Fluid Mechanics: Fundamentals and Applications, 2nd Edition
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Chapter 2
PROPERTIES OF FLUIDS

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PROPERTIES OF FLUIDS

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Capillary Effect

A drop forms when liquid is forced out of a small tube. The shape of the drop is determined by a balance of pressure, gravity, and surface tension forces.

Objectives

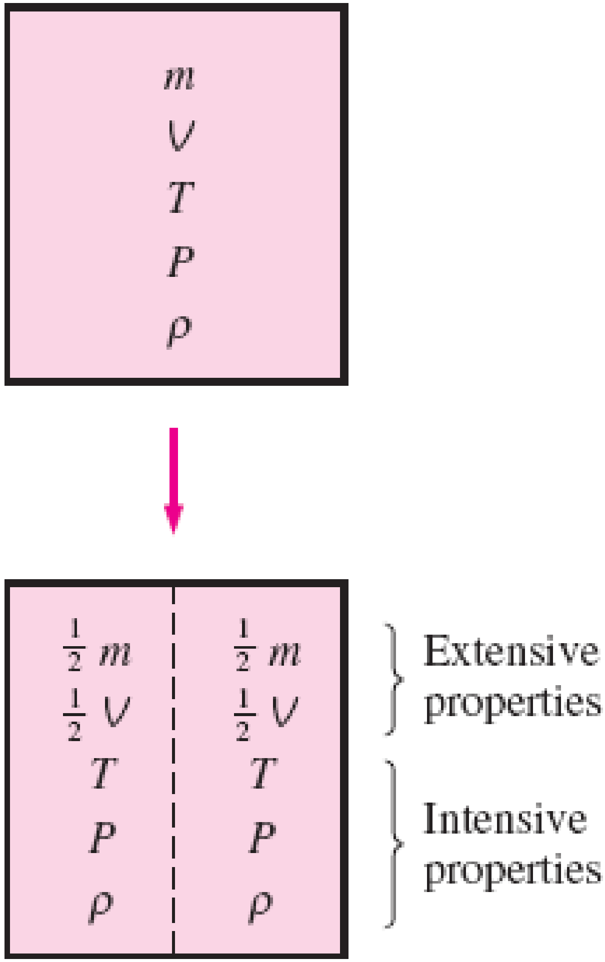
- Have a working knowledge of the basic properties of fluids and understand the continuum approximation.
- Have a working knowledge of viscosity and the consequences of the frictional effects it causes in fluid flow.
- Calculate the capillary rise (or drop) in tubes due to the surface tension effect.

2–1 INTRODUCTION

- **Property:** Any characteristic of a system.
- Some familiar properties are pressure *P*, temperature *T*, volume *V*, and mass *m*.
- Properties are considered to be either *intensive* or *extensive*.
- **Intensive properties:** Those that are independent of the mass of a system, such as temperature, pressure, and density.
- **Extensive properties:** Those whose values depend on the size -or extent - of the system.
- **Specific properties:** Extensive properties per unit mass.

specific volume (v=V/m)

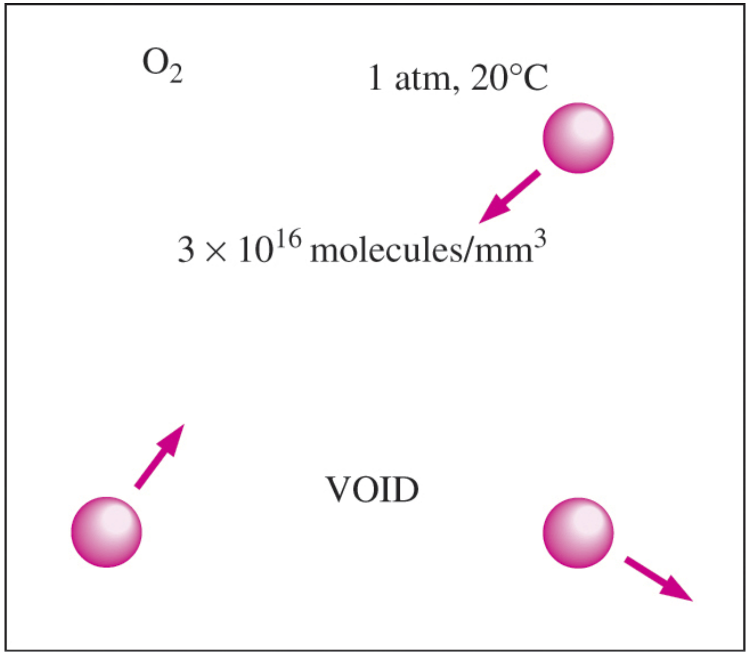
specific energy (e=E/m)



Criterion to differentiate intensive and extensive properties.

Continuum

- Matter is made up of atoms that are widely spaced in the gas phase. Yet it is very convenient to disregard the atomic nature of a substance and view it as a continuous, homogeneous matter with no holes, that is, a **continuum**.
- The continuum idealization allows us to treat properties as point functions and to assume the properties vary continually in space with no jump discontinuities.
- This idealization is valid as long as the size of the system we deal with is large relative to the space between the molecules.
- This is the case in practically all problems.
- In this text we will limit our consideration to substances that can be modeled as a continuum.



Despite the relatively large gaps between molecules, a substance can be treated as a continuum because of the very large number of molecules even in an extremely small volume.

- The diameter of the oxygen molecule is about 3×10^{-10} m and its mass is 5.3×10^{-26} kg. Also, the *mean free path* of oxygen at 1 atm pressure and 20°C is 6.3×10^{-8} m.

- That is, an oxygen molecule travels, on average, a distance of 6.3×10^{-8} m (about 200 times its diameter) before it collides with another molecule.

- Also, there are about 2.5×10^{-16} molecules of oxygen in the tiny volume of 1 mm^3 at 1 atm pressure and 20°C. The continuum model is applicable as long as the characteristic length of the system (such as its diameter) is much larger than the mean free path of the molecules.

- At very high vacuums or very high elevations, the mean free path may become large (for example, it is about 0.1 m for atmospheric air at an elevation of 100 km).

[redacted]

The length scale associated with most flows, such as seagulls in flight, is orders of magnitude larger than the mean free path of the air molecules. Therefore, here, and for all fluid flows considered in this book, the continuum idealization is appropriate.

Density

Density is mass per unit volume; specific volume is volume per unit mass.

$$\rho = \frac{m}{V} \qquad (\mathrm{kg/m^3})$$

$$V = 12\ \mathrm{m^3}$$
$$m = 3\ \mathrm{kg}$$
$$\Downarrow$$
$$\rho = 0.25\ \mathrm{kg/m^3}$$
$$v = \frac{1}{\rho} = 4\ \mathrm{m^3/kg}$$

Specific volume

$$v = \frac{V}{m} = \frac{1}{\rho}$$

Specific weight: The weight of a unit volume of a substance.

$$\gamma_s = \rho g \qquad (\mathrm{N/m^3})$$

Specific gravity: The ratio of the density of a substance to the density of some standard substance at a specified temperature (usually water at 4°C).

$$\mathrm{SG} = \frac{\rho}{\rho_{H_2O}}$$

Specific gravities of some substances at 0°C

Substance	SG
Water	1.0
Blood	1.05
Seawater	1.025
Gasoline	0.7
Ethyl alcohol	0.79
Mercury	13.6
Wood	0.3–0.9
Gold	19.2
Bones	1.7–2.0
Ice	0.92
Air (at 1 atm)	0.0013

Density of Ideal Gases

Property tables provide very accurate and precise information about the properties, but sometimes it is convenient to have some simple relations among the properties that are sufficiently general and accurate.

Equation of state: Any equation that relates the pressure, temperature, and density (or specific volume) of a substance.

Ideal-gas equation of state: The simplest and best-known equation of state for substances in the gas phase.

$$Pv = RT \quad \text{or} \quad P = \rho RT \qquad \text{where} \qquad R = R_u / M$$

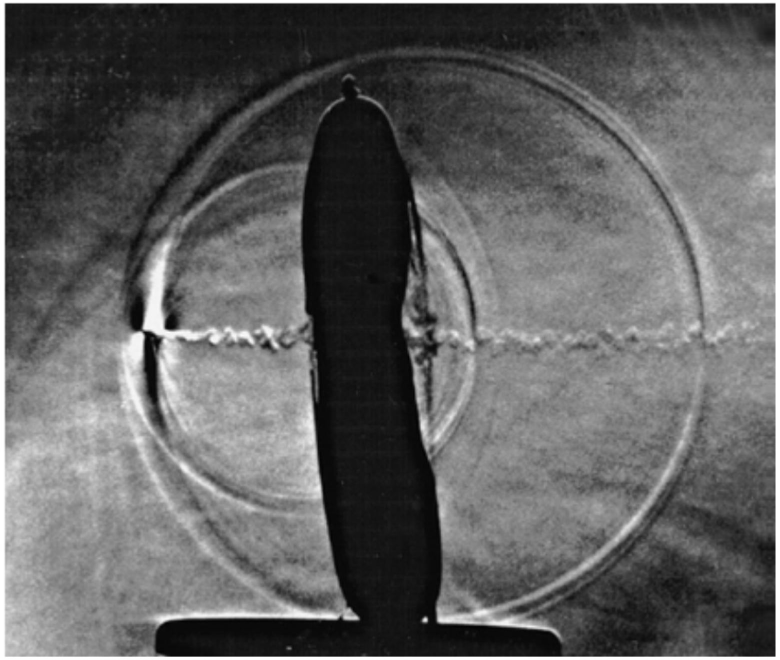
P is the absolute pressure, *v* is the specific volume, *T* is the thermodynamic (absolute) temperature, ρ is the density, and *R* is the gas constant.

The thermodynamic temperature scale in the SI is the Kelvin scale.
In the English system, it is the Rankine scale.

$$T(\text{K}) = T(^\circ\text{C}) + 273.15$$

$$T(\text{R}) = T(^\circ\text{F}) + 459.67$$

[redacted]



Air behaves as an ideal gas, even at very high speeds. In this schlieren image, a bullet traveling at about the speed of sound bursts through both sides of a balloon, forming two expanding shock waves. The turbulent wake of the bullet is also visible.

An ideal gas is a hypothetical substance that obeys the relation $Pv = RT$.

The ideal-gas relation closely approximates the *P-v-T* behavior of real gases at low densities.

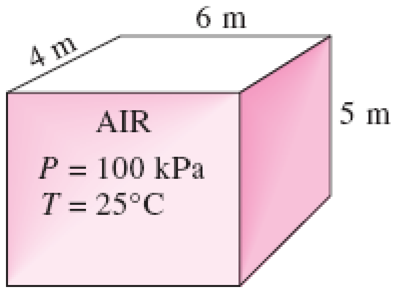
At low pressures and high temperatures, the density of a gas decreases and the gas behaves like an ideal gas.

In the range of practical interest, many familiar gases such as air, nitrogen, oxygen, hydrogen, helium, argon, neon, and krypton and even heavier gases such as carbon dioxide can be treated as ideal gases with negligible error.

Dense gases such as water vapor in steam power plants and refrigerant vapor in refrigerators, however, should not be treated as ideal gases since they usually exist at a state near saturation.

EXAMPLE 2-1

Determine the density, specific gravity, and mass of the air in a room whose dimensions are 4x5x6m at 100 kPa and 25°C.



Solution: The density, specific gravity, and mass of the air in a room are to be determined.

Assumptions: At specified conditions, air can be treated as an ideal gas.

Properties: The gas constant of air is R= 0.287 kPa.m^3/kg.K.

$$\rho = \frac{P}{RT} = \frac{100\ \text{kPa}}{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(25+273)\ \text{K}} = \mathbf{1.17\ kg/m^3}$$

$$\text{SG} = \frac{\rho}{\rho_{H_2O}} = \frac{1.17\ \text{kg/m}^3}{1000\ \text{kg/m}^3} = \mathbf{0.00117}$$

$$V = (4\ \text{m})(5\ \text{m})(6\ \text{m}) = 120\ \text{m}^3$$

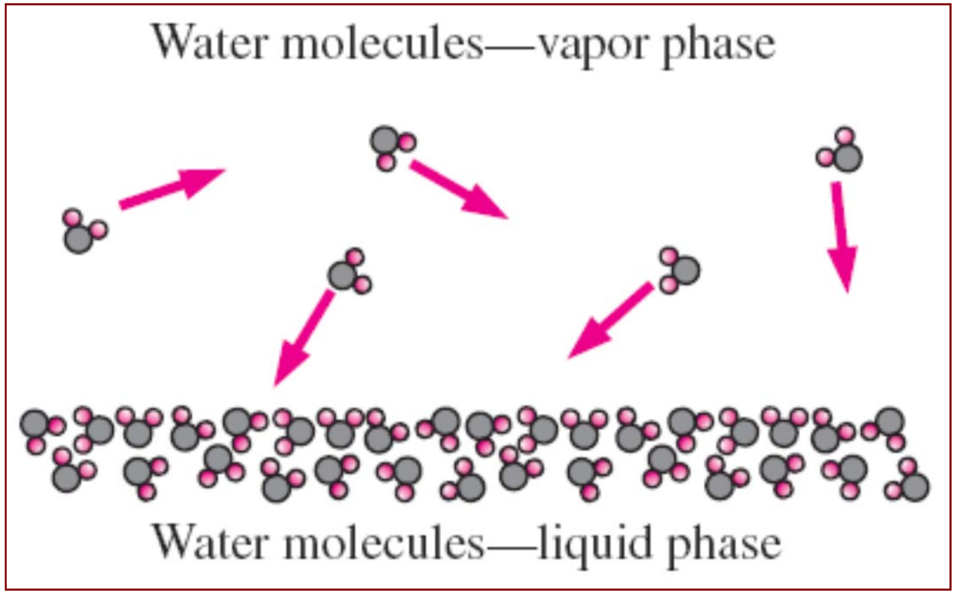
$$m = \rho V = (1.17\ \text{kg/m}^3)(120\ \text{m}^3) = \mathbf{140\ kg}$$

Discussion Note that we converted the temperature to the unit K from °C before using it in the ideal-gas relation.

