Fluid Mechanics: Fundamentals and Applications, 2nd Edition
Yunus A. Cengel, John M. Cimbala
McGraw-Hill, 2010

Chapter 11
EXTERNAL FLOW: DRAG AND LIFT

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FLOW OVER BODIES: DRAG AND LIFT

The wake of a Boeing 767 disrupts the top of a cumulus cloud and clearly shows the counter-rotating trailing vortices.

- Have an intuitive understanding of the various physical phenomena associated with external flow such as drag, friction and pressure drag, drag reduction, and lift

- Calculate the drag force associated with flow over common geometries

- Understand the effects of flow regime on the drag coefficients associated with flow over cylinders and spheres

- Understand the fundamentals of flow over airfoils, and calculate the drag and lift forces acting on airfoils

Fluid flow over solid bodies frequently occurs in practice, and it is responsible for numerous physical phenomena such as

- the _**drag force**_ acting on automobiles, power lines, trees, and underwater pipelines;

- the _**lift**_ developed by airplane wings;

- _**upward draft**_ of rain, snow, hail, and dust particles in high winds;

- the transportation of red blood cells by blood flow;

- the entrainment and disbursement of liquid droplets by sprays;

- the vibration and noise generated by bodies moving in a fluid; and

- the power generated by wind turbines.

A fluid moving over a stationary body (such as the wind blowing over a building), and a body moving through a quiescent fluid (such as a car moving through air) are referred to as **flow over bodies** or **external flow.**

Flow over bodies is commonly encountered in practice.

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Chapter 11: EXTERNAL FLOW: DRAG AND LIFT

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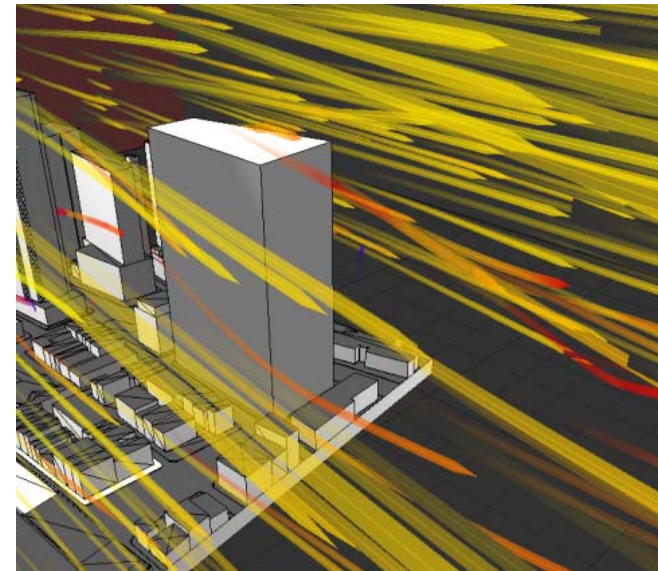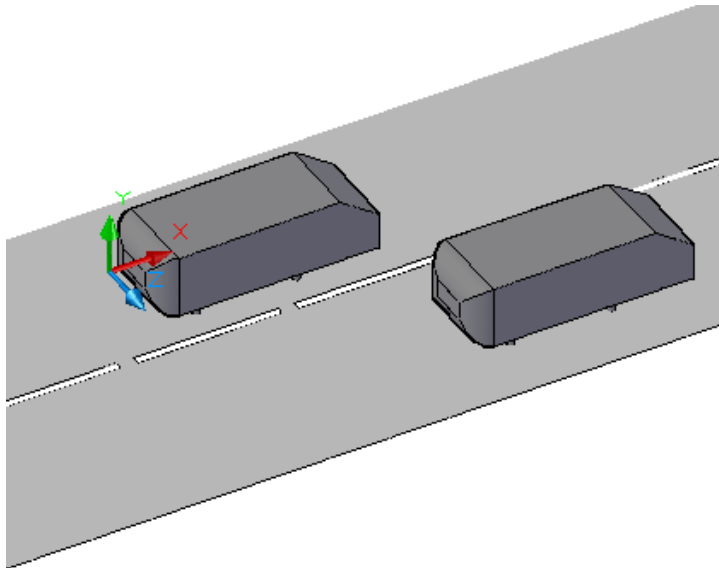
The flow fields and geometries for most external flow problems are too complicated and we have to rely on correlations based on experimental data.

Free-stream velocity: The velocity of the fluid approaching a body (V or u_∞ or U_∞)

Two-dimensional flow: When the body is very long and of constant cross section and the flow is normal to the body.

Axisymmetric flow: When the body possesses rotational symmetry about an axis in the flow direction. The flow in this case is also two-dimensional.

Three-dimensional flow: Flow over a body that cannot be modeled as two-dimensional or axisymmetric such as flow over a car.

Incompressible flows: (e.g., flows over automobiles, submarines, and buildings)

Compressible flows: (e.g., flows over high-speed aircraft, rockets, and missiles).

Compressibility effects are negligible at low velocities (flows with Ma < 0.3).

Streamlined body: If a conscious effort is made to align its shape with the anticipated streamlines in the flow.

Streamlined bodies such as race cars and airplanes appear to be contoured and sleek.
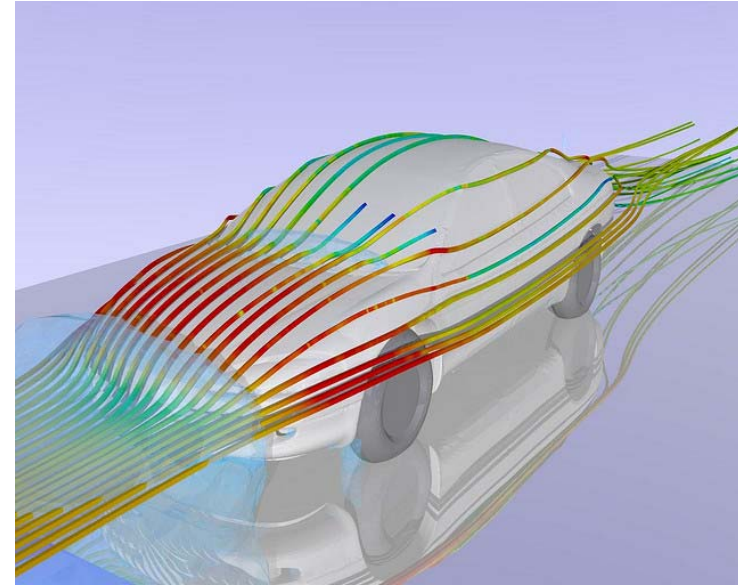
Bluff or **blunt body:** If a body (such as a building) tends to block the flow.

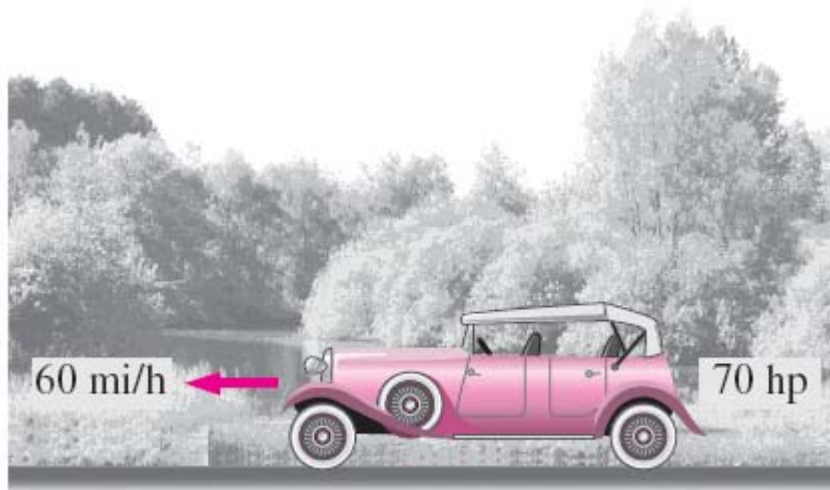Usually it is much easier to force a streamlined body through a fluid.

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3-D, axisymmetric, and three-dimensional flows.

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60 mi/h — 70 hp

60 mi/h — 50 hp

It is much easier to force a streamlined body than a blunt body through a fluid.

11–2 DRAG AND LIFT

- A body meets some resistance when it is forced to move through a fluid, especially a liquid.

- A fluid may exert forces and moments on a body in and about various directions.

- **Drag:** The force a flowing fluid exerts on a body in the flow direction.

- The drag force can be measured directly by simply attaching the body subjected to fluid flow to a calibrated spring and measuring the displacement in the flow direction.

- Drag is usually an undesirable effect, like friction, and we do our best to minimize it.

- But in some cases drag produces a very beneficial effect and we try to maximize it (e.g., automobile brakes).

High winds knock down trees, power lines, and even people as a result of the drag force.

Lift: The components of the pressure and wall shear forces in the direction *normal* to the flow tend to move the body in that direction, and their sum is called lift.

The fluid forces may generate moments and cause the body to rotate.

Rolling moment: The moment about the flow direction.

Yawing moment: The moment about the lift direction.

Pitching moment: The moment about the side force direction.

$$F_D = F_R \cos \phi$$
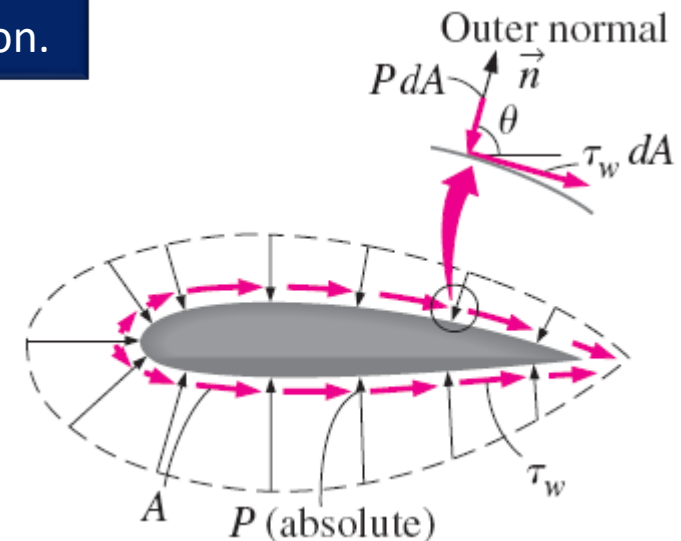$$F_L = F_R \sin \phi$$
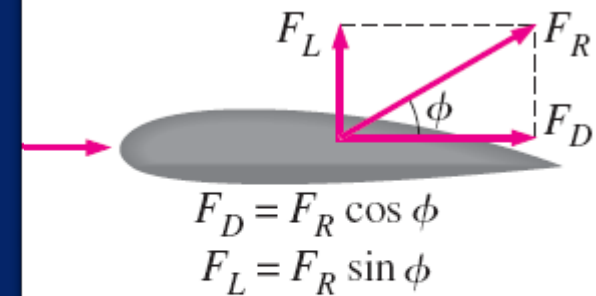
Drag force:
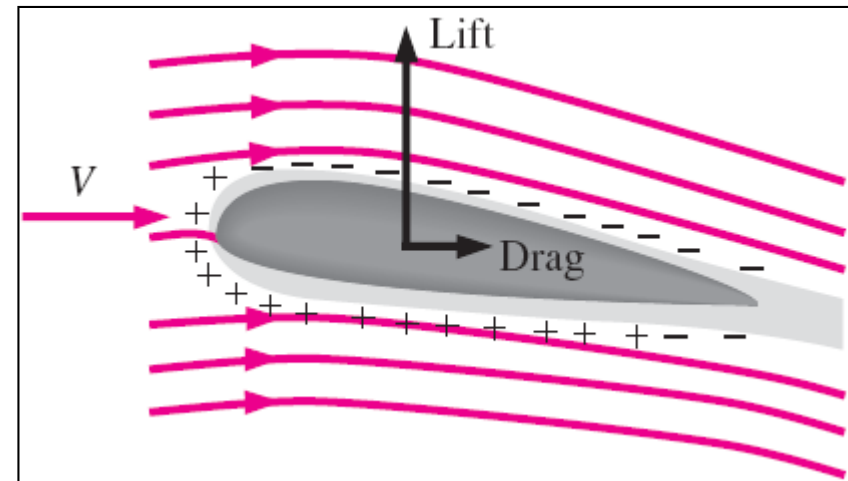$$dF_D = -P\, dA \cos \theta + \tau_w\, dA \sin \theta$$
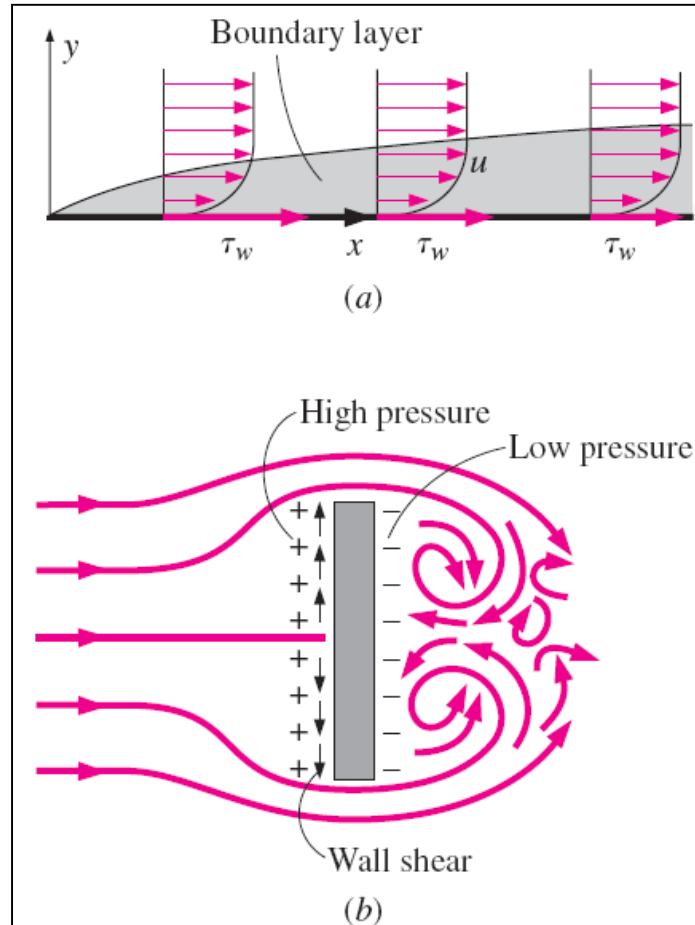
Lift force:
$$dF_L = -P\, dA \sin \theta - \tau_w\, dA \cos \theta$$

$$F_D = \int_A dF_D = \int_A (-P \cos \theta + \tau_w \sin \theta)\, dA$$

$$F_L = \int_A dF_L = -\int_A (P \sin \theta + \tau_w \cos \theta)\, dA$$

The pressure and viscous forces acting on a two-dimensional body and the resultant lift and drag forces.

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(a)

High pressure — Low pressure

Wall shear

(b)

Lift

V

Drag

Airplane wings are shaped and positioned to generate sufficient lift during flight while keeping drag at a minimum. Pressures above and below atmospheric pressure are indicated by plus and minus signs, respectively.

(a) Drag force acting on a flat plate parallel to the flow depends on wall shear only.
(b) Drag force acting on a flat plate normal to the flow depends on the pressure only and is independent of the wall shear, which acts normal to the free-stream flow.

The drag and lift forces depend on the density of the fluid, the upstream velocity, and the size, shape, and orientation of the body.

It is more convenient to work with appropriate dimensionless numbers that represent the drag and lift characteristics of the body.

These numbers are the **drag coefficient C_D**, and the **lift coefficient C_L**.

Drag coefficient: $$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

Lift coefficient: $$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$$
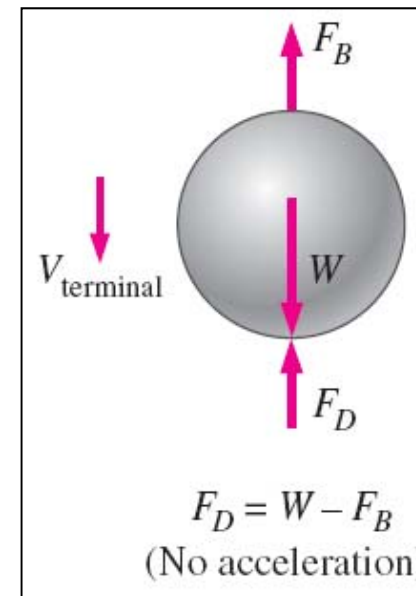
Dynamic pressure $\frac{1}{2}\rho V^2$

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\, dx$$

$$C_L = \frac{1}{L}\int_0^L C_{L,x}\, dx$$

In lift and drag calculations of some thin bodies, such as airfoils, *A* is taken to be the **planform area**, which is the area seen by a person looking at the body from above in a direction normal to the body.

A: Frontal area

F_B

$V_{terminal}$ W

F_D

$$F_D = W - F_B$$
(No acceleration)

During a free fall, a body reaches its *terminal velocity* when the drag force equals the weight of the body minus the buoyant force.

EXAMPLE 11–1

The drag coefficient of a car at the design conditions of 1 atm, 70°F, and 60 mi/h is to be determined experimentally in a large wind tunnel in a full-scale test. The frontal area of the car is 22.26 ft². If the force acting on the car in the flow direction is measured to be 68 lbf, determine the drag coefficient of this car.

