 10^{12}\ m = 1\ Tm$	$1\ picometre = 10^{-12}\ m = 1\ pm$

b) The SI unit for area is the derived unit square meter (m^2). Using the mentioned prefixes one obtains:

$1\ are = 10^2\ m^2 = 1\ a$	$1\ square\ decimetre = 10^{-2}\ m^2 = 1\ dm^2$
$1\ hectare = 10^4\ m^2 = 1\ ha$	$1\ square\ centimetre = 10^{-4}\ m^2 = 1\ cm^2$
$1\ square\ kilometre = 10^6\ m^2 = 1\ km^2$	$1\ square\ millimetre = 10^{-6}\ m^2 = 1\ mm^2$

c) The SI unit for volume is the derived unit cubic meter (m^3). With the respective prefixes, one obtains dm^3, cm^3, mm^3. One litre has been retained as a specific denotation for 1 cubic decimeter. The formal connection between the litre and the unit for mass has been discontinued (1 L equals the volume of 1 kg pure water at maximum density under 1 atm pressure).

Conversion	
1 m^3	1000 dm^3
1 dm^3	1000 cm^3
1 cm^3	1000 mm^3
1 lt=1 dm^3	1000 cm^3

SI Units for Plane Angles

There are three systems in use for angular units, namely: sexagesimal graduation, centesimal graduation, and radiant (arc definition). The first two are as follows:

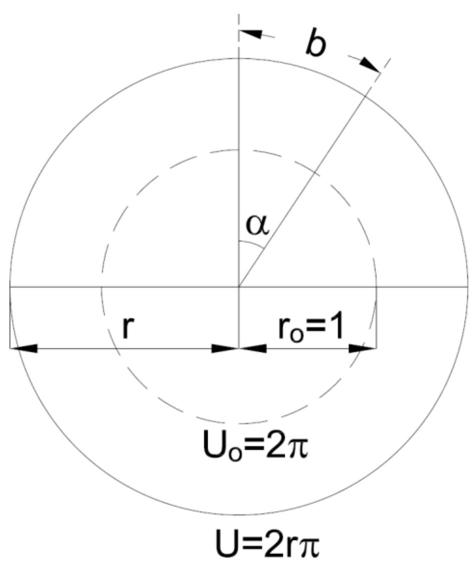
- *Sexagesimal Graduation:*

  $$1\ full\ circle = 360^o\ (degrees)$$
  $$1^o = 60^\iota\quad (minutes)$$
  $$1^\iota = 60^{\iota\iota}\quad (seconds)$$

- *Centesimal Graduation:*

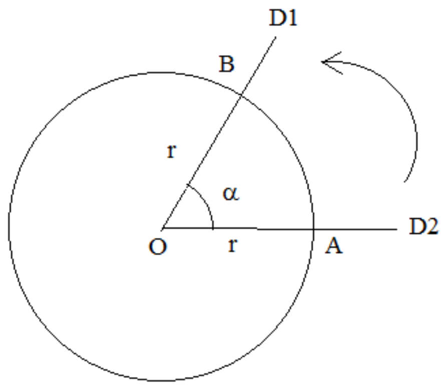
  $$1\ full\ circle = 400\ gon$$
  $$1\ gon\ = 100\ cgon\quad (centigon)$$
  $$1\ cgon = 100\ mgon\quad (milligon)$$

The arc of an angle is given by the ratio between the arc *b* as bisected by the sides of the angle α with vertex in its center, and the radius r of the circle. The unit of the arc is the angle for which this ratio equals 1, for b=r. This angle is called the "radian", because it is obtained by the length of the radius on the circumferences of the circle. The arc of the full circle is therefore 2π, while it is π /2 for a right angle. In the below representation, the definition of the arc of an angle may be found.



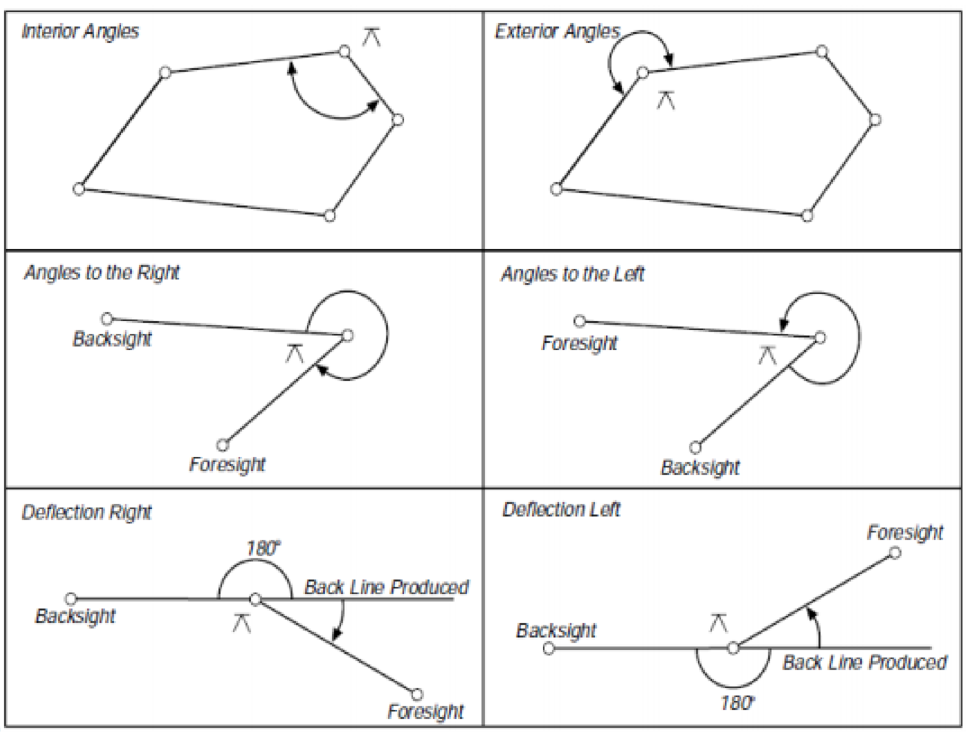
Specifically, one radian equals the plane angle, which, as center angle of a circle with radius 1 m, cuts an arc of 1 m length from the circumference. The illustration of the unit circle, where the center angle a, the corresponding arc b, and the radius r (indexed with zero) may be found as follow.

What is an angle: An angle is a combination of two rays (half-lines) with a common endpoint. The latter is known as the *vertex* of the angle and the rays as the *sides*, sometimes as the *legs* and sometimes the *arms* of the angle.



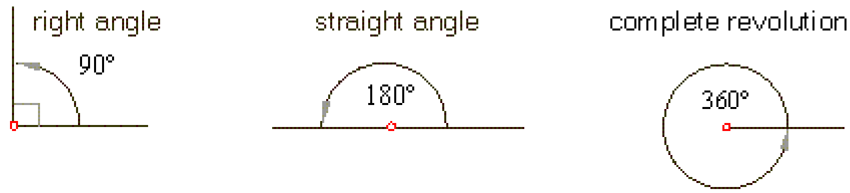
Measured Angles:

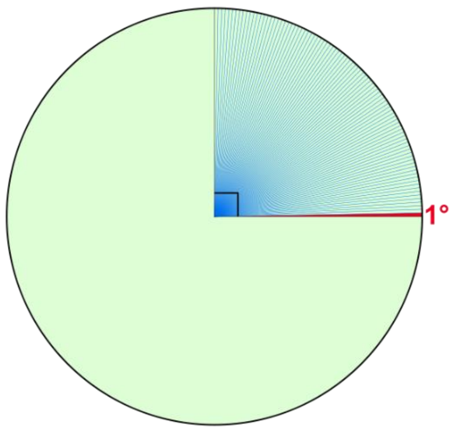
Interior angles are measured clockwise or counter-clockwise between two adjacent lines on the inside of a closed polygon figure. Exterior angles are measured clockwise or counter-clockwise between two adjacent lines on the outside of a closed polygon figure. Deflection angles, right or left, are measured from an extension of the preceding course and the ahead line. It must be noted when the deflection is right (R) or left (L).



Degree

Degree usually denoted by ° (the degree symbol), is a measurement of plane angle, representing 1⁄360 of a full rotation.





1 Degree	**1^0**	**60 minutes**	**3600 second**
1 Minute	**1 '**	**1/60 degree**	**60 seconds**
1 Second	**1 "**	**1/360 degree**	**1/60 minute**

1° = 60'= 3600" 1' = 60"

As an example of preferred notation of angles with sexagesimal system; 380^0 28' 43".6

Notice that minutes and seconds equal to or greater than 60 are carried over to the next larger unit and that degrees and minutes do not have decimals. Decimal seconds are acceptable.

For performing certain mathematical operations with angles, it is sometimes easier to convert to decimal degrees first, perform the necessary math, then convert back to degrees, minutes, and seconds.