2–3 VAPOR PRESSURE AND CAVITATION

- Saturation temperature T_{sat}: The temperature at which a pure substance changes phase at a given pressure.
- Saturation pressure P_{sat}: The pressure at which a pure substance changes phase at a given temperature.
- Vapor pressure (P_v): The pressure exerted by its vapor in phase equilibrium with its liquid at a given temperature. It is identical to the saturation pressure P_{sat} of the liquid ($P_v = P_{sat}$).
- Partial pressure: The pressure of a gas or vapor in a mixture with other gases. For example, atmospheric air is a mixture of dry air and water vapor, and atmospheric pressure is the sum of the partial pressure of dry air and the partial pressure of water vapor.

The partial pressure of a vapor must be less than or equal to the vapor pressure if there is no liquid present. However, when both vapor and liquid are present and the system is in phase equilibrium, the partial pressure of the vapor must equal the vapor pressure, and the system is said to be saturated.



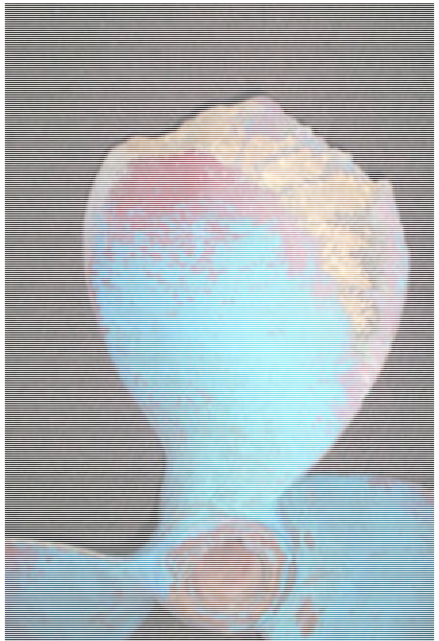
The vapor pressure (saturation pressure) of a pure substance (e.g., water) is the pressure exerted by its vapor molecules when the system is in phase equilibrium with its liquid molecules at a given temperature.

Saturation (or vapor) pressure of water at various temperatures

Temperature T, °C	Saturation Pressure P_{sat}, kPa
−10	0.260
−5	0.403
0	0.611
5	0.872
10	1.23
15	1.71
20	2.34
25	3.17
30	4.25
40	7.38
50	12.35
100	101.3 (1 atm)
150	475.8
200	1554
250	3973
300	8581

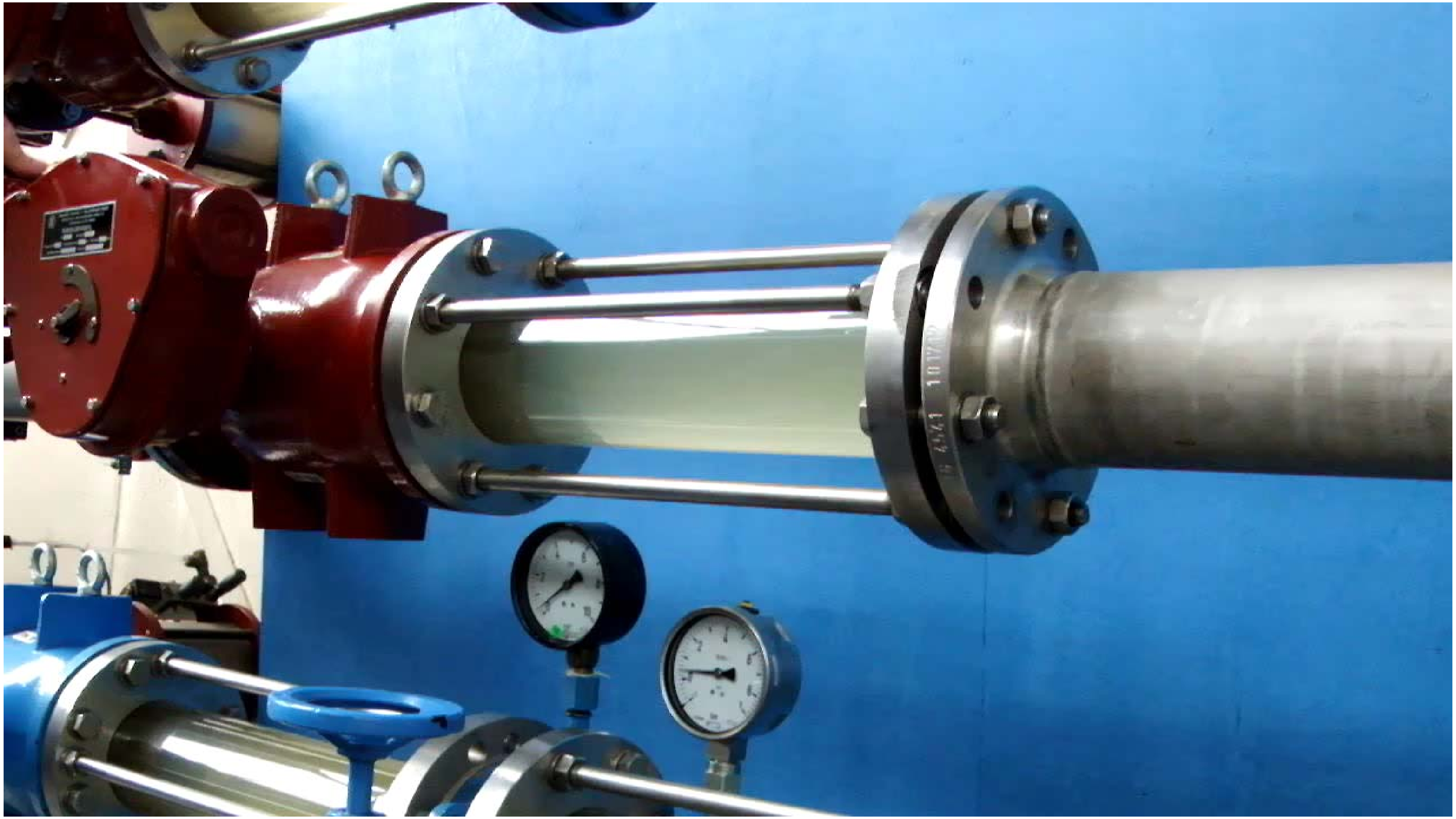
[redacted]

- There is a possibility of the liquid pressure in liquid-flow systems dropping below the vapor pressure at some locations, and the resulting unplanned vaporization.
- The vapor bubbles (called cavitation bubbles since they form “cavities” in the liquid) collapse as they are swept away from the low-pressure regions, generating highly destructive, extremely high-pressure waves.
- This phenomenon, which is a common cause for drop in performance and even the erosion of impeller blades, is called cavitation, and it is an important consideration in the design of hydraulic turbines and pumps.



Cavitation damage on a 16-mm by 23-mm aluminum sample tested at 60 m/s for 2.5 h. The sample was located at the cavity collapse region downstream of a cavity generator specifically designed to produce high damage potential.

Cavitation



EXAMPLE 2-2

In a water distribution system, the temperature of water is observed to be as high as 30°C. Determine the minimum pressure allowed in the system to avoid cavitation.

Solution: The minimum pressure in a water distribution system to avoid cavitation is to be determined.
Properties: The vapor pressure of water at 30°C is 4.25 kPa.
Analysis: To avoid cavitation, the pressure anywhere in the flow should not be allowed to drop below the vapor (or saturation) pressure at the given temperature.

$$P_{\min} = P_{\text{sat@30°C}} = \mathbf{4.25\ kPa}$$

Discussion: Note that the vapor pressure increases with increasing temperature, and thus the risk of cavitation is greater at higher fluid temperatures.

2–4 ENERGY AND SPECIFIC HEATS

- Energy can exist in numerous forms such as thermal, mechanical, kinetic, potential, electric, magnetic, chemical, and nuclear, and their sum constitutes the **total energy, *E*** of a system.
- Thermodynamics deals only with the ***change*** of the total energy.
- **Macroscopic forms of energy**: Those a system possesses as a whole with respect to some outside reference frame, such as kinetic and potential energies.
- **Microscopic forms of energy**: Those related to the molecular structure of a system and the degree of the molecular activity.
- **Internal energy, *U***: The sum of all the microscopic forms of energy.

Kinetic energy, KE: The energy that a system possesses as a result of its motion relative to some reference frame.

Potential energy, PE: The energy that a system possesses as a result of its elevation in a gravitational field.

The macroscopic energy of an object changes with velocity and elevation.

[redacted]

Enthalpy

$$h = u + Pv = u + \frac{P}{\rho}$$

$$du = c_v\, dT \qquad \text{and} \qquad dh = c_p\, dT$$

Energy of a flowing fluid

$$e_{\text{flowing}} = P/\rho + e = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz$$

$$\Delta u = c_{v,\text{ave}}\, \Delta T \qquad \text{and} \qquad \Delta h = c_{p,\text{ave}}\, \Delta T$$

For *incompressible substances*, the constant-volume and pressure specific heats are identical. Therefore, c_p *and* c_v for liquids, and the change in the internal energy of liquids can be expressed as $\Delta u = c_{ave}\Delta T$

P/ρ is the *flow energy*, also called the *flow work*, which is the energy per unit mass needed to move the fluid and maintain flow.

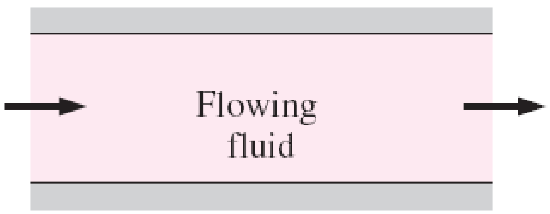
$$\Delta h = \Delta u + \Delta P/\rho \cong c_{\text{ave}}\, \Delta T + \Delta P/\rho$$

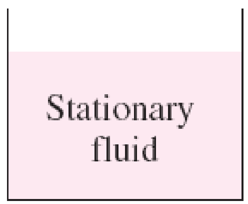
$$\Delta h = \Delta u \cong c_{\text{ave}}\, \Delta T$$

for a P=cnst. proc.

$$\Delta h = \Delta P/\rho$$

For a *T* = const. process

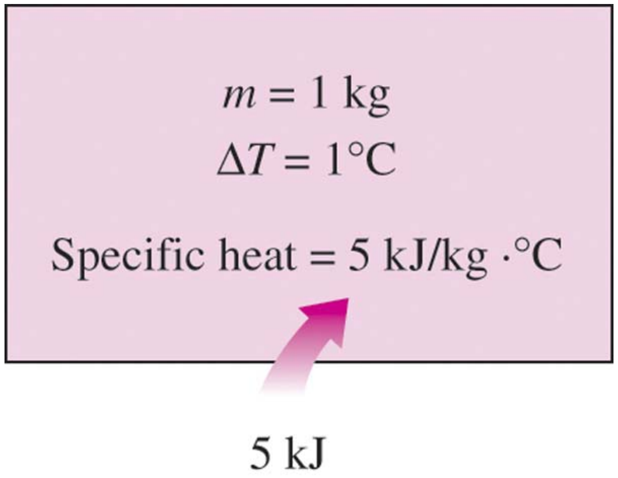




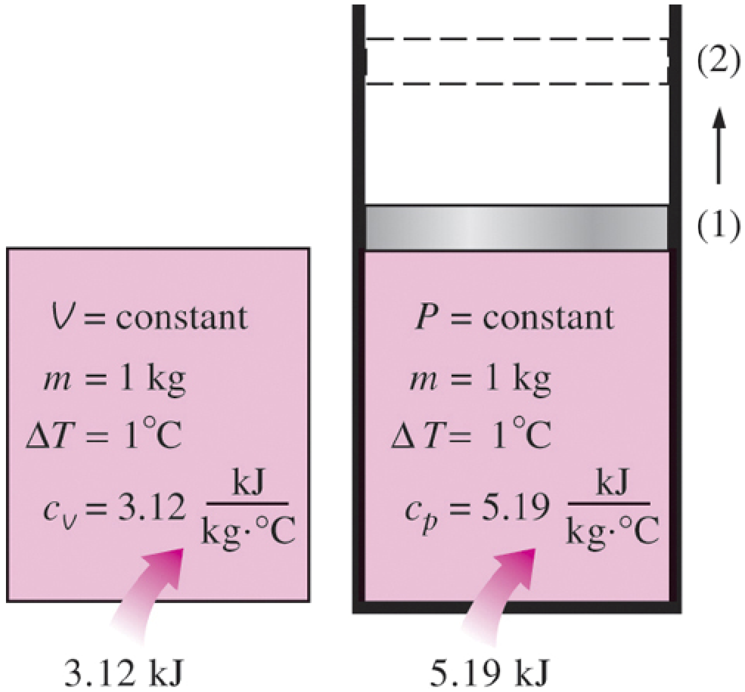
The ***internal energy u*** represents the microscopic energy of a nonflowing fluid per unit mass, whereas *enthalpy h* represents the microscopic energy of a flowing fluid per unit mass.

Specific heat at constant volume, c_v: The energy required to raise the temperature of the unit mass of a substance by one degree as the volume is maintained constant.

Specific heat at constant pressure, c_p: The energy required to raise the temperature of the unit mass of a substance by one degree as the pressure is maintained constant.



Specific heat is the energy required to raise the temperature of a unit mass of a substance by one degree in a specified way.



Constant-volume and constant-pressure specific heats c_v and c_p (values are for helium gas).

2–5 COMPRESSIBILITY AND SPEED OF SOUND

Coefficient of Compressibility

We know from experience that the volume (or density) of a fluid changes with a change in its temperature or pressure.

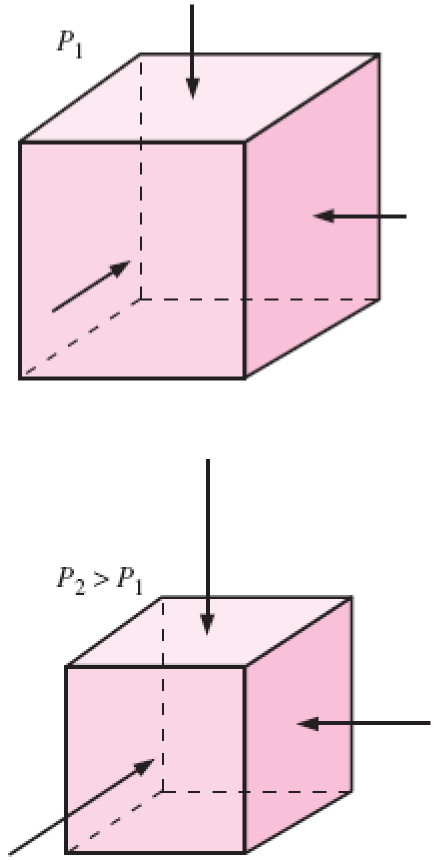
Fluids usually expand as they are heated or depressurized and contract as they are cooled or pressurized.