Wind tunnel
60 mi/h

F_D

1 mi/h = 1.467 ft/s
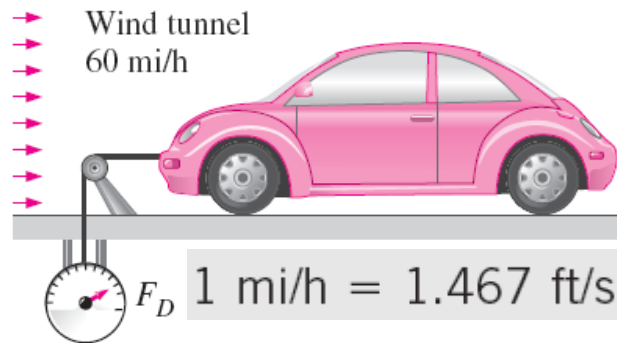
SOLUTION The drag force acting on a car is measured in a wind tunnel. The drag coefficient of the car at test conditions is to be determined.
***Assumptions* 1** The flow of air is steady and incompressible. **2** The cross section of the tunnel is large enough to simulate free flow over the car. **3** The bottom of the tunnel is moving at the speed of air to approximate actual driving conditions, this effect is negligible.

$$F_D = C_D A \frac{\rho V^2}{2} \quad \text{and} \quad C_D = \frac{2F_D}{\rho A V^2}$$

$$C_D = \frac{2 \times (68 \text{ lbf})}{(0.07489 \text{ lbm/ft}^3)(22.26 \text{ ft}^2)(60 \times 1.467 \text{ ft/s})^2} \left(\frac{32.2 \text{ lbm} \cdot \text{ft/s}^2}{1 \text{ lbf}} \right) = 0.34$$

Note that the drag coefficient depends on the design conditions, and its value may be different at different conditions such as the Reynolds number. Therefore, the published drag coefficients of different vehicles can be compared meaningfully only if they are determined under similar conditions. This shows the importance of developing standard testing procedures in industry.

- The drag force is the net force exerted by a fluid on a body in the direction of flow due to the combined effects of wall shear and pressure forces.

- The part of drag that is due directly to wall shear stress is called the **skin friction drag** (or *friction drag*) since it is caused by frictional effects, and the part that is due directly to pressure is called the **pressure drag** (also called the *form drag* because of its strong dependence on the form or shape of the body)

$$C_{D,\ \text{friction}} = \frac{F_{D,\ \text{friction}}}{\frac{1}{2}\rho V^2 A} \qquad \text{and} \qquad C_{D,\ \text{pressure}} = \frac{F_{D,\ \text{pressure}}}{\frac{1}{2}\rho V^2 A}$$

$$C_D = C_{D,\ \text{friction}} + C_{D,\ \text{pressure}} \qquad \text{and} \qquad F_D = F_{D,\ \text{friction}} + F_{D,\ \text{pressure}}$$

- The *friction drag* is the component of the wall shear force in the direction of flow, and thus it depends on the orientation of the body as well as the magnitude of the wall shear stress.

- **For parallel flow over a flat surface**, the drag coefficient is equal to the *friction drag coefficient*.

- **Friction drag is a strong function of viscosity, and increases with increasing viscosity.**
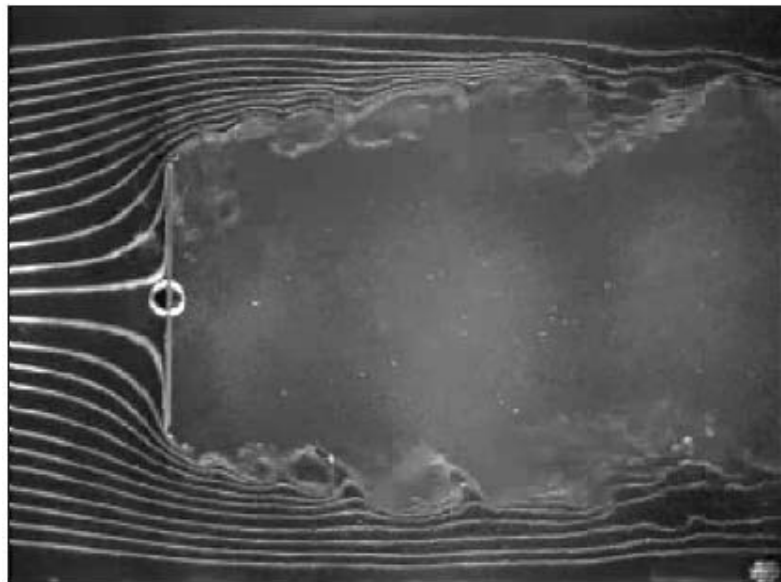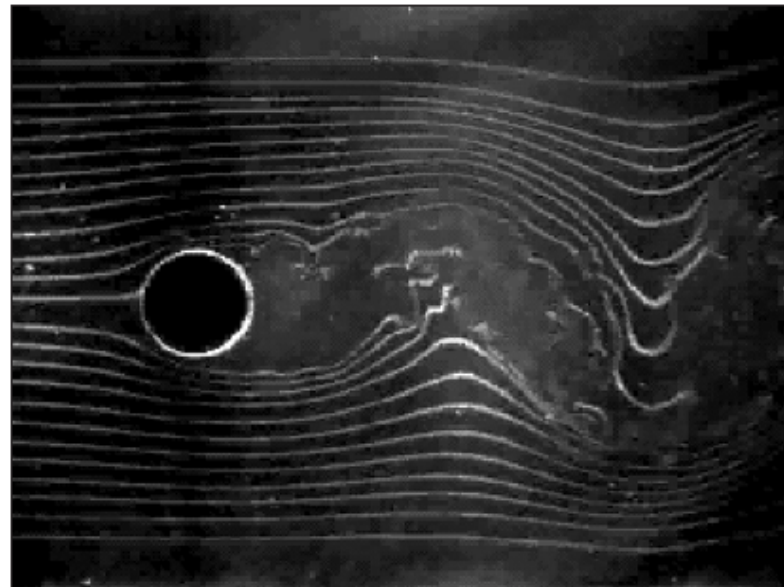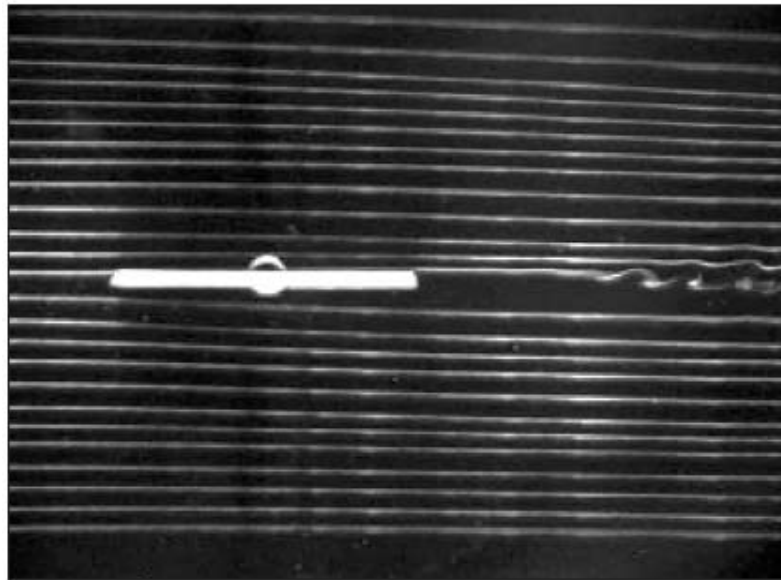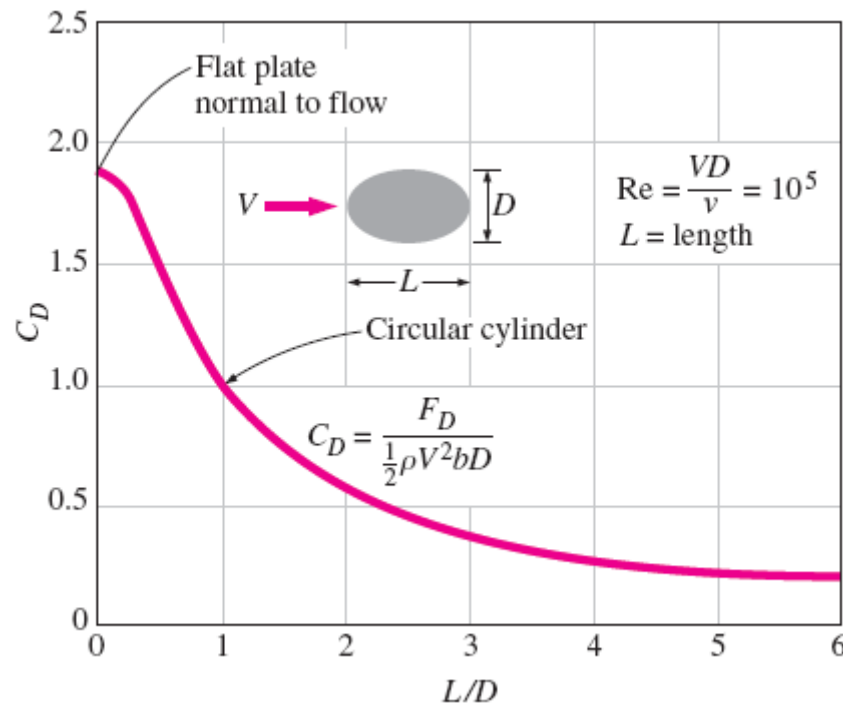
Drag is due entirely to *friction drag* for a flat plate parallel to the flow; it is due entirely to pressure drag for a flat plate normal to the flow; and it is due to *both* (but mostly *pressure drag*) for a cylinder normal to the flow. The total drag coefficient C_D is lowest for a parallel flat plate, highest for a vertical flat plate, and in between (but close to that of a vertical flat plate) for a cylinder.

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Reducing Drag by Streamlining

Streamlining decreases pressure drag by delaying boundary layer separation and thus reducing the pressure difference between the front and back of the body but increases the friction drag by increasing the surface area. The end result depends on which effect dominates.

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 LD}$$
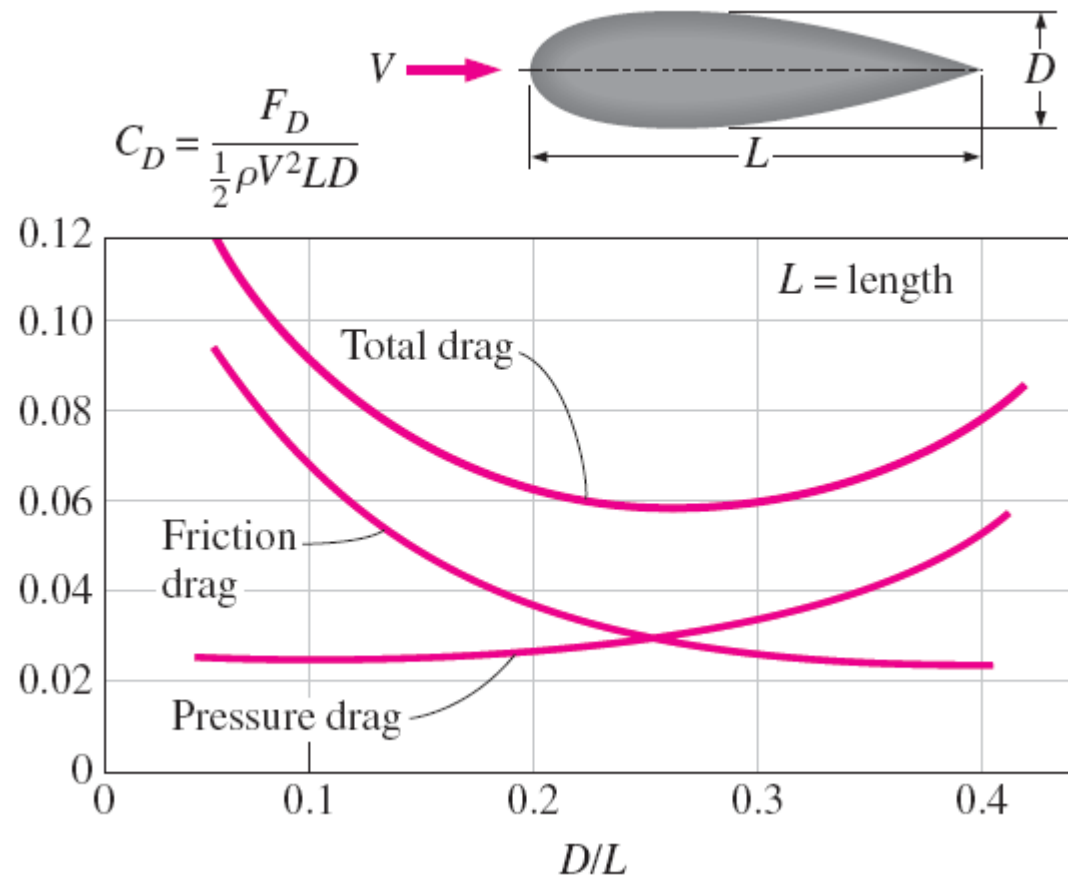
The variation of friction, pressure, and total drag coefficients of a streamlined strut with thickness-to-chord length ratio for Re = 4×10⁴. Note that C_D for airfoils and other thin bodies is based on *planform* area rather than frontal area.

L = length

Total drag

Friction drag

Pressure drag

D/L

The graph shows the variation of C_D versus L/D:

- y-axis: C_D from 0 to 2.5
- x-axis: L/D from 0 to 6

Flat plate normal to flow

$V \rightarrow$ (ellipse with D)

$\leftarrow L \rightarrow$

$\mathrm{Re} = \dfrac{VD}{v} = 10^5$
$L = \text{length}$

Circular cylinder

$$C_D = \dfrac{F_D}{\frac{1}{2}\rho V^2 bD}$$

The variation of the drag coefficient of a long elliptical cylinder with aspect ratio. Here C_D is based on the frontal area bD where b is the width of the body.

The drag coefficient decreases drastically as the ellipse becomes slimmer.

The reduction in the drag coefficient at high aspect ratios is primarily due to the boundary layer staying attached to the surface longer and the resulting pressure recovery.

Streamlining has the added benefit of *reducing vibration and noise*.

Streamlining should be considered only for blunt bodies that are subjected to high-velocity fluid flow (and thus high Reynolds numbers) for which flow separation is a real possibility.

Streamlining is not necessary for bodies that typically involve low Reynolds number flows.
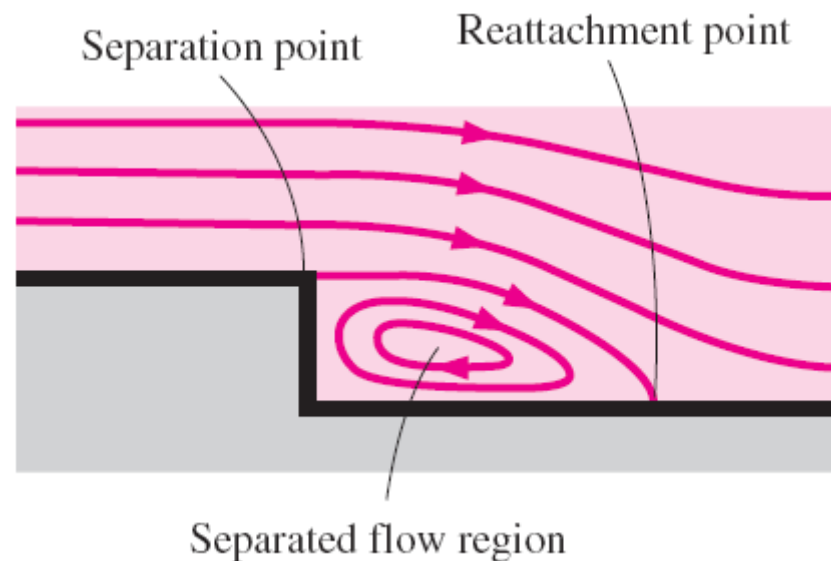
Flow Separation

Flow separation: At sufficiently high velocities, the fluid stream detaches itself from the surface of the body.

The location of the separation point depends on several factors such as the Reynolds number, the surface roughness, and the level of fluctuations in the free stream, and it is usually difficult to predict exactly where separation will occur.


Flow separation in a waterfall.

Flow separation over a backward-facing step along a wall.

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- **Separated region:** When a fluid separates from a body, it forms a separated region between the body and the fluid stream.

- This is a low-pressure region behind the body where recirculating and backflows occur.

- The larger the separated region, the larger the pressure drag.

- The effects of flow separation are felt far downstream in the form of reduced velocity (relative to the upstream velocity).

- **Wake:** The region of flow trailing the body where the effects of the body on velocity are felt.

- Viscous and rotational effects are the most significant in the boundary layer, the separated region, and the wake.
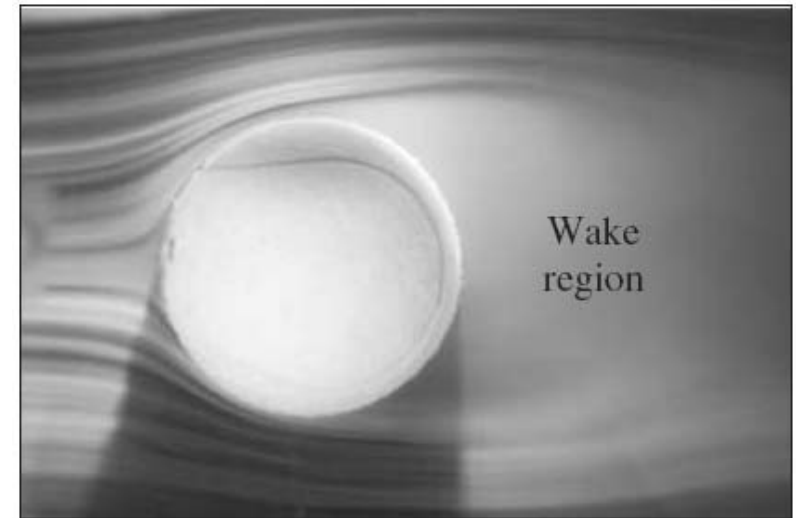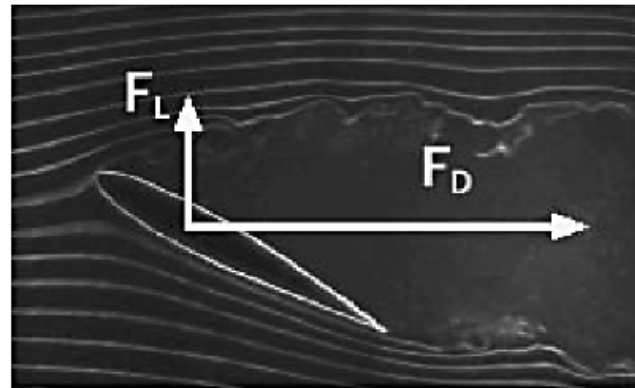

Wake region

Flow separation and the wake region during flow over a tennis ball.
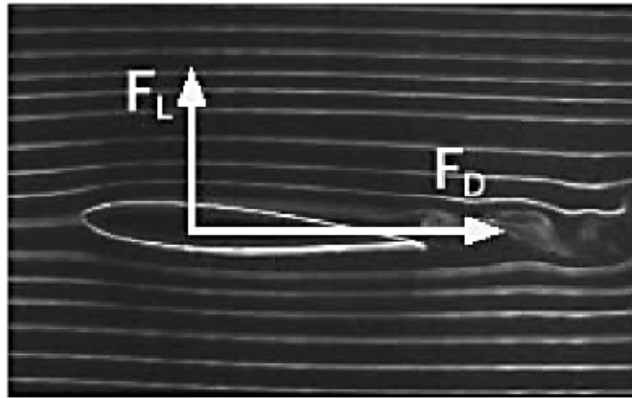
(a) 5°
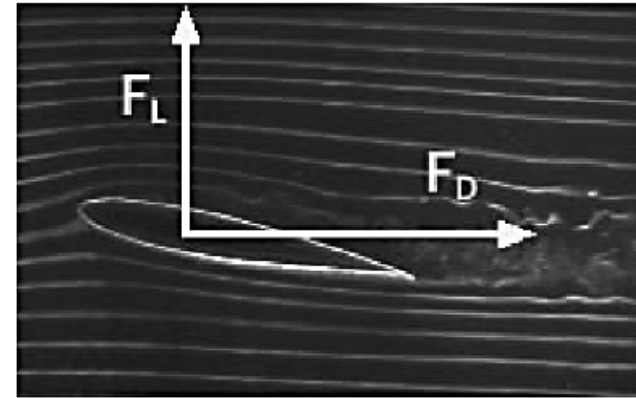
(b) 15°

(c) 30°

At large angles of attack (usually larger than 15°), flow may separate completely from the top surface of an airfoil, reducing lift drastically and causing the airfoil to stall.