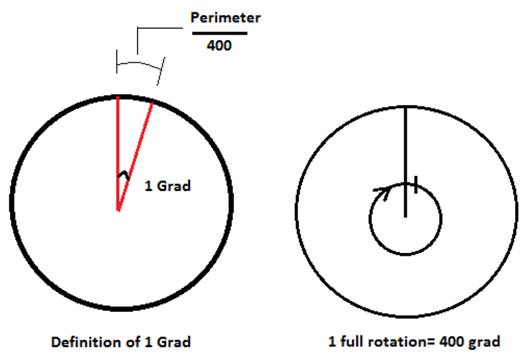
degrees – minutes – seconds	**decimal degrees**
87° 58 '48"	87.98^0

87° + (58/60) + (48/3600) = 87.98 °

Gradian (Grad/ Gon)

The gon is a unit of plane angle, equivalent to 1⁄400 of a turn.

- A grad is defined as 1/400 of a circle.
- A grad is dividing into 100 centigrad, centigrad into 100 centicentigrad.
- Grad is represented by the symbol (g), centigrad by (c), centicentigrad by (cc)

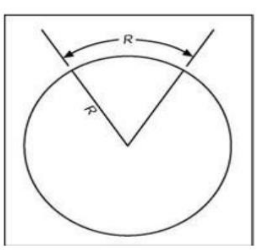


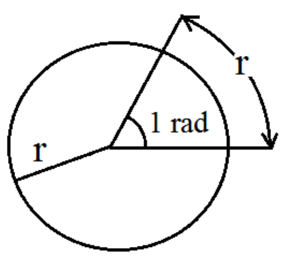
1^{g}	Grad	100 centigon	100^{c}
1^{c}	Centigon	100 centicentigon	100^{cc}
1^{cc}	Centicentigon	0.0001 grad	
1^{g}		100 cgon	
1^{c}		10 mgon (milligon)	

$58^{g}\ 62^{c}\ 73^{cc} = 58^{g}\ 62{,}73^{c} = 58{,}6273^{g}$

Radian

The radian is the standard unit of angular measure, used in many areas of mathematics. An angle's measurement in radians is numerically equal to the length of a corresponding arc of a unit circle; one radian is just under 57.3 degrees (when the arc length is equal to the radius).





The circumference of a circle is twice the radius length times π, or $C = 2\pi r$.

Therefore, 1 circle=2 π radians.

Conversion between Angular Units

Conversion of units is the conversion between different units of measurement for the same quantity, typically through multiplicative conversion factors.

$$360^o = 400^g = 2.\pi$$

$$\frac{D}{180} = \frac{G}{200} = \frac{R}{\pi}$$

Exercise: Please transform $45^o 17^{'} 58^{''}$ into gon.

$$\alpha = 45^o + \frac{17^{'}}{60} + \frac{58^{''}}{3600} = 45^o.29944$$

$$\frac{45^o.29944}{180} = \frac{G}{200}$$

$$G = \frac{45^o.29944 \times 200}{180} = 50^g.3327$$

Exercise: Please transform $60^g.2735$ into degree.

$$\frac{D}{180} = \frac{60^g.2735}{200}$$

$$D = \frac{60^g.2735 \times 180}{200} = 54^o.24615$$

$$D^o = 54^o.24615 \ \ (decimal \deg rees)$$

$$D^o = 54^o + (0.24615 \times 60) = 54^o \ 14^{'}.769$$

$$D^o = 54^o + 14^{'} + (0.769 \times 60) = 54^o \ 14^{'} \ 46^{''}.14$$

Exercise: Please transform 1.055221 radian into degree.

$$\frac{D}{180} = \frac{1.055221}{\pi}$$

$$D = \frac{1.055221 \times 180}{\pi} = 60^o.45970974$$

$$D^o = 60^o.45970974 \ \ (decimal \deg rees)$$

$$D^o = 60^o + (0.45970974 \times 60) = 60^o \; 27^{'}.5825844$$

$$D^o = 60^o + 27^{'} + (0.5825844 \times 60) = 60^o \; 27^{'} \; 34^{''}.96$$

Exercise: Please transform $149^g.5824$ into radian.

$$\frac{149^g.5824}{200} = \frac{R}{\pi}$$

$$R = \frac{149^g.5824 \times \pi}{200} = 2.349634\, rad$$

Conversion between radian to angle

$$360^o = 2.\pi$$

$$180^o = \pi$$

$$1^o = \frac{\pi}{180}\, rad = \frac{1}{\frac{180}{\pi}} = \frac{1}{57.2957795}\, rad$$

$$1^o = 0.01745329 \;\; rad$$

$$60^{'} = \frac{\pi}{180}\, rad \rightarrow 1^{'} = \frac{\pi}{180 \times 60} = \frac{1}{\frac{180 \times 60}{\pi}} = \frac{1}{3437.7467} = 0.00029088\, rad$$

$$60^{''} = \frac{\pi}{180 \times 60} \rightarrow 1^{''} = \frac{\pi}{180 \times 60 \times 60} = \frac{1}{\frac{180 \times 60 \times 60}{\pi}} = \frac{1}{206264.80} = 0.00000484\, rad$$

To convert:

$$1^o = 0.01745329 \;\; rad; \quad 1^{'} = 0.00029088 \quad rad; \quad 1^{''} = 0.00000484 \quad rad$$

3 TOPOGRAPHIC UNIT CIRCLE in SURVEYING

In geodesy, the azimuth angle is computed considering the topographic unit circle. The angle system bases on “gon” instead of “degree”. Comparing to the regular case the axis have been adapted in order that x blends to the top and y to the right. X axis also represents the north-south direction and y axis represents east-west direction. Azimuth starts with north direction and increase clockwise. Topographic unit circle is defined with its radius, which is 1. On a unit circle, OP line from 0 to 100 gon attends to 1st quadrant. Similarly, from 100 to 200 gon attends 2nd quadrant, from 200 to 300 gon attends 3rd quadrant and finally, from 300 to 400 gon attends 4th quadrant.

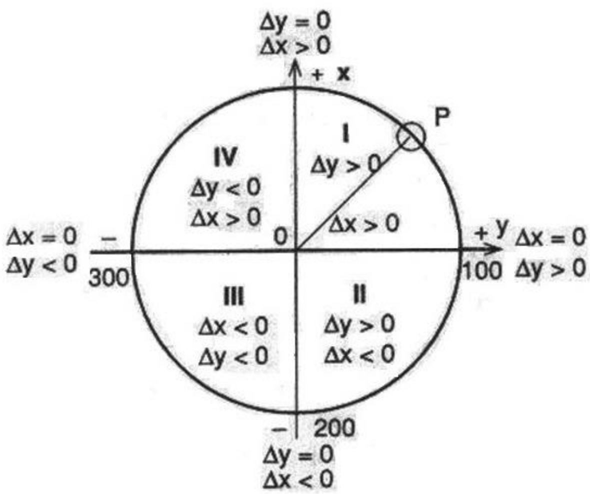


Figure 3.1 Topographic Unit Circle

Quadrants and limits

1st Quadrant	2nd Quadrant	3rd Quadrant	4th Quadrant
0^g-100^g	100^g-200^g	200^g-300^g	300^g-400^g
0^o-90^o	90^o-180^o	180^o-270^o	270^o-360^o

Most useable functions of Trigonometry

All trigonometric functions are simply ratios of one side of a right triangle to a second side of the same triangle, or one side over another side. The distinction between functions is which two sides are compared in the ratio. The figure below illustrates the side opposite from and the side adjacent to Angle A, and the hypotenuse (the side opposite the right angle).

The trigonometric functions of any angle are by definition:

$$Sine\ A = \sin(A) = \frac{Opposite\ Side}{\text{Hypotenuse}}$$

$$Cosine\ A = \cos(A) = \frac{\text{Adjacent Side}}{\text{Hypotenuse}}$$

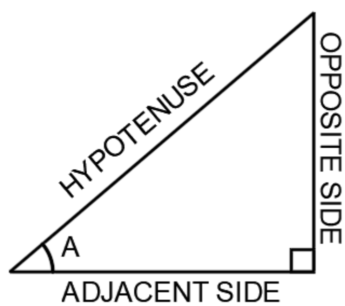
$$Tangent\ A = \tan(A) = \frac{\text{Opposite Side}}{\text{Adjacent Side}}$$

and inverting each ratio, we have;

