Thermodynamics: An Engineering Approach, 5th Edition
Yunus A. Cengel, Michael A. Boles
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Chapter 5

MASS AND ENERGY ANALYSIS OF CONTROL VOLUMES

Prof. Dr. Ali PINARBAŞI
Yildiz Technical University
Mechanical Engineering Department
Yildiz, ISTANBUL

Objectives

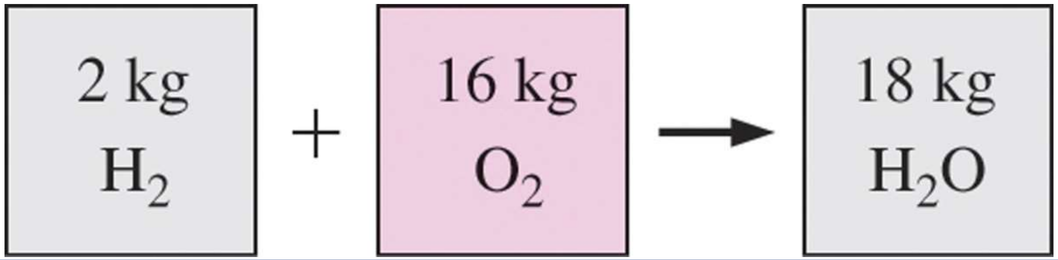
- Develop the conservation of mass principle.
- Apply the conservation of mass principle to various systems including steady- and unsteady-flow control volumes.
- Apply the first law of thermodynamics as the statement of the conservation of energy principle to control volumes.
- Identify the energy carried by a fluid stream crossing a control surface as the sum of internal energy, flow work, kinetic energy, and potential energy of the fluid and to relate the combination of the internal energy and the flow work to the property enthalpy.
- Solve energy balance problems for common steady-flow devices such as nozzles, compressors, turbines, throttling valves, mixers, heaters, and heat exchangers.
- Apply the energy balance to general unsteady-flow processes with particular emphasis on the uniform-flow process as the model for commonly encountered charging and discharging processes.

CONSERVATION OF MASS

Conservation of mass: Mass, like energy, is a conserved property, and it cannot be created or destroyed during a process.

Closed systems: The mass of the system remain constant during a process.

Control volumes: Mass can cross the boundaries, and so we must keep track of the amount of mass entering and leaving the control volume.



Mass is conserved even during chemical reactions.

Conservation of Mass

The conservation of mass relation for a closed system undergoing a change is expressed as m_{sys}=Const. or dm_{sys}/dt =0, which is the statement that the mass of the system remains constant during a process.

Mass balance for a control volume (CV) in rate form:

$$\boxed{\dot{m}_{\text{in}} - \dot{m}_{\text{out}} = \frac{dm_{\text{CV}}}{dt}}$$

$\dot{m}_{\text{in}}$ and $\dot{m}_{\text{out}}$ — the total rates of mass flow into and out of the control volume

dm_{CV}/dt — the rate of change of mass within the control volume boundaries.

Continuity equation: In fluid mechanics, the conservation of mass relation written for a differential control volume is usually called the ***continuity** equation*.

Conservation of momentum

Linear momentum: The product of the mass and the velocity of a body is called the *linear momentum* or just the *momentum* of the body.

The momentum of a rigid body of mass m moving with a velocity V is mV.

Newton's second law: The acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass, and that the rate of change of the momentum of a body is equal to the net force acting on the body.

Conservation of Momentum

Conservation of momentum principle: The momentum of a system remains constant only when the net force acting on it is zero, and thus the momentum of such systems is conserved.

Linear momentum equation: In fluid mechanics, Newton's second law is usually referred to as the *linear momentum equation*.

Mass flow rate: The amount of mass flowing through a cross section per unit time.

The differential mass flow rate

$$\delta \dot{m} = \rho V_n \, dA_c$$

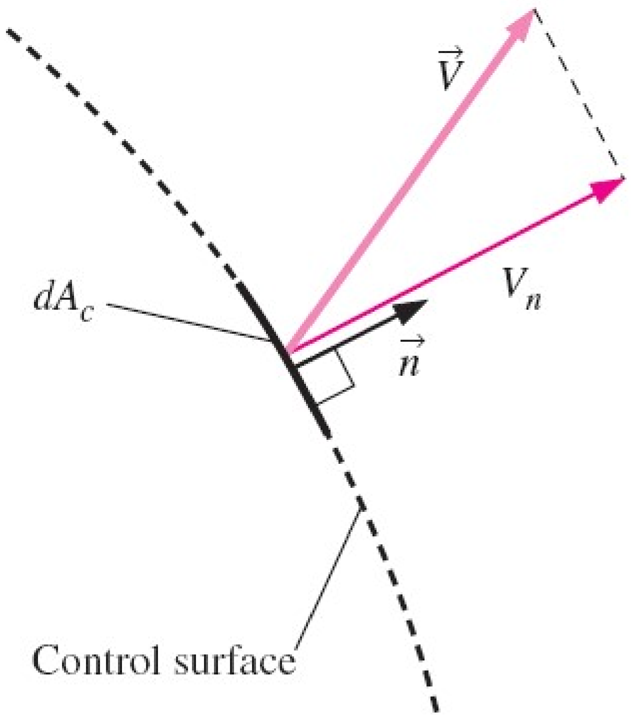
Point functions have *exact differentials*

$$\int_1^2 dA_c = A_{c2} - A_{c1} = \pi(r_2^2 - r_1^2)$$

Path functions have *inexact differentials*

$$\int_1^2 \delta \dot{m} = \dot{m}_{\text{total}}$$

$$\text{not } \dot{m}_2 - \dot{m}_1$$



The normal velocity V_n for a surface is the component of velocity perpendicular to the surface.

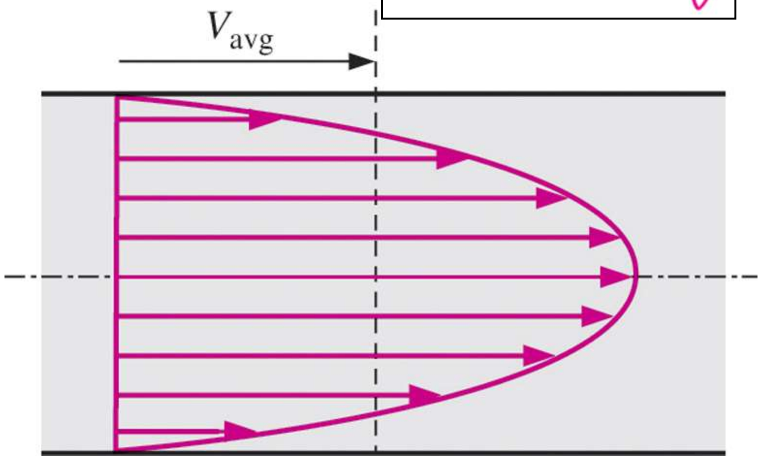
$$\delta \dot{m} = \rho V_n \, dA_c$$

$$\dot{m} = \int_{A_c} \delta \dot{m} = \int_{A_c} \rho V_n \, dA_c \qquad (\text{kg/s})$$