But the amount of volume change is different for different fluids, and we need to define properties that relate volume changes to the changes in pressure and temperature.

Two such properties are:

the bulk modulus of elasticity κ

the coefficient of volume expansion β.



Fluids, like solids, compress when the applied pressure is increased from P_1 to P_2.

[redacted]

$$\kappa = -v\left(\frac{\partial P}{\partial v}\right)_T = \rho\left(\frac{\partial P}{\partial \rho}\right)_T \qquad \text{(Pa)}$$

Coefficient of compressibility (also called the bulk modulus of compressibility or bulk modulus of elasticity) for fluids

$$\kappa \cong -\frac{\Delta P}{\Delta v/v} \cong \frac{\Delta P}{\Delta \rho/\rho} \qquad (T = \text{constant})$$

The coefficient of compressibility represents the change in pressure corresponding to a fractional change in volume or density of the fluid while the temperature remains constant.

What is the coefficient of compressibility of a truly incompressible substance (*v* = constant)?

A large value of κ indicates that a large change in pressure is needed to cause a small fractional change in volume, and thus **a fluid with a large κ is essentially incompressible.**

This is typical for liquids, and explains why liquids are usually considered to be ***incompressible***.

[redacted]

For an ideal gas, $P = \rho RT$ and $(\partial P/\partial \rho)_T = RT = P/\rho$, and thus

$$\kappa_{\text{ideal gas}} = P \qquad \text{(Pa)}$$

The coefficient of compressibility of an ideal gas is equal to its absolute pressure, and the coefficient of compressibility of the gas increases with increasing pressure.

$$\textit{Ideal gas:} \qquad \frac{\Delta \rho}{\rho} = \frac{\Delta P}{P} \qquad (T = \text{constant})$$

The percent increase of density of an ideal gas during isothermal compression is equal to the percent increase in pressure.

Isothermal compressibility: The inverse of the coefficient of compressibility.

The isothermal compressibility of a fluid represents the fractional change in volume or density corresponding to a unit change in pressure.

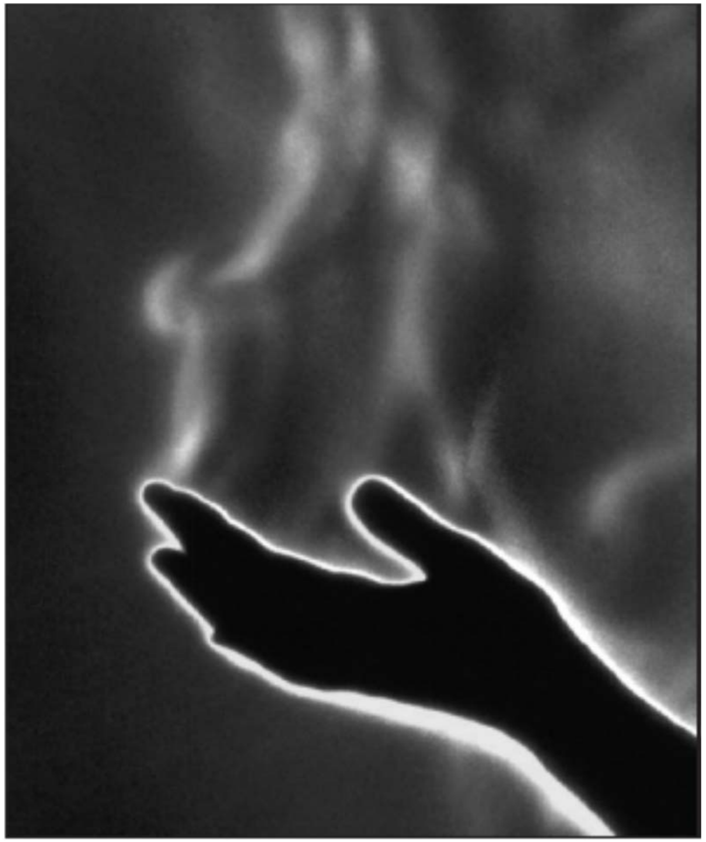
$$\alpha = \frac{1}{\kappa} = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_T = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\right)_T \qquad \text{(1/Pa)}$$

Coefficient of Volume Expansion

The density of a fluid depends more strongly on temperature than it does on pressure.

The variation of density with temperature is responsible for numerous natural phenomena such as winds, currents in oceans, rise of plumes in chimneys, the operation of hot-air balloons, heat transfer by natural convection, and even the rise of hot air and thus the phrase "heat rises".

To quantify these effects, we need a property that represents the ***variation of the density of a fluid with temperature at constant pressure*.**



Natural convection over a woman's hand.

[redacted]

The coefficient of volume expansion (or *volume expansivity*): The variation of the density of a fluid with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\mathrm{K})$$

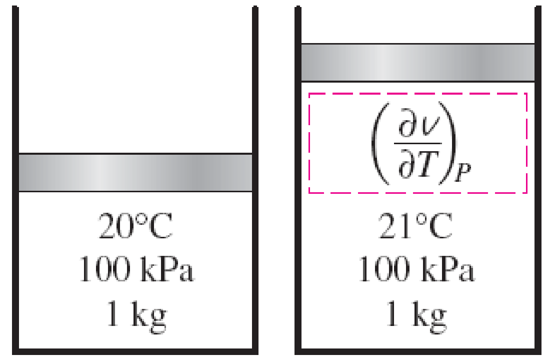
It can also be expressed approximately in terms of finite changes as

$$\beta \approx \frac{\Delta v/v}{\Delta T} = -\frac{\Delta \rho/\rho}{\Delta T} \qquad (\text{at constant } P)$$

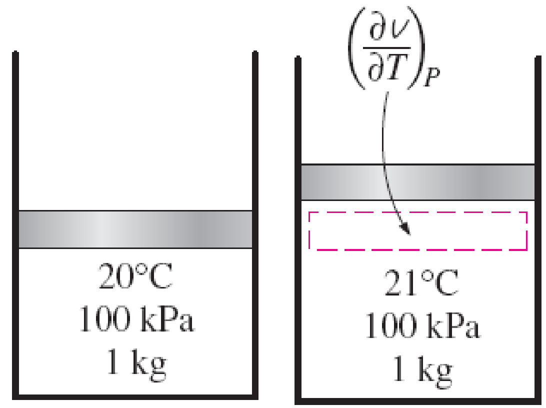
A large value of β for a fluid means a large change in density with temperature, and the product $\beta \Delta T$ represents the fraction of volume change of a fluid that corresponds to a temperature change of T at constant pressure.

The volume expansion coefficient of an *ideal gas* ($P = \rho RT$) at a absolute temperature T is equivalent to the inverse of the temperature:

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\mathrm{K})$$



(*a*) A substance with a large β



(*b*) A substance with a small β

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

[redacted]

In the study of natural convection currents, the condition of the main fluid body that surrounds the finite hot or cold regions is indicated by the subscript “infinity” to serve as a reminder that this is the value at a distance where the presence of the hot or cold region is not felt. In such cases, the volume expansion coefficient can be expressed approximately as

$$\beta \approx -\frac{(\rho_\infty - \rho)/\rho}{T_\infty - T} \qquad \text{or} \qquad \rho_\infty - \rho = \rho\beta(T - T_\infty)$$

The combined effects of pressure and temperature changes on the volume change of a fluid can be determined by taking the specific volume to be a function of *T* and *P*.

$$d\mathrm{v} = \left(\frac{\partial \mathrm{v}}{\partial T}\right)_P dT + \left(\frac{\partial \mathrm{v}}{\partial P}\right)_T dP = (\beta\, dT - \alpha\, dP)\mathrm{v}$$

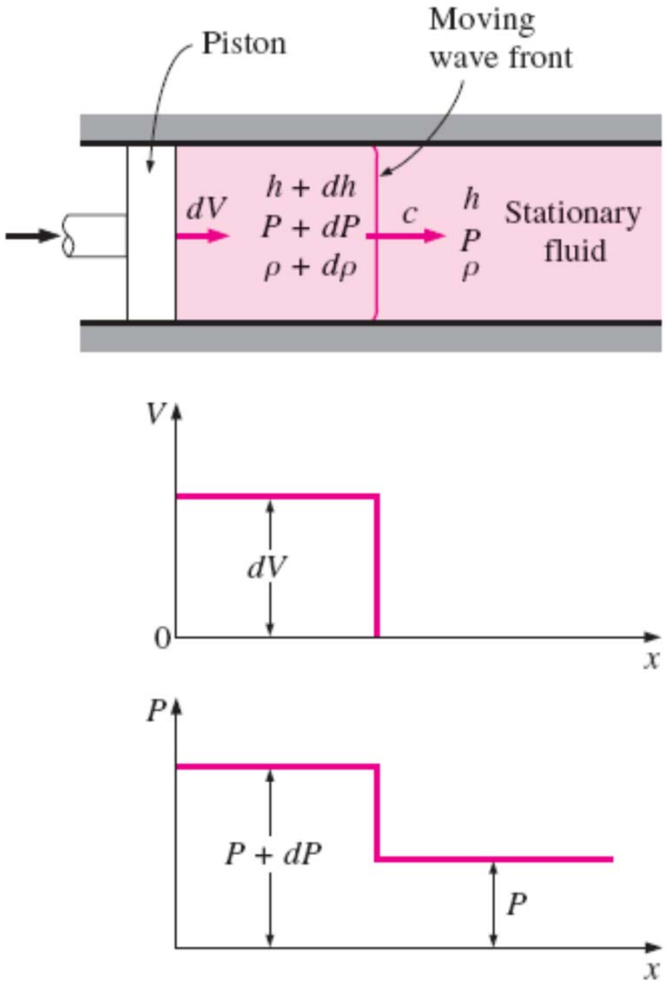
The fractional change in volume (or density) due to changes in pressure and temperature can be expressed approximately as

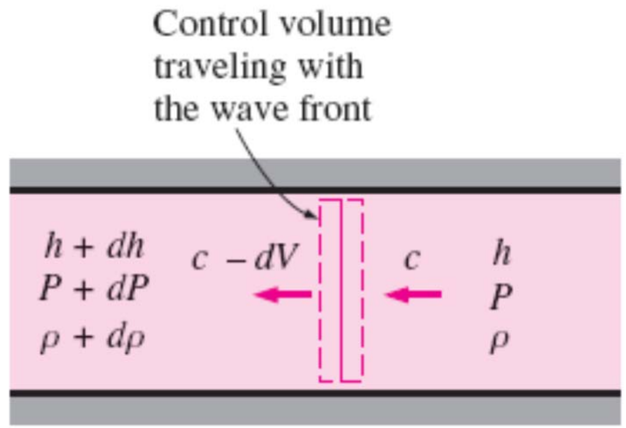
$$\frac{\Delta \mathrm{v}}{\mathrm{v}} = -\frac{\Delta \rho}{\rho} \cong \beta\, \Delta T - \alpha\, \Delta P$$

Speed of Sound and Mach Number

Speed of sound (sonic speed): The speed at which an infinitesimally small pressure wave travels through a medium.

Propagation of a small pressure wave along a duct.





Control volume moving with the small pressure wave along a duct.

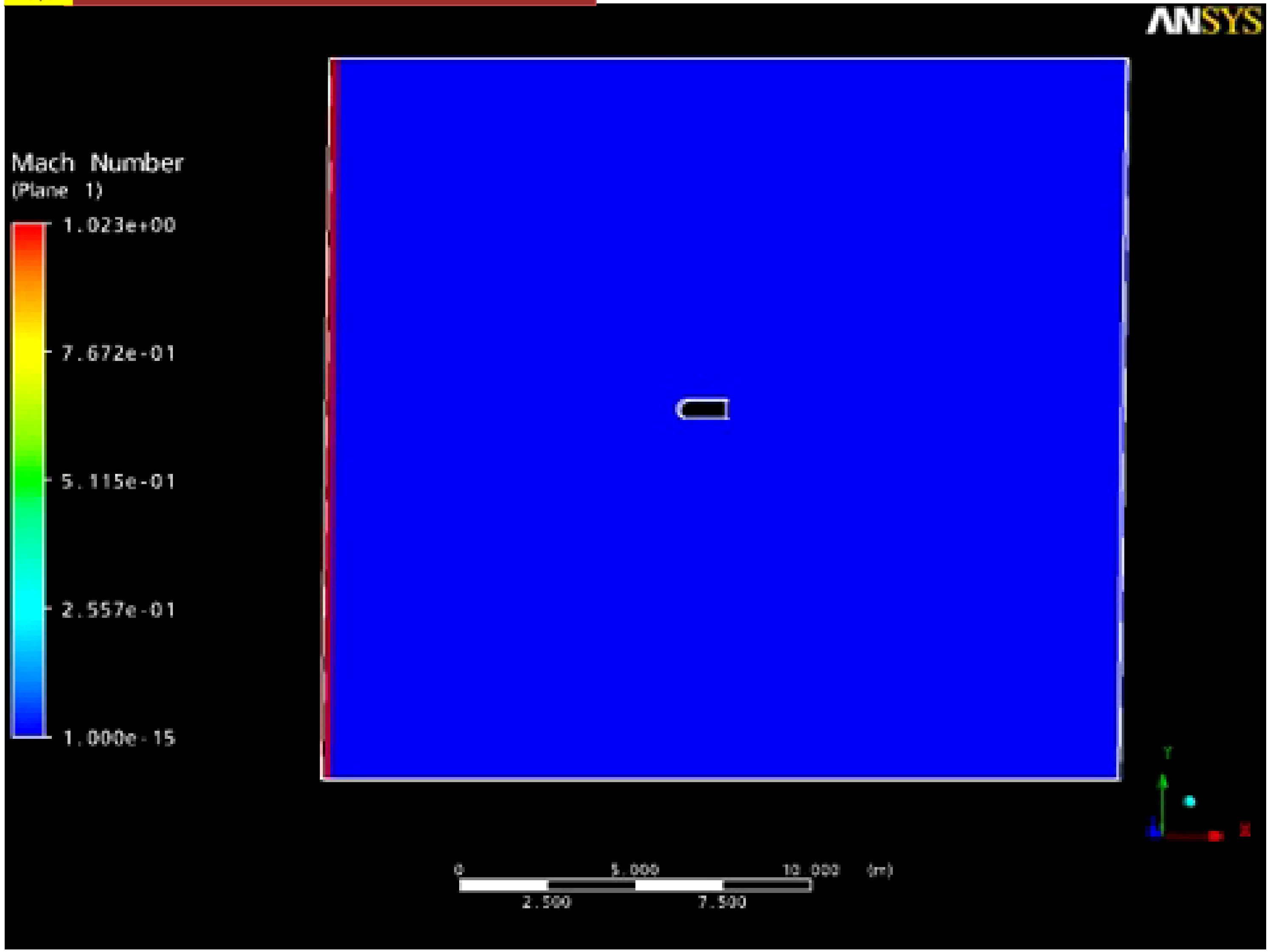
$$c^2 = k\left(\frac{\partial P}{\partial \rho}\right)_T$$