Cosecant A = Hypotenuse / Opposite Side = 1/sine A

Secant A = Hypotenuse / Adjacent Side = 1/cosine A

Cotangent A = Adjacent Side / Opposite Side = 1/tangent A



Examination of the trigonometric functions on unit circle

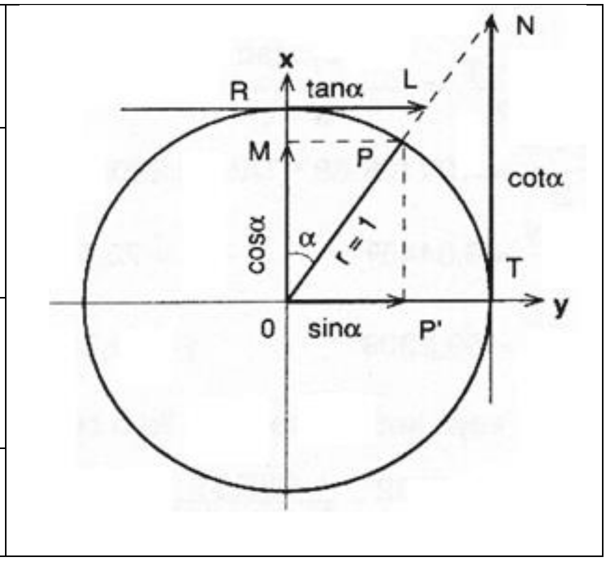
1- Angle α for 1st quadrant ($0^g < \alpha < 100^g$)

$$sin\alpha = \frac{\overline{MP}}{\overline{OP}} = \frac{\overline{OP'}}{r} = \overline{OP'}$$

$$cos\alpha = \frac{\overline{OM}}{\overline{OP}} = OM$$

$$tan\alpha = \frac{\overline{MP}}{\overline{OM}} = \frac{\overline{RL}}{\overline{OR}} = \overline{RL}$$

$$cot\alpha = \frac{\overline{OR}}{\overline{RL}} = \frac{\overline{NT}}{\overline{OT}} = \overline{NT}$$



If angle α is altered between 0 to 100 while OR side is taken constant, the dimensional values of trigonometric function can be defined as given above, which may be represented as;

The value of **sine** function is on the axes of **y**,

The value of **cosine** function is on the axes of **x**,

The value of **tangent** function is on the tangent of **R** point,

The value of **cotangent** function is on the tangent of **T** point.

The axes that the trigonometric functions have been defined are so called as “trigonometric function axis”.

The definitions of axes and +/- directions are illustrated as below.

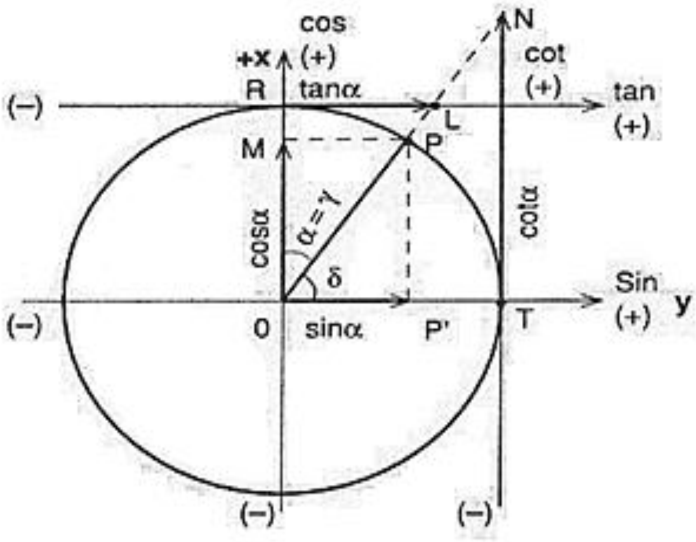


Figure 3.2 The trigonometric function estimations for 1st quadrant

Function	Sign	α=100-δ	α=γ
$sin\alpha$	+	$sin\alpha = +cos\delta$	$sin\alpha = +sin\gamma$
$cos\alpha$	+	$cos\alpha = +sin\delta$	$cos\alpha = +cos\gamma$
$tan\alpha$	+	$tan\alpha = +cot\delta$	$tan\alpha = +tan\gamma$
$cot\alpha$	+	$cot\alpha = +tan\delta$	$cot\alpha = +cot\gamma$

$$sin\delta = \frac{OM}{r} = OM = cos\alpha$$

$$cos\delta = \frac{OP'}{OP} = OP' = sin\alpha$$

$$tan\delta = \frac{NT}{OT} = NT = cot\alpha$$

$$cot\delta = \frac{RL}{OR} = RL = tan\alpha$$

Example:

$\alpha = 58.7234^g, then\ \delta = 41.2766^g$
$sin58.7234^g = cos41.2766^g$
$cos58.7234^g = sin41.2766^g$
$tan58.7234^g = cot41.2766^g$
$cot58.7234^g = tan41.2766^g$

2- Angle α for 2^{nd} quadrant ($100^g < \alpha < 200^g$)

As known, we can calculate the value of any trigonometric function using a calculator. However, since the value of trigonometric function is known, it is not estimated in which quadrant it attends except if it is in 1^{st} quadrant. Calculator gives the value for between -100^g and $+100^g$ (-90^o to $+90^o$). By the help of this, we should compute the desired exact angle.

The Angle computed by a calculator for between 0^g and 100^g should be converted to a value between 0^g and 400^g considering the signs and values of ΔY and ΔX. It is crucial to know the specifications of unit circle for solving the problems especially about ***Fundamental Computations-2***. Figure shows the trigonometric function estimations for 2^{nd} quadrant.

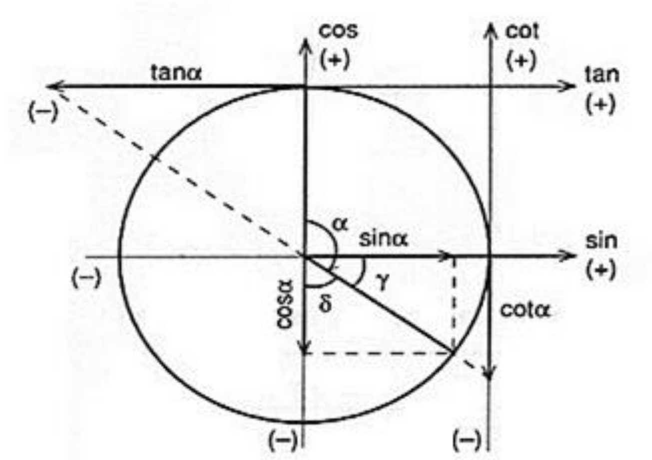


Figure 3.3 The trigonometric function estimations for 2^{nd} quadrant

Function	Sign	α=100+γ	α=200-δ
$sin\alpha$	+	$sin\alpha = +cos\gamma$	$sin\alpha = +sin\delta$
$cos\alpha$	-	$cos\alpha = -sin\gamma$	$cos\alpha = -cos\delta$
$tan\alpha$	-	$tan\alpha = -cot\gamma$	$tan\alpha = -tan\delta$
$cot\alpha$	-	$cot\alpha = -tan\gamma$	$cot\alpha = -cot\delta$

Example:

$\alpha = 147.7857^g, then\ \gamma = 47.7857^g \quad \delta = 52.2143^g$

$sin147.7857^g = +cos47.7857^g = +sin52.2143^g$

$cot147.7857^g = -tan47.7857^g = -cot52.2143^g$

3- Angle α for 3rd quadrant ($200^g < \alpha < 300^g$)

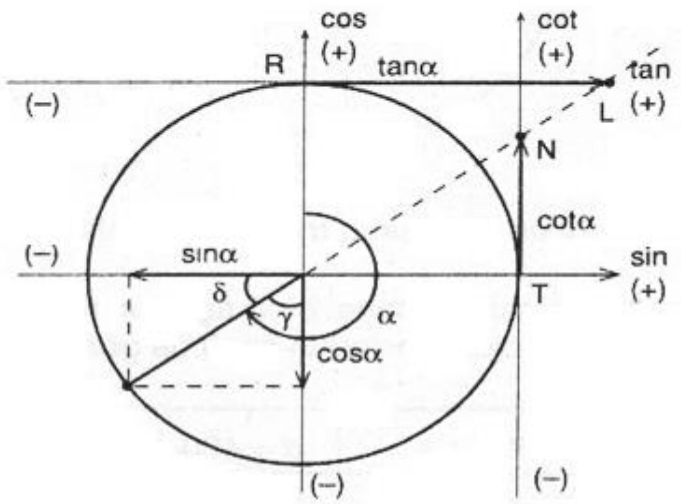


Figure 3.4 Trigonometric description of any α angle in 3rd Quadrant.