Mass flow rate

$$\dot{m} = \rho V_{avg} A_c \qquad (\text{kg/s})$$

$$\dot{m} = \rho \dot{V} = \frac{\dot{V}}{v}$$



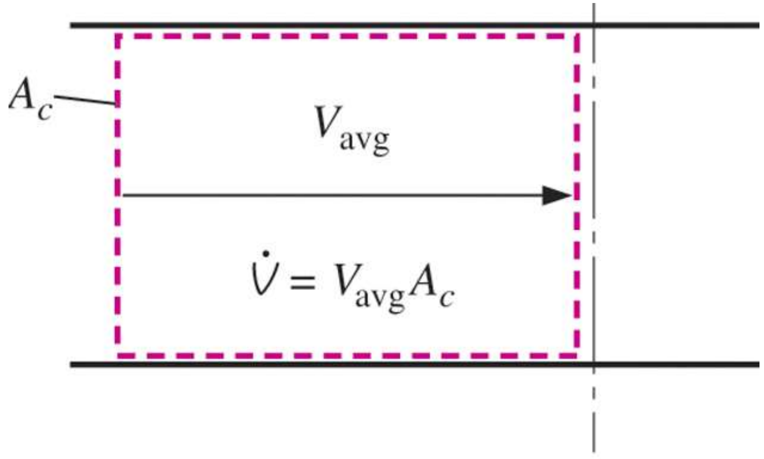
The average velocity V_{avg} is defined as the average speed through a cross section.

Average velocity

$$V_{avg} = \frac{1}{A_c} \int_{A_c} V_n \, dA_c$$

Volume flow rate

$$\dot{V} = \int_{A_c} V_n \, dA_c = V_{avg} A_c = V A_c \qquad (\text{m}^3/\text{s})$$



The volume flow rate is the volume of fluid flowing through a cross section per unit time.

The conservation of mass principle for a control volume: The net mass transfer to or from a control volume during a time interval Δt is equal to the net change (increase or decrease) in the total mass within the control volume during Δt.

$$\begin{pmatrix}\text{Total mass entering}\\ \text{the CV during } \Delta t\end{pmatrix} - \begin{pmatrix}\text{Total mass leaving}\\ \text{the CV during } \Delta t\end{pmatrix} = \begin{pmatrix}\text{Net change in mass}\\ \text{within the CV during } \Delta t\end{pmatrix}$$

Conservation of mass principle for an ordinary bathtub.

$$m_{in} - m_{out} = \Delta m_{CV} \qquad \text{(kg)}$$

$$\dot{m}_{in} - \dot{m}_{out} = dm_{CV}/dt \qquad \text{(kg/s)}$$

$\dot{m}_{in}$ and $\dot{m}_{out}$: the total rates of mass flow into and out of the control volume

dm_{CV}/dt: the rate of change of mass within the control volume boundaries.

Mass balance is applicable to any control volume undergoing any kind of process.

$$dm = \rho\, dV$$

Total mass within the CV: $$m_{CV} = \int_{CV} \rho\, dV$$

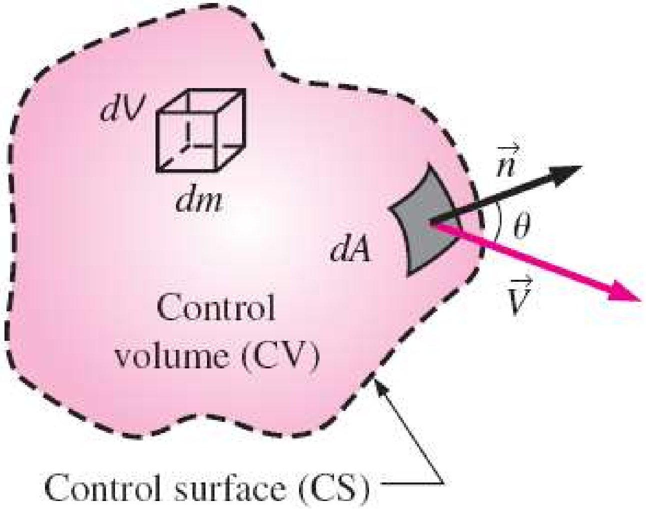
Rate of change of mass within the CV: $$\frac{dm_{CV}}{dt} = \frac{d}{dt}\int_{CV} \rho\, dV$$

Normal component of velocity: $$V_n = V\cos\theta = \vec{V}\cdot\vec{n}$$

Differential mass flow rate: $$\delta\dot{m} = \rho V_n\, dA = \rho(V\cos\theta)\, dA = \rho(\vec{V}\cdot\vec{n})\, dA$$

Net mass flow rate: $$\dot{m}_{net} = \int_{CS} \delta\dot{m} = \int_{CS} \rho V_n\, dA = \int_{CS} \rho(\vec{V}\cdot\vec{n})\, dA$$

The differential control volume *dV* and the differential control surface *dA* used in the derivation of the conservation of mass relation.



General conservation of mass

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho \, dV + \int_{\mathrm{CS}} \rho(\vec{V}\cdot\vec{n})\, dA = 0$$

The time rate of change of mass within the control volume plus the net mass flow rate through the control surface is equal to zero.

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho \, dV + \sum_{\mathrm{out}} \int_{A} \rho V_n \, dA - \sum_{\mathrm{in}} \int_{A} \rho V_n \, dA = 0$$

$$\frac{d}{dt}\int_{\mathrm{CV}} \rho \, dV = \sum_{\mathrm{in}} \dot{m} - \sum_{\mathrm{out}} \dot{m}$$

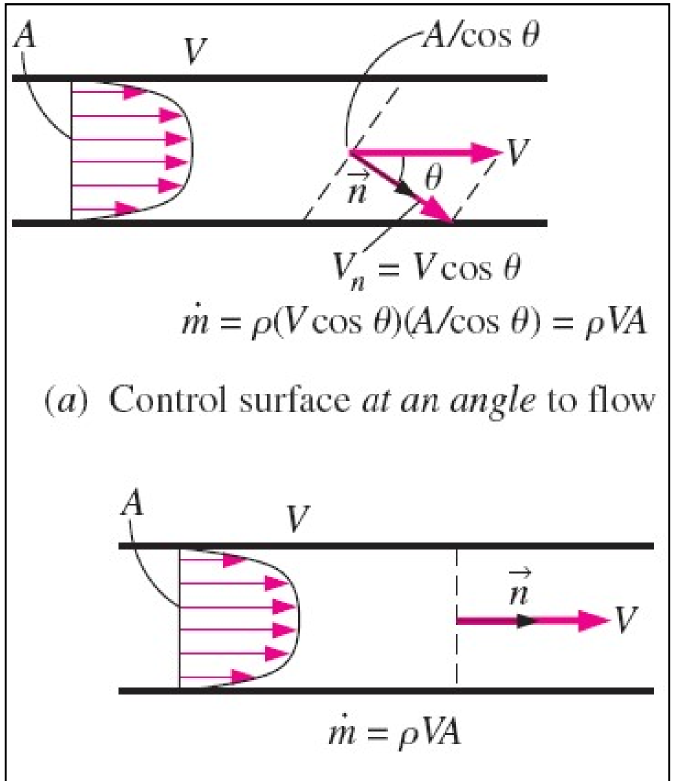
$$\frac{dm_{\mathrm{CV}}}{dt} = \sum_{\mathrm{in}} \dot{m} - \sum_{\mathrm{out}} \dot{m}$$