For any fluid

$$c = \sqrt{kRT}$$

For an ideal gas

Mach Number Bullet



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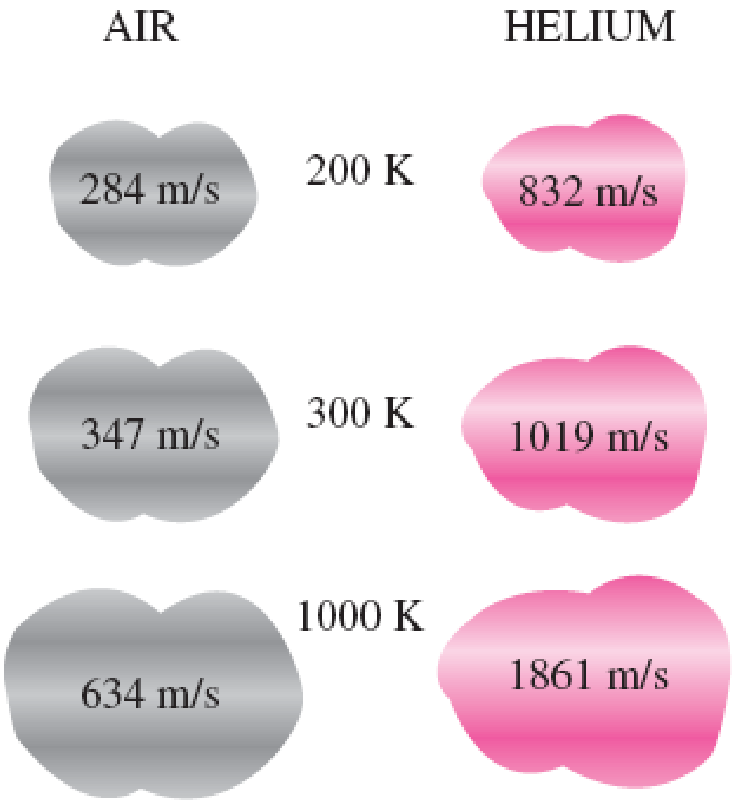


The speed of sound changes with temperature and varies with the fluid.

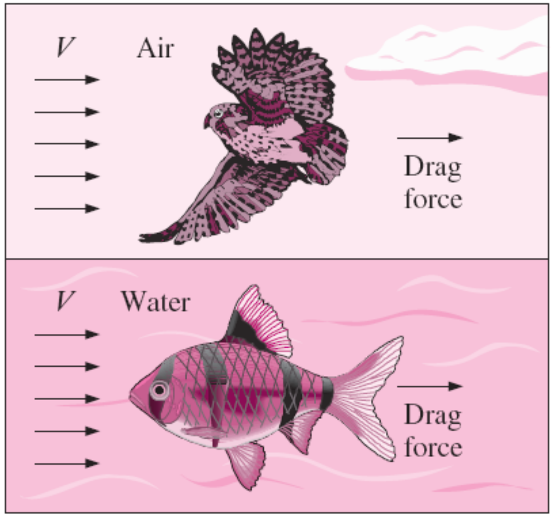
Mach number Ma: The ratio of the actual speed of the fluid (or an object in still fluid) to the speed of sound in the same fluid at the same state.

$$\text{Ma} = \frac{V}{c}$$

The Mach number depends on the speed of sound, which depends on the state of the fluid.

The Mach number can be different at different temperatures even if the flight speed is the same.

2–6 VISCOSITY



A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.

When two solid bodies in contact move relative to each other, a friction force develops at the contact surface in the direction opposite to motion.
To move a table on the floor, for example, we have to apply a force to the table in the horizontal direction large enough to overcome the friction force.
The magnitude of the force needed to move the table depends on the ***friction coefficient*** between the table and the floor.

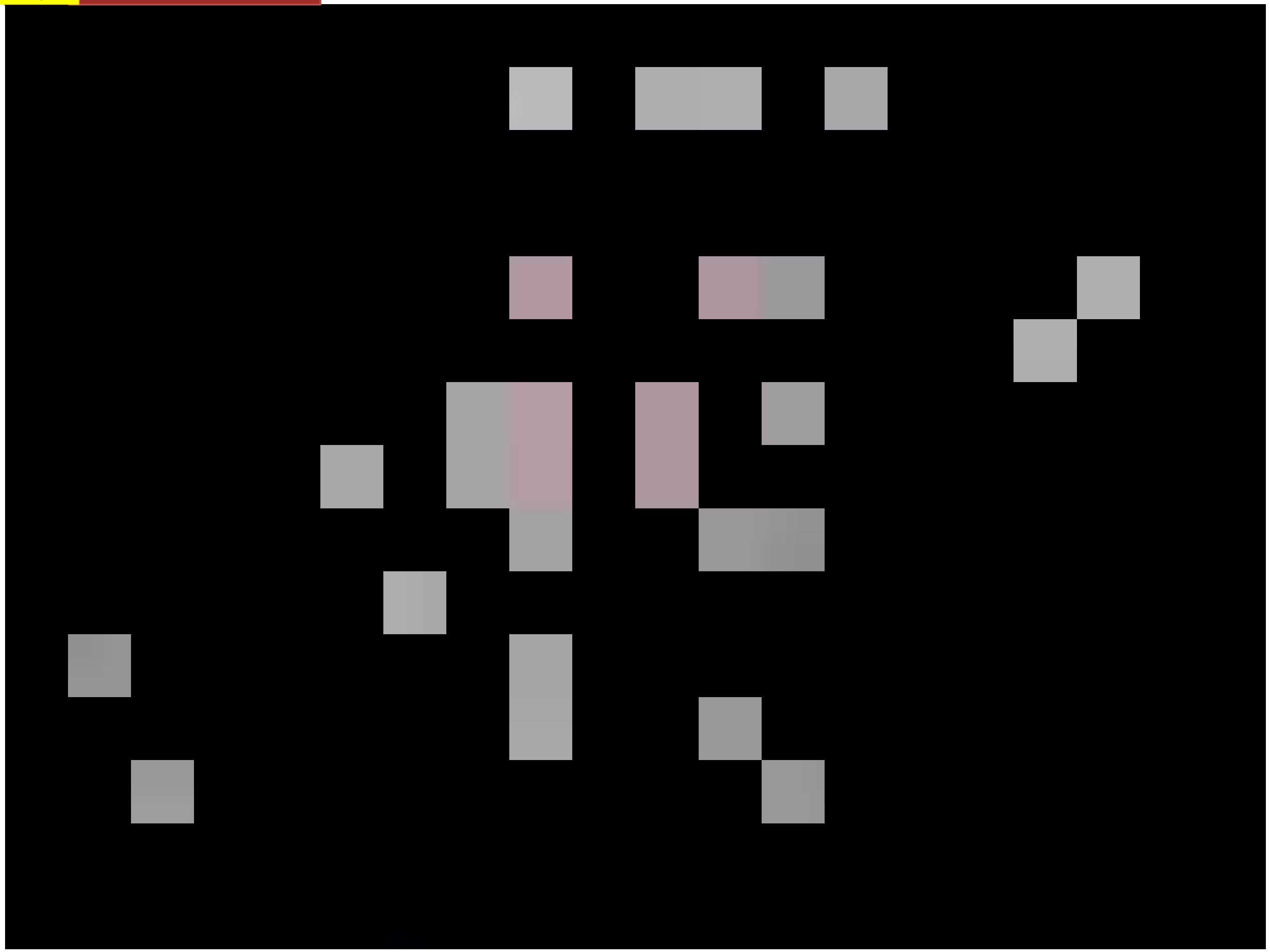
Viscosity: A property that represents the internal resistance of a fluid to motion or the “fluidity”.

Drag force: The force a flowing fluid exerts on a body in the flow direction. The magnitude of this force depends, in part, on viscosity

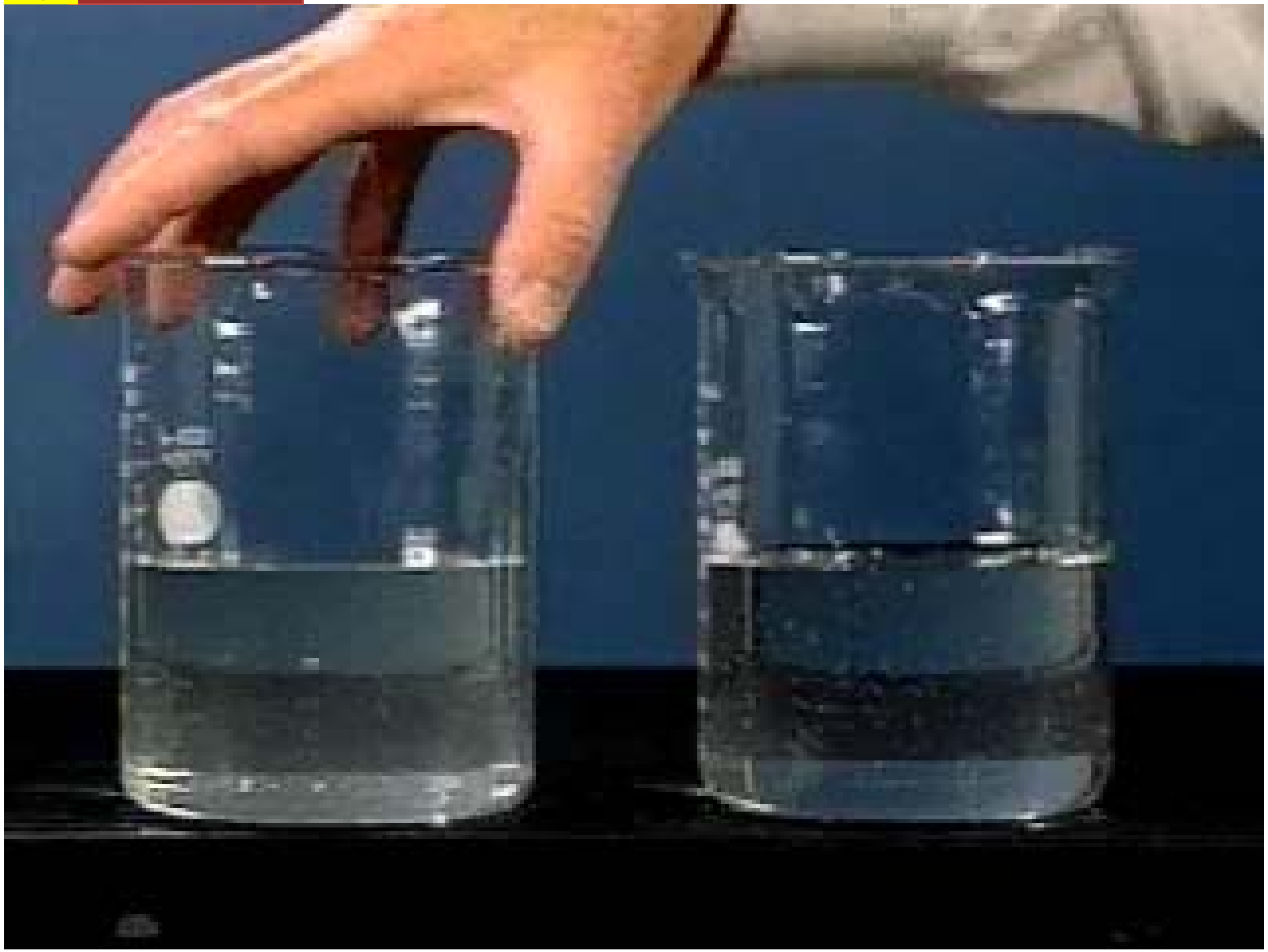
The viscosity of a fluid is a measure of its ***“resistance to deformation.”***

Viscosity is due to the internal frictional force that develops between different layers of fluids as they are forced to move relative to each other.

Viscosity



Viscosity



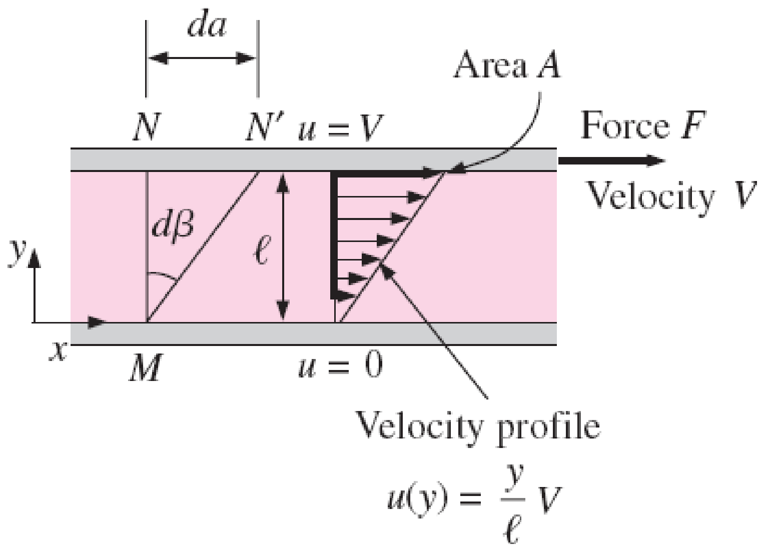
[redacted]

Newtonian fluids: Fluids for which the rate of deformation is proportional to the shear stress.