Function	Sign	α=200+γ	α=300-δ
$sin\alpha$	-	$sin\alpha = -sin\gamma$	$sin\alpha = -cos\delta$
$cos\alpha$	-	$cos\alpha = -cos\gamma$	$cos\alpha = -sin\delta$
$tan\alpha$	+	$tan\alpha = +tan\gamma$	$tan\alpha = +cot\delta$
$cot\alpha$	+	$cot\alpha = +cot\gamma$	$cot\alpha = +tan\delta$

Example:

$\alpha = 281.4351^g, then\ \gamma = 81.4351^g \quad \delta = 18.5649^g$
$cos281.4351^g = -cos81.4351^g = -sin18.5649^g$
$tan281.4351^g = +tan81.4351^g = +cot18.5649^g$

4- Angle α for 4th quadrant ($300^g < \alpha < 400^g$)

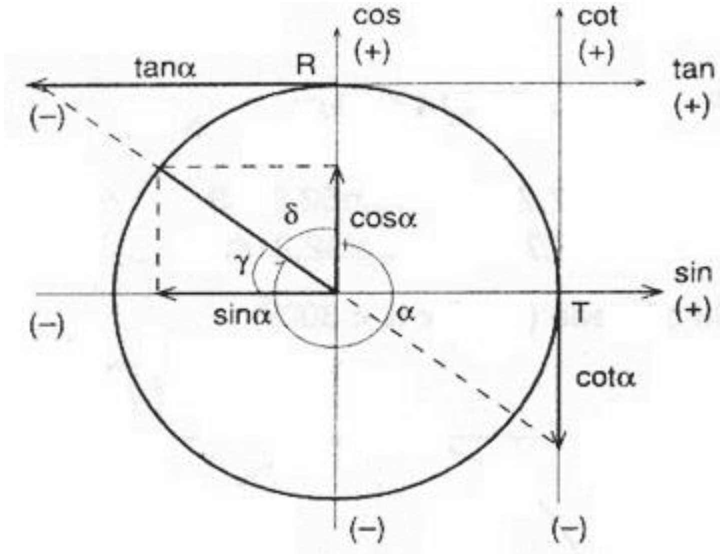


Figure 3.5 Trigonometric description of any α angle in 4th Quadrant.

Function	Sign	α=300+γ	α=400-δ
$sin\alpha$	-	$sin\alpha = -cos\gamma$	$sin\alpha = -sin\delta$
$cos\alpha$	+	$cos\alpha = +sin\gamma$	$cos\alpha = +cos\delta$
$tan\alpha$	-	$tan\alpha = -cot\gamma$	$tan\alpha = -tan\delta$
$cot\alpha$	-	$cot\alpha = -tan\gamma$	$cot\alpha = -cot\delta$

Example:

$\alpha = 343.8275^g, then\ \gamma = 43.8275^g \quad \delta = 56.1725^g$
$cos343.8275^g = +sin43.8275^g = +cos56.1725^g$
$cot343.8275^g = -tan43.8275^g = -cot56.1725^g$

Trigonometric Function	1st Quadrant	2nd Quadrant	3rd Quadrant	4th Quadrant
	$\gamma = \alpha$	$\gamma = \alpha - 100$	$\gamma = \alpha - 200$	$\gamma = \alpha - 300$
	$\delta = 100 - \alpha$	$\delta = 200 - \alpha$	$\delta = 300 - \alpha$	$\delta = 400 - \alpha$
$sin\alpha$	$+sin\gamma$	$+cos\gamma$	$-sin\gamma$	$-cos\gamma$
	$+cos\delta$	$+sin\delta$	$-cos\delta$	$-sin\delta$
$cos\alpha$	$+cos\gamma$	$-sin\gamma$	$-cos\gamma$	$+sin\gamma$
	$+sin\delta$	$-cos\delta$	$-sin\delta$	$+cos\delta$
$tan\alpha$	$+tan\gamma$	$-cot\gamma$	$+tan\gamma$	$-cot\gamma$
	$+cot\delta$	$-tan\delta$	$+cot\delta$	$-tan\delta$
$cot\alpha$	$+cot\gamma$	$-tan\gamma$	$+cot\gamma$	$-tan\gamma$
	$+tan\delta$	$-cot\delta$	$+tan\delta$	$-cot\delta$

α / Trig. Function	0^g-100^g	100^g-200^g	200^g-300^g	300^g-400^g
$sin\alpha$	$0 \rightarrow +1$	$+1 \rightarrow 0$	$0 \rightarrow -1$	$-1 \rightarrow 0$
$cos\alpha$	$+1 \rightarrow 0$	$0 \rightarrow -1$	$-1 \rightarrow 0$	$0 \rightarrow +1$
$tan\alpha$	$0 \rightarrow +\infty$	$-\infty \rightarrow 0$	$0 \rightarrow +\infty$	$-\infty \rightarrow 0$
$cot\alpha$	$+\infty \rightarrow 0$	$0 \rightarrow -\infty$	$+\infty \rightarrow 0$	$0 \rightarrow -\infty$

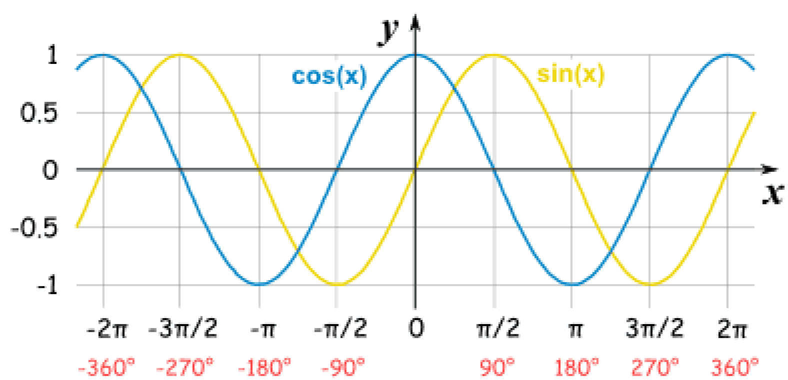


Figure 3.6 Graphical representation of sine and cosine functions

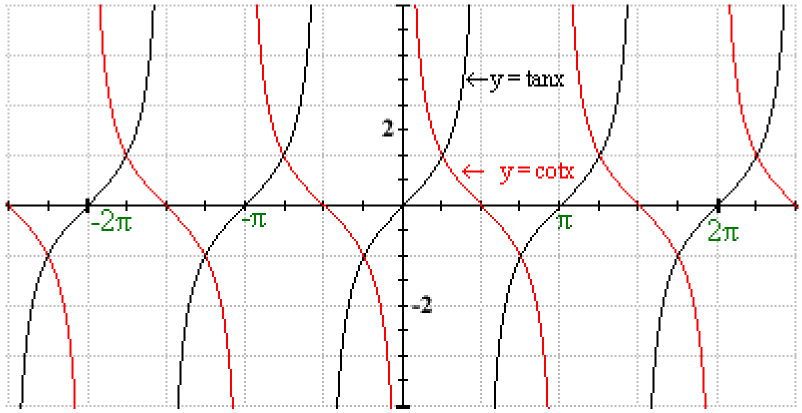


Figure 3.7 Graphical representation of tangent and cotangent functions

4 RECTANGULAR COORDINATE SYSTEM

Rectangular coordinates are the most convenient method available for describing the horizontal positions of survey points. Dams, highways, industrial plants, and mass-transit systems are located, planned, designed, and constructed on the basis of computerized information, which includes coordinates as well as other information concerning topography, geology, drainage, population and so on. As a result of these factors, it is absolutely necessary for the surveyors to be familiar with, able to use, coordinates. The coordinates of a particular point are defined as the distance measured to that point from a pair of mutually perpendicular axes. The axes are usually labelled x and y, the perpendicular distance from the y-axis to a point is called the x coordinate, and the perpendicular distance from the x-axis to a point is called the y coordinate.

In our country, also in Europe, it is common to have the x-axis coincide with the north-south direction and y-axis with the east-west direction. Just the opposite system is used in some countries, particularly in the United States.

In plane surveys, it is convenient to perform the work in a rectangular *XY* coordinate system.

Direction of +x refers to north,

Direction of +y refers to east,

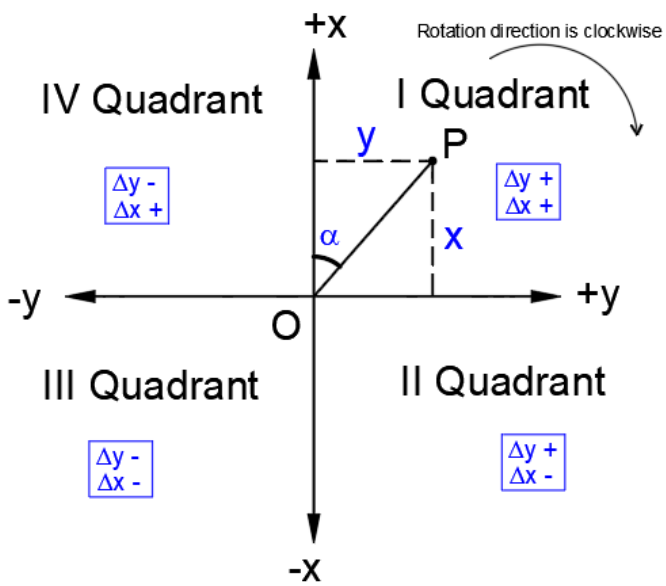
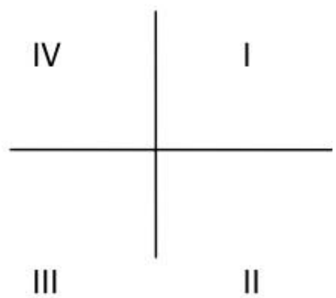


Figure 4.1 Rectangular Coordinate System

A vertical directed line (y-axis) crosses the horizontal directed line (x-axis) at the origin point. This system uses an ordered pair of coordinates to locate a point. The coordinates are always expressed as (x,y).