Flow separation is the formation and sheeding of circulating fluid structures, called **vortices**, in the wake region.
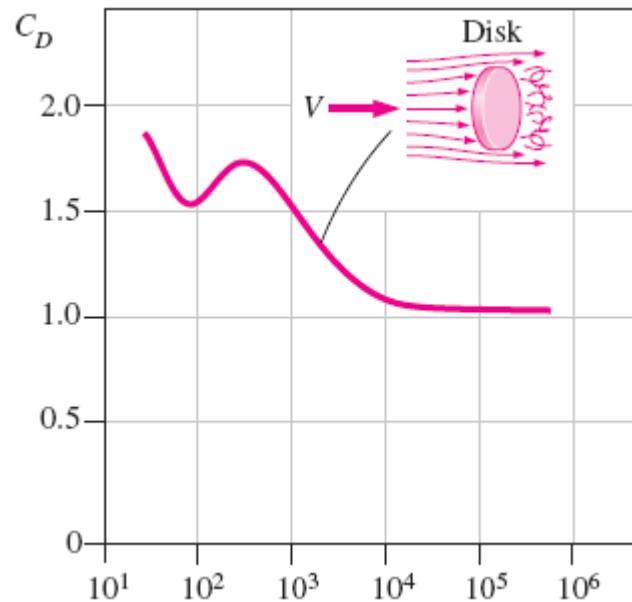The periodic generation of these vortices downstream is referred to as **vortex shedding.**
The vibrations generated by vortices near the body may cause the body to resonate to dangerous levels.

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The drag behavior of various natural and human-made bodies is characterized by their drag coefficients measured under typical operating conditions.

Usually the *total* (friction+pressure) drag coefficient is reported.

The drag coefficient exhibits different behavior in the low (creeping), moderate (laminar), and high (turbulent) regions of the Reynolds number.

The inertia effects are negligible in low Reynolds number flows (Re < 1), called *creeping flows*, and the fluid wraps around the body smoothly.
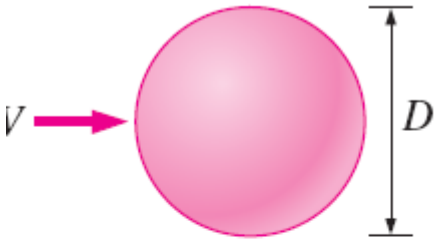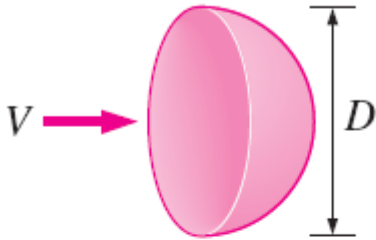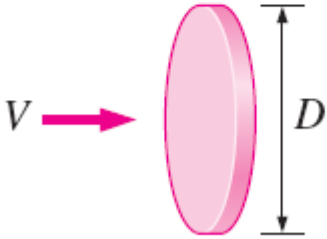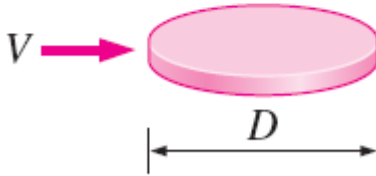
$$C_D = \frac{24}{Re} \qquad (Re \lesssim 1)$$

Creeping flow, sphere

Stokes law

$$F_D = C_D A \frac{\rho V^2}{2} = \frac{24}{Re} A \frac{\rho V^2}{2} = \frac{24}{\rho V D/\mu} \frac{\pi D^2}{4} \frac{\rho V^2}{2} = 3\pi\mu VD$$

The drag coefficient for many (but not all) geometries remains essentially constant at Reynolds numbers above about 10^4.

Stokes law is often applicable to dust particles in the air and suspended solid particles in water.

Sphere

$V \rightarrow$ D

$C_D = 24/\text{Re}$

Hemisphere

$V \rightarrow$ D

$C_D = 22.2/\text{Re}$

Circular disk
(normal to flow)

$V \rightarrow$ D

$C_D = 20.4/\text{Re}$

Circular disk
(parallel to flow)

$V \rightarrow$

D

$C_D = 13.6/\text{Re}$

Drag coefficients C_D at low velocities

(Re$\leq$1 where Re=VD/ν and $A=\pi D^2/4$).

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Drag coefficients C_D of various two-dimensional bodies for Re $> 10^4$ based on the frontal area $A = bD$, where b is the length in direction normal to the page (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream veloci

Square rod

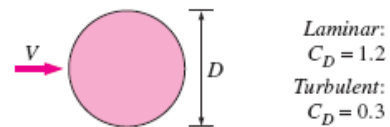Sharp corners: $C_D = 2.2$

Round corners ($r/D = 0.2$): $C_D = 1.2$

Rectangular rod

Sharp corners:
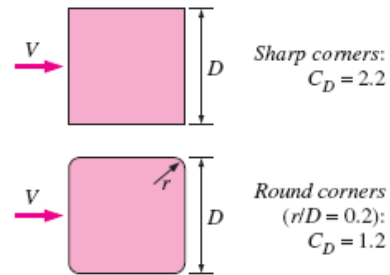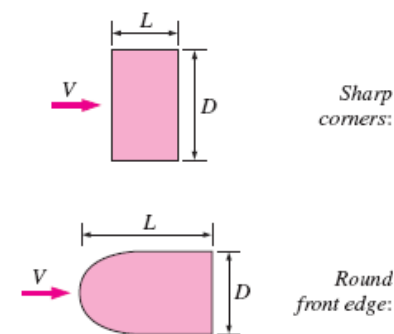
L/D	C_D
0.0*	1.9
0.1	1.9
0.5	2.5
1.0	2.2
2.0	1.7
3.0	1.3

* Corresponds to thin plate

Round front edge:

L/D	C_D
0.5	1.2
1.0	0.9
2.0	0.7
4.0	0.7

Circular rod (cylinder)

Laminar: $C_D = 1.2$
Turbulent: $C_D = 0.3$

Elliptical rod
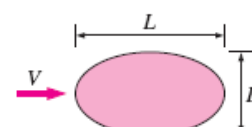
| | C_D | |
L/D	Laminar	Turbule
2	0.60	0.20
4	0.35	0.15
8	0.25	0.10

Equilateral triangular rod

$C_D = 1.5$

$C_D = 2.0$

Semicircular shell

$C_D = 2.3$

$C_D = 1.2$

Semicircular rod

$C_D = 1.2$

$C_D = 1.7$

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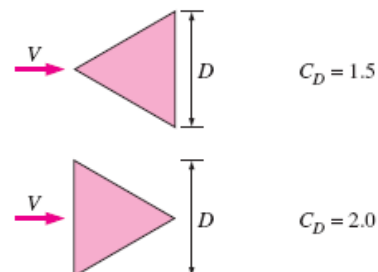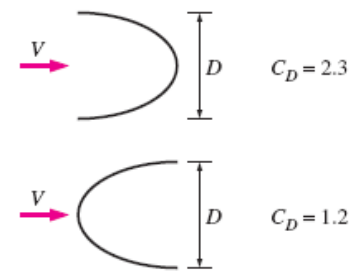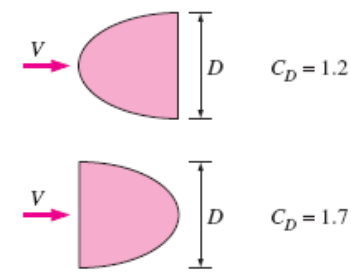
Representative drag coefficients C_D for various three-dimensional bodies for Re $> 10^4$ based on the frontal area (for use in the drag force relation $F_D = C_D A \rho V^2/2$ where V is the upstream velocity)
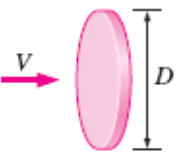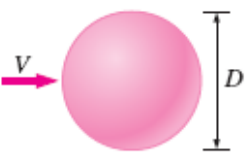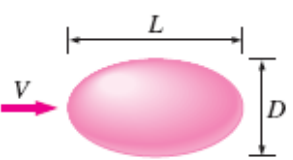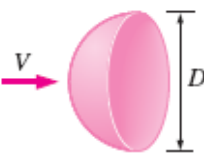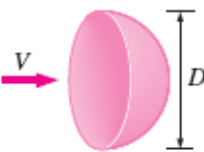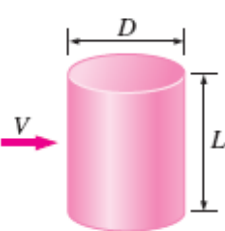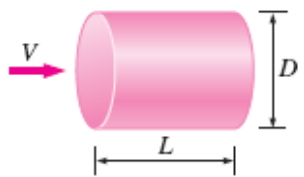
Cube, $A = D^2$

$C_D = 1.05$

Thin circular disk, $A = \pi D^2/4$

$C_D = 1.1$

Cone (for $\theta = 30°$), $A = \pi D^2/4$

$C_D = 0.5$

Sphere, $A = \pi D^2/4$

Laminar:
$C_D = 0.5$
Turbulent:
$C_D = 0.2$

See Fig. 11–36 for C_D vs. Re for smooth and rough spheres.

Ellipsoid, $A = \pi D^2/4$

	C_D	
L/D	Laminar	Turbulent
0.75	0.5	0.2
1	0.5	0.2
2	0.3	0.1
4	0.3	0.1
8	0.2	0.1

Hemisphere, $A = \pi D^2/4$

$C_D = 0.4$

$C_D = 1.2$

Short cylinder, vertical, $A = LD$

L/D	C_D
1	0.6
2	0.7
5	0.8
10	0.9
40	1.0
∞	1.2

Values are for laminar flow

Short cylinder, horizontal, $A = \pi D^2/4$

L/D	C_D
0.5	1.1
1	0.9
2	0.9
4	0.9
8	1.0

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TABLE 11–2 (Continued)

Streamlined body, $A = \pi D^2/4$

V $\rightarrow$ $\bigcirc$ ↕D $C_D = 0.04$

Parachute, $A = \pi D^2/4$

$C_D = 1.3$

Tree, $A =$ frontal area

V →

V, m/s	C_D
10	0.4–1.2
20	0.3–1.0
30	0.2–0.7

Person (average)

Standing: $C_D A = 9$ ft$^2 = 0.84$ m^2
Sitting: $C_D A = 6$ ft$^2 = 0.56$ m^2

Bikes

$C_D = 0.5$ $C_D = 0.9$

Upright:
$A = 5.5$ ft$^2 = 0.51$ m^2
$C_D = 1.1$

Drafting:
$A = 3.9$ ft$^2 = 0.36$ m^2
$C_D = 0.50$

Racing:
$A = 3.9$ ft$^2 = 0.36$ m^2
$C_D = 0.9$

With fairing:
$A = 5.0$ ft$^2 = 0.46$ m^2
$C_D = 0.12$
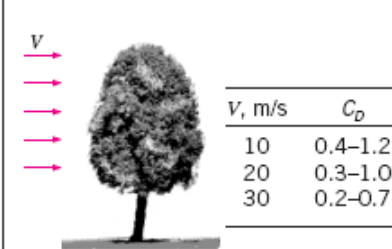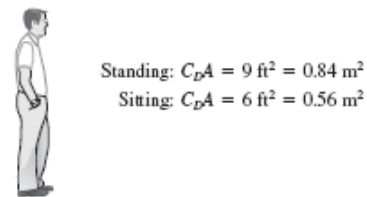
Semitrailer, $A =$ frontal area

Without fairing:
$C_D = 0.96$

With fairing:
$C_D = 0.76$

Automotive, $A =$ frontal area

Minivan:
$C_D = 0.4$

Passenger car:
$C_D = 0.3$
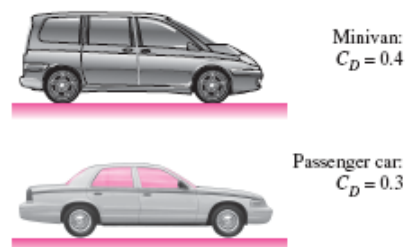
High-rise buildings, $A =$ frontal area

$C_D = 1.4$

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Flow @ RE = 10^4

Potential Flow

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Flow @ RE = 10⁴

Potential Flow

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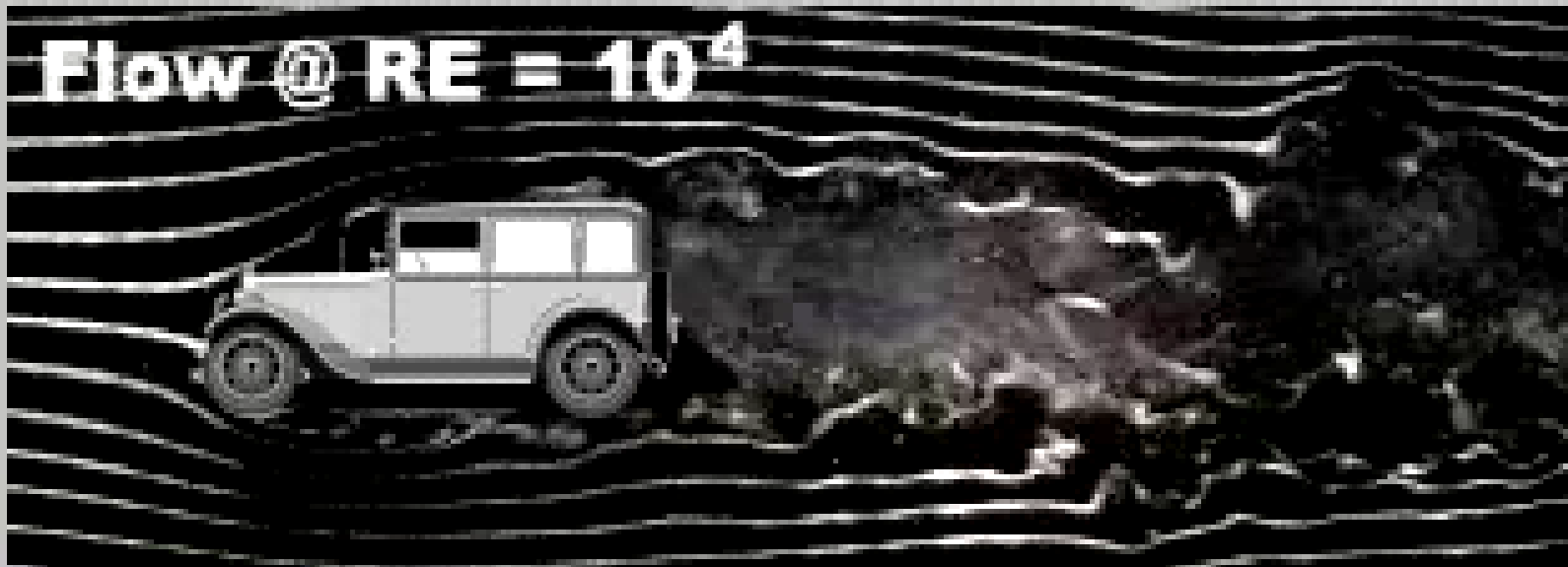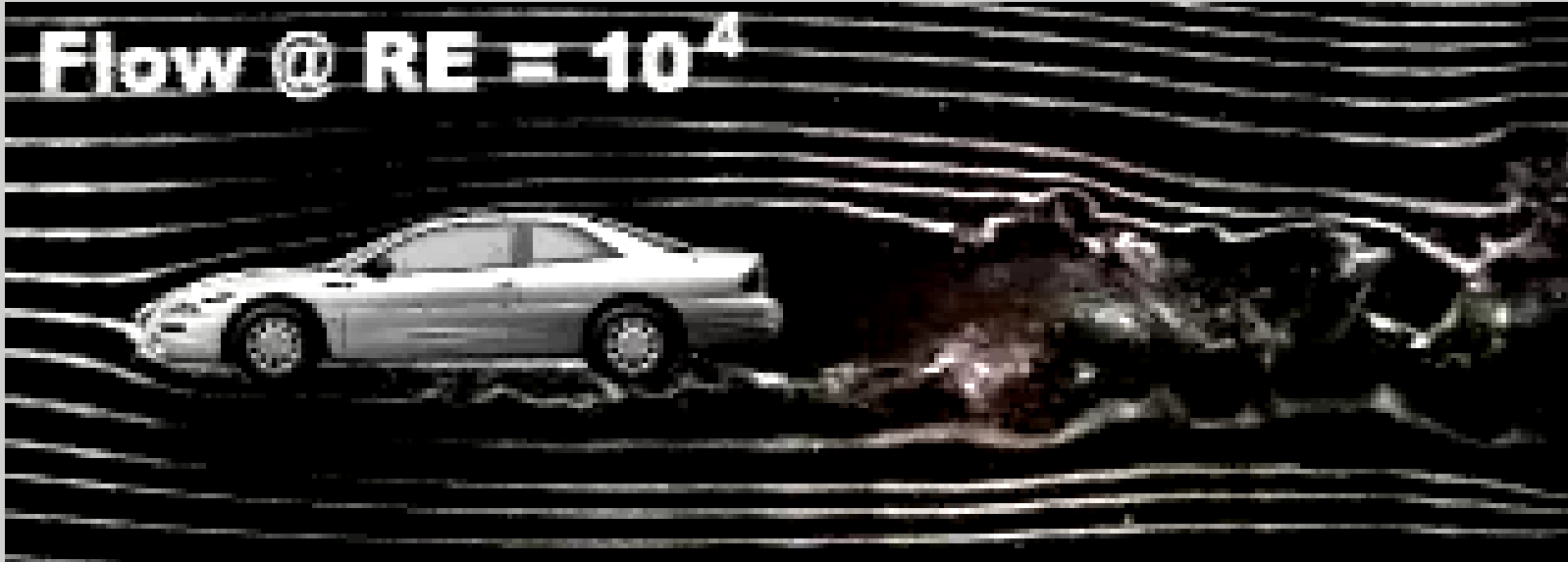
Observations from the drag coefficient tables

The *orientation* of the body relative to the direction of flow has a major influence on the drag coefficient.