Shear stress

$$\tau \propto \frac{d\beta}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy} \qquad (\mathrm{N/m^2})$$



Shear force

$$F = \tau A = \mu A \frac{du}{dy} \qquad (\mathrm{N})$$

The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

μ coefficient of viscosity, Dynamic (absolute) viscosity
(kg/m·s or N·s/m^2 or Pa·s) 1 poise = 0.1 Pa·s

The force *F* required to move the upper plate at a constant velocity of *V* while the lower plate remains stationary is;

$$F = \mu A \frac{V}{\ell} \qquad \text{(N)}$$

the shear stress

$$\tau = \frac{F}{A}$$

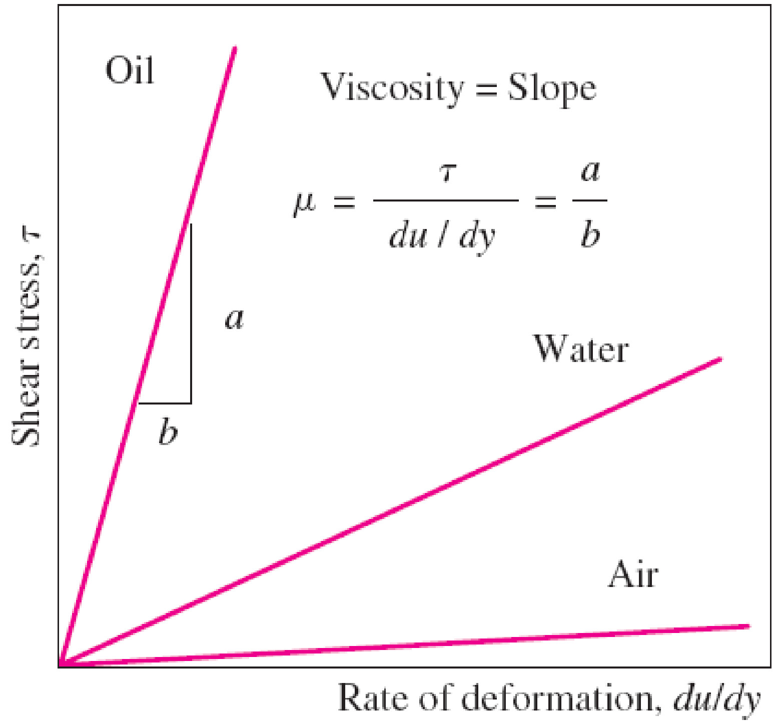
velocity profile and the *velocity gradient*

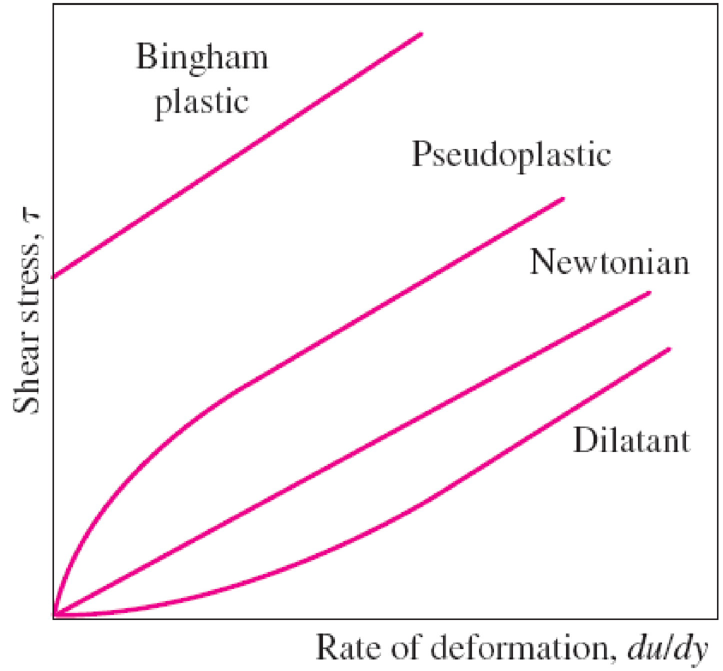
$$\boxed{u(y) = \frac{y}{\ell} V \qquad \text{and} \qquad \frac{du}{dy} = \frac{V}{\ell}}$$

$$\boxed{d\beta \approx \tan \beta = \frac{da}{\ell} = \frac{V\,dt}{\ell} = \frac{du}{dy} dt}$$

$$\boxed{\frac{d\beta}{dt} = \frac{du}{dy}}$$

[redacted]





The rate of deformation (velocity gradient) of a Newtonian fluid is proportional to shear stress, and the constant of proportionality is the viscosity.

Variation of shear stress with the rate of deformation for Newtonian and non-Newtonian fluids (the slope of a curve at a point is the apparent viscosity of the fluid at that point).

For non-Newtonian fluids, the relationship between shear stress and rate of deformation is not linear. The slope of the curve on the τ versus *du*/*dy* chart is referred to as the ***apparent viscosity*** of the fluid.

Non_Newtoinan Fluid

Fluids for which shearing stress is not linearly related to the rate of shearing strain are designated as non-Newtonian fluids.

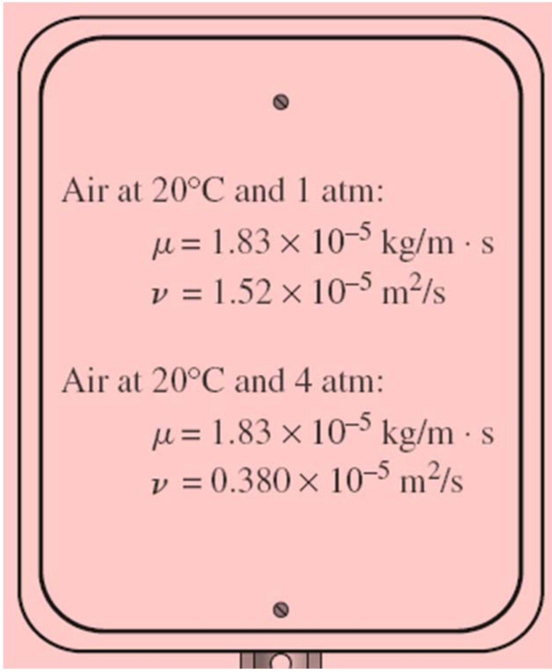
Kinematic viscosity

$\nu = \mu/\rho$

m²/s or stoke 1 stoke =1 cm²/s

For *liquids*, both the dynamic and kinematic viscosities are practically independent of pressure and any small variation with pressure is usually disregarded, except at extremely high pressures.

For *gases*, this is also the case for dynamic viscosity (at low to moderate pressures), but not for kinematic viscosity since the density of a gas is proportional to its pressure.



Dynamic viscosity, in general, does not depend on pressure, but kinematic viscosity does.

For gases:

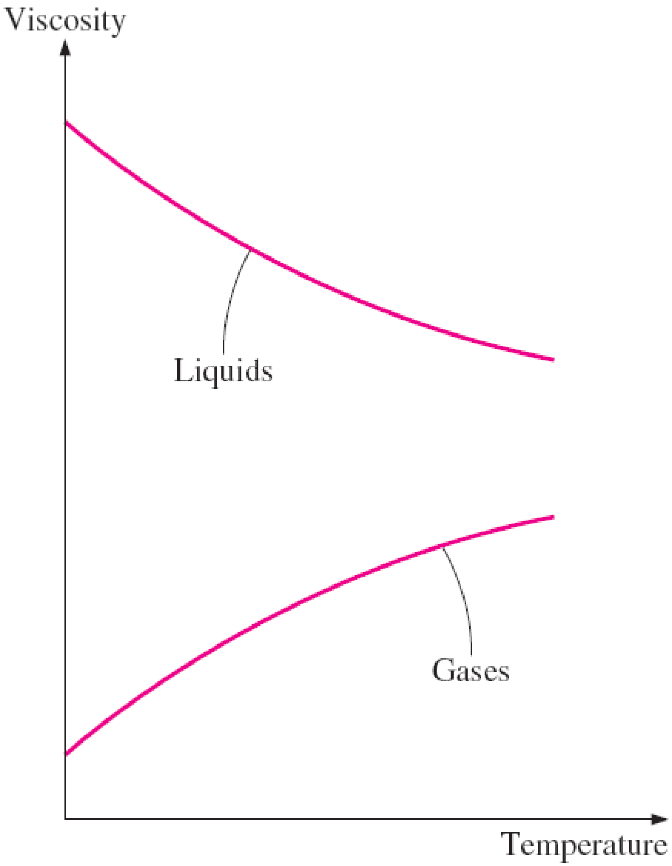
$$\mu = \frac{aT^{1/2}}{1 + b/T}$$

For liquids

$$\mu = a10^{b/(T-c)}$$

T is absolute temperature and *a, b,* and *c* are experimentally determined constants. For **water**, using the values *a*= 2.414x10⁻⁵ N s/m², *b*=247.8 K, and *c*=140 K results in less than 2.5 % error in viscosity in the temperature range of 0°C to 370°C

[redacted]



The viscosity of liquids decreases and the viscosity of gases increases with temperature.

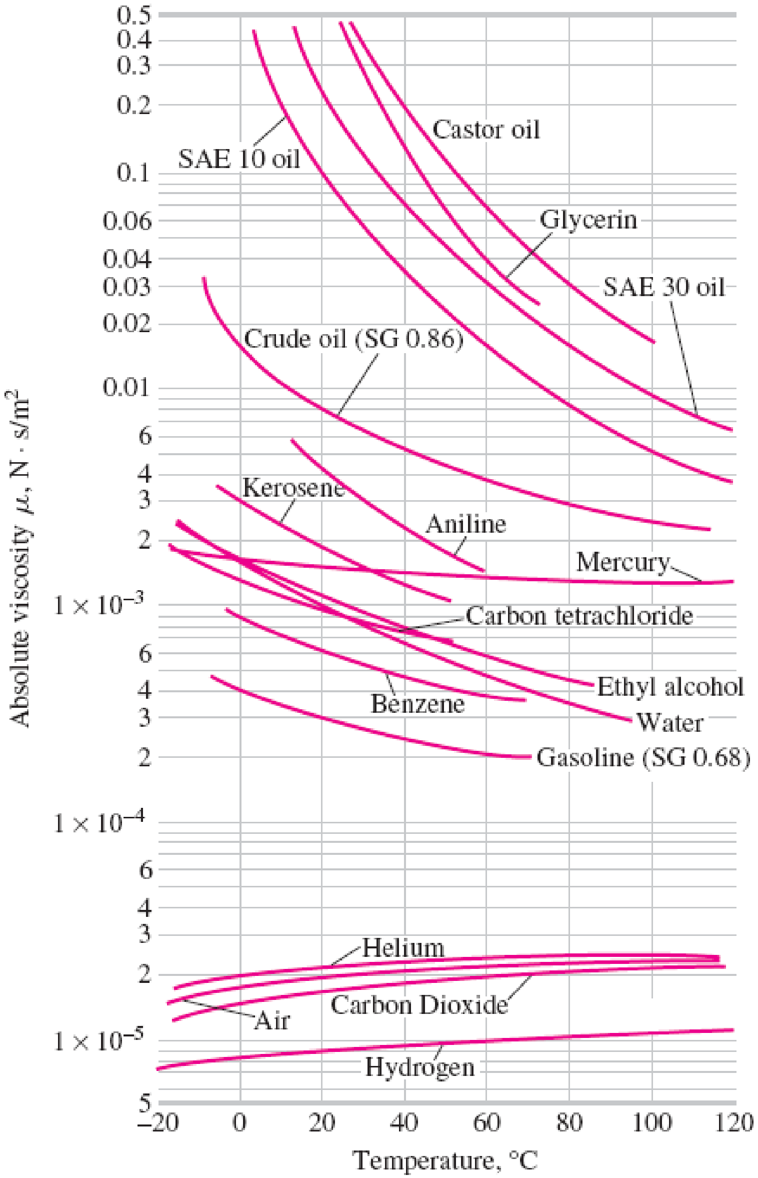
The viscosity of a fluid is directly related to the pumping power needed to transport a fluid in a pipe or to move a body through a fluid.

Viscosity is caused by the cohesive forces between the molecules in liquids and by the molecular collisions in gases, and it varies greatly with temperature.

In a liquid, the molecules possess more energy at higher temperatures, and they can oppose the large cohesive intermolecular forces more strongly. As a result, the energized liquid molecules can move more freely.

In a gas, the intermolecular forces are negligible, and the gas molecules at high temperatures move randomly at higher velocities. This results in more molecular collisions per unit volume per unit time and therefore in greater resistance to flow.