The x and y-axes divide the plane into four parts, numbered in a counter-clockwise direction as shown in the figure above. Signs of the coordinates of points in each quadrant are also shown in this figure. Note: In surveying, the quadrants are numbered clockwise starting with the upper right quadrant and the normal way of denoting coordinates (in the United States) is the opposite (y,x) or more appropriately North, East.

The *x* and *y-axes* divide the plane into four parts. The quadrants are numbered clockwise starting with the upper right quadrant.



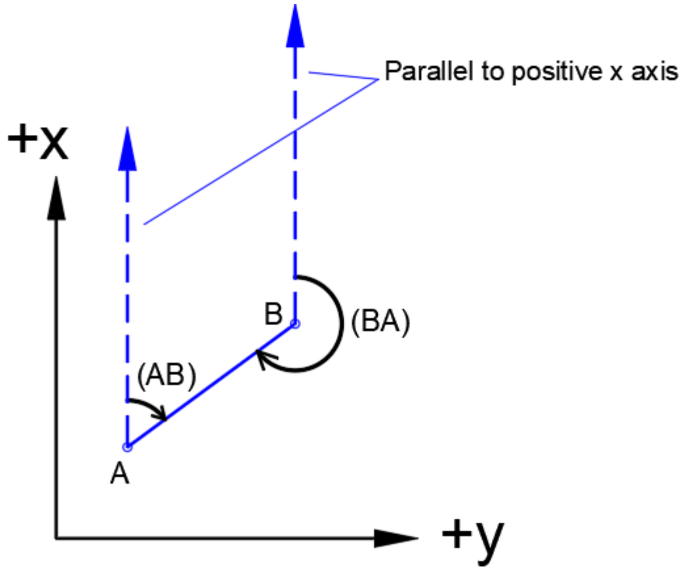
QUADRANT	ΔX	ΔY
I. Quadrant	+	+
II. Quadrant	-	+
III. Quadrant	-	-
IV. Quadrant	+	-

AZIMUTH

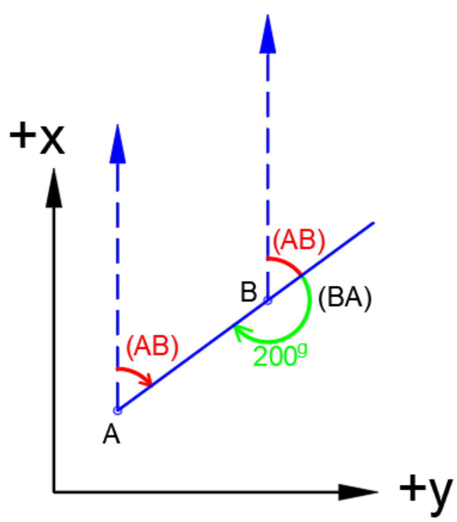
The azimuth of a line is its direction, given by the angle between the meridian and the line, measured in a clockwise direction. It ranges from 0 to 400 gon. Azimuths can be either true, magnetic or grid, depending on the reference meridian. Azimuths can be indicated from either the south point or the north point of a meridian, but they are always measured in a clockwise direction.

- The azimuth of a line on the ground is its horizontal angle measured clockwise from the meridian to the line.
- Azimuth gives the direction of the line with respect to the meridian.
- In plan surveying azimuths are generally measured from the north.
- Azimuths may have values between 0^g and 400^g (0 – 360 degrees).

Every line has two azimuths (forward and back) - and their values differ by 200^g.



α_{AB} = (AB) = forward azimuth of AB line
α_{BA} = (BA) = back azimuth of AB line

	$(BA) = (AB) + 200^g$

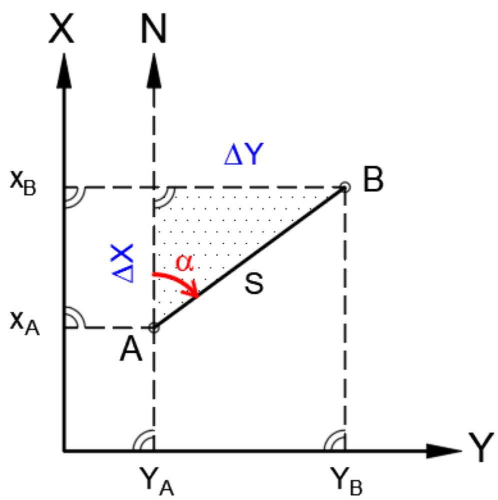
In plane surveying, forward azimuths are converted to back azimuths, and vice versa, by adding or subtracting 200^g .

$\alpha_{AB} < 200^g \longrightarrow \alpha_{BA} = \alpha_{AB} + 200^g$
$\alpha_{AB} > 200^g \longrightarrow \alpha_{BA} = \alpha_{AB} - 200^g$

FUNDAMENTAL COMPUTATION – 1

Rectangular Coordinates using Distance and Azimuth-***First Geodetic Problem***

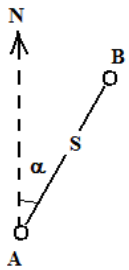
Given:	**Wanted:**
A(X_A, Y_A), S, α	B(X_B, Y_B)



$$\boxed{\begin{aligned} Sin\alpha &= \frac{\Delta Y}{S} \rightarrow \Delta Y = S \cdot Sin\alpha \\ Cos\alpha &= \frac{\Delta X}{S} \rightarrow \Delta X = S \cdot Cos\alpha \end{aligned}}$$

$$\boxed{\begin{aligned} Y_B &= Y_A + \Delta Y = Y_A + S \cdot Sin\alpha \\ X_B &= X_A + \Delta X = X_A + S \cdot Cos\alpha \end{aligned}}$$

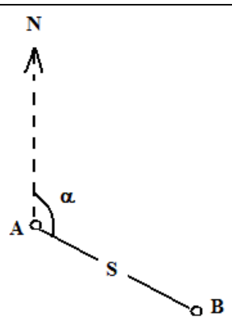
Example for Quadrant 1:

	Given:	To be computed:
	$Y_A = 120.48\,m$ $X_A = 230.51\,m$ $\alpha = 57.^{g}6248$ $S = 121.58\,m$	$Y_B = ?$ $X_B = ?$

Solution:

Pt.	α	S	Δy Δx	Y	X
	(gon)	(m)	(m)	(m)	(m)
A				**120.48**	**230.51**
	$57.^{g}6248$	121.58	**+95.63** **+75.08**	**+95.63**	**+75.08**
B				**216.11**	**305.59**

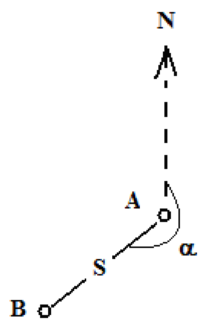
Example for Quadrant 2:

	Given:	To be computed:
	$Y_A = 250.00\,m$ $X_A = 250.00\,m$ $\alpha = 161.^{g}8675$ $S = 154.15\,m$	$Y_B = ?$ $X_B = ?$

Solution:

Pt.	α	S	Δy Δx	Y	X
	(gon)	(m)	(m)	(m)	(m)
A				**250.00**	**250.00**
	$161.^{g}8675$	154.15	**+86.91** **-127.31**	**+86.91**	**-127.31**
B				**336.91**	**122.69**

Example for Quadrant 3:

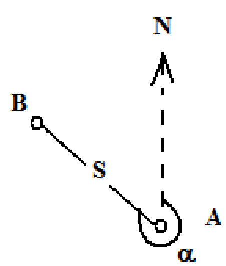


Given:	To be computed:
$Y_A = 300.00\ m$	$Y_B = ?$
$X_A = 300.00\ m$	$X_B = ?$
$\alpha = 240.^g1275$	
$S = 159.22\ m$	

Solution:

Pt.	α	S	Δy Δx	Y	X
	(gon)	(m)	(m)	(m)	(m)
A				**300.00**	**300.00**
	$240.^g1275$	159.22	**-93.84** **-128.62**	**-93.84**	**-128.62**
B				**206.16**	**171.38**

Example for Quadrant 4:



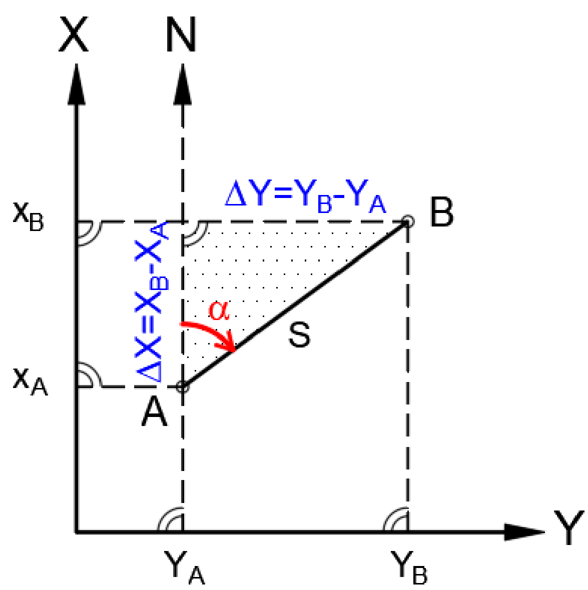
Given:	To be computed:
$Y_A = 400.00\ m$	$Y_B = ?$
$X_A = 500.00\ m$	$X_B = ?$
$\alpha = 328.^g5361$	
$S = 167.15\ m$	

Solution:

Pt.	α	S	Δy Δx	Y	X
	(gon)	(m)	(m)	(m)	(m)
A				**400.00**	**500.00**
	$328.^g5361$	167.15	**-150.64** **+72.44**	**-150.64**	**+72.44**
B				**249.36**	**572.44**

FUNDAMENTAL COMPUTATION – 2

Given:	**Wanted:**
A(X_A, Y_A), B(X_B, Y_B)	S, α
	(The azimuth angle can be expressed as (AB) for α_{AB}, α)



$$\tan(\alpha) = \frac{Y_B - Y_A}{X_B - X_A}$$

$$\Delta Y = Y_B - Y_A$$

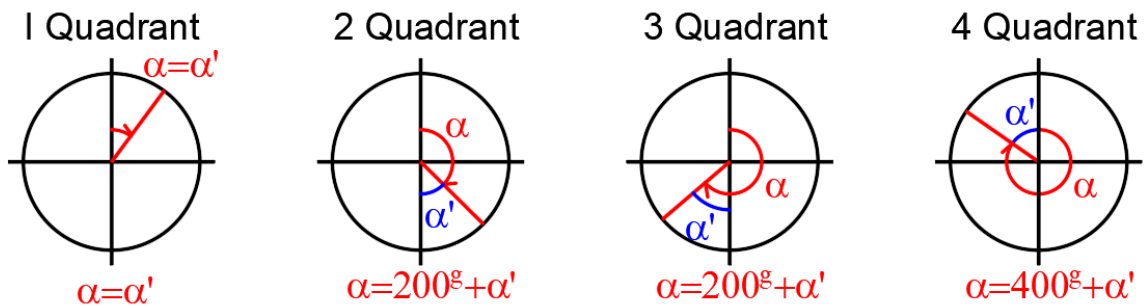
$$\Delta X = X_B - X_A$$

$$(\alpha) = atan\frac{\Delta_Y}{\Delta_X}$$

$$S = \sqrt{\Delta Y^2 + \Delta X^2}$$

The equation of (α) is used for calculating the azimuth in first quadrant. The azimuth angle α can have any value between 0 and 400 grad. To determine the exact value, differential values of coordinate differences should be considered.

Assume that the calculated azimuth angle from a calculator is represented as α^I, then the actual value can be computed as given table, depending on the signs of differential values of coordinates.

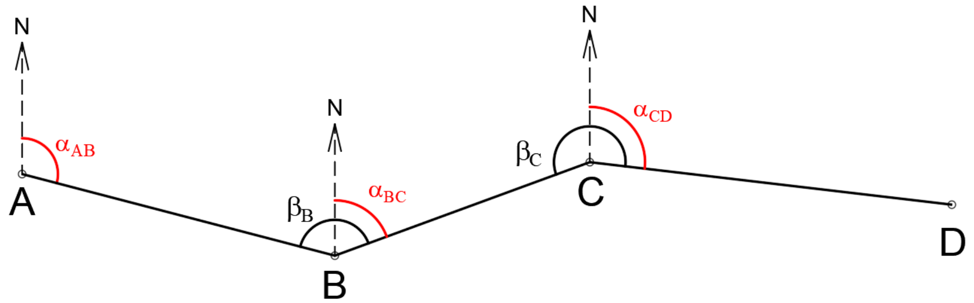


QUADRANT	Δy / Δx	Azimuth
First quadrant	+ / +	$\alpha = \alpha'$
Second quadrant	+ / -	$\alpha = 200^g + \alpha'$
Third quadrant	- / -	$\alpha = 200^g + \alpha'$
Fourth quadrant	- / +	$\alpha = 400^g + \alpha'$

FUNDAMENTAL COMPUTATION – 3

Relation between Azimuth and Traverse Angles

Given:	**Wanted:**
α_{AB} , β_B	α_{BC}



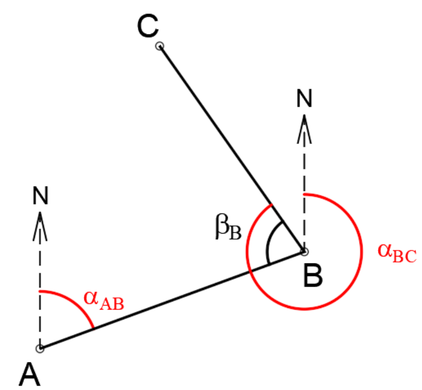
$$\alpha_{AB} + \beta_{B=}\alpha_{BC}$$

$$\alpha_{BC} < 200^g \rightarrow \alpha_{BC} + 200^g \Rightarrow \alpha_{BC} = \alpha_{AB} + \beta_B + 200^g$$

$$200^g < \alpha_{BC} < 600^g \rightarrow \alpha_{BC} - 200^g \Rightarrow \alpha_{BC} = \alpha_{AB} + \beta_B - 200^g$$

$$\alpha_{BC} > 600^g \rightarrow \alpha_{BC} - 200^g \Rightarrow \alpha_{BC} = \alpha_{AB} + \beta_B - 600^g$$

To solve this problem; three situations should be considered:

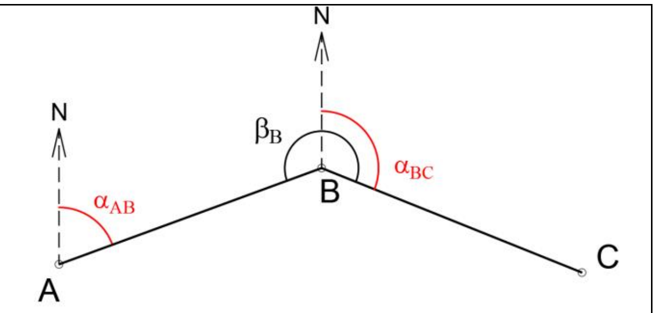
<table>
<tr><td>Situation 1st; $\alpha + \beta < 200^g$</td><td rowspan="2">
</td></tr>
<tr><td>Solution:
$(BC) = \alpha + \beta + 200^g$</td></tr>
</table>

Situation 2nd; $200^g \le \alpha + \beta \le 600^g$

Solution:

$$(BC) = \beta - (200^g - \alpha)$$

$$(BC) = \alpha + \beta - 200^g$$

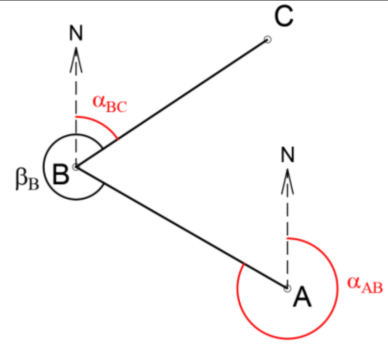


Situation 3rd: $\alpha + \beta \ge 600^g$

Solution:

$$(BC) = \beta - 200^g - (400^g - \alpha)$$

$$(BC) = \alpha + \beta - 600^g$$



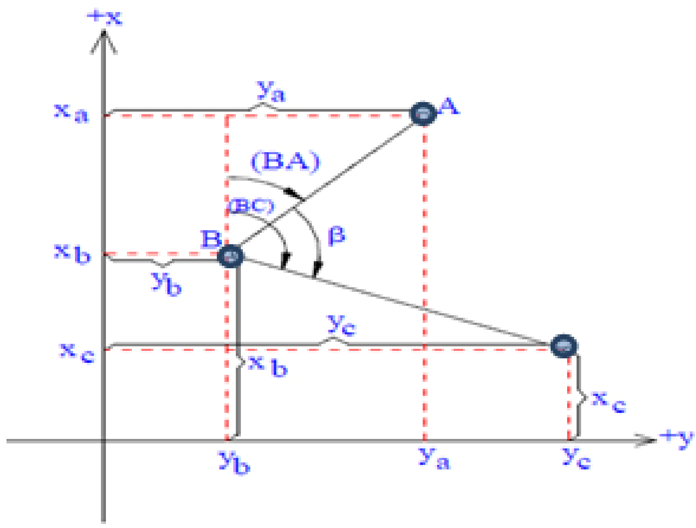
The brief description of «*Fundamental Computation-3*» may be written as follows:

If	$\alpha + \beta < 200^g$	Thus	$(BC) = \alpha + \beta + 200^g$
If	$200^g \le \alpha + \beta \le 600^g$	Thus	$(BC) = \alpha + \beta - 200^g$
If	$\alpha + \beta \ge 600^g$	Thus	$(BC) = \alpha + \beta - 600^g$

FUNDAMENTAL COMPUTATION – 4

Relation between Azimuth and Traverse Angles

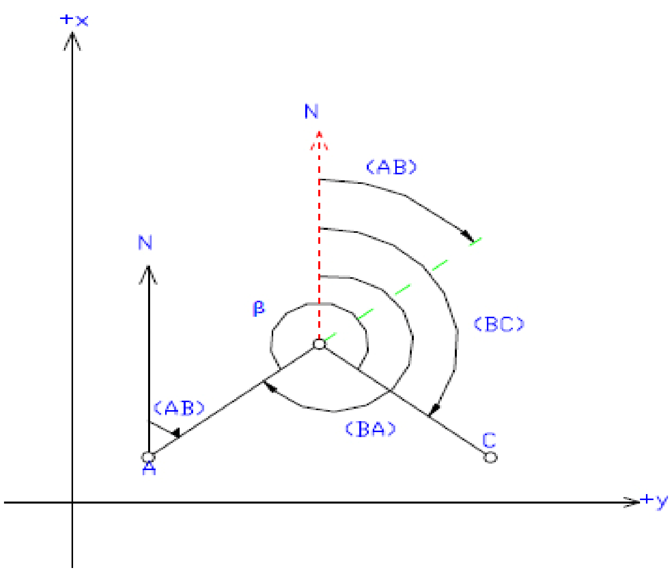
Given:	**Wanted:**
$A(X_A, Y_A)$ $B(X_B, Y_B)$ $C(X_C, Y_C)$	β_B



$$\beta = (BC) - (BA)$$

$$(BC) = atn\frac{Y_C - Y_B}{X_C - X_B}$$

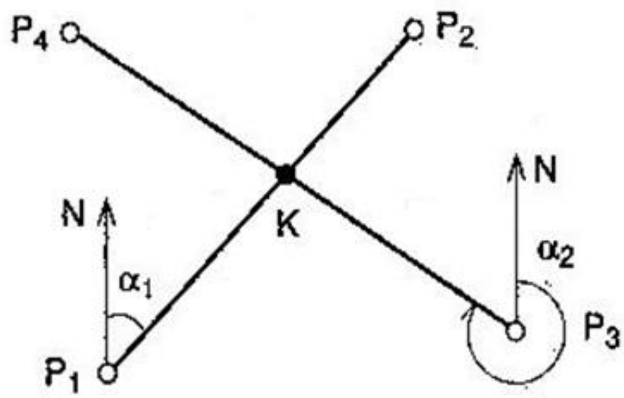
$$(BA) = atn\frac{Y_A - Y_B}{X_A - X_B}$$



5 INTERSECTION of TWO LINES

Computing the Coordinates of the Intersection Point

Assume that two lines with starting and ending points are given, and they intersect in a point. For computation of the coordinates of this point, the azimuth angle may be used for the equations.



From the given illustration above, it can be said that the direction of P_1P_2 is equal to the direction of P_1P_K.

Then, below equations can be written as follow;

$$\tan\alpha_1 = \frac{Y_2 - Y_1}{X_2 - X_1} = \frac{Y_K - Y_1}{X_K - X_1}$$

$$Y_K - Y_1 = \tan\alpha_1 * (X_K - X_1) \quad \text{[Equation 1]}$$

$$\tan\alpha_2 = \frac{Y_4 - Y_3}{X_4 - X_3} = \frac{Y_K - Y_3}{X_K - X_3}$$

$$Y_K - Y_3 = \tan\alpha_2 * (X_K - X_3) \quad \text{[Equation 2]}$$

Subtract Eq.2 from Eq.1,

$$Y_K - Y_3 - Y_K + Y_1 = \tan\alpha_2 * (X_K - X_3) - \tan\alpha_1 * (X_K - X_1)$$

$$-Y_3 + Y_1 = \tan\alpha_2 * (X_K - X_3) - \tan\alpha_1 * (X_K - X_1)$$

Multiply both sides with (-1);

$$Y_3 - Y_1 = \tan\alpha_1 * (X_K - X_1) - \tan\alpha_2 * (X_K - X_3)$$

$$Y_3 - Y_1 = \tan\alpha_1 * X_K - \tan\alpha_1 * X_1 - \tan\alpha_2 * X_K + \tan\alpha_2 * X_3$$

$$Y_3 - Y_1 = X_K * (\tan\alpha_1 - \tan\alpha_2) - tan\alpha_1 * X_1 + \tan\alpha_2 * X_3$$

$$X_k = \frac{Y_3 - Y_1 + tan\alpha_1 * X_1 - \tan\alpha_2 * X_3}{\tan\alpha_1 - \tan\alpha_2}$$

Subtract X_1 from both sides of the equation;

$$X_k - X_1 = \frac{Y_3 - Y_1 + tan\alpha_1 * X_1 - \tan\alpha_2 * X_3}{\tan\alpha_1 - \tan\alpha_2} - X_1$$

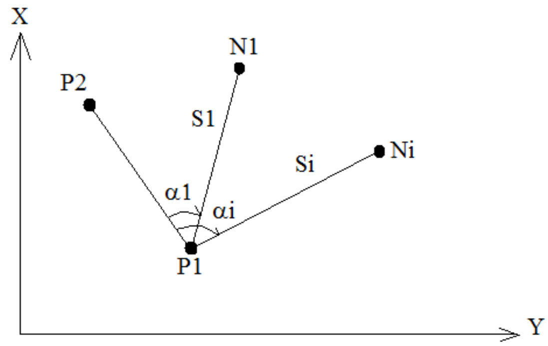
$$X_k - X_1 = \frac{Y_3 - Y_1 - (X_3 - X_1) * \tan\alpha_2}{\tan\alpha_1 - \tan\alpha_2}$$

Similarly, from the Equation 2;

$$X_k - X_3 = \frac{Y_3 - Y_1 - (X_3 - X_1) * \tan\alpha_1}{\tan\alpha_1 - \tan\alpha_2}$$

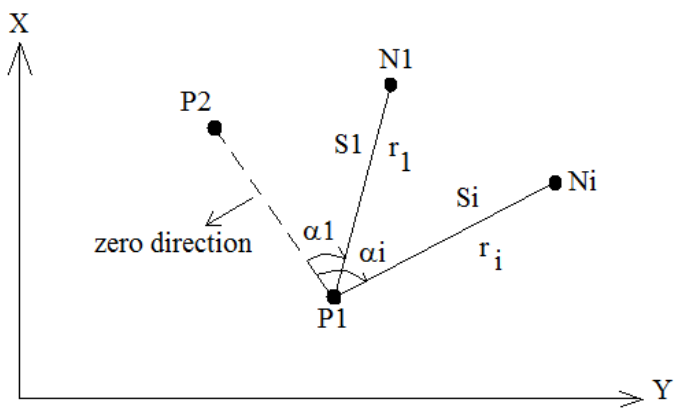
6 POLAR SURVEYS of OBJECT POINTS

For detailed work, polar surveys are common. New points N_i are determined by measuring distances S_i and angles α_i with respect to a known reference direction. These polar coordinates are either used directly for plotting the survey plan, or else transformed into rectangular coordinates for further use.



Observation stations are placed at 300 to 700 m intervals, if the terrain is reasonably open. The influence of refraction on the accuracy of the directions can be neglected for these distances. In more difficult terrain, i.e., dense vegetation or densely built-up areas, the stations are closer together.

Given: Two fixed points, P_1 and P_2 plus the directions r_{P1P2}, $r_1,\ldots,r_i$ and the distances $S_1,\ldots S_i$.



First, the azimuth angle between known points is computed as follow:

$$(P_1P_2) = \arctan\left(\frac{Y_2 - Y_1}{X_2 - X_1}\right)$$

Then, the reduced directions are computed to provide zero direction:

$$Reduced\ Directions = \beta_i = r_i - r_{P1P2} \qquad for\ i = 1, 2, \dots, i$$

For all new points to be coordinated, the azimuths should be computed with respect to the known point, P1, where the instrument is established.