For blunt bodies with sharp corners, such as flow over a rectangular block or a flat plate normal to flow, separation occurs at the edges of the front and back surfaces, with no significant change in the character of flow.

Therefore, the drag coefficient of such bodies is nearly independent of the Reynolds number.

The drag coefficient of a long rectangular rod can be reduced almost by half from 2.2 to 1.2 by rounding the corners.

A hemisphere at two different orientations for $Re > 10^4$

$V \longrightarrow \quad C_D = 0.4$

$V \longrightarrow \quad C_D = 1.2$
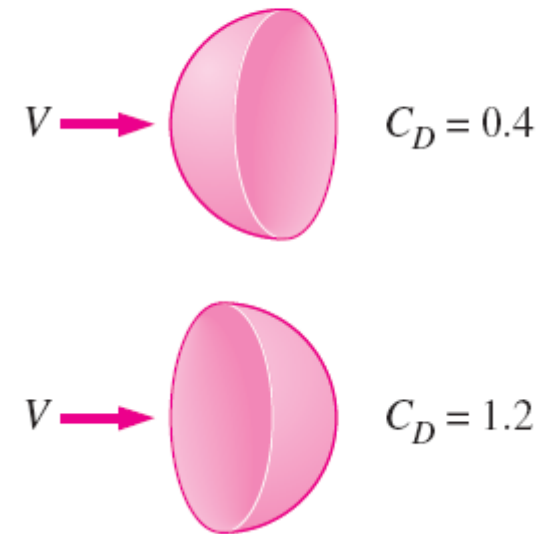
The drag coefficient of a body may change drastically by changing the body's orientation (and thus shape) relative to the direction of flow.

Biological Systems and Drag

The concept of drag also has important consequences for biological systems.

The bodies of *fish*, especially the ones that swim fast for long distances (such as dolphins), are highly streamlined to minimize drag (the drag coefficient of dolphins based on the wetted skin area is about 0.0035, comparable to the value for a flat plate in turbulent flow).

Airplanes, which look somewhat like big birds, retract their wheels after takeoff in order to reduce drag and thus fuel consumption.

The flexible structure of plants enables them to reduce drag at high winds by changing their shapes.

Large flat leaves, for example, curl into a low-drag conical shape at high wind speeds, while tree branches cluster to reduce drag.

Flexible trunks bend under the influence of the wind to reduce drag, and the bending moment is lowered by reducing frontal area.

Horse and bicycle riders lean forward as much as they can to reduce drag.

Feet folded back

Beak extended forward
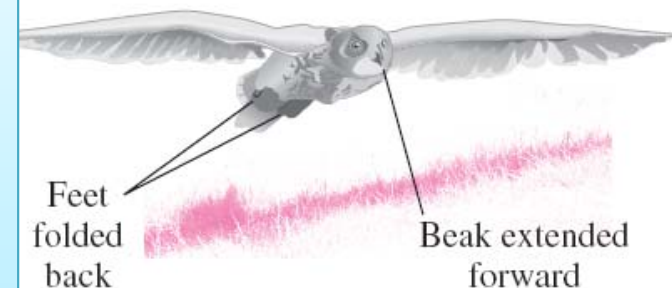
Birds teach us a lesson on drag reduction by extending their beak forward and folding their feet backward during flight.

Drag Coefficients of Vehicles

The drag coefficients of vehicles range from about 1.0 for large semitrailers to 0.4 for minivans, 0.3 for passenger cars, and 0.2 for race cars. The theoretical lower limit is about 0.1.

In general, the more blunt the vehicle, the higher the drag coefficient.

Installing a fairing reduces the drag coefficient of tractor-trailer rigs by about 20 percent by making the frontal surface more streamlined.

As a rule of thumb, the percentage of fuel savings due to reduced drag is about half the percentage of drag reduction at highway speeds.

This sleek-looking Toyota Prius has a drag coefficient of 0.26—one of the lowest for a passenger car.

Flow @ Re = 10^4 | Potential Flow
Flow @ Re = 10^4 | Potential Flow
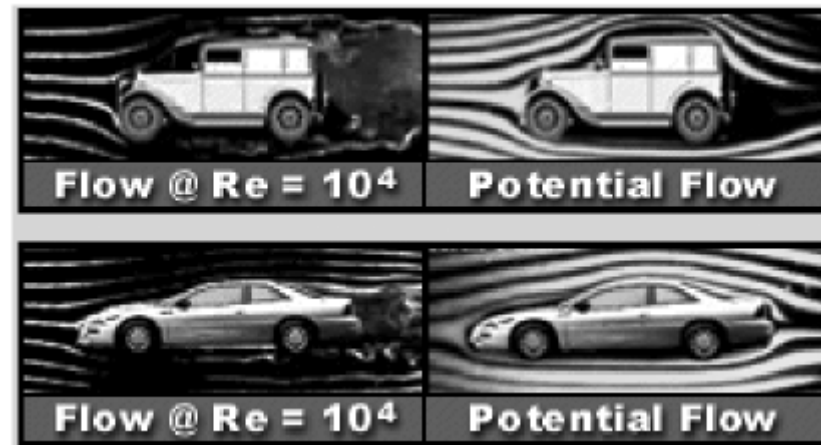
Streamlines around an aerodynamically designed modern car closely resemble the streamlines around the car in the ideal potential flow, except near the rear end, resulting in a low drag coefficient.

The aerodynamic drag is negligible at low speeds, but becomes significant at speeds above about 50 km/h.

At highway speeds, a driver can often save fuel in hot weather by running the air conditioner instead of driving with the windows rolled down.

The turbulence and additional drag generated by open windows consume more fuel than does the air conditioner.

$C_D = 0.5$ Low pressure $C_D = 0.9$ High pressure
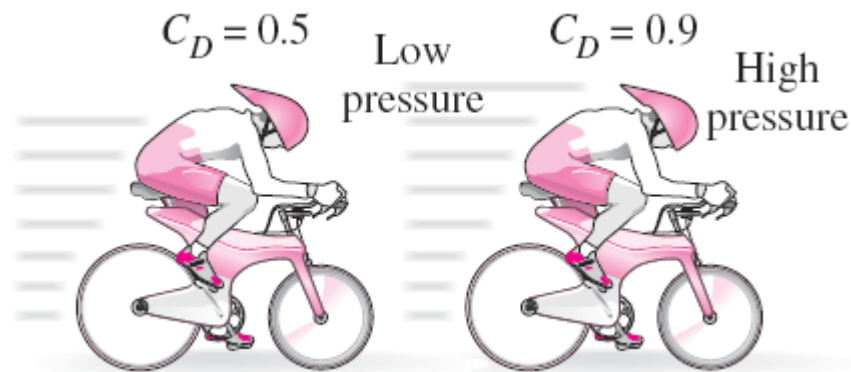
The drag coefficients of bodies following other moving bodies closely can be reduced considerably due to drafting (i.e., falling into the low pressure region created by the body in front).

The shapes of many bodies encountered in practice are not simple.

But such bodies can be treated conveniently in drag force calculations by considering them to be composed of two or more simple bodies.

A satellite dish mounted on a roof with a cylindrical bar, for example, can be considered to be a combination of a hemispherical body and a cylinder.

Then the drag coefficient of the body can be determined approximately by using superposition.

EXAMPLE 11-2

As part of the continuing efforts to reduce the drag coefficient and thus to improve the fuel efficiency of cars, the design of side rearview mirrors has changed drastically from a simple circular plate to a streamlined shape. Determine the amount of fuel and money saved per year as a result of replacing a 13-cm-diameter flat mirror by one with a hemispherical back. Assume the car is driven 24,000 km a year at an average speed of 95 km/h. Take the density and price of gasoline to be 0.8 kg/L and $0.60/L, respectively; the heating value of gasoline to be 44,000 kJ/kg; and the overall efficiency of the engine to be 30 percent.

Solution The flat mirror of a car is replaced by one with a hemispherical back. The amount of fuel and money saved per year as a result are to be determined.

Assumptions **1** The car is driven 24,000 km a year at an average speed of 95 km/h. **2** The effect of the car body on the flow around the mirror is negligible (no interference).

Flat mirror
95 km/h

$D = 13$ cm

Rounded mirror
95 km/h

$D = 13$ cm

$$F_D = C_D A \frac{\rho V^2}{2}$$

$$F_D = 1.1 \frac{\pi(0.13 \text{ m})^2}{4} \frac{(1.20 \text{ kg/m}^3)(95 \text{ km/h})^2}{2} \left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right)^2 \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 6.10 \text{ N}$$
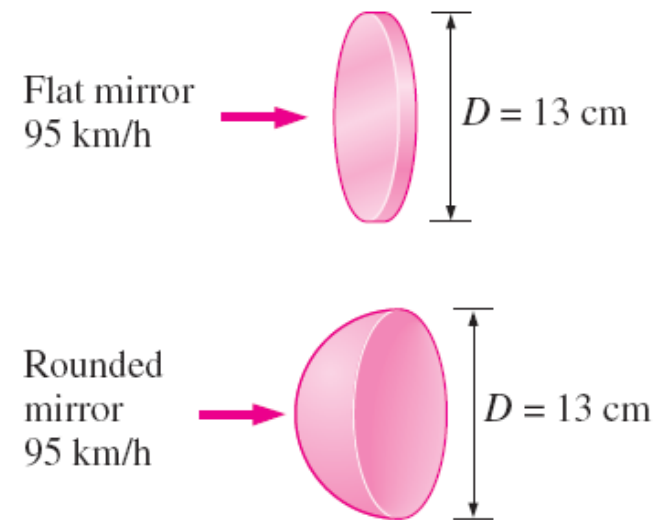
$$W_{\text{drag}} = F_D \times L = (6.10\ \text{N})(24{,}000\ \text{km/year}) = 146{,}400\ \text{kJ/year}$$

$$E_{\text{in}} = \frac{W_{\text{drag}}}{\eta_{\text{car}}} = \frac{146{,}400\ \text{kJ/year}}{0.3} = 488{,}000\ \text{kJ/year}$$

$$\text{Amount of fuel} = \frac{m_{\text{fuel}}}{\rho_{\text{fuel}}} = \frac{E_{\text{in}}/\text{HV}}{\rho_{\text{fuel}}} = \frac{(488{,}000\ \text{kJ/year})/(44{,}000\ \text{kJ/kg})}{0.8\ \text{kg/L}} = 13.9\ \text{L/year}$$

$$\text{Cost} = (\text{Amount of fuel})(\text{Unit cost}) = (13.9\ \text{L/year})(\$0.60/\text{L}) = \$8.32/\text{year}$$

$$\text{Reduction ratio} = \frac{C_{D,\text{ flat}} - C_{D,\text{ hemisp}}}{C_{D,\text{ flat}}} = \frac{1.1 - 0.4}{1.1} = 0.636$$

$$\text{Fuel reduction} = (\text{Reduction ratio})(\text{Amount of fuel})$$

$$= 0.636(13.9\ \text{L/year}) = \textbf{8.84 L/year}$$

$$\text{Cost reduction} = (\text{Reduction ratio})(\text{Cost}) = 0.636(\$8.32/\text{year}) = \textbf{\$5.29/year}$$

Significant reductions in drag and fuel consumption can be achieved by streamlining the shape of various components and the entire car. The sharp corners are replaced in late model cars by rounded contours. This also explains why large airplanes retract their wheels after takeoff and small airplanes use contoured fairings around their wheels.

Velocity boundary layer: The region of the flow above the plate bounded by δ in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

The **boundary layer thickness δ** is typically defined as the distance y from the surface at which $u = 0.99V$.

The hypothetical curve of $u = 0.99V$ divides the flow into two regions:

Boundary layer region: The viscous effects and the velocity changes are significant.

Irotational flow region: The frictional effects are negligible and the velocity remains essentially constant.
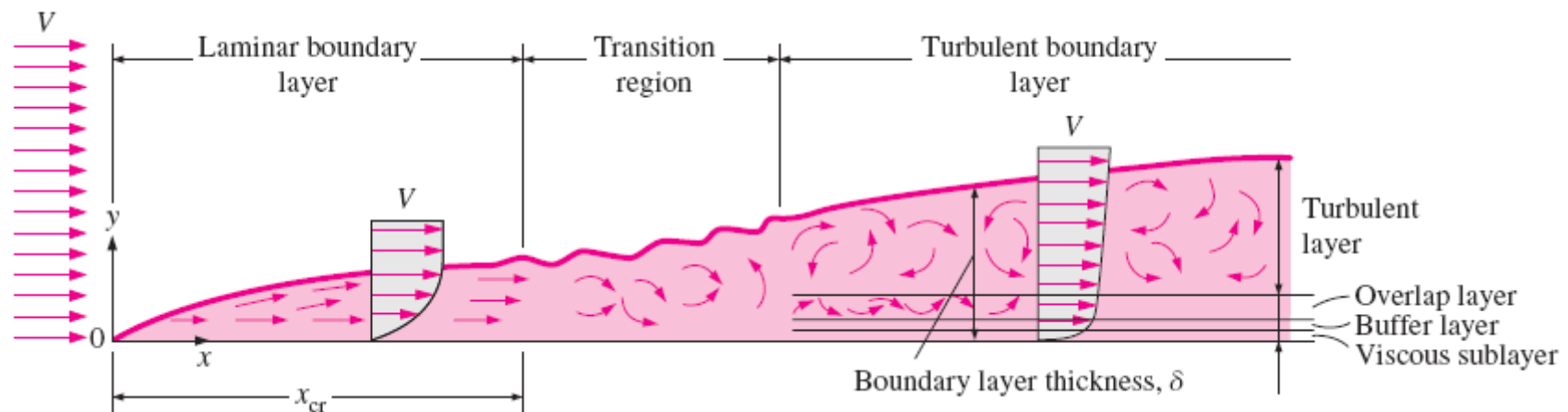
The development of the boundary layer for flow over a flat plate and the different flow regimes

The turbulent boundary layer can be considered to consist of four regions, characterized by the distance from the wall:

- **viscous sublayer**
- **buffer layer**
- **overlap layer**
- **turbulent layer**

Relative velocities of fluid layers

V

V

$0.99V$

Zero velocity at the surface

δ
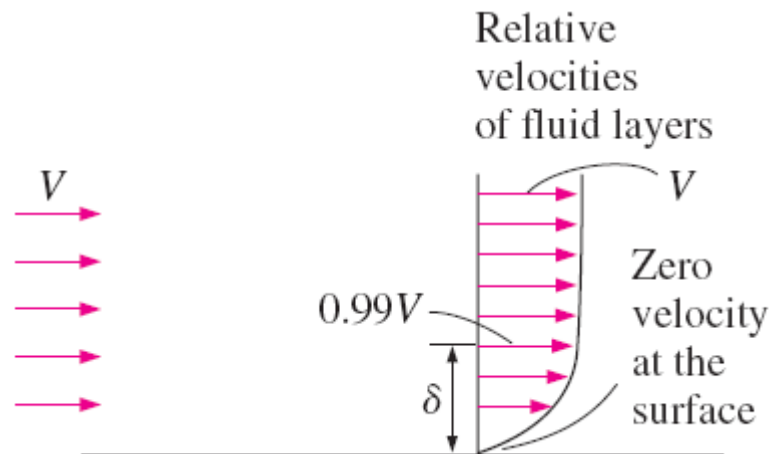
The development of a boundary layer on a surface is due to the no-slip condition and friction.

Friction coefficient on a flat plate

$$C_D = C_{D,\,\text{friction}} = C_f$$

Friction force on a flat plate

$$F_D = F_f = \tfrac{1}{2} C_f A \rho V^2$$

Flow over a flat plate

$$C_{D,\,\text{pressure}} = 0$$

$$C_D = C_{D,\,\text{friction}} = C_f$$

$$F_{D,\,\text{pressure}} = 0$$

$$F_D = F_{D,\,\text{friction}} = F_f = C_f A \frac{\rho V^2}{2}$$

For parallel flow over a flat plate, the pressure drag is zero, and thus the drag coefficient is equal to the friction coefficient and the drag force is equal to the friction force.

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The transition from laminar to turbulent flow depends on the *surface geometry, surface roughness, upstream velocity, surface temperature*, and the *type of fluid*, among other things, and is best characterized by the Reynolds number.

The Reynolds number at a distance *x* from the leading edge of a flat plate is expressed as

$$\text{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$

V   upstream velocity

x   characteristic length of the geometry (for a flat plate, it is the length of the plate in the flow direction)

For flow over a smooth flat plate, transition from laminar to turbulent begins at about Re $\approx 1\times10^5$, but does not become fully turbulent before the Reynolds number reaches much higher values, typically around 3×10^6.