[redacted]

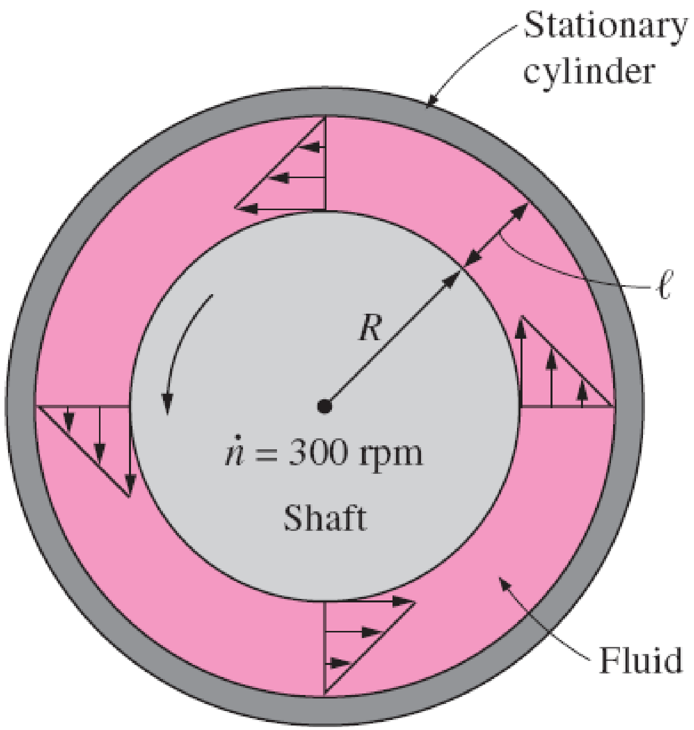


The variation of dynamic (absolute) viscosity of common fluids with emperature at 1 atm (1 N·s/m² = 1kg/m·s = 0.020886 lbf·s/ft²)

The viscosities of different fluids differ by several orders of magnitude. Also it is more difficult to move an object in a higher-viscosity fluid such as engine oil than it is in a lower-viscosity fluid such as water. **Liquids, in general, are much more viscous than gases.**

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

Fluid	Dynamic Viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088



Consider a fluid layer of thickness within a small gap between two concentric cylinders, such as the thin layer of oil in a journal bearing. The gap between the cylinders can be modeled as two parallel flat plates separated by a fluid. Noting that torque is T= *FR* (force times the moment arm, which is the radius *R* of the inner cylinder in this case), the tangential velocity is *V*=w*R* (angular velocity times the radius), and taking the wetted surface area of the inner cylinder to be A=$2\pi RL$ by disregarding the shear stress acting on the two ends of the inner cylinder, torque can be expressed as

L length of the cylinder

$\dot{n}$ number of revolutions per unit time

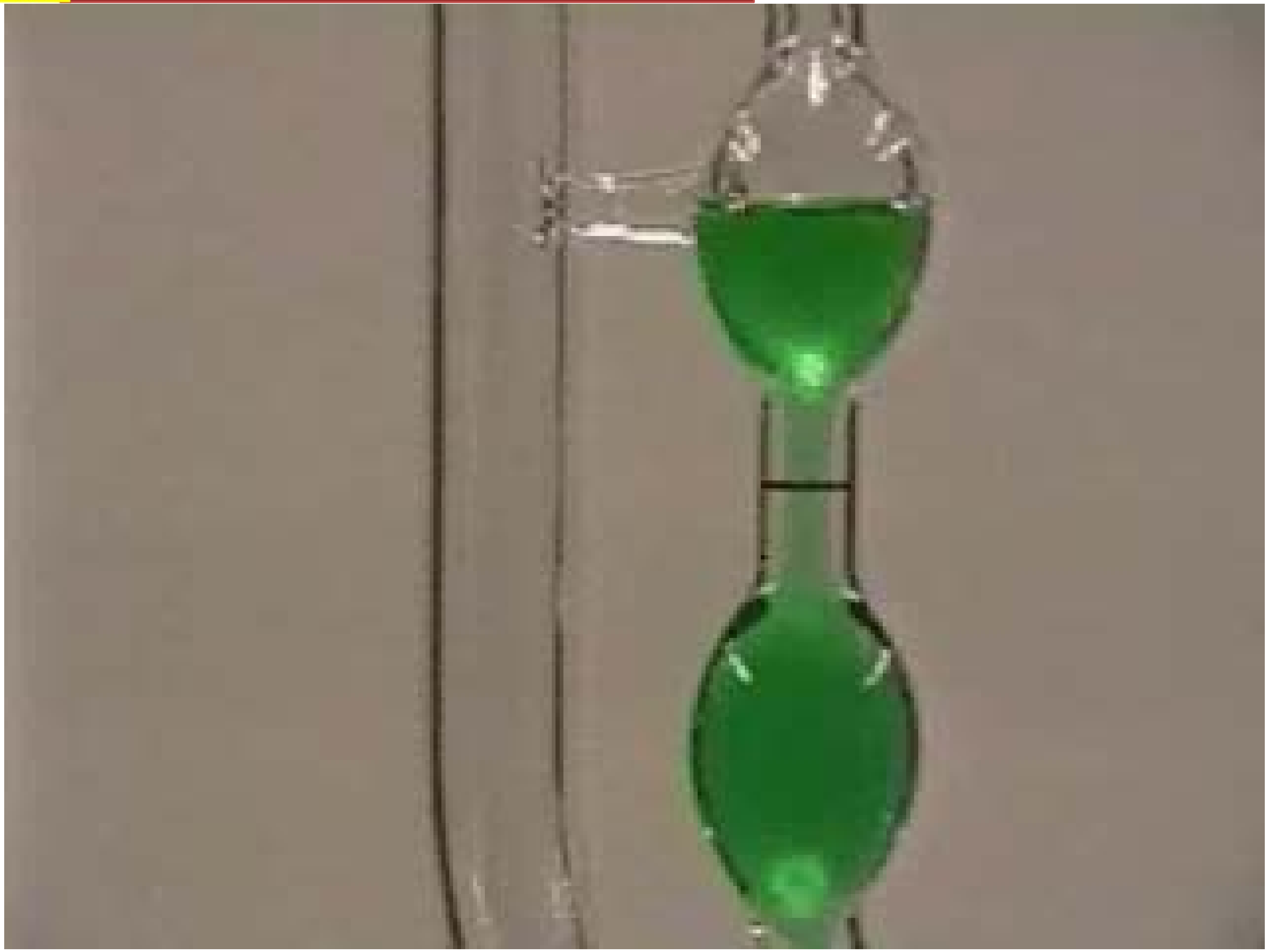
$$\mathrm{T} = FR = \mu \frac{2\pi R^3 \omega L}{\ell} = \mu \frac{4\pi^2 R^3 \dot{n} L}{\ell}$$

This equation can be used to calculate the viscosity of a fluid by measuring torque at a specified angular velocity. Therefore, two concentric cylinders can be used as a ***viscometer***, a device that measures viscosity.

[redacted]

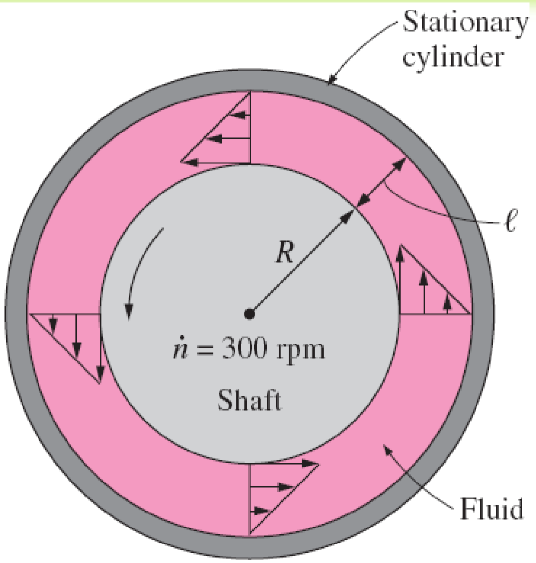
Capilar tube Viscometer

Most devices (called viscometers) used to determine viscosity do not measure it directly, but instead measure some characteristic with a known relationship to viscosity. The capillary tube viscometer involves the laminar flow of a fixed volume of fluid through a capillary tube. The time required for the fluid to pass through the tube is a measure of the kinematic viscosity of the fluid. As shown with the four tubes, the drain times can vary depending on the viscosity of the fluid and the diameter of the capillary tube.



EXAMPLE 2–4

The viscosity of a fluid is to be measured by a viscometer constructed of two 40-cm-long concentric cylinders. The outer diameter of the inner cylinder is 12 cm, and the gap between the two cylinders is 0.15 cm. The inner cylinder is rotated at 300 rpm, and the torque is measured to be 1.8 N m. Determine the viscosity of the fluid.



Solution: The torque and the rpm of a double cylinder viscometer are given. The viscosity of the fluid is to be determined.

Assumptions **1** The inner cylinder is completely submerged in oil. **2** The viscous effects on the two ends of the inner cylinder are negligible.

Analysis The velocity profile is linear only when the curvature effects are negligible, and the profile can be approximated as being linear in this case since $l/R \ll 1$. Solving for viscosity and substituting the given values, the viscosity of the fluid is determined to be;

$$\mu = \frac{T\ell}{4\pi^2 R^3 \dot{n} L} = \frac{(1.8\ \text{N}\cdot\text{m})(0.0015\ \text{m})}{4\pi^2(0.06\ \text{m})^3(300/60\ 1/\text{s})(0.4\ \text{m})} = \mathbf{0.158\ N\cdot s/m^2}$$

Discussion Viscosity is a strong function of temperature, and a viscosity value without a corresponding temperature is of little value. Therefore, the temperature of the fluid should have also been measured during this experiment, and reported with this calculation.

2–7 SURFACE TENSION AND CAPILLARY EFFECT

A drop of blood forms a hump on a horizontal glass;
A drop of mercury forms a near-perfect sphere and can be rolled just like a steel ball over a smooth surface;
Water droplets from rain or dew hang from branches or leaves of trees;
A liquid fuel injected into an engine forms a mist of spherical droplets;
Water dripping from a leaky faucet falls as spherical droplets;
A soap bubble released into the air forms a spherical water beads up into small drops on flower petals.

Liquid droplets behave like small balloons filled with the liquid on a solid surface, and the surface of the liquid acts like a stretched elastic membrane under tension.

The pulling force that causes this tension acts parallel to the surface and is due to the attractive forces between the molecules of the liquid.

The magnitude of this force per unit length is called **surface tension** (or *coefficient of surface tension*) and is usually expressed in the unit N/m.

This effect is also called ***surface energy*** [per unit area] and is expressed in the equivalent unit of $N \cdot m/m^2$.

[redacted]

Some consequences of surface tension.

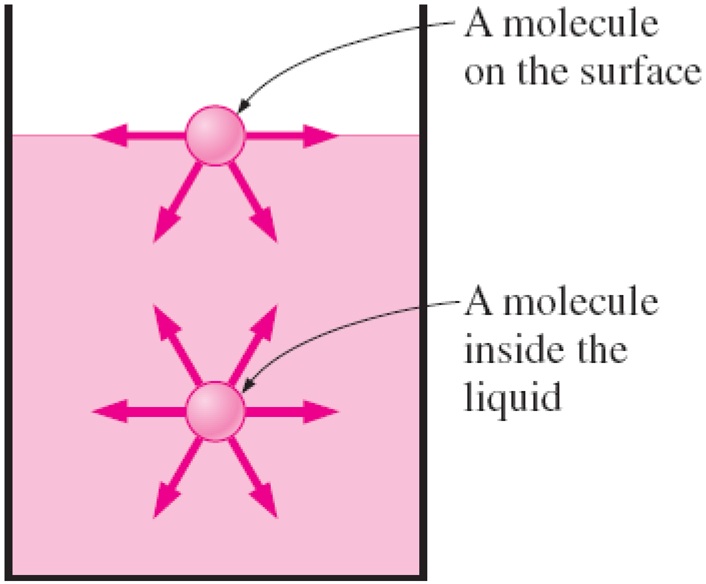
Some insects can land on water or even walk on water and that small steel needles can float on water. These phenomena are again made possible by surface tension that balances the weights of these objects.

Surface tension depends on the nature of the liquid, the surrounding environment and emperature. Liquids were molecules have large attractive intermolecular forces will have a large surface tension

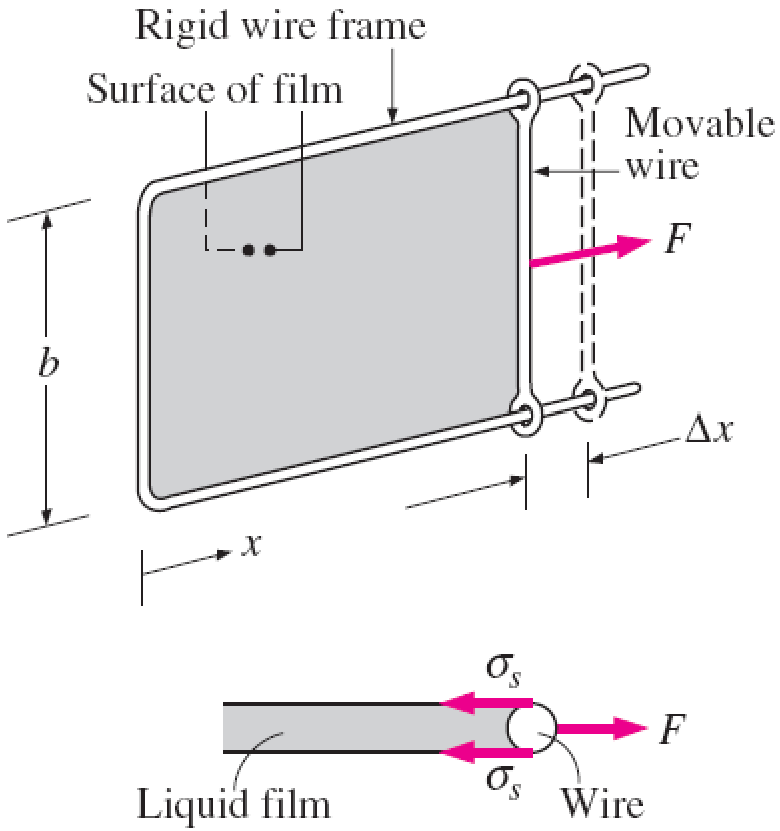
This can be observed by slightly overfilling a drinking glass; the water will stand above the rim without spilling.

Surface Tension

[redacted]



Attractive forces acting on a liquid molecule at the surface and deep inside the liquid.



Stretching a liquid film with a U-shaped wire, and the forces acting on the movable wire of length *b*.

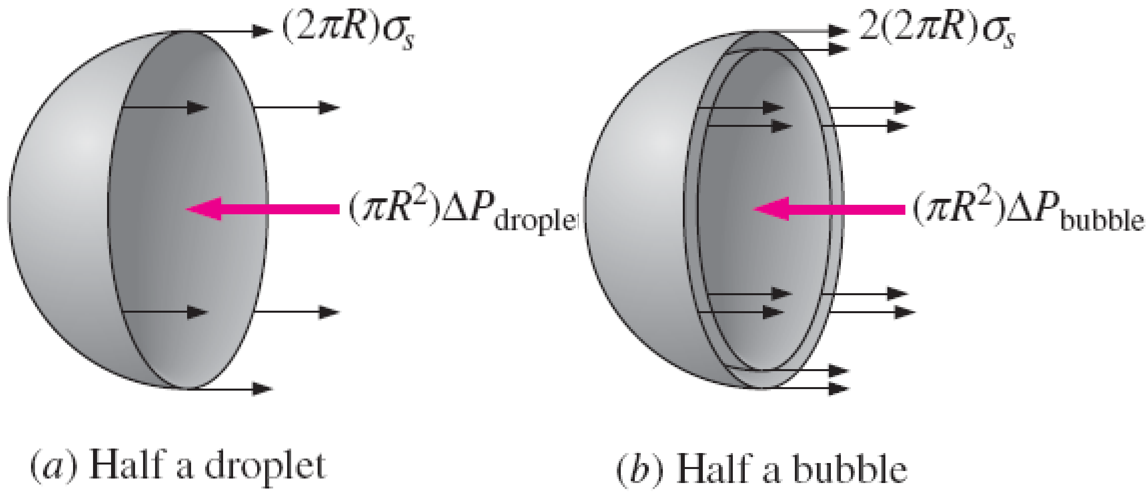
$$\sigma_s = \frac{F}{2b}$$

Surface tension: The work done per unit increase in the surface area of the liquid.

$$W = \text{Force} \times \text{Distance} = F\,\Delta x = 2b\sigma_s\,\Delta x = \sigma_s\,\Delta A$$

Surface tension of some fluids in air at 1 atm and 20°C (unless otherwise stated)

Fluid	Surface Tension σ_s, N/m*
Water:	
0°C	0.076
20°C	0.073
100°C	0.059
300°C	0.014
Glycerin	0.063
SAE 30 oil	0.035
Mercury	0.440
Ethyl alcohol	0.023
Blood, 37°C	0.058
Gasoline	0.022
Ammonia	0.021
Soap solution	0.025
Kerosene	0.028



The free-body diagram of half a droplet or air bubble and half a soap bubble.