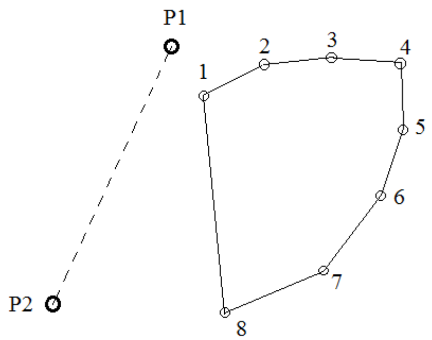
$$(P_1N_i) = (P_1P_2) + r_i$$

Then, the problem turns into the Fundamental Computation-1, which includes computation of the coordinates of the new points with azimuths and distances.

$$Y_{Ni} = Y_{P1} + S_i * \sin((P_1N_i))$$

$$X_{Ni} = X_{P1} + S_i * \cos((P_1N_i))$$

Exercise: The parcel given in sketch below was surveyed with respect to the polar survey and the instrument was established at P2 (station point) during the survey job. According to the observations, compute the new points' rectangular coordinates (for point 1, 2, …).



Sketch of the polar survey

S.N.	Y (m)	X(m)
P_1	7600.73	5066.84
P_2	7561.58	5093.10

Observations from field:

Station No (SN)	Target Point (TP)	Horizontal Direction (gon)	Horizontal Distance (m)
P2	P1	38.465	47.14
	1	58.631	18.55
	2	65.539	22.71
	3	77.026	26.35
	4	97.965	21.20
	5	102.965	19.97
	6	110.582	18.08
	7	126.477	14.98
	8	162.273	11.44
	9	164.144	10.40

Solution:

$(P_2P_1) = \arctan\left(\frac{Y_1 - Y_2}{X_1 - X_2}\right) = \arctan\left(\frac{39.15}{-26.26}\right)$ ➔ 2$^{\text{nd}}$ Quadrant

$(P_2P_1) = 137.6132\ gon$

SN	TP	Hor. Dir. (gon)	Reduced Dir. (gon)	Hor. Dist. (m)	Azimuth (gon)	Y (m)	X (m)
P2	P1	38.465	**0.000**	47.14	**137.6132**	**7600.73**	**5066.84**
	1	58.631	20.166	18.55	157.7792	7573.00	5078.48
	2	65.539	27.074	22.71	164.6872	7573.54	5073.80
	3	77.026	38.561	26.35	176.1742	7571.21	5068.57
	4	97.965	59.500	21.20	197.1132	7562.54	5071.92
	5	102.965	64.500	19.97	202.1132	4560.92	5073.14
	6	110.582	72.117	18.08	209.7302	7558.83	5075.23
	7	126.477	88.012	14.98	225.6252	7555.71	5079.32
	8	162.273	123.808	11.44	261.4212	7552.18	5086.58
	9	164.144	125.679	10.40	263.2922	7552.86	5087.43

7 COORDINATE TRANSFORMATION

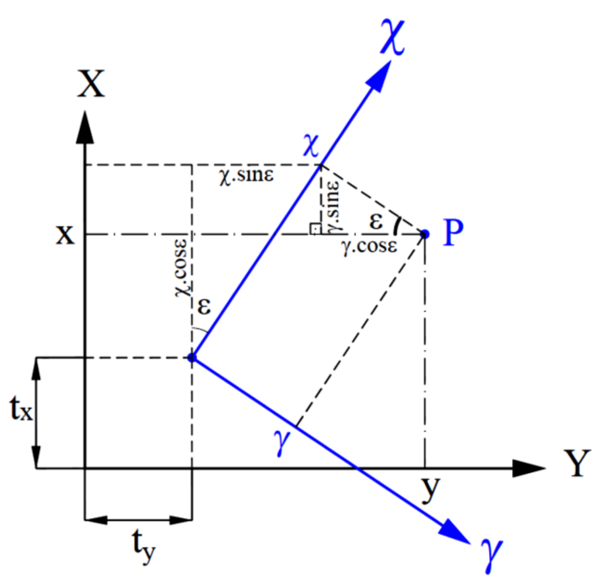
Similarity Transformation

Coordinate transformation is one of the most common issues in geodesy phenomena. It is used to transform coordinates one datum to other by using parameters as translation terms, scale and rotation angle. The increase of application areas in engineering surveys and integration of layouts with different datum have been increased the necessity of accurate datum transformation. The problem in datum transformation is to compute the transformation parameters using common points with known coordinates into two different datum.

The array of points that is processed through a similarity transformation undergoes the following coordinate changes:

1- Shifts in both coordinate directions (translation terms: tx and ty)
2- Rotation by an angle (rotation angle: ε)
3- Multiplication of the scale of a factor k such that the scale becomes equal to the one of the other system (scale factor).

The first system is $\chi\gamma$ coordinates, and the second system is XY coordinate system. Once transformation parameters between two systems are estimated by common points, χ and γ coordinates are converted to second system namely X and Y coordinates



Assume that coordinates of P is changed in a ratio, namely, k. Then, coordinates of P in second system can be obtained by the above equations:

$$X = t_\chi + k.cos\varepsilon.\chi - k.sin\varepsilon.\gamma \quad (1)$$

$$Y = t_\gamma + k.sin\varepsilon.\chi + k.cos\varepsilon.\gamma \quad (2)$$

Here, tx, ty, k and ε are the translation parameters.

with; $o = k.cos\varepsilon$

$a = k.sin\varepsilon$

The equations can be rewritten as follows:

$$X = t_\chi + o.\chi - a.\gamma$$

$$Y = t_\gamma + a.\chi + o.\gamma$$

Here, using points, which are common on two systems, the scale factor and rotational angle, can be estimated.

Rotation angle: $\varepsilon = \arctan\left(\frac{a}{o}\right)$

Scale Factor: $k = \sqrt{o^2 + a^2}$

To estimate the translation parameters, at least two common points should be known in both systems.

In brief, for unique solution:

The scale factor: k;

$$k = \frac{S_{XY}}{S_{xy}} = \frac{\sqrt{\Delta Y^2 + \Delta X^2}}{\sqrt{\Delta y^2 + \Delta x^2}}$$

Here,

$$\Delta Y = Y_2 - Y_1$$

$$\Delta X = X_2 - X_1$$

$$\Delta y = y_2 - y_1$$

$$\Delta x = x_2 - x_1$$

The rotation angle; ε;

$$\varepsilon = \alpha_{XY} - \alpha_{xy} = \arctan\left(\frac{\Delta Y}{\Delta X}\right) - \arctan\left(\frac{\Delta y}{\Delta x}\right)$$

$$a = k.\sin(\varepsilon)$$

$$o = k.\cos(\varepsilon)$$

If the rotational center is placed at the origin Po of the local coordinate system, then we get;

$$t_\chi = X - x.o + y.a$$

$$t_y = Y - x.a - y.o$$

Then, the coordinates of the points to be converted into second system can be computed as follows:

$$X = t_\chi + x.o - y.a$$

$$Y = t_\gamma + x.a + y.o$$

Exercise: Given are the coordinates of P1 and P2 in both the XY and X'Y' systems. The points 1 and 2, coordinated in the XY system are to be transformed into the X'Y' system.

Pt. No	XY System		X'Y' system	
	Y (m)	X (m)	Y' (m)	X' (m)
P1	97319.35	8802.06	96935.27	8922.55
P2	98858.81	9717.54	98511.77	9772.69
1	98338.99	8586.69	?	?
2	97918.31	9538.01	?	?

Solution:

$$\Delta Y = Y_2 - Y_1 = 98858.81 - 97319.35 = 1539.46$$

$$\Delta X = X_2 - X_1 = 9717.54 - 8802.06 = 915.48$$

$$\Delta Y' = Y'_2 - Y'_1 = 98511.77 - 96935.27 = 1576.50$$

$$\Delta X' = X'_2 - X'_1 = 9772.69 - 8922.55 = 850.14$$

$$\varepsilon = \alpha_{X'Y'} - \alpha_{XY} = \arctan\left(\frac{1576.50}{850.14}\right) - \arctan\left(\frac{1539.46}{915.48}\right) = 68.5154 - 65.8456 = 2.6698\ g$$

$$k = \frac{S_{XY}}{S_{xy}} = \frac{\sqrt{\Delta Y^2 + \Delta X^2}}{\sqrt{\Delta y^2 + \Delta x^2}} = \frac{\sqrt{1576.50^2 + 850.14^2}}{\sqrt{1539.46^2 + 915.48^2}} = \frac{1791.114}{1791.100} = 1.000007816$$

$$a = k.\sin(\varepsilon) = +0.041925$$

$$o = k.\cos(\varepsilon) = +0.999129$$

$$t_\chi = X' - X.o + Y.a = 8922.55 - 8802.06 * 0.999129 + 97319.35 * 0.041925 = 4208.27$$

$$t_y = Y' - X.a - Y.o = 96935.27 - 8802.06 * 0.041925 - 97319.35 * 0.999129 = -668.34$$

$$X' = t_\chi + X.o - Y.a = 4208.27 + 8586.69 * 0.999129 - 98338.99 * 0.041925 = 8664.62$$

$$Y' = t_\gamma + X.a + Y.o = -668.34 + 8586.69 * 0.041925 + 98338.99 * 0.999129 = 97944.99$$