The conservation of mass equation is obtained by replacing *B* in the Reynolds transport theorem by mass *m*, and *b* by 1

$$\frac{dB_{\mathrm{sys}}}{dt} = \frac{d}{dt}\int_{\mathrm{CV}} \rho b \, dV + \int_{\mathrm{CS}} \rho b(\vec{V}\cdot\vec{n})\, dA$$

$B = m \qquad b = 1 \qquad b = 1$

$$\frac{dm_{\mathrm{sys}}}{dt} = \frac{d}{dt}\int_{\mathrm{CV}} \rho \, dV + \int_{\mathrm{CS}} \rho(\vec{V}\cdot\vec{n})\, dA$$



(a) Control surface *at an angle* to flow

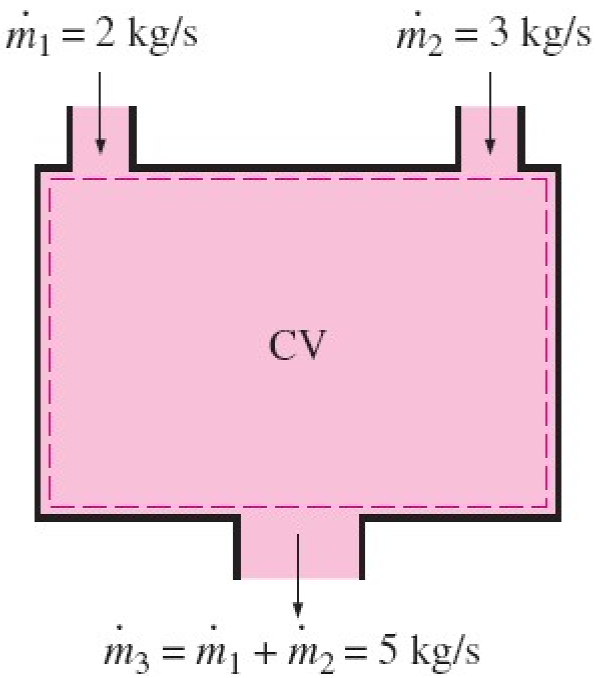
(b) Control surface *normal* to flow

A control surface should always be selected *normal to the flow* at all locations where it crosses the fluid flow to avoid complications, even though the result is the same.

Mass Balance for Steady-Flow Processes

During a steady-flow process, the total amount of mass contained within a control volume does not change with time (m_{CV} = constant).

Then the conservation of mass principle requires that the total amount of mass entering a control volume equal the total amount of mass leaving it.



Conservation of mass principle for a two-inlet–one-outlet steady-flow system.

For steady-flow processes, we are interested in the amount of mass flowing per unit time, that is, *the mass flow rate*.

$$\sum_{\text{in}} \dot{m} = \sum_{\text{out}} \dot{m} \qquad \text{(kg/s)}$$

Multiple inlets and exits

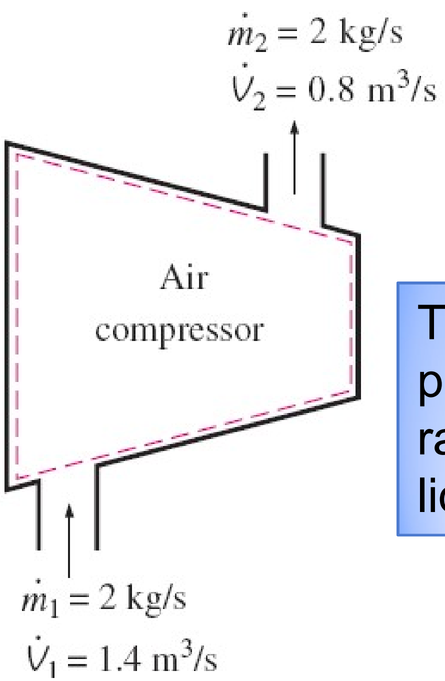
$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$$

Single stream

Many engineering devices such as nozzles, diffusers, turbines, compressors, and pumps involve a single stream (only one inlet and one outlet).

SPECIAL CASE: INCOMPRESSIBLE FLOW

The conservation of mass relations can be simplified even further when the fluid is incompressible, which is usually the case for liquids.



$$\sum_{in} \dot{V} = \sum_{out} \dot{V} \qquad (m^3/s)$$

Steady, incompressible

$$\dot{V}_1 = \dot{V}_2 \rightarrow V_1 A_1 = V_2 A_2$$

Steady, incompressible flow (single stream)

There is no such thing as a "conservation of volume" principle. However, for steady flow of liquids the volume flow rates, as well as the mass flow rates, remain constant since liquids are essentially incompressible substances.

During a steady-flow process, volume flow rates are not necessarily conserved although mass flow rates are.

The conservation of mass principle is based on experimental observations and requires every bit of mass to be accounted for during a process. If you can balance your checkbook (by keeping track of deposits and withdrawals, or by simply observing the "conservation of money" principle), you should have no difficulty applying the conservation of mass principle to engineering systems.

EXAMPLE 5–1

A garden hose attached with a nozzle is used to fill a 10-gal bucket. The inner diameter of the hose is 2 cm, and it reduces to 0.8 cm at the nozzle exit. If it takes 50 s to fill the bucket with water, determine (*a*) the volume and mass flow rates of water through the hose, and (*b*) the average velocity of water at the nozzle exit.

Solution A garden hose is used to fill a water bucket. The volume and mass flow rates of water and the exit velocity are to be determined.

Assumptions **1** Water is an incompressible substance. **2** Flow through the hose is steady. **3** There is no waste of water by splashing.

(*a*) Noting that 10 gal of water are discharged in 50 s, the volume and mass flow rates of water are

$$\dot{V} = \frac{V}{\Delta t} = \frac{10\text{ gal}}{50\text{ s}}\left(\frac{3.7854\text{ L}}{1\text{ gal}}\right) = \mathbf{0.757\ L/s}$$

$$\dot{m} = \rho\dot{V} = (1\text{ kg/L})(0.757\text{ L/s}) = \mathbf{0.757\ kg/s}$$

(*b*) The cross-sectional area of the nozzle exit is

$$A_e = \pi r_e^2 = \pi(0.4\text{ cm})^2 = 0.5027\text{ cm}^2 = 0.5027 \times 10^{-4}\text{ m}^2$$

$$V_e = \frac{\dot{V}}{A_e} = \frac{0.757\text{ L/s}}{0.5027 \times 10^{-4}\text{ m}^2}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right) = \mathbf{15.1\ m/s}$$

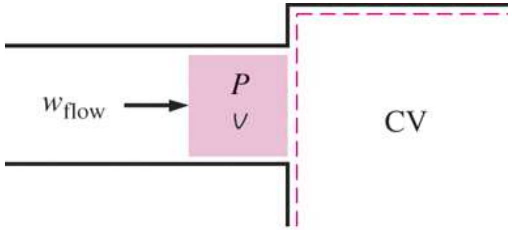
Discussion It can be shown that the average velocity in the hose is 2.4 m/s. Therefore, the nozzle increases the water velocity by over six times.