In engineering analysis, a generally accepted value for the critical Reynolds number is

$$\text{Re}_{x,\,cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from about 10^5 to 3×10^6 depending on the **surface roughness**, the **turbulence level**, and the **variation of pressure along the surface**.

Friction Coefficient

- The friction coefficient for **laminar flow** over a flat plate can be determined theoretically by solving the conservation of mass and momentum equations numerically.

- For **turbulent flow**, it must be determined experimentally and expressed by empirical correlations.

$$C_f = \frac{1}{L} \int_0^L C_{f,x}\, dx$$

$$\mathrm{Re}_x = Vx/\nu$$
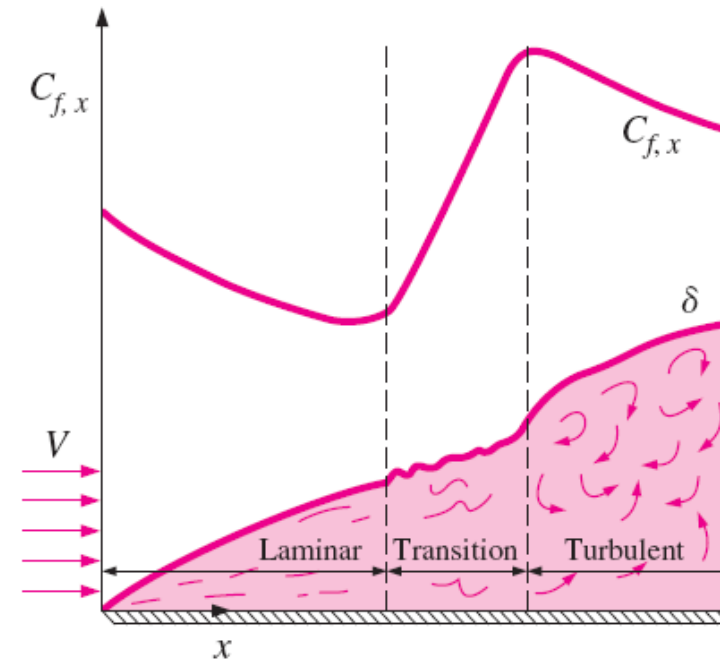
The variation of the local friction coefficient for flow over a flat plate..

$$\text{Laminar:} \qquad \delta = \frac{4.91x}{\mathrm{Re}_x^{1/2}} \qquad \text{and} \qquad C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}, \qquad \mathrm{Re}_x < 5 \times 10^5$$

$$\text{Turbulent:}\ \delta = \frac{0.38x}{\mathrm{Re}_x^{1/5}} \qquad \text{and} \qquad C_{f,x} = \frac{0.059}{\mathrm{Re}_x^{1/5}}, \qquad 5 \times 10^5 \leq \mathrm{Re}_x \leq 10^7$$

Average friction coefficient over the entire plate

$$C_f = \frac{1}{L} \int_0^L C_{f,x}\, dx$$

$$= \frac{1}{L} \int_0^L \frac{0.664}{\mathrm{Re}_x^{1/2}}\, dx$$

$$= \frac{0.664}{L} \int_0^L \left(\frac{Vx}{\nu}\right)^{-1/2} dx$$

$$= \frac{0.664}{L} \left(\frac{V}{\nu}\right)^{-1/2} \left.\frac{x^{1/2}}{\frac{1}{2}}\right|_0^L$$

$$= \frac{2 \times 0.664}{L} \left(\frac{V}{\nu L}\right)^{-1/2}$$

$$= \frac{1.328}{\mathrm{Re}_L^{1/2}}$$

The average friction coefficient over a surface is determined by integrating the local friction coefficient over the entire surface. The values shown here are for a laminar flat plate boundary layer.

Laminar: $C_f = \dfrac{1.33}{\mathrm{Re}_L^{1/2}}$ $\mathrm{Re}_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{\mathrm{Re}_L^{1/5}}$ $5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

$$\mathrm{Re}_{cr} = 5 \times 10^5 = Vx_{cr}/\nu$$

When the laminar flow region is not disregarded

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x,\,\text{laminar}}\, dx + \int_{x_{cr}}^L C_{f,x,\,\text{turbulent}}\, dx \right)$$

$$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L} \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

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For laminar flow, the friction coefficient depends only on the Reynolds number, and the surface roughness has no effect.

For turbulent flow, surface roughness causes the friction coefficient to increase severalfold, to the point that in the fully rough turbulent regime the friction coefficient is a function of surface roughness alone and is independent of the Reynolds number.

Fully rough turbulent regime: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

Relative Roughness, ε/L	Friction Coefficient, C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

ε the surface roughness
L the length of the plate in the flow direction.
This relation can be used for turbulent flow on rough surfaces for Re > 10^6, especially when $\varepsilon/L > 10^4$.

For turbulent flow, surface roughness may cause the friction coefficient to increase several fold.

C_f increases several fold with roughness in turbulent flow.

C_f is independent of the Reynolds number in the fully rough region.

This chart is the flat-plate analog of the Moody chart for pipe flows.
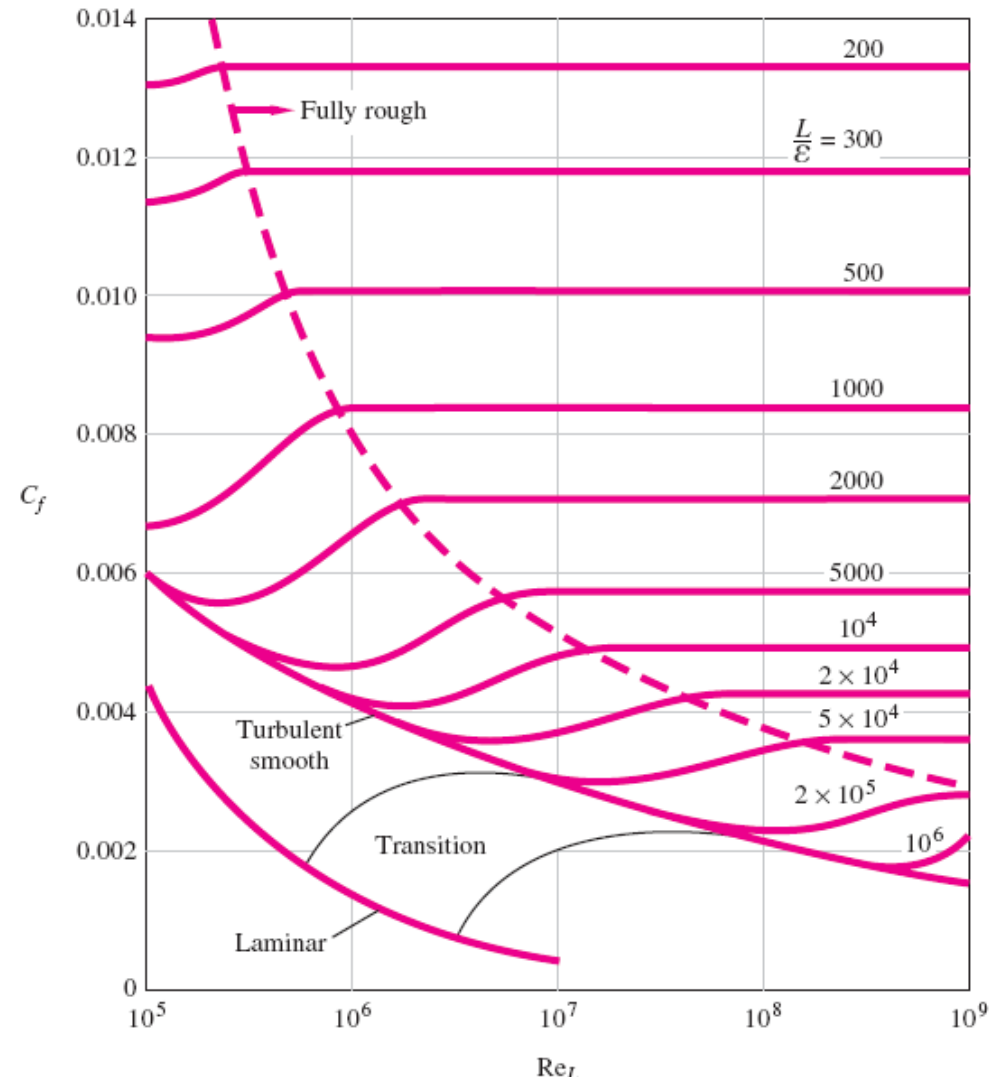
Friction coefficient for parallel flow over smooth and rough flat plates for both laminar and turbulent flows.

EXAMPLE 11-3

Engine oil at 40°C flows over a 5-m-long flat plate with a free-stream velocity of 2 m/s. Determine the drag force acting on the plate per unit width.
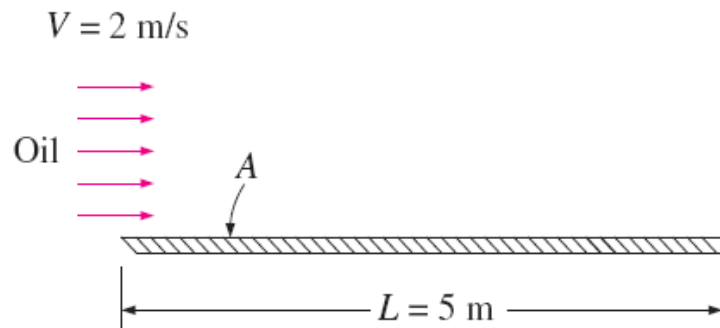
Solution Engine oil flows over a flat plate. The drag force per unit width of the plate is to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The critical Reynolds number is $R_{ecr} = 5 \times 10^5$.

$$Re_L = \frac{VL}{\nu} = \frac{(2 \text{ m/s})(5 \text{ m})}{2.485 \times 10^{-4} \text{ m}^2/\text{s}} = 4.024 \times 10^4$$

$$C_f = 1.328 Re_L^{-0.5} = 1.328 \times (4.024 \times 10^4)^{-0.5} = 0.00662$$

$$F_D = C_f A \frac{\rho V^2}{2} = 0.00662(5 \times 1 \text{ m}^2) \frac{(876 \text{ kg/m}^3)(2 \text{ m/s})^2}{2} \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 58.0 \text{ N}$$

The force per unit width corresponds to the weight of a mass of about 6 kg. Therefore, a person who applies an equal and opposite force to the plate to keep it from moving will feel like he or she is using as much force as is necessary to hold a 6-kg mass from dropping.

Flow over cylinders and spheres is frequently encountered in practice.

The tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes.

Many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D.* Thus, the Reynolds number is defined as Re VD/n where V is the uniform velocity of the fluid as it approaches the cylinder or sphere. The critical Reynolds number for flow across a circular cylinder or sphere is about $R_{ecr} \cong 2\times10^5$. That is, the boundary layer remains laminar
for about Re $\lesssim 2\times10^5$ and becomes turbulent for Re $\gtrsim 2\times10^5$.
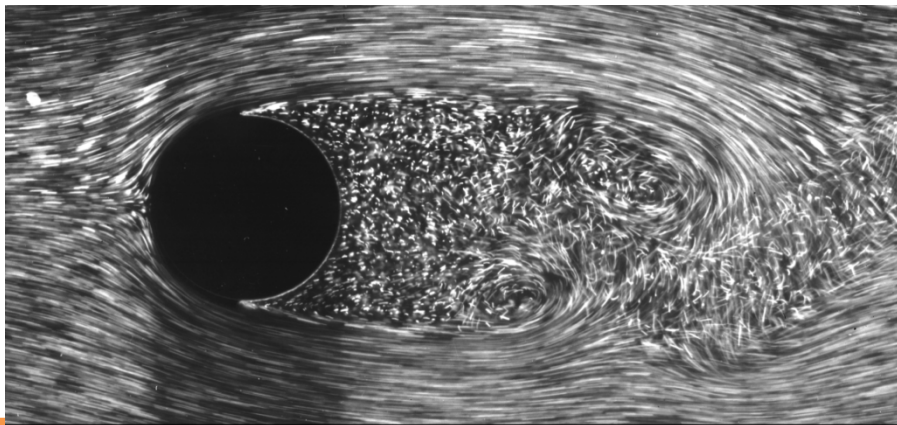
Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder at Re=2000.

At very low velocities, the fluid completely wraps around the cylinder. Flow in the wake region is characterized by periodic vortex formation and low pressures.

For flow over cylinder or sphere, both the *friction drag* and the *pressure drag* can be significant.

The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow.

The drag force is primarily due to friction drag at low Reynolds numbers (Re<10) and to pressure drag at high Reynolds numbers (Re>5000).

Both effects are significant at intermediate Reynolds numbers.
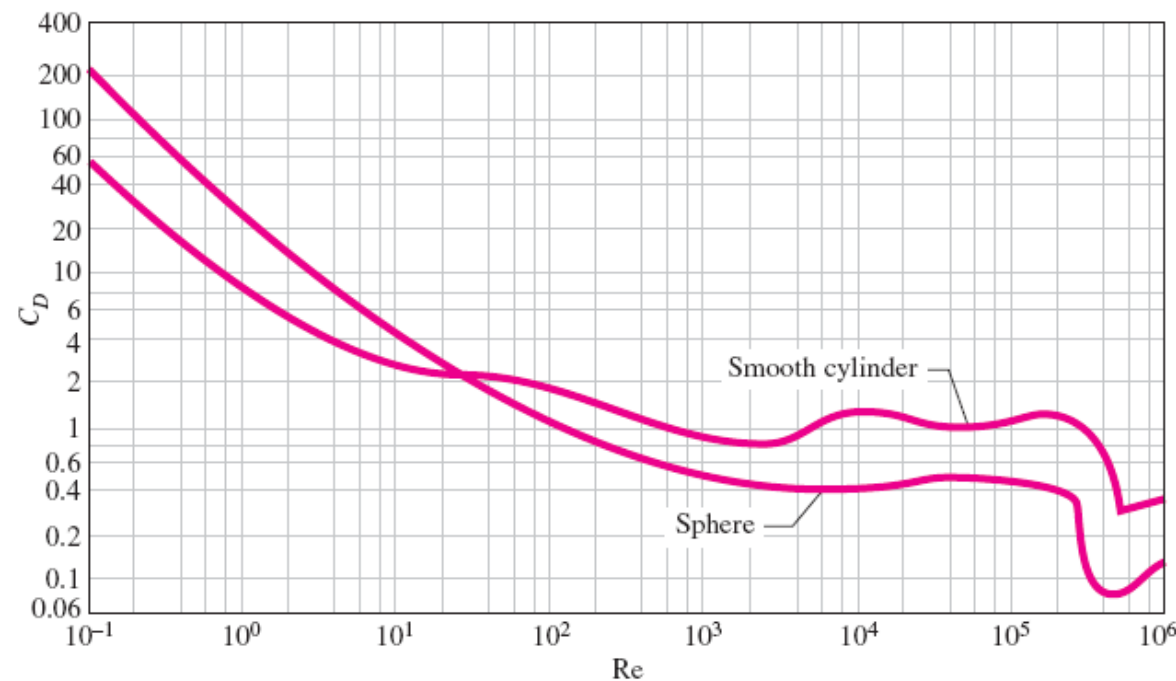
Average drag coefficient for cross-flow over a smooth circular cylinder and a smooth sphere.

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Observations from C_D curves

- For Re<1, we have creeping flow, and the drag coefficient decreases with increasing Reynolds number. For a sphere, it is C_D=24/Re. There is no flow separation in this regime.

- At about Re=10, separation starts occurring on the rear of the body with vortex shedding starting at about Re=90. The region of separation increases with increasing Reynolds number up to about Re=10^3. At this point, the drag is mostly (about 95 %) due to pressure drag. The drag coefficient continues to decrease with increasing Reynolds number in this range of 10<Re<10^3.

- In the moderate range of 10^3<Re<10^5, the drag coefficient remains relatively constant. This behavior is characteristic of bluff bodies. The flow in the boundary layer is laminar in this range, but the flow in the separated region past the cylinder or sphere is highly turbulent with a wide turbulent wake.

- There is a sudden drop in the drag coefficient somewhere in the range of 10^5<Re<10^6 (usually, at about 2×10^5). This large reduction in C_D is due to the flow in the boundary layer becoming *turbulent*, which moves the separation point further on the rear of the body, reducing the size of the wake and thus the magnitude of the pressure drag. This is in contrast to streamlined bodies, which experience an increase in the drag coefficient (mostly due to friction drag) when the boundary layer becomes turbulent.

- There is a "transitional" regime for 2×10^5<Re<2×10^6, in which C_D dips to a minimum value and then slowly rises to its final turbulent value.

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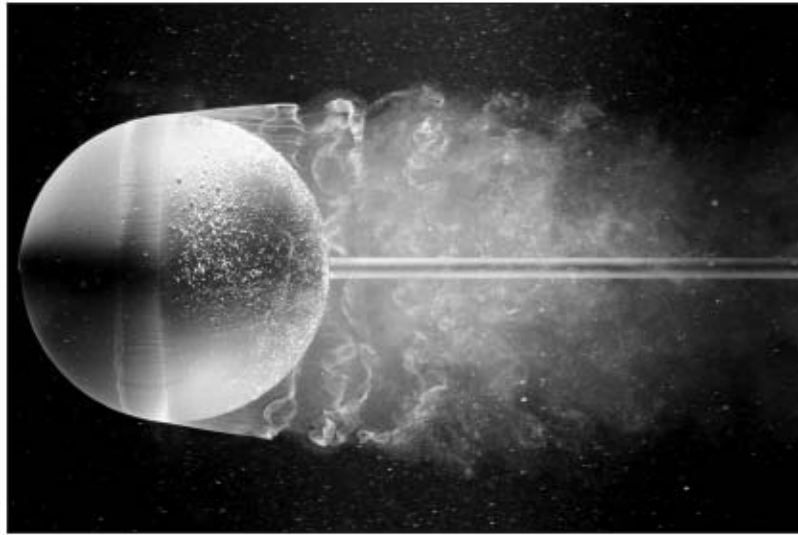
(a)
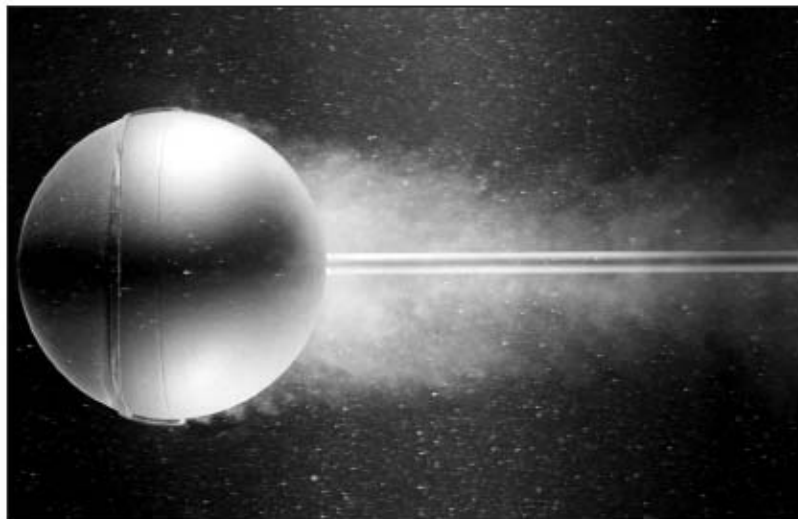
Flow separation occurs at about $\theta = 80°$ (measured from the front stagnation point of a cylinder) when the boundary layer is *laminar* and at about $\theta = 140°$ when it is *turbulent*.