Droplet:

$$(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{droplet}} \rightarrow \Delta P_{\text{droplet}} = P_i - P_o = \frac{2\sigma_s}{R}$$

Bubble:

$$2(2\pi R)\sigma_s = (\pi R^2)\Delta P_{\text{bubble}} \rightarrow \Delta P_{\text{bubble}} = P_i - P_o = \frac{4\sigma_s}{R}$$

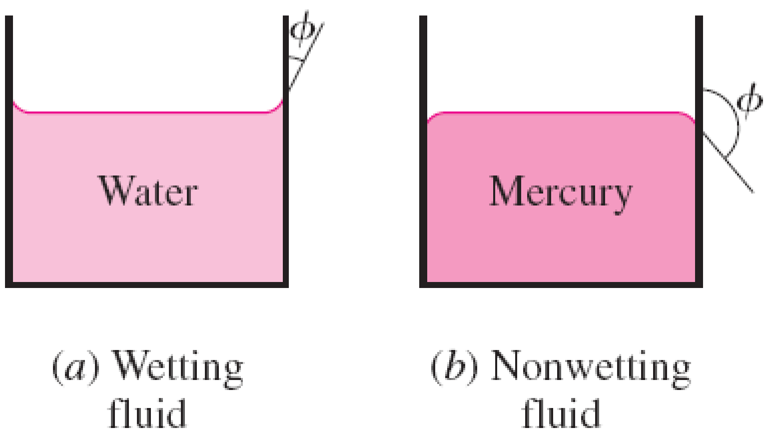
Capillary Effect

Capillary effect: The rise or fall of a liquid in a small-diameter tube inserted into the liquid.

Capillaries: Such narrow tubes or confined flow channels. The capillary effect is partially responsible for the rise of water to the top of tall trees.

Meniscus: The curved free surface of a liquid in a capillary tube.

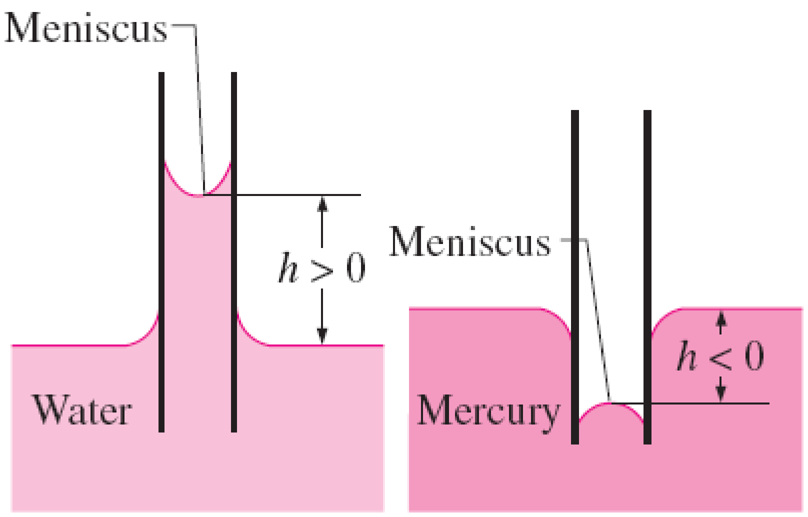
The strength of the capillary effect is quantified by the contact (or *wetting*) angle, defined as *the angle that the tangent to the liquid surface makes with the solid surface at the point of contact*.



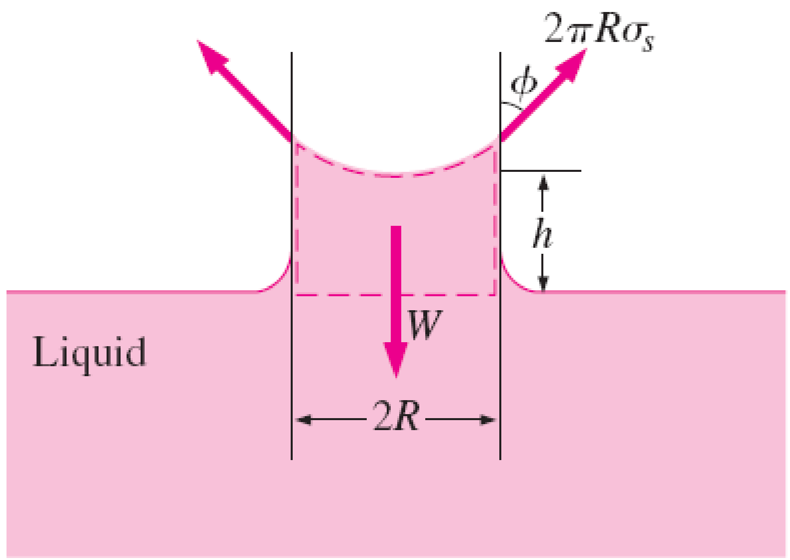
The contact angle for wetting and nonwetting fluids.

The meniscus of colored water in a 4-mm-inner-diameter glass tube. Note that the edge of the meniscus meets the wall of the capillary tube at a very small contact angle.

[redacted]



The capillary rise of water and the capillary fall of mercury in a small-diameter glass tube.



The forces acting on a liquid column that has risen in a tube due to the capillary effect.

$$W = mg = \rho V g = \rho g(\pi R^2 h)$$

$$W = F_{\text{surface}} \rightarrow \rho g(\pi R^2 h) = 2\pi R \sigma_s \cos\phi$$

Capillary rise: $$h = \frac{2\sigma_s}{\rho g R}\cos\phi \qquad (R = \text{constant})$$

Capillary rise is inversely proportional to the radius of the tube and density of the liquid.

DENSITY

If the relative density of mercury is 13.6, the actual density of mercury is

(a) 13.6 kg/m^3
(b) 136 kg/m^3
(c) 1/13.6 kg/m^3
(d) 1360 kg/m^3
(e) 13600 kg/m^3

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VISCOSITY

The **dynamic** viscosity of pure glycerol is 1.5 Pa.s and its density 1262 kg/m^3.
The **kinematic** viscosity of pure glycerol is

(a) 1.5 m^2/s
(b) 1893 Pa.s
(c) 1.19 x 10^{-3} m^2/s
(d) 1.19 x 10^{-3} Pa.s

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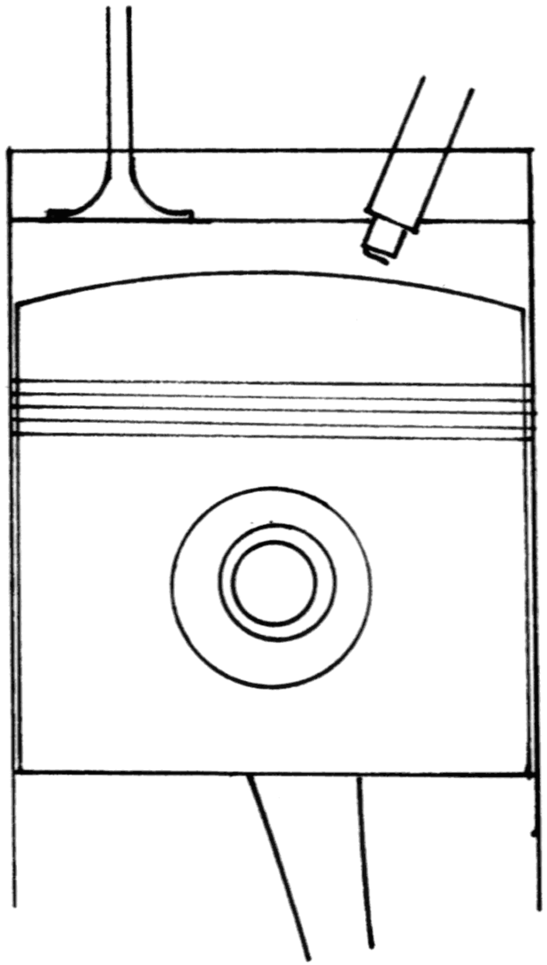
VISCOSITY

Shear stress $\tau = \mu$ x velocity gradient



Peak rpm for a racing engine is 18000 rpm which corresponds to a maximum piston speed of 25 m/s. The viscosity of the lubricating oil in the gap between a piston and the cylinder wall is 0.016 Pa.s. If the gap width is 100 μm, is the shear stress acting on the piston at maximum speed

(a) 400 Pa
(b) 0.04 bar
(c) 0.4 bar

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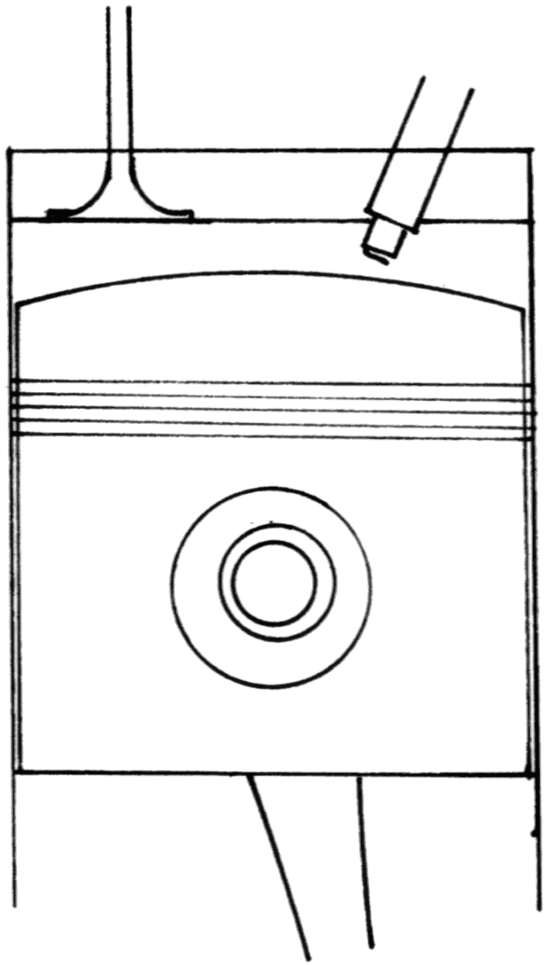
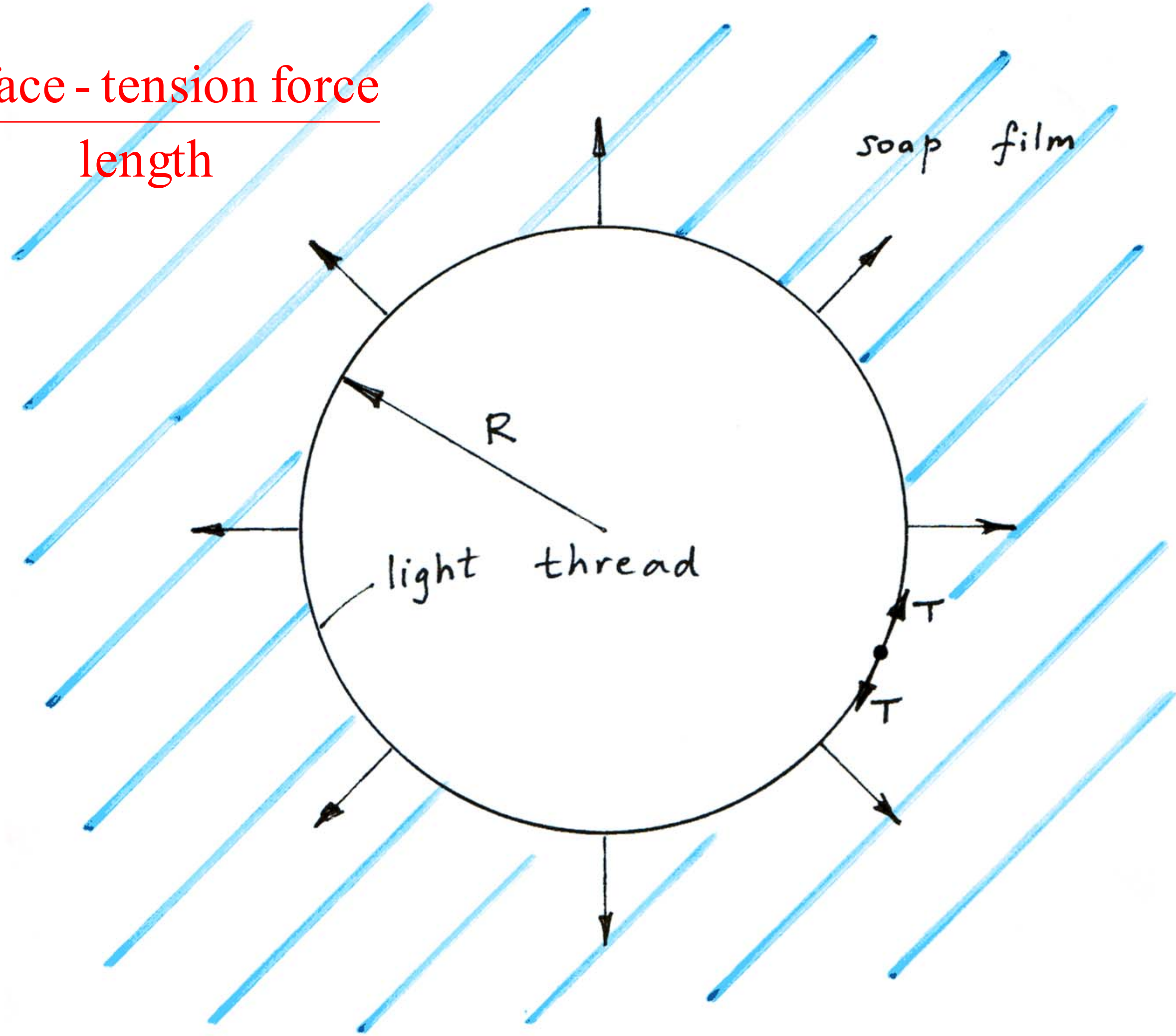