Flow work, or flow energy: The work (or energy) required to push the mass into or out of the control volume. This work is necessary for maintaining a continuous flow through a control volume.

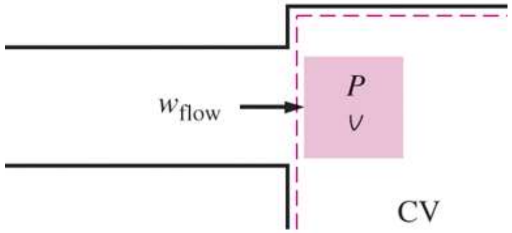
$$F = PA$$

$$W_{\text{flow}} = FL = PAL = PV \qquad (\text{kJ})$$

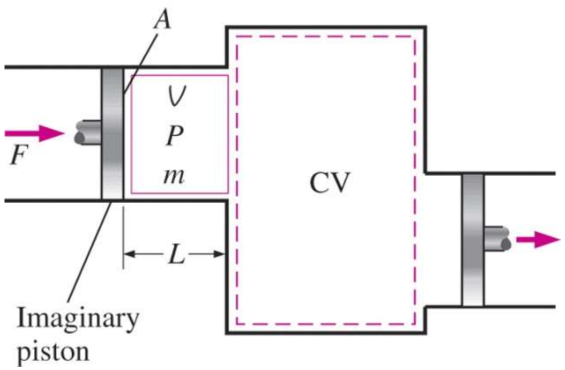
$$w_{\text{flow}} = Pv \qquad (\text{kJ/kg})$$



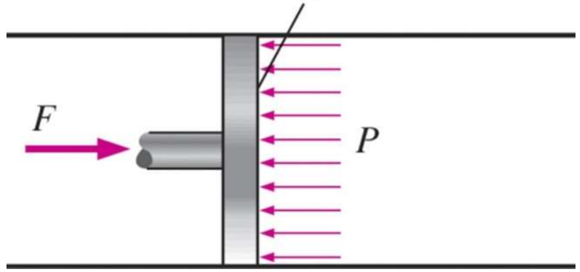
(a) Before entering



(b) After entering



Schematic for flow work.



In the absence of acceleration, the force applied on a fluid by a piston is equal to the force applied on the piston by the fluid.

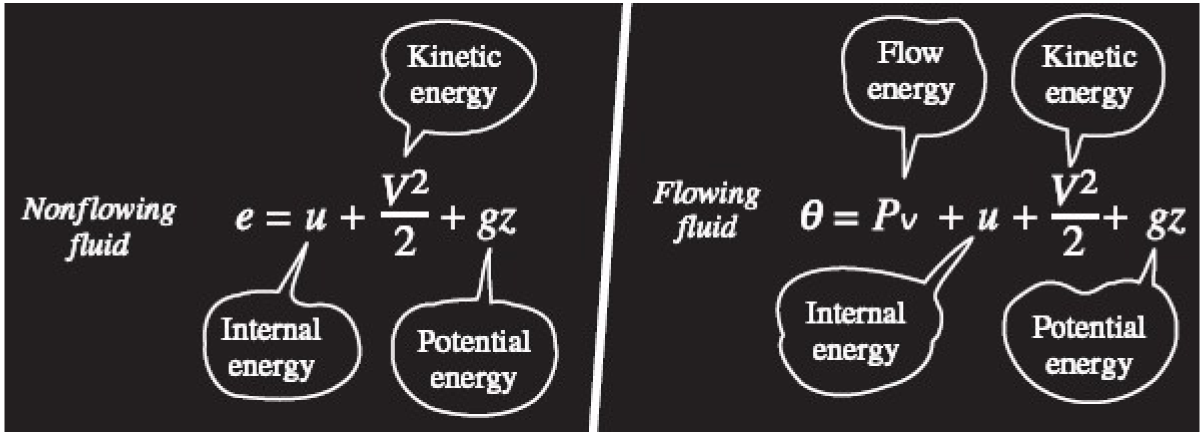
$$e = u + \text{ke} + \text{pe} = u + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)}$$

$$\theta = Pv + e = Pv + (u + \text{ke} + \text{pe})$$

$$h = u + Pv$$

$$\theta = h + \text{ke} + \text{pe} = h + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)}$$

The flow energy is automatically taken care of by enthalpy. In fact, this is the main reason for defining the property enthalpy.



The total energy consists of three parts for a nonflowing fluid and four parts for a flowing fluid.

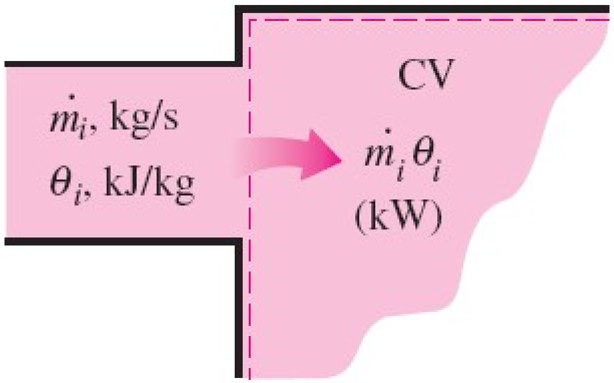
ENERGY TRANSPORT BY MASS

$$\textit{Amount of energy transport:} \quad E_{\text{mass}} = m\theta = m\left(h + \frac{V^2}{2} + gz\right) \quad \text{(kJ)}$$

$$\textit{Rate of energy transport:} \quad \dot{E}_{\text{mass}} = \dot{m}\theta = \dot{m}\left(h + \frac{V^2}{2} + gz\right) \quad \text{(kW)}$$

When the kinetic and potential energies of a fluid stream are negligible

$$E_{\text{mass}} = mh \qquad \dot{E}_{\text{mass}} = \dot{m}h$$



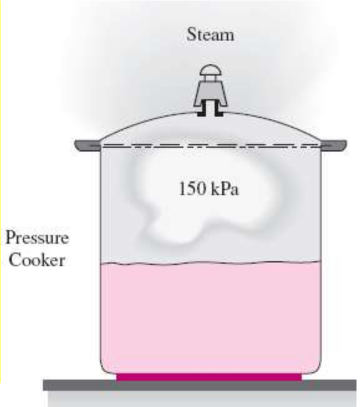
When the properties of the mass at each inlet or exit change with time as well as over the cross section

$$E_{\text{in, mass}} = \int_{m_i} \theta_i \, \delta m_i = \int_{m_i} \left(h_i + \frac{V_i^2}{2} + gz_i\right) \delta m_i$$

The product $\dot{m}_i\theta_i$ is the energy transported into control volume by mass per unit time.