The delay of separation in turbulent flow is caused by the rapid fluctuations of the fluid in the transverse direction, which enables the turbulent boundary layer to travel farther along the surface before separation occurs, resulting in a narrower wake and a smaller pressure drag.

Flow visualization of flow over (*a*) a smooth sphere at Re = 15,000, and (*b*) a sphere at Re = 30,000 with a trip wire. The delay of boundary layer separation is clearly seen by comparing the two photographs.

Effect of Surface Roughness

Surface roughness, in general, increases the drag coefficient in turbulent flow.

This is especially the case for streamlined bodies.

For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *increase* or *decrease* the drag coefficient depending on Reynolds number.
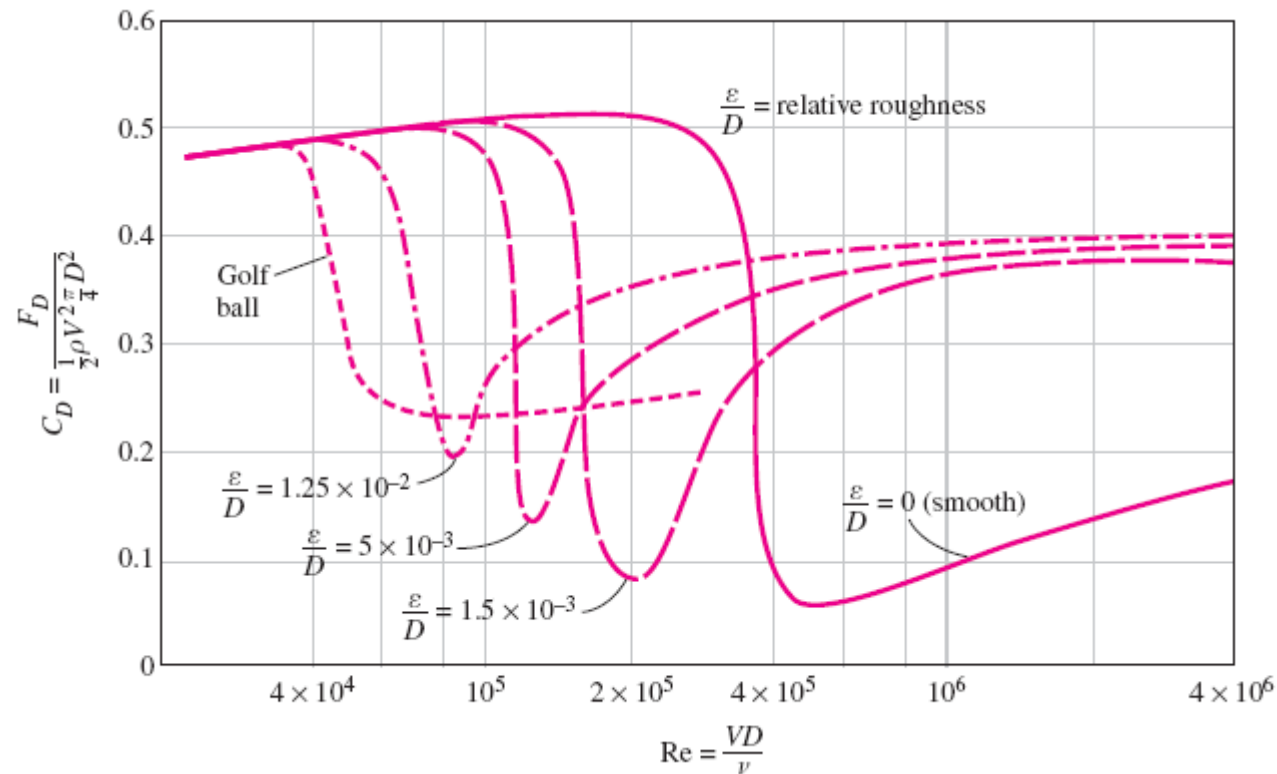
The effect of surface roughness on the drag coefficient of a sphere.

	C_D	
Re	Smooth Surface	Rough Surface, $\varepsilon/D = 0.0015$
2×10^5	0.5	0.1
10^6	0.1	0.4

Surface roughness may increase or decrease the drag coefficient of a spherical object, depending on the value of the Reynolds number.

Drag force relation

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

Frontal area for a cylinder and sphere

$$A = LD \qquad A = \pi D^2/4$$

Roughening the surface can be used to great advantage in reducing drag.

Golf balls are intentionally roughened to induce *turbulence* at a lower Reynolds number to take advantage of the sharp *drop* in the drag coefficient at the onset of turbulence in the boundary layer (the typical velocity range of golf balls is 15 to 150 m/s, and the Reynolds number is less than 4×10^5). The occurrence of turbulent flow at this Reynolds number reduces the drag coefficient of a golf ball by about half. For a given hit, this means a longer distance for the ball.

For a table tennis ball, however, the speeds are slower and the ball is smaller—it never reaches speeds in the turbulent range. Therefore, the surfaces of table tennis balls are made smooth.

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EXAMPLE 11-4

A 2.2-cm-outer-diameter pipe is to span across a river at a 30-m-wide section while being completely immersed in water. The average flow velocity of water is 4 m/s and the water temperature is 15°C. Determine the drag force exerted on the pipe by the river.
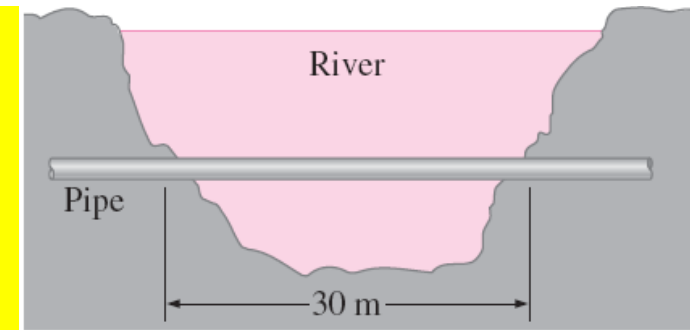
Solution A pipe is submerged in a river. The drag force acts on the pipe is to be determined.

Assumptions 1 The outer surface of the pipe is smooth so that Fig. 11–34 can be used to determine the drag coefficient. 2 Water flow in the river is steady. 3 The direction of water flow is normal to the pipe. 4 Turbulence in river flow is not considered.

$$Re = \frac{VD}{\nu} = \frac{\rho VD}{\mu} = \frac{(999.1 \text{ kg/m}^3)(4 \text{ m/s})(0.022 \text{ m})}{1.138 \times 10^{-3} \text{ kg/m} \cdot \text{s}} = 7.73 \times 10^4$$

$$F_D = C_D A \frac{\rho V^2}{2} = 1.0(30 \times 0.022 \text{ m}^2)\frac{(999.1 \text{ kg/m}^3)(4 \text{ m/s})^2}{2}\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 5275 \text{ N} \cong \textbf{5300 N}$$

Note that this force is equivalent to the weight of a mass over 500 kg. Therefore, the drag force the river exerts on the pipe is equivalent to hanging a total of over 500 kg in mass on the pipe supported at its ends 30 m apart. The necessary precautions should be taken if the pipe cannot support this force. If the river were to flow at a faster speed or if turbulent fluctuations in the river were more significant, the drag force would be even larger. *Unsteady* forces on the pipe might then be significant.

Lift: The component of the net force (due to viscous and pressure forces) that is perpendicular to the flow direction.

Lift coefficient

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A}$$

A planform area: the area that would be seen by a person looking at the body from above in a direction normal to the body
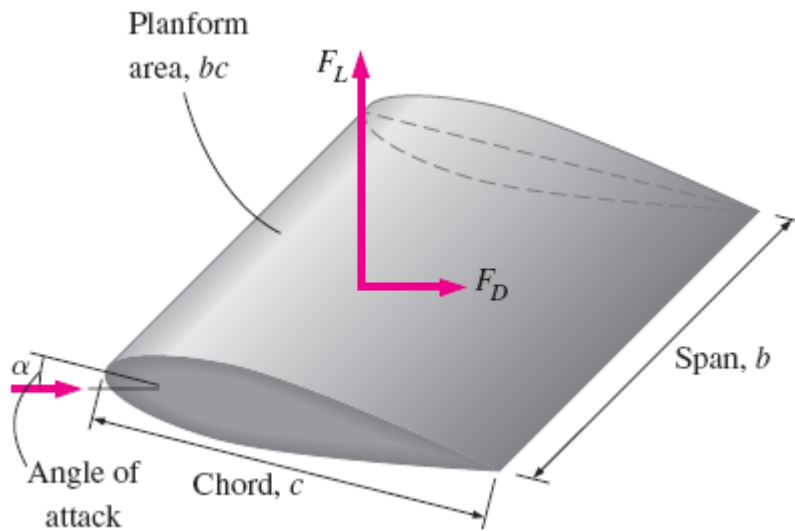
Definition of various terms associated with an airfoil.

For an aircraft, the wingspan is the total distance between the tips of the two wings, which includes the width of the fuselage between the wings.

The average lift per unit planform area F_L/A is called the **wing loading**, which is simply the ratio of the weight of the aircraft to the planform area of the wings (since lift equals the weight during flying at constant altitude).

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Chapter 11: EXTERNAL FLOW: DRAG AND LIFT

Airfoils are designed to generate lift while keeping the drag at a minimum.

Some devices such as the *spoilers* and *inverted airfoils* on racing cars are designed for avoiding lift or generating negative lift to improve traction and control.

Lift in practice can be taken to be due entirely to the pressure distribution on the surfaces of the body, and thus the shape of the body has the primary influence on lift.

Then the primary consideration in the design of airfoils is minimizing the average pressure at the upper surface while maximizing it at the lower surface.

Pressure is low at locations where the flow velocity is high, and pressure is high at locations where the flow velocity is low.

Lift at moderate angles of attack is practically independent of the surface roughness since roughness affects the wall shear, not the pressure.

Direction of lift

F_L

F_D

V

α

Direction of wall shear
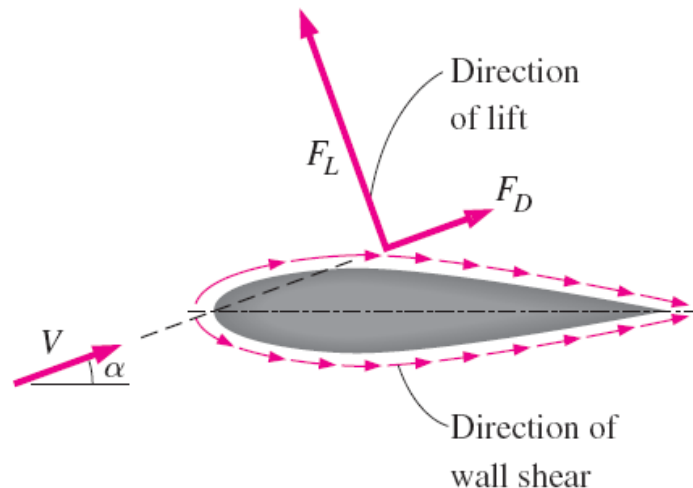
For airfoils, the contribution of viscous effects to lift is usually negligible since wall shear is parallel to the surfaces and thus nearly normal to the direction of lift.

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(a) Irrotational flow past a symmetrical airfoil (zero lift)

(b) Irrotational flow past a nonsymmetrical airfoil (zero lift)

(c) Actual flow past a nonsymmetrical airfoil (positive lift)
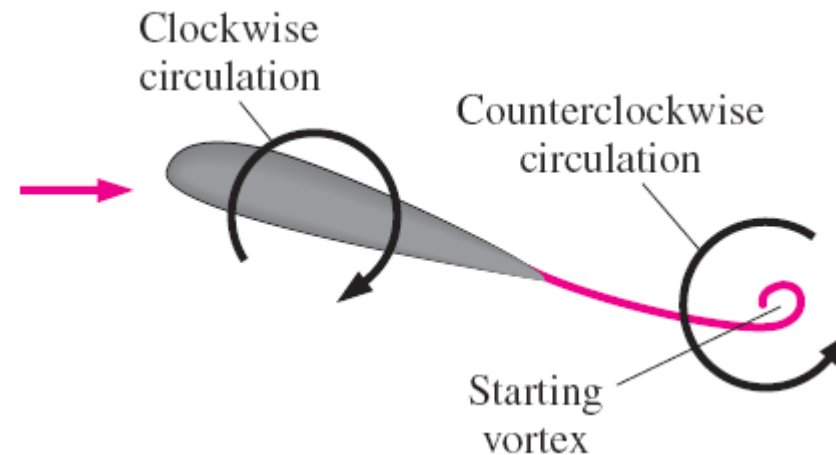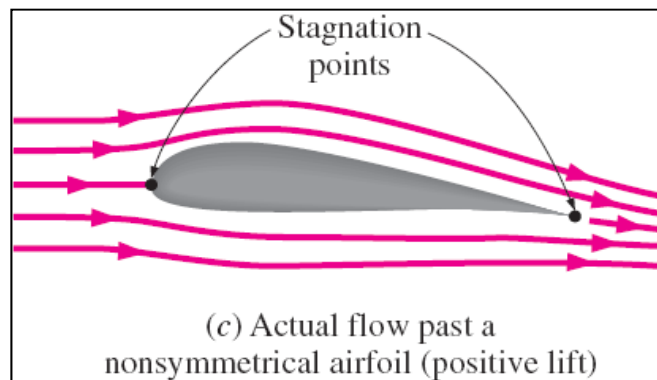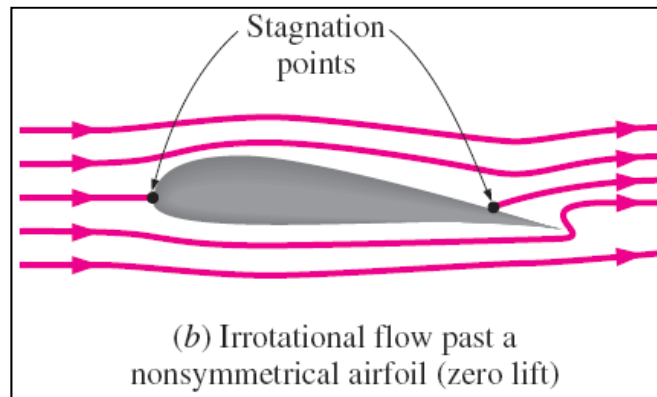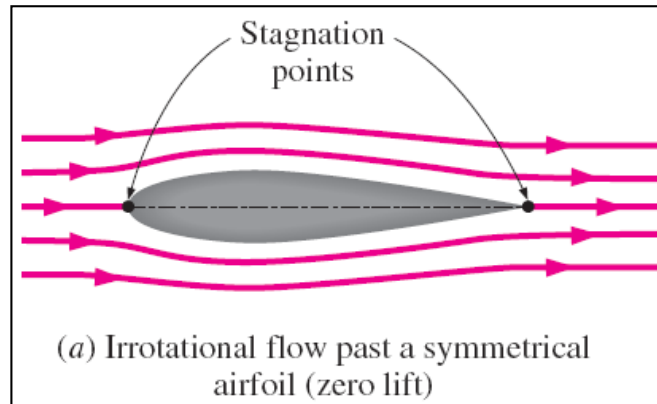
Shortly after a sudden increase in angle of attack, a counterclockwise starting vortex is shed from the airfoil, while clockwise circulation appears around the airfoil, causing lift to be generated.

Irrotational and actual flow past symmetrical and nonsymmetrical 2-D airfoils.

The variation of the lift-to-drag ratio with angle of attack for a 2-D airfoil.
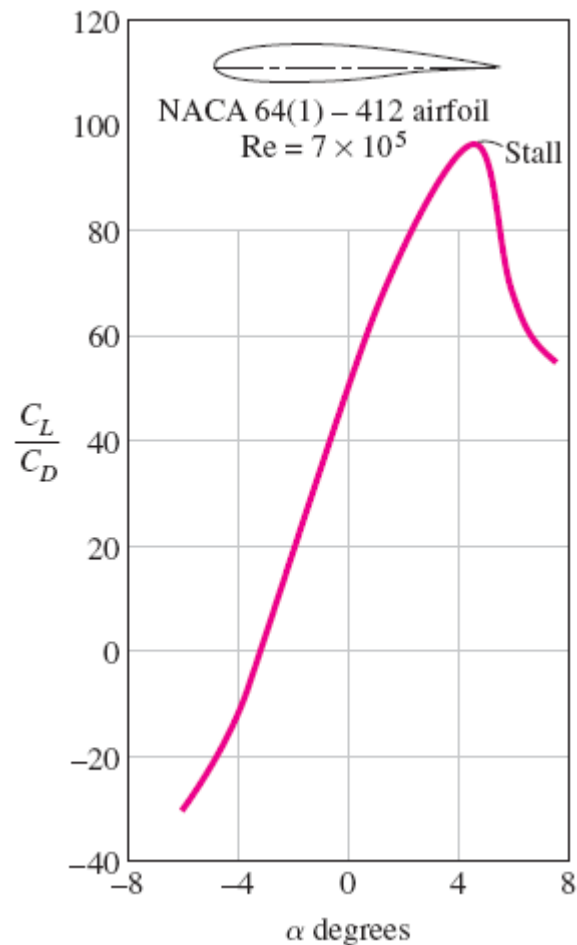
It is desirable for airfoils to generate the most lift while producing the least drag.