SURFACE TENSION

$$\text{surface tension } \sigma \equiv \frac{\text{surface - tension force}}{\text{length}}$$

The tension force T in the thread due to surface tension is

(a) $2\pi\,\sigma R$
(b) $\pi\,\sigma R$
(c) $\sigma\,R$



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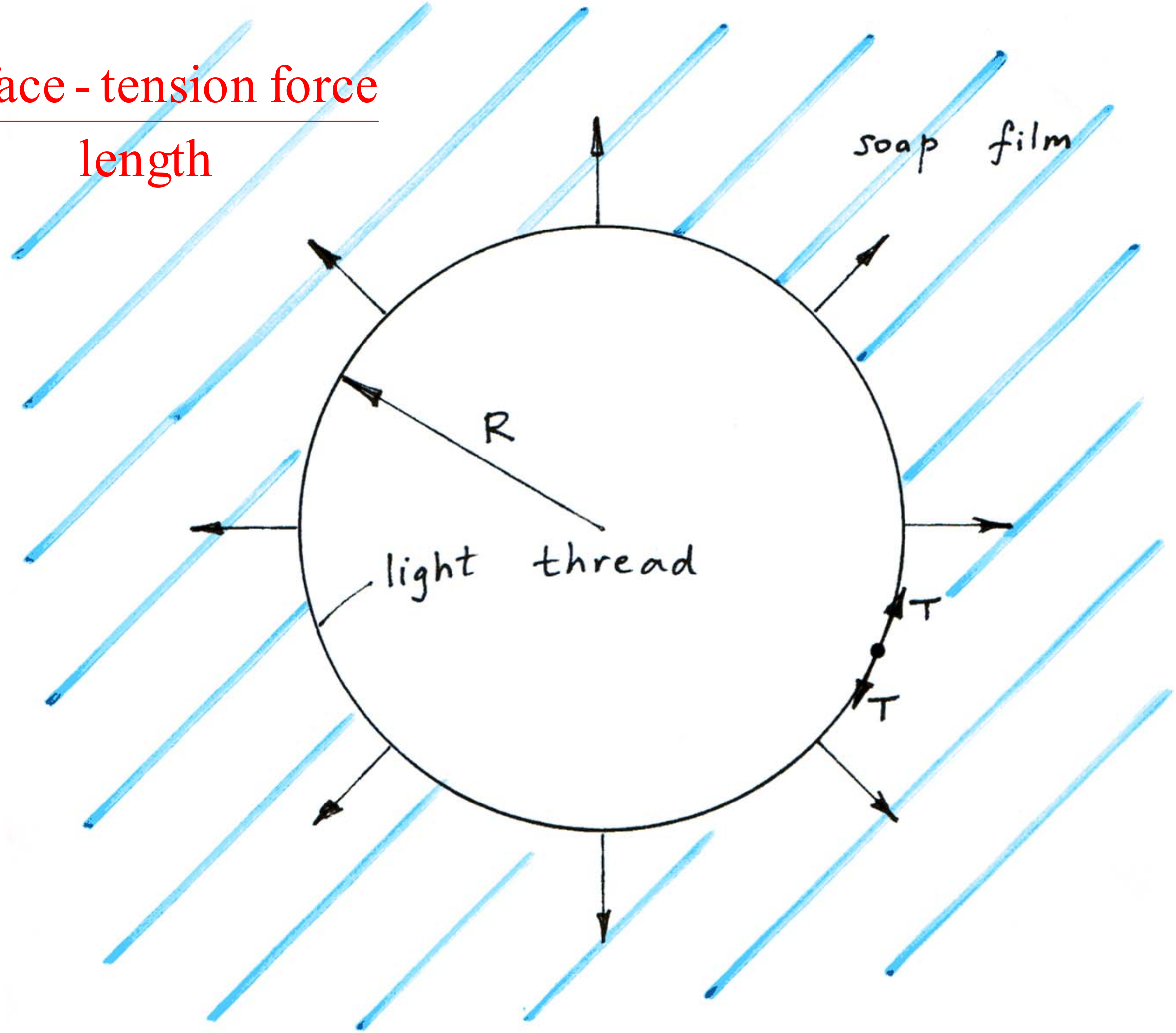
(b) $\pi \sigma R$
(c) σR



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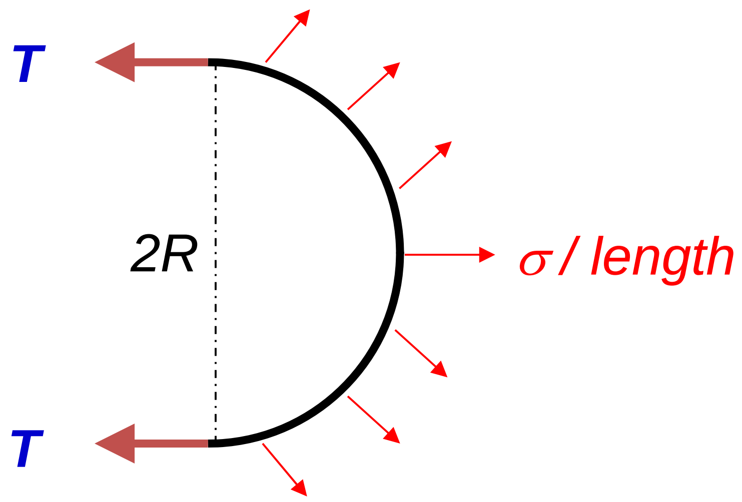
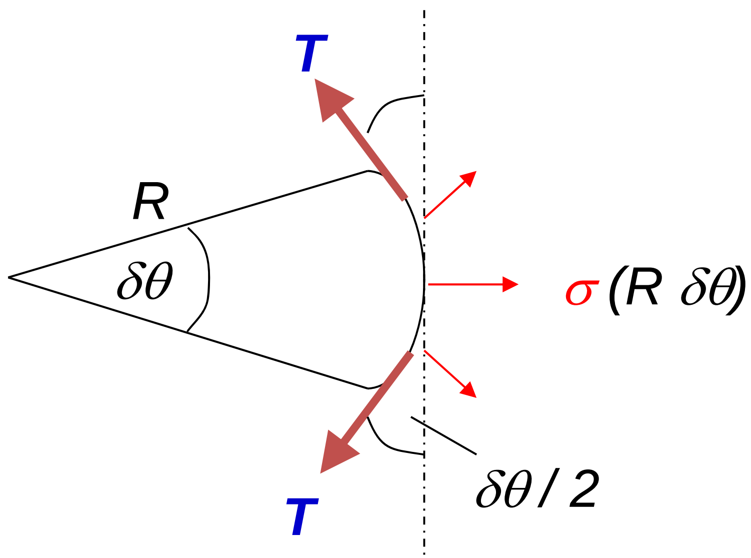




force balance in horizontal direction

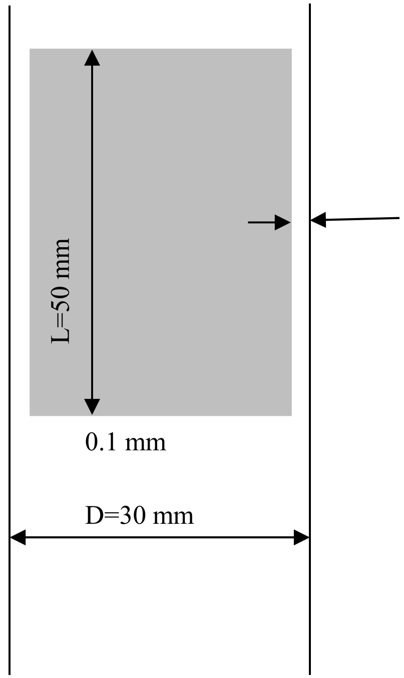
$$2T = \sigma\, 2R$$

$$T = \sigma R$$

$$-2T \sin(\delta\theta / 2) + \sigma (R\ \delta\theta) = 0$$

$$T\ \delta\theta = \sigma R\ \delta\theta \qquad T = \sigma R$$

A solid cylinder of weight W slides down in a vertical pipe which has a diameter of D=30 mm. The length of solid cylinder is L=50 mm, the clearance between the pipe and the block is 0.1 mm and filled with oil (ρ_{oil}=890 kg/m^3, μ_{oil}=0.40 kg/m s). Assuming linear velocity distribution in the film and neglecting the air drag, find the terminal velocity of the solid cylinder if the density of the solid cylinder is ρ_s=1300 kg/m^3.



$$\tau = \frac{F}{A} = \mu \frac{V}{l} \rightarrow V = \frac{F * l}{\mu * A}$$

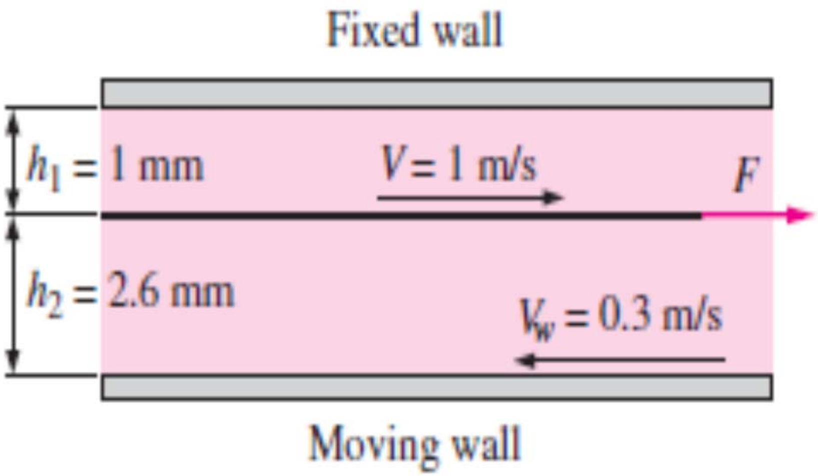
$$F = G = \rho g \frac{\pi d^2}{4} L \rightarrow G = 1300 * 9{,}81 * \frac{\pi * (0{,}03 - 0{,}0002)^2}{4} 0{,}05 = 0.44 \text{ N}$$

$$V = \frac{0.44 * 0{,}0001}{0{,}40 * 3.14(0{,}03 - 0{,}0002) * 0{,}05} \rightarrow V = 0.0234\, m/s$$

A thin 20-cm 20-cm flat plate is pulled at 1 m/s horizontally through a 3.6-mm-thick oil layer sandwiched between two plates, one stationary and the other moving at a constant velocity of 0.3 m/s, as shown in Figure. The dynamic viscosity of oil is 0.027 Pa.s. Assuming the velocity in each oil layer to vary linearly,

(*a*) plot the velocity profile and find the location where the oil velocity is zero and

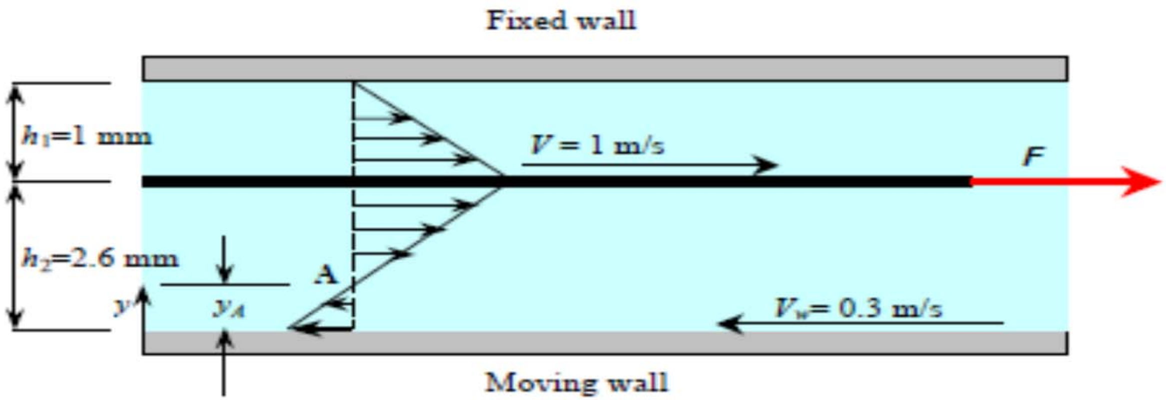
(*b*) determine the force that needs to be applied on the plate to maintain this motion.



[redacted]

(*a*) plot the velocity profile and find the location where the oil velocity is zero and

$$\frac{2.6 - y_A}{y_A} = \frac{1}{0.3} \quad \rightarrow \quad y_A = 0.60 \text{ mm}$$



(*b*) determine the force that needs to be applied on the plate to maintain this motion.

$$F_{shear,upper} = \tau_{w,up} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V-0}{h_1} = 0.027 \cdot (0.2 \cdot 0.2) \frac{1}{1 \cdot 10^{-3}} = 1.08\ N$$

$$F_{shear,lower} = \tau_{w,low} A_s = \mu A_s \left|\frac{du}{dy}\right| = \mu A_s \frac{V - V_w}{h_2} = 0.027 \cdot (0.2 \cdot 0.2) \frac{1 - (-0.3)}{2.6 \cdot 10^{-3}} = 0.54\ N$$

$$F = F_{shear,upper} + F_{shear,lower} = 1.08 + 0.54 = 1.62\ N$$

Summary

- Introduction
  - Continuum
- Density and Specific Gravity
  - Density of Ideal Gases
- Vapor Pressure and Cavitation
- Energy and Specific Heats
- Compressibility and Speed of Sound
  - Coefficient of Compressibility
  - Coefficient of Volume Expansion
  - Speed of Sound and Mach Number
- Viscosity
- Surface Tension and Capillary Effect