EXAMPLE 5–3

Steam is leaving a 4-L pressure cooker whose operating pressure is 150 kPa. It is observed that the amount of liquid in the cooker has decreased by 0.6 L in 40 min after the steady operating conditions are established, and the cross-sectional area of the exit opening is 8 mm^2. Determine (*a*) the mass flow rate of the steam and the exit velocity, (*b*) the total and flow energies of the steam per unit mass, and (*c*) the rate at which energy is leaving the cooker by steam.



Solution Steam is leaving a pressure cooker at a specified pressure. The velocity, flow rate, the total and flow energies, and the rate of energy transfer by mass are to be determined.

Assumptions **1** The flow is steady, and the initial start-up period is disregarded. **2** The kinetic and potential energies are negligible and thus they are not considered. **3** Saturation conditions exist within the cooker at all times so that steam leaves the cooker as a saturated vapor at the cooker pressure.

(*a*) the mass flow rate of the steam and the exit velocity,

$$m = \frac{\Delta V_{\text{liquid}}}{v_f} = \frac{0.6\text{ L}}{0.001053\text{ m}^3/\text{kg}}\left(\frac{1\text{ m}^3}{1000\text{ L}}\right) = 0.570\text{ kg}$$

$$\dot{m} = \frac{m}{\Delta t} = \frac{0.570\text{ kg}}{40\text{ min}} = 0.0142\text{ kg/min} = \mathbf{2.37 \times 10^{-4}\ kg/s}$$

$$V = \frac{\dot{m}}{\rho_g A_c} = \frac{\dot{m} v_g}{A_c} = \frac{(2.37 \times 10^{-4}\text{ kg/s})(1.1594\text{ m}^3/\text{kg})}{8 \times 10^{-6}\text{ m}^2} = \mathbf{34.3\ m/s}$$

(*b*) the total and flow energies of the steam per unit mass

$$e_{\text{flow}} = Pv = h - u = 2693.1 - 2519.2 = \mathbf{173.9\ kJ/kg}$$

$$\theta = h + \text{ke} + \text{pe} \cong h = \mathbf{2693.1\ kJ/kg}$$

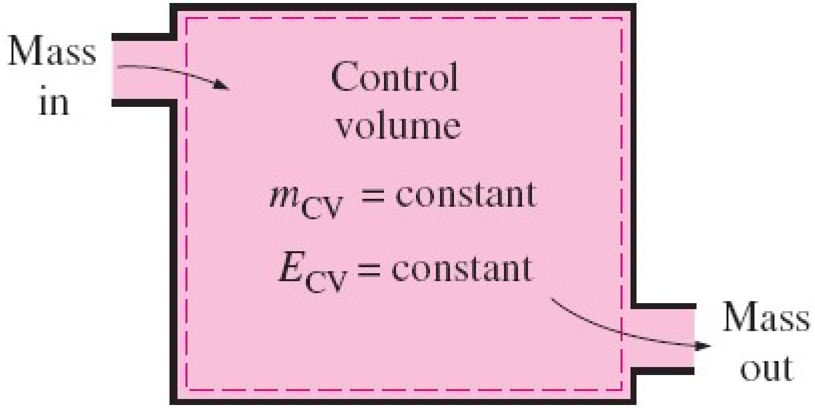
The kinetic energy in this case is ke=$V^2/2$ = $(34.3\ \text{m/s})^2/2$ = 588 m^2/s^2 =0.588 kJ/kg, which is small compared to enthalpy.

(*c*) The rate at which energy is leaving the cooker by mass is simply the product of the mass flow rate and the total energy of the exiting steam per unit mass,

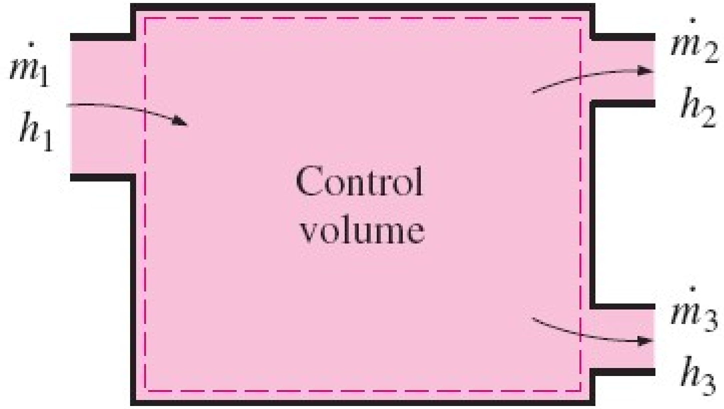
$$\dot{E}_{\text{mass}} = \dot{m}\theta = (2.37 \times 10^{-4}\ \text{kg/s})(2693.1\ \text{kJ/kg}) = 0.638\ \text{kJ/s} = \mathbf{0.638\ kW}$$

Discussion The numerical value of the energy leaving the cooker with steam alone does not mean much since this value depends on the reference point selected for enthalpy (it could even be negative). The significant quantity is the difference between the enthalpies of the exiting vapor and the liquid inside (which is h_{fg}) since it relates directly to the amount of energy supplied to the cooker.

Many engineering systems such as power plants operate under steady conditions.



Under steady-flow conditions, the mass and energy contents of a control volume remain constant.

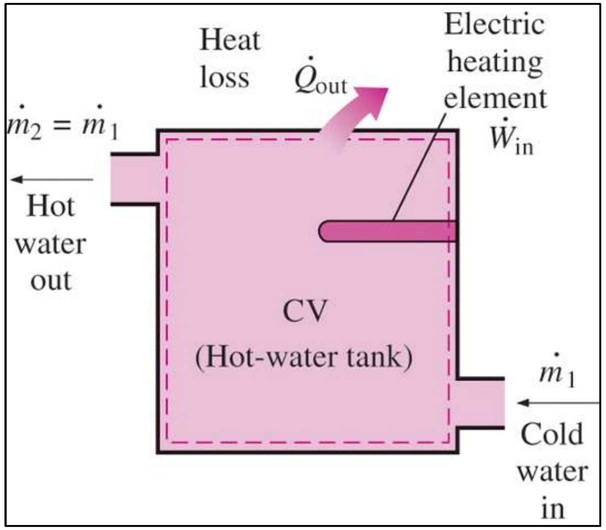


Under steady-flow conditions, the fluid properties at an inlet or exit remain constant (do not change with time).