Therefore, a measure of performance for airfoils is the **lift-to-drag ratio**, which is equivalent to the ratio of the lift-to-drag coefficients C_L/C_D.

The C_L/C_D ratio increases with the angle of attack until the airfoil stalls, and the value of the lift-to-drag ratio can be of the order of 100 for a two-dimensional airfoil.

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One way to change the lift and drag characteristics of an airfoil is to change the angle of attack.

On an airplane, the entire plane is pitched up to increase lift, since the wings are fixed relative to the fuselage.

Another approach is to change the shape of the airfoil by the use of movable *leading edge* and *trailing edge flaps*.

The flaps are used to alter the shape of the wings during takeoff and landing to maximize lift at low speeds.

Once at cruising altitude, the flaps are retracted, and the wing is returned to its "normal" shape with minimal drag coefficient and adequate lift coefficient to minimize fuel consumption while cruising at a constant altitude.

Note that even a small lift coefficient can generate a large lift force during normal operation because of the large cruising velocities of aircraft and the proportionality of lift to the square of flow velocity.

(a) Flaps extended (takeoff)

(b) Flaps retracted (cruising)

The lift and drag characteristics of an airfoil during takeoff and landing can be changed by changing the shape of the airfoil by the use of movable flaps.
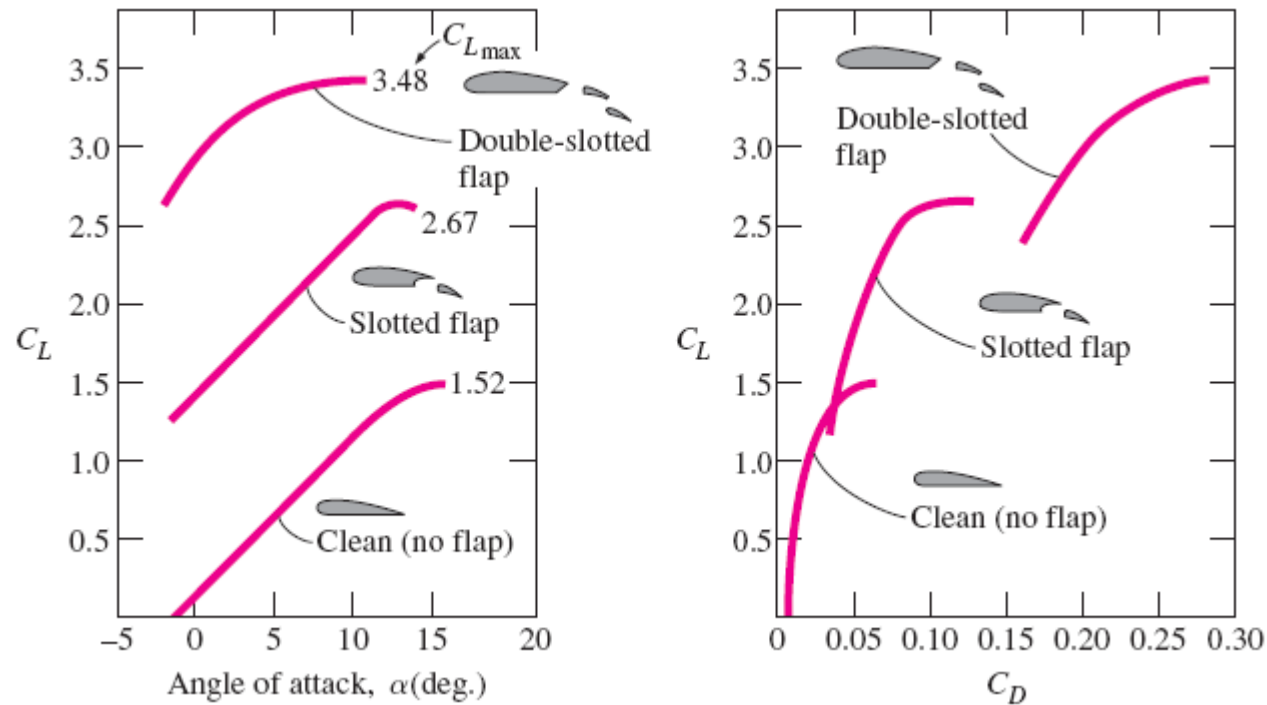
Effect of flaps on the lift and drag coefficients of an airfoil.

The maximum lift coefficient increases from about 1.5 for the airfoil with no flaps to 3.5 for the double-slotted flap case.

The maximum drag coefficient increases from about 0.06 for the airfoil with no flaps to about 0.3 for the double-slotted flap case.

The angle of attack of the flaps can be increased to maximize the lift coefficient.

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The minimum flight velocity can be determined from the requirement that the total weight W of the aircraft be equal to lift and $C_L = C_{L, max}$:

$$W = F_L = \tfrac{1}{2} C_{L, max} \rho V_{min}^2 A \qquad \rightarrow \qquad V_{min} = \sqrt{\frac{2W}{\rho C_{L, max} A}}$$

For a given weight, the landing or takeoff speed can be minimized by maximizing the product of the lift coefficient and the wing area, $C_{L, max}A$.

One way of doing that is to use flaps. Another way is to control the boundary layer, which can be accomplished simply by leaving flow sections (slots) between the flaps.

Slots are used to prevent the separation of the boundary layer from the upper surface of the wings and the flaps.

This is done by allowing air to move from the high-pressure region under the wing into the low-pressure region at the top surface.

Wing
Slot
Flap
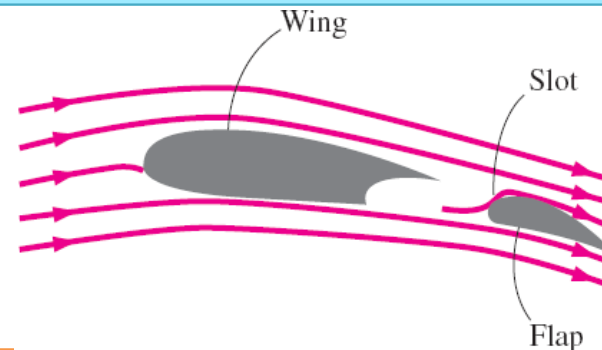
A flapped airfoil with a slot to prevent the separation of the boundary layer from the upper surface and to increase the lift coefficient.
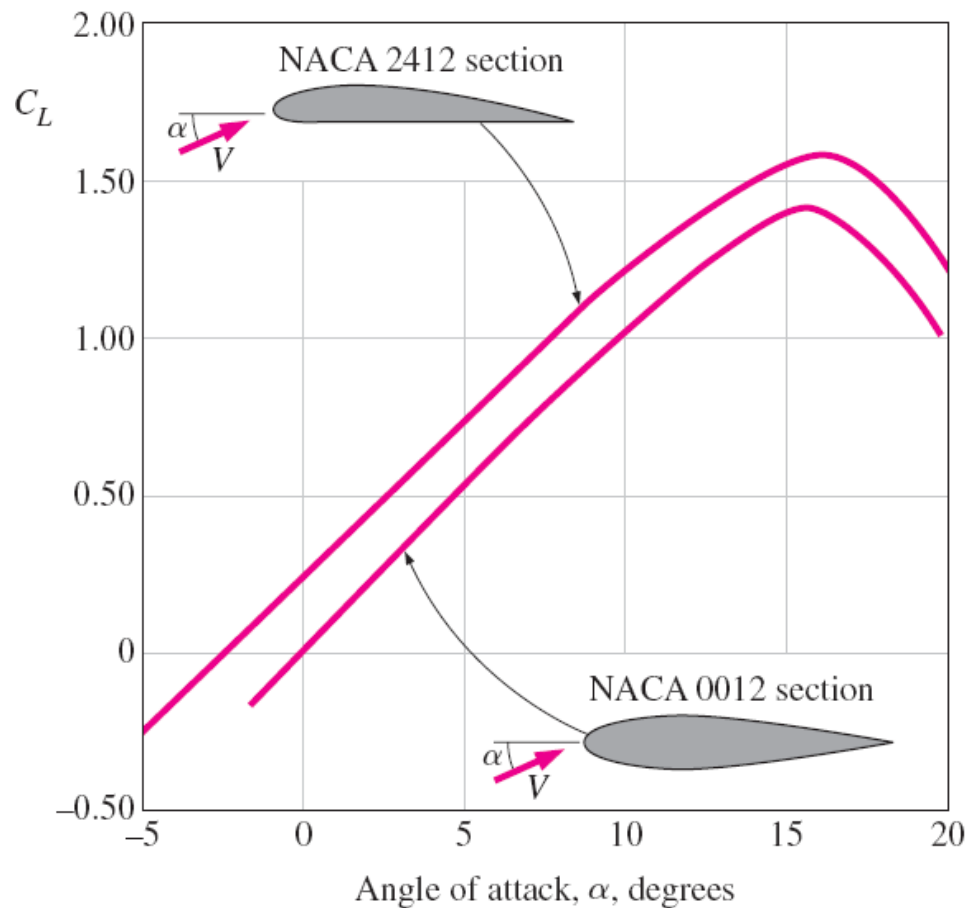
The variation of the lift coefficient with the angle of attack for a symmetrical and a nonsymmetrical airfoil.

C_L increases almost linearly with the angle of attack α, reaches a maximum at about $\alpha=16°$, and then starts to decrease sharply. This decrease of lift with further increase in the angle of attack is called **stall**, and it is caused by flow separation and the formation of a wide wake region over the top surface of the airfoil. Stall is highly undesirable since it also increases drag.

At zero angle of attack ($\alpha = 0°$), the lift coefficient is zero for symmetrical airfoils but nonzero for nonsymmetrical ones with greater curvature at the top surface. Therefore, planes with symmetrical wing sections must fly with their wings at higher angles of attack in order to produce the same lift.

The lift coefficient can be increased severalfold by adjusting the angle of attack (from 0.25 at α =0° for the nonsymmetrical airfoil to 1.25 at α =10°).

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The drag coefficient increases with the angle of attack, often exponentially.

Therefore, large angles of attack should be used sparingly for short periods of time for fuel efficiency.
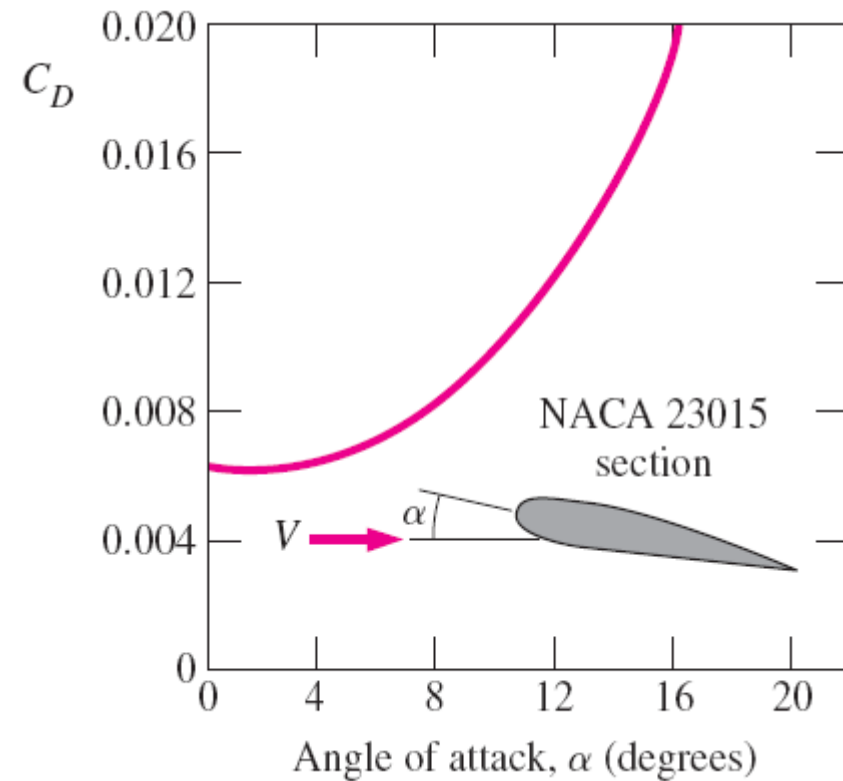
The variation of the drag coefficient of an airfoil with the angle of attack.

For airplane wings and other airfoils of finite span, the end effects at the tips become important because of the fluid leakage between the lower and upper surfaces.

The pressure difference between the lower surface (high pressure region) and the upper surface (low-pressure region) drives the fluid at the tips upward while the fluid is swept toward the back because of the relative motion between the fluid and the wing.

This results in a swirling motion that spirals along the flow, called the tip vortex, at the tips of both wings.

Vortices are also formed along the airfoil between the tips of the wings.

These distributed vortices collect toward the edges after being shed from the trailing edges of the wings and combine with the tip vortices to form two streaks of powerful **trailing vortices** along the tips of the wings
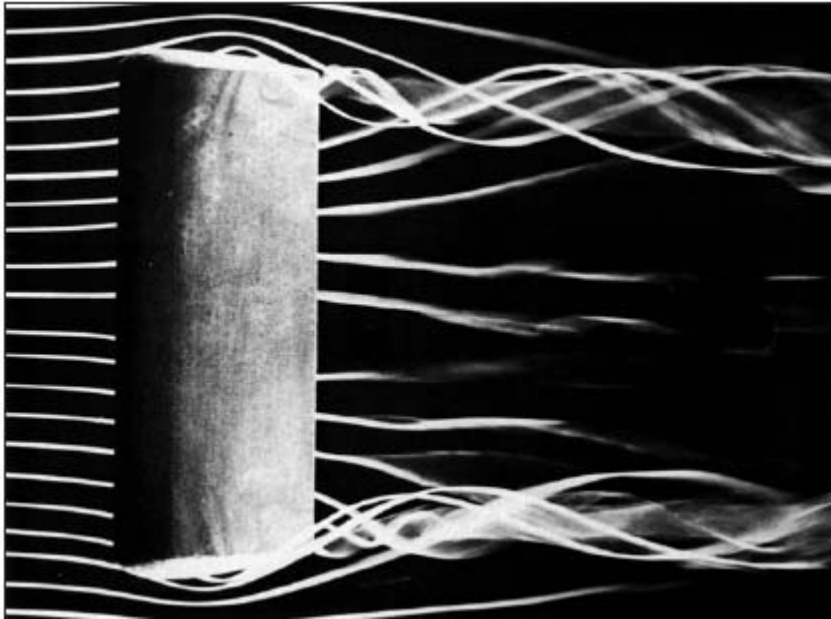
(b)

Trailing vortices visualized in various ways: (*a*) Smoke streaklines in a wind tunnel show vortex cores leaving the trailing edge of a rectangular wing; (*b*) Four contrails initially formed by condensation of water vapor in the low pressure region behind the jet engines eventually merge into the two counter-rotating trailing vortices that persist very far downstream; (*c*) A crop duster flies through smoky air which swirls around in one of the tip vortices from the aircraft's wing.

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(a) A bearded vulture with its wing feathers fanned out during flight.

(b) Winglets are used on this sailplane to reduce induced drag.

Induced drag is reduced by (*a*) wing tip feathers on bird wings and (*b*) endplates or other disruptions on airplane wings.

It has been determined that the birds in a typical flock can fly to their destination in V-formation with one-third less energy (by utilizing the updraft generated by the bird in front).

Military jets also occasionally fly in V-formation for the same reason

Tip vortices that interact with the free stream impose forces on the wing tips in all directions, including the flow direction.

The component of the force in the flow direction adds to drag and is called induced drag.

The total drag of a wing is then the sum of the induced drag (3-D effects) and the drag of the airfoil section (2-D effects).

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Aspect ratio: The ratio of the square of the average span of an airfoil to the planform area. For an airfoil with a rectangular planform of chord *c* and span *b*,

$$AR = \frac{b^2}{A} = \frac{b^2}{bc} = \frac{b}{c}$$

The aspect ratio is a measure of how narrow an airfoil is in the flow direction.

The lift coefficient of wings, in general, increases while the drag coefficient decreases with increasing aspect ratio.

Bodies with large aspect ratios fly more efficiently, but they are less maneuverable because of their larger moment of inertia (owing to the greater distance from the center).

Magnus effect: The phenomenon of producing lift by the rotation of a solid body.

When the ball is not spinning, the lift is zero because of top–bottom symmetry. But when the cylinder is rotated about its axis, the cylinder drags some fluid around because of the no-slip condition and the flow field reflects the superposition of the spinning and nonspinning flows.
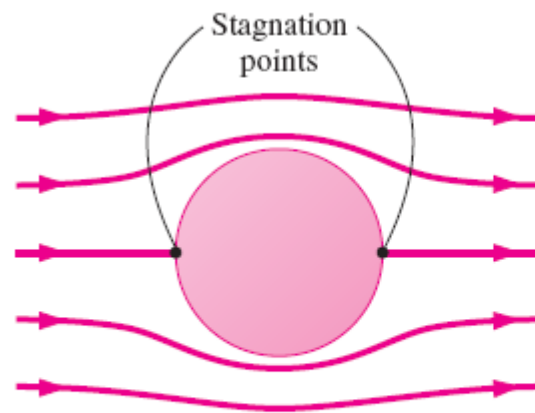
(a) Potential flow over a stationary cylinder   (b) Potential flow over a rotating cylinder
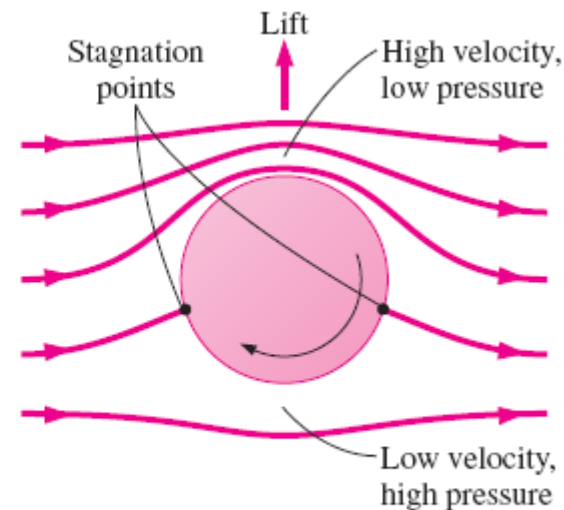
Generation of lift on a rotating circular cylinder for the case of "idealized" potential flow (the actual flow involves flow separation in the wake region).