Mass balance for steady-flow systems: $\dot{m}_{\text{in}} = \dot{m}_{\text{out}} \quad \text{(kg/s)}$

Multiple inlets and exits: $\sum \dot{m}_i = \sum \dot{m}_e \quad \text{(kg/s)}$

One inlet and one exit: $\dot{m}_1 = \dot{m}_2 \quad \text{or} \quad \rho_1 V_1 A_1 = \rho_2 V_2 A_2$



A water heater in steady operation.

Energy balance

$$\underbrace{\dot{E}_{\text{in}} - \dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer}\\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{\text{system}}^{\nearrow 0 \text{ (steady)}}}_{\substack{\text{Rate of change in internal, kinetic,}\\ \text{potential, etc., energies}}} = 0$$

$$\underbrace{\dot{E}_{\text{in}}}_{\substack{\text{Rate of net energy transfer in}\\ \text{by heat, work, and mass}}} = \underbrace{\dot{E}_{\text{out}}}_{\substack{\text{Rate of net energy transfer out}\\ \text{by heat, work, and mass}}} \quad \text{(kW)}$$

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum \dot{m}_i \theta_i = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum \dot{m}_e \theta_e$$

$$\dot{Q}_{\text{in}} + \dot{W}_{\text{in}} + \sum \underbrace{\dot{m}_i \left(h_i + \frac{V_i^2}{2} + g z_i \right)}_{\text{for each inlet}} = \dot{Q}_{\text{out}} + \dot{W}_{\text{out}} + \sum \underbrace{\dot{m}_e \left(h_e + \frac{V_e^2}{2} + g z_e \right)}_{\text{for each exit}}$$

$$\dot{Q} - \dot{W} = \underbrace{\sum \dot{m}_e\left(h_e + \frac{V_e^2}{2} + gz_e\right)}_{\text{for each exit}} - \underbrace{\sum \dot{m}_i\left(h_i + \frac{V_i^2}{2} + gz_i\right)}_{\text{for each inlet}}$$

Energy balance relations with sign conventions

(heat input and work output are positive)

$$\dot{Q} - \dot{W} = \dot{m}\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$q - w = h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)$$

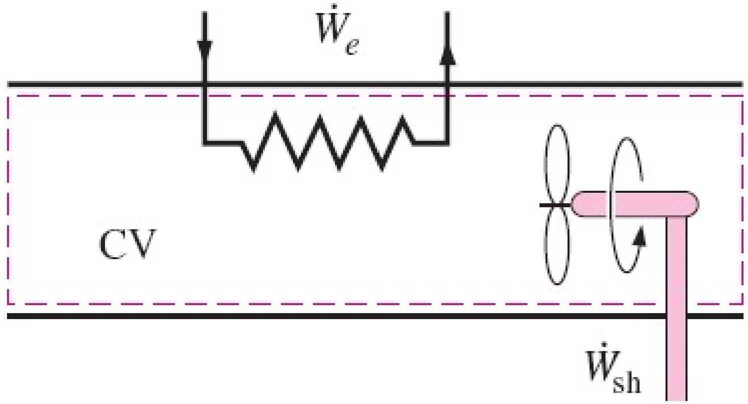
when kinetic and potential energy changes are negligible

$$q - w = h_2 - h_1$$

$$\frac{\mathrm{J}}{\mathrm{kg}} \equiv \frac{\mathrm{N \cdot m}}{\mathrm{kg}} \equiv \left(\mathrm{kg}\frac{\mathrm{m}}{\mathrm{s}^2}\right)\frac{\mathrm{m}}{\mathrm{kg}} \equiv \frac{\mathrm{m}^2}{\mathrm{s}^2}$$

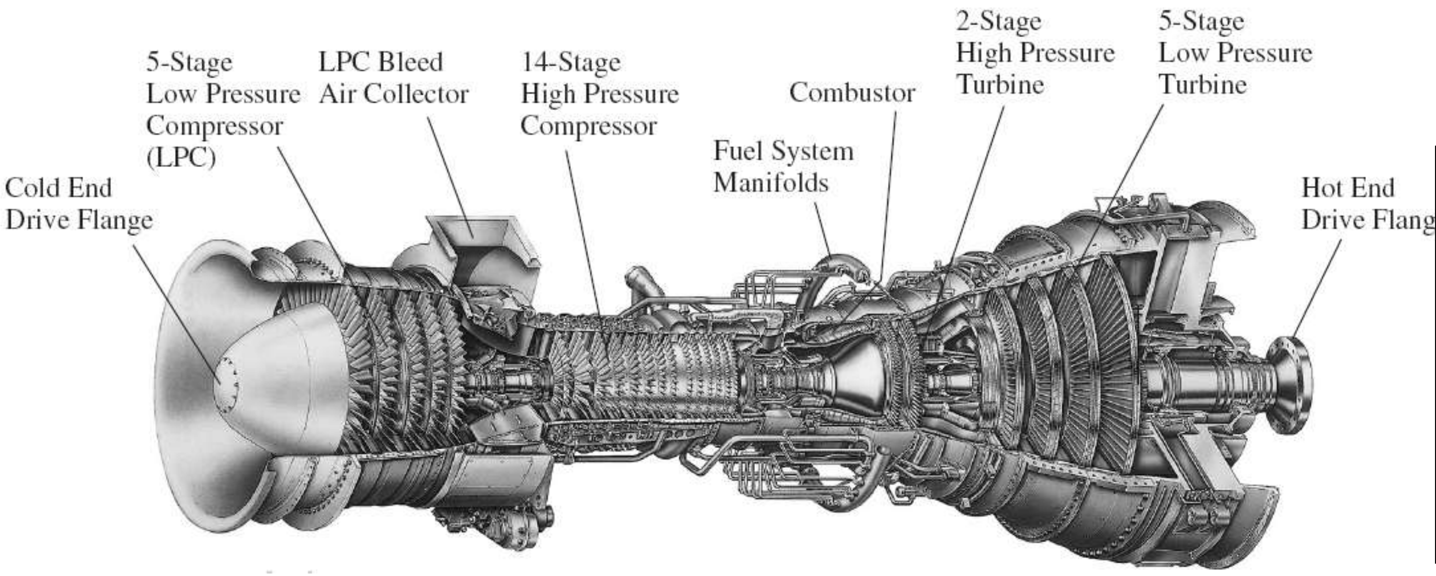
$$\left(\text{Also, } \frac{\mathrm{Btu}}{\mathrm{lbm}} \equiv 25{,}037\ \frac{\mathrm{ft}^2}{\mathrm{s}^2}\right)$$

Some energy unit equivalents



Under steady operation, shaft work and electrical work are the only forms of work a simple compressible system may involve.

Many engineering devices operate essentially under the same conditions for long periods of time. The components of a steam power plant (turbines, compressors, heat exchangers, and pumps), for example, operate nonstop for months before the system is shut down for maintenance. Therefore, these devices can be conveniently analyzed as steady-flow devices.