The variation of lift and drag coefficients of a smooth sphere with the nondimensional rate of rotation for Re = $VD/\nu = 6\times10^4$.
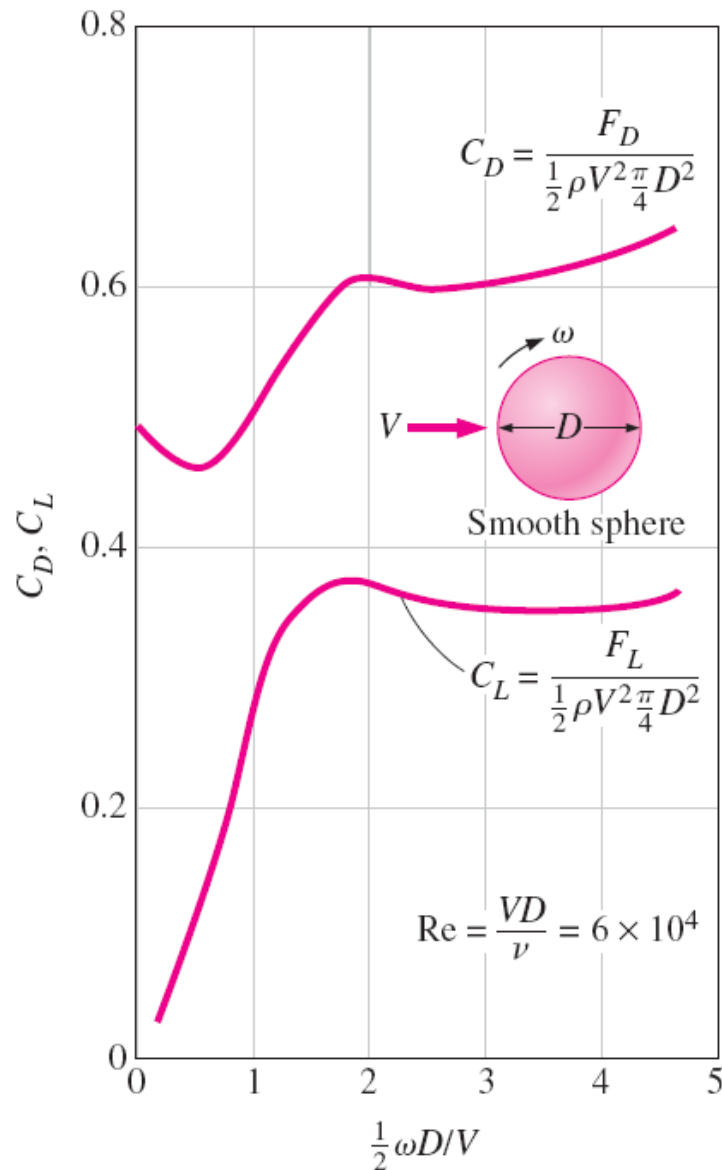
Note that the lift coefficient strongly depends on the rate of rotation, especially at low angular velocities.

The effect of the rate of rotation on the drag coefficient is small. Roughness also affects the drag and lift coefficients.

In a certain range of Reynolds number, roughness produces the desirable effect of increasing the lift coefficient while decreasing the drag coefficient.

Therefore, golf balls with the right amount of roughness travel higher and farther than smooth balls for the same hit.

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 \frac{\pi}{4} D^2}$$

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 \frac{\pi}{4} D^2}$$

$$Re = \frac{VD}{\nu} = 6\times10^4$$

Smooth sphere

EXAMPLE 11-5

A commercial airplane has a total mass of 70,000 kg and a wing planform area of 150 m^2 . The plane has a cruising speed of 558 km/h and a cruising altitude of 12,000 m, where the air density is 0.312 kg/m^3. The plane has double-slotted flaps for use during takeoff and landing, but it cruises with all flaps retracted. Assuming the lift and the drag characteristics of the wings can be approximated by NACA 23012, determine (a) the minimum safe speed for takeoff and landing with and without extending the flaps, (b) the angle of attack to cruise steadily at the cruising altitude, and (c) the power that needs to be supplied to provide enough thrust to overcome wing drag.

558 km/h

70,000 kg

150 m², double-flapped

12,000 m
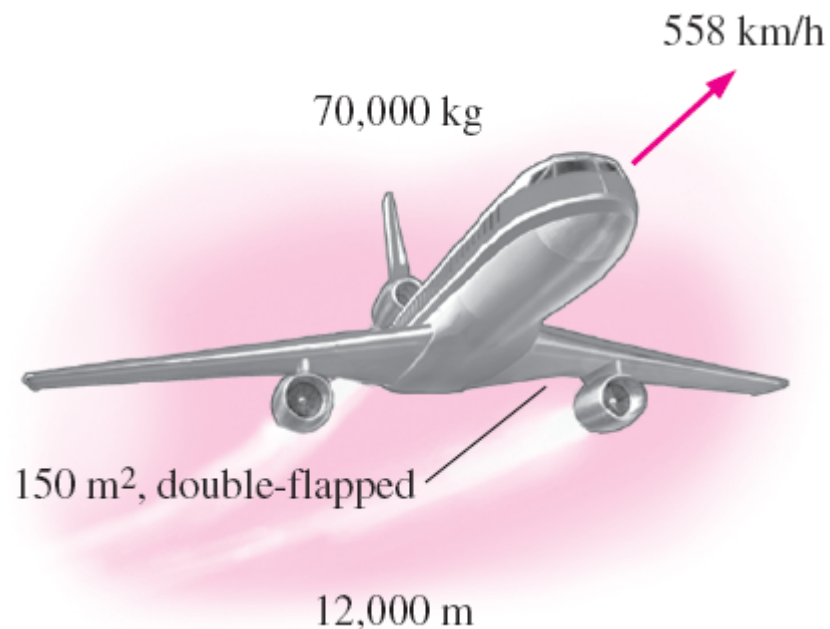
Solution The cruising conditions of a passenger plane and its wing characteristics are given. The minimum safe landing and takeoff speeds, the angle of attack during cruising, and the power required are to be determined.

Assumptions **1** The drag and lift produced by parts of the plane other than the wings, such as the fuselage drag, are not considered. **2** The wings are assumed to be 2-D airfoil sections, and the tip effects of the wings are not considered. **3** The lift and the drag characteristics of the wings can be approximated by NACA 23012. **4** The average density of air on the ground is 1.20 kg/m^3.

$$W = mg = (70{,}000 \text{ kg})(9.81 \text{ m/s}^2)\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = 686{,}700 \text{ N}$$

$$V = (558 \text{ km/h})\left(\frac{1 \text{ m/s}}{3.6 \text{ km/h}}\right) = 155 \text{ m/s}$$

$$V_{\text{min 1}} = \sqrt{\frac{2W}{\rho C_{L,\text{ max 1}}A}} = \sqrt{\frac{2(686{,}700 \text{ N})}{(1.2 \text{ kg/m}^3)(1.52)(150 \text{ m}^2)}}\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 70.9 \text{ m/s}$$

$$V_{\text{min 2}} = \sqrt{\frac{2W}{\rho C_{L,\text{ max 2}}A}} = \sqrt{\frac{2(686{,}700 \text{ N})}{(1.2 \text{ kg/m}^3)(3.48)(150 \text{ m}^2)}}\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 46.8 \text{ m/s}$$

Without flaps: $V_{\text{min 1, safe}} = 1.2V_{\text{min 1}} = 1.2(70.9 \text{ m/s}) = 85.1 \text{ m/s} = \textbf{306 km/h}$

With flaps: $V_{\text{min 2, safe}} = 1.2V_{\text{min 2}} = 1.2(46.8 \text{ m/s}) = 56.2 \text{ m/s} = \textbf{202 km/h}$

since 1 m/s 3.6 km/h. Note that the use of flaps allows the plane to take off and land at considerably lower velocities, and thus on a shorter runway.

(*b*) When an aircraft is cruising steadily at a constant altitude, the lift must be equal to the weight of the aircraft, $F_L = W$. Then the lift coefficient is determined to be

$$C_L = \frac{F_L}{\frac{1}{2}\rho V^2 A} = \frac{686{,}700 \text{ N}}{\frac{1}{2}(0.312 \text{ kg/m}^3)(155 \text{ m/s})^2(150 \text{ m}^2)}\left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}}\right) = 1.22$$

For the case with no flaps, the angle of attack corresponding to this value of C_L is determined from Figure to be $\alpha \cong \mathbf{10°}$

(*c*) When the aircraft is cruising steadily at a constant altitude, the net force acting on the aircraft is zero, and thus thrust provided by the engines must be equal to the drag force. The drag coefficient corresponding to the cruising lift coefficient of 1.22 is determined from Figure to be $C_D \cong 0.03$ for the case with no flaps. Then the drag force acting on the wings becomes
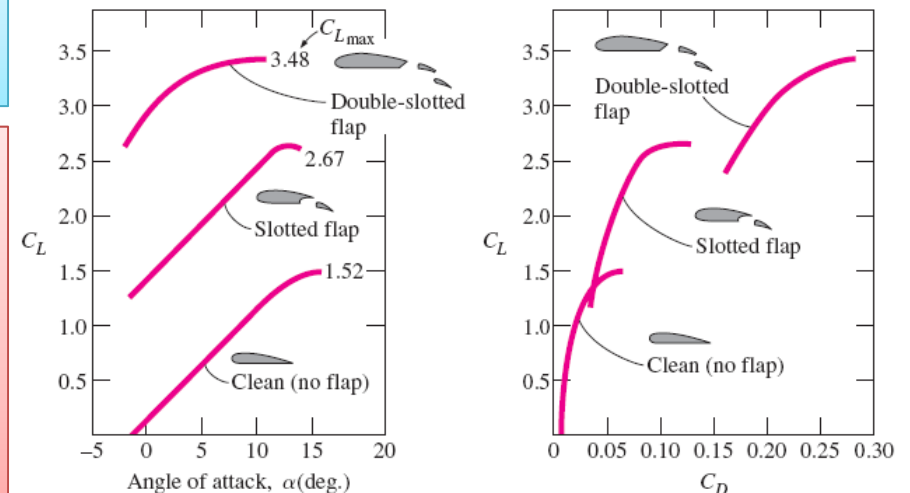
$$F_D = C_D A \frac{\rho V^2}{2} = (0.03)(150 \text{ m}^2)\frac{(0.312 \text{ kg/m}^3)(155 \text{ m/s})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 16.9 \text{ kN}$$

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Noting that power is force times velocity (distance per unit time), the power required to overcome this drag is equal to the thrust times the cruising velocity:

$$\text{Power} = \text{Thrust} \times \text{Velocity} = F_D V = (16.9 \text{ kN})(155 \text{ m/s})\left(\frac{1 \text{ kW}}{1 \text{ kN} \cdot \text{m/s}}\right)$$
$$= \mathbf{2620 \text{ kW}}$$

Therefore, the engines must supply 2620 kW of power to overcome the drag on the wings during cruising. For a propulsion efficiency of 30 percent (i.e., 30% of the energy of the fuel is utilized to propel the aircraft), the plane requires energy input at a rate of 8733 kJ/s.

The power determined is the power to overcome the drag that acts on the wings only and does not include the drag that acts on the remaining parts of the aircraft (the fuselage, the tail, etc.). Therefore, the total power required during cruising will be much greater. Also, it does not consider induced drag, which can be dominant during takeoff when the angle of attack is high.

EXAMPLE 11-6

A tennis ball with a mass of 0.125 lbm and a diameter of 2.52 in is hit at 45 mi/h with a backspin of 4800 rpm. Determine if the ball will fall or rise under the combined effect of gravity and lift due to spinning shortly after being hit in air at 1 atm and 80°F.

4800 rpm

45 mi/h ← Ball
$m = 0.125$ lbm
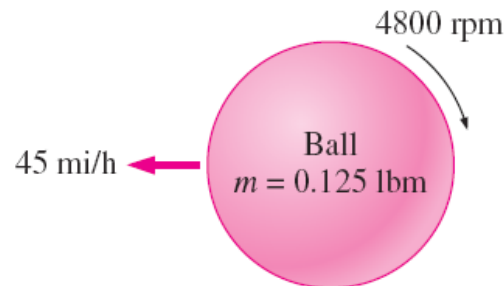
Solution A tennis ball is hit with a backspin. It is to be determined whether the ball will fall or rise after being hit.
Assumptions **1** The surfaces of the ball are smooth enough for Fig. 11–53 to be applicable. **2** The ball is hit horizontally so that it starts its motion horizontally.

$$F_L = C_L A \frac{\rho V^2}{2}$$

$$V = (45 \text{ mi/h})\left(\frac{5280 \text{ ft}}{1 \text{ mi}}\right)\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = 66 \text{ ft/s}$$

$$\omega = (4800 \text{ rev/min})\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = 502 \text{ rad/s}$$

$$\frac{\omega D}{2V} = \frac{(502 \text{ rad/s})(2.52/12 \text{ ft})}{2(66 \text{ ft/s})} = 0.80 \text{ rad}$$

$$F_L = (0.21)\frac{\pi(2.52/12 \text{ ft})^2}{4}\frac{(0.0735 \text{ lbm/ft}^3)(66 \text{ ft/s})^2}{2}\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right)$$

$$= 0.036 \text{ lbf}$$

$$W = mg = (0.125 \text{ lbm})(32.2 \text{ ft/s}^2)\left(\frac{1 \text{ lbf}}{32.2 \text{ lbm} \cdot \text{ft/s}^2}\right) = 0.125 \text{ lbf}$$

which is more than the lift. Therefore, the ball will **drop** under the combined effect of gravity and lift due to spinning with a net force of 0.125 - 0.036 = 0.089 lbf.

This example shows that the ball can be hit much farther by giving it a backspin. Note that a topspin has the opposite effect (negative lift) and speeds up the drop of the ball to the ground. Also, the Reynolds number for this problem is 8×10^4, which is sufficiently close to the 6×10^4 for which Fig. 11–53 is prepared. Also keep in mind that although some spin may increase the distance traveled by a ball, there is an optimal spin that is a function of launch angle, as most golfers are now more aware. Too much spin decreases distance by introducing more induced drag.

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Chapter 11: EXTERNAL FLOW: DRAG AND LIFT

The Wright Brothers are truly the most impressive engineering team of all time. **Self-taught, they were well informed of the contemporary theory and practice in aeronautics.**

They both corresponded with other leaders in the field and published in technical journals.
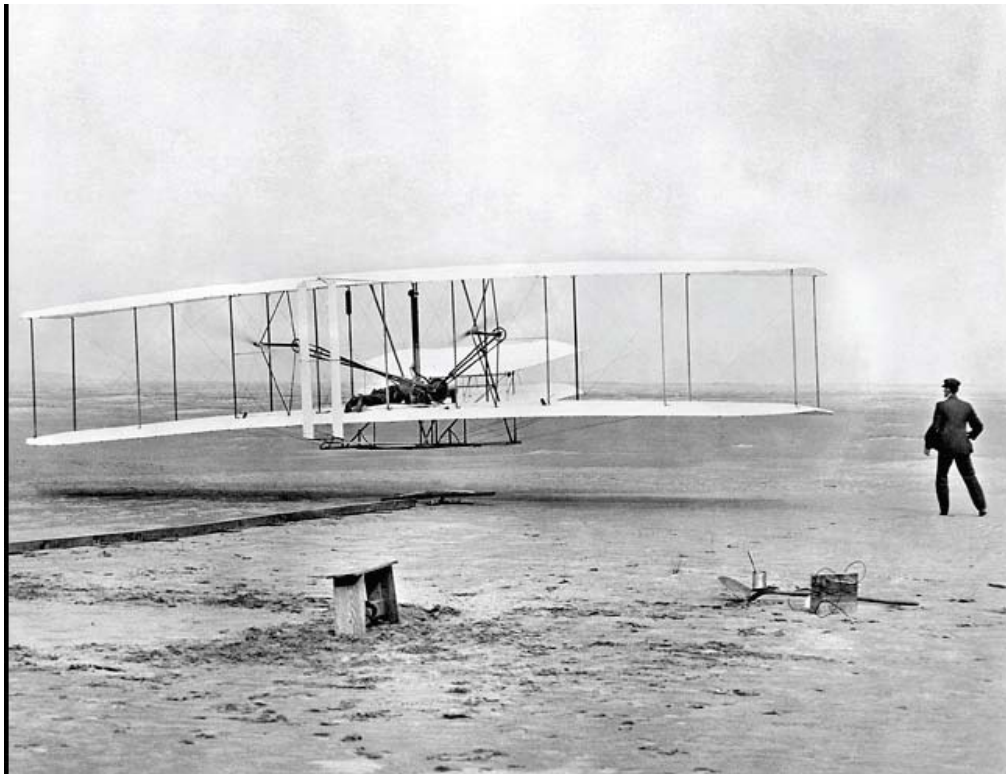
The Wright Brothers take flight at Kitty Hawk.

While they cannot be credited with developing the concepts of lift and drag, they used them to achieve the first powered, manned, heavier-than-air, controlled flight.

They succeeded, while so many before them failed, because they evaluated and designed parts separately.

Before the Wrights, experimenters were building and testing whole airplanes.

- Introduction
- Drag and Lift
- Friction and Pressure Drag
  - Reducing Drag by Streamlining
  - Flow Separation
- Drag Coefficients of Common Geometries
  - Biological Systems and Drag
  - Drag Coefficients of Vehicles
  - Superposition
- Parallel Flow Over Flat Plates
  - Friction Coefficient
- Flow Over Cylinders and Spheres
  - Effect of Surface Roughness
- Lift
  - Finite-Span Wings and Induced Drag
  - Lift Generated by Spinning