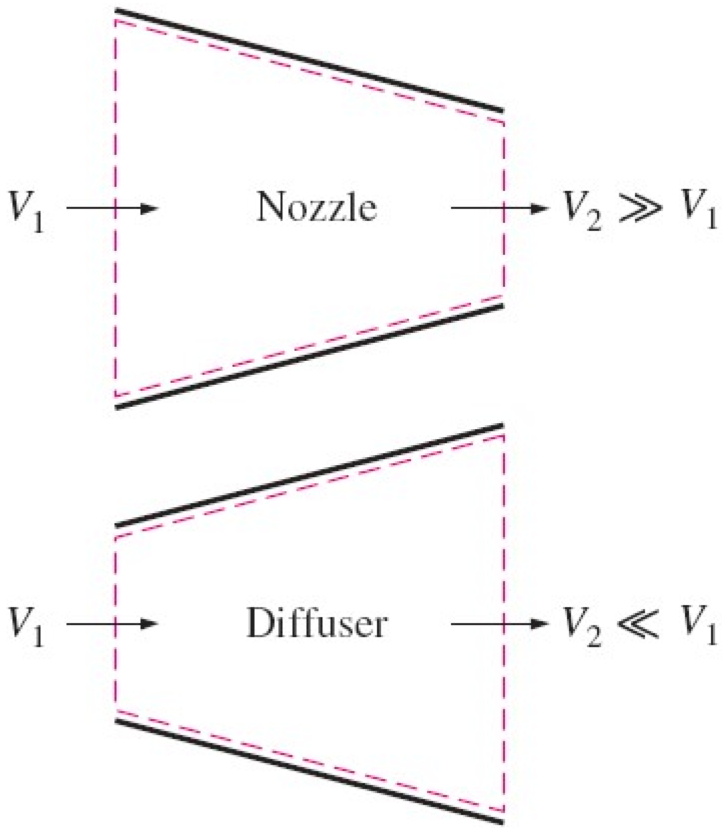


V_1 m/s	V_2 m/s	Δke kJ/kg
0	40	1
50	67	1
100	110	1
200	205	1
500	502	1

A modern land-based gas turbine used for electric power production. This is a General Electric LM5000 turbine. It has a length of 6.2 m, it weighs 12.5 tons, and produces 55.2 MW at 3600 rpm with steam injection.

At very high velocities, even small changes in velocities can cause significant changes in the kinetic energy of the fluid.

Nozzles and Diffusers



Nozzles and diffusers are shaped so that they cause large changes in fluid velocities and thus kinetic energies.

Nozzles and diffusers are commonly utilized in jet engines, rockets, spacecraft, and even garden hoses.

A **nozzle** is a device that *increases the velocity of a fluid* at the expense of pressure.

A **diffuser** is a device that *increases the pressure of a fluid* by slowing it down.

The cross-sectional area of a nozzle decreases in the flow direction for subsonic flows and increases for supersonic flows. The reverse is true for diffusers.

Energy balance for a nozzle or diffuser:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

Conservation of Energy

The conservation of energy principle (the energy balance): The net energy transfer to or from a system during a process be equal to the change in the energy content of the system.

Energy can be transferred to or from a closed system by heat or work.

Control volumes also involve energy transfer via mass flow.

Conservation of energy: $$\dot{E}_{\text{in}} - \dot{E}_{\text{out}} = \frac{dE_{\text{CV}}}{dt}$$

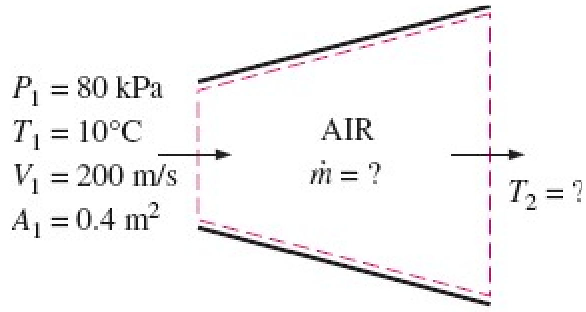
$\dot{E}_{\text{in}}$ and $\dot{E}_{\text{out}}$ the total rates of energy transfer into and out of the control volume

dE_{CV}/dt the rate of change of energy within the control volume boundaries

In fluid mechanics, we usually limit our consideration to mechanical forms of energy only.

EXAMPLE 5–4

Air at 10°C and 80 kPa enters the diffuser of a jet engine steadily with a velocity of 200 m/s. The inlet area of the diffuser is 0.4 m^2. The air leaves the diffuser with a velocity that is very small compared with the inlet velocity. Determine (*a*) the mass flow rate of the air and (*b*) the temperature of the air leaving the diffuser.



Solution Air enters the diffuser of a jet engine steadily at a specified velocity. The mass flow rate of air and the temperature at the diffuser exit are to be determined.
Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV}=0$ and $\Delta E_{CV}=0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** The potential energy change is zero, $\Delta pe=0$. **4** Heat transfer is negligible. **5** Kinetic energy at the diffuser exit is negligible. **6** There are no work interactions.

(*a*) To determine the mass flow rate, we need to find the specific volume of the air first. This is determined from the ideal-gas relation at the inlet conditions:

$$v_1 = \frac{RT_1}{P_1} = \frac{0.287\ \mathrm{kPa \cdot m^3/kg \cdot K})(283\ \mathrm{K})}{80\ \mathrm{kPa}} = 1.015\ \mathrm{m^3/kg}$$

$$\dot{m} = \frac{1}{v_1} V_1 A_1 = \frac{1}{1.015\ \mathrm{m^3/kg}} (200\ \mathrm{m/s})(0.4\ \mathrm{m^2}) = \mathbf{78.8\ kg/s}$$

(*b*) Under stated assumptions and observations, the energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}\left(h_1 + \frac{V_1^2}{2}\right) = \dot{m}\left(h_2 + \frac{V_2^2}{2}\right) \qquad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0, \text{ and } \Delta\text{pe} \cong 0)$$

$$h_2 = h_1 - \frac{V_2^2 - V_1^2}{2} \qquad h_1 = h_{@\,283\text{ K}} = 283.14 \text{ kJ/kg}$$

The exit velocity of a diffuser is usually small compared with the inlet velocity ($V_2 \ll V_1$); thus, the kinetic energy at the exit can be neglected. The enthalpy of air at the diffuser inlet is determined from the air table

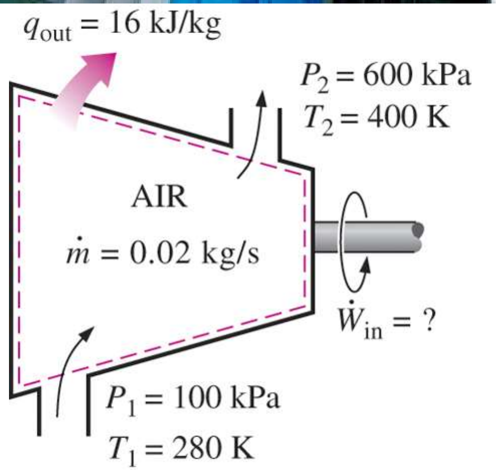
$$h_2 = 283.14 \text{ kJ/kg} - \frac{0 - (200 \text{ m/s})^2}{2}\left(\frac{1 \text{ kJ/kg}}{1000 \text{ m}^2/\text{s}^2}\right) = 303.14 \text{ kJ/kg}$$

the temperature corresponding to this enthalpy value is

$$T_2 = \mathbf{303\ K}$$

Discussion This result shows that the temperature of the air increased by about 20°C as it was slowed down in the diffuser. The temperature rise of the air is mainly due to the conversion of kinetic energy to internal energy.

Turbines and Compressors



Energy balance for the compressor in this figure:

Turbine drives the electric generator In steam, gas, or hydroelectric power plants.

As the fluid passes through the turbine, work is done against the blades, which are attached to the shaft. As a result, the shaft rotates, and the turbine produces work.

Compressors, as well as **pumps** and **fans**, are devices used to increase the pressure of a fluid. Work is supplied to these devices from an external source through a rotating shaft.

A ***fan*** increases the pressure of a gas slightly and is mainly used to mobilize a gas.

A ***compressor*** is capable of compressing the gas to very high pressures.

Pumps work very much like compressors except that they handle liquids instead of gases.

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

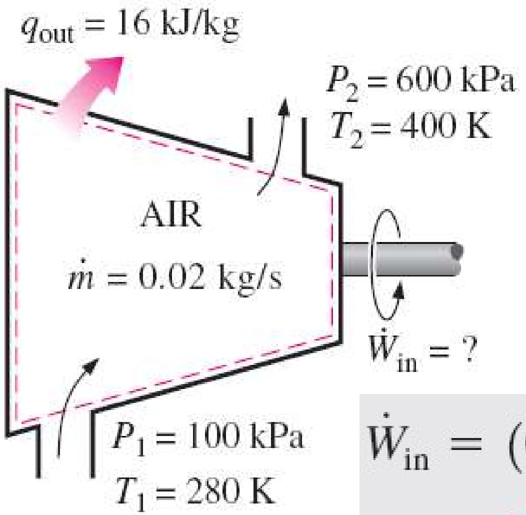
$$(\text{since } \Delta\text{ke} = \Delta\text{pe} \cong 0)$$

EXAMPLE 5–6

Air at 100 kPa and 280 K is compressed steadily to 600 kPa and 400 K. The mass flow rate of the air is 0.02 kg/s, and a heat loss of 16 kJ/kg occurs during the process. Assuming the changes in kinetic and potential energies are negligible, determine the necessary power input to the compressor.

Solution Air is compressed steadily by a compressor to a specified temperature and pressure. The power input to the compressor is to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV}=0$ and $\Delta E_{CV}=0$. **2** Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. **3** $\Delta ke = \Delta pe = 0$.



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \qquad (\text{since } \Delta\text{ke} = \Delta\text{pe} \cong 0)$$

$$\dot{W}_{in} = \dot{m}q_{out} + \dot{m}(h_2 - h_1)$$

$$h_1 = h_{@\,280\text{ K}} = 280.13 \text{ kJ/kg}$$

$$h_2 = h_{@\,400\text{ K}} = 400.98 \text{ kJ/kg}$$

$$\dot{W}_{in} = (0.02 \text{ kg/s})(16 \text{ kJ/kg}) + (0.02 \text{ kg/s})(400.98 - 280.13) \text{ kJ/kg}$$
$$= \mathbf{2.74\ kW}$$

Discussion Note that the mechanical energy input to the compressor manifests itself as a rise in enthalpy of air and heat loss from the compressor.

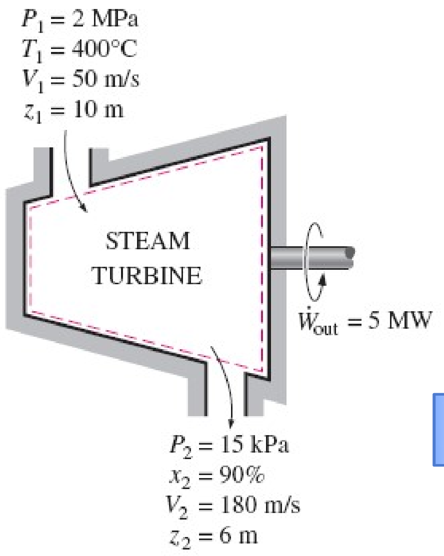
EXAMPLE 5-7

The power output of an adiabatic steam turbine is 5 MW, and the inlet and the exit conditions of the steam are;

(*a*) Compare the magnitudes of Δh, Δke, and Δpe.

(*b*) Determine the work done per unit mass of the steam flowing through the turbine.

(*c*) Calculate the mass flow rate of the steam.



Solution The inlet and exit conditions of a steam turbine and its power output are given. The changes in kinetic energy, potential energy, and enthalpy of steam, as well as the work done per unit mass and the mass flow rate of steam are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV}=0$ and $\Delta E_{CV}=0$. **2** The system is adiabatic and thus there is no heat transfer.

(*a*) At the inlet, steam is in a superheated vapor state, and its enthalpy is

$$\left.\begin{array}{l} P_1 = 2 \text{ MPa} \\ T_1 = 400°\text{C} \end{array}\right\} \quad h_1 = 3248.4 \text{ kJ/kg} \qquad \text{(Table A–6)}$$

At the turbine exit, we obviously have a saturated liquid–vapor mixture at 15-kPa pressure. The enthalpy at this state is

$$h_2 = h_f + x_2 h_{fg} = [225.94 + (0.9)(2372.3)] \text{ kJ/kg} = 2361.01 \text{ kJ/kg}$$

$$\Delta h = h_2 - h_1 = (2361.01 - 3248.4)\ \text{kJ/kg} = \mathbf{-887.39\ kJ/kg}$$

$$\Delta ke = \frac{V_2^2 - V_1^2}{2} = \frac{(180\ \text{m/s})^2 - (50\ \text{m/s})^2}{2}\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{14.95\ kJ/kg}$$

$$\Delta pe = g(z_2 - z_1) = (9.81\ \text{m/s}^2)[(6 - 10)\ \text{m}]\left(\frac{1\ \text{kJ/kg}}{1000\ \text{m}^2/\text{s}^2}\right) = \mathbf{-0.04\ kJ/kg}$$

(*b*) The energy balance for this steady-flow system can be expressed in the rate form as

$$\dot{m}\left(h_1 + \frac{V_1^2}{2} + gz_1\right) = \dot{W}_{out} + \dot{m}\left(h_2 + \frac{V_2^2}{2} + gz_2\right) \qquad (\text{since } \dot{Q} = 0)$$

$$w_{out} = -\left[(h_2 - h_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right] = -(\Delta h + \Delta ke + \Delta pe)$$

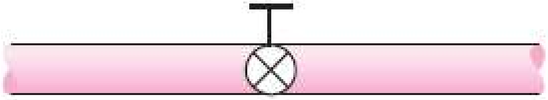
$$= -[-887.39 + 14.95 - 0.04]\ \text{kJ/kg} = \mathbf{872.48\ kJ/kg}$$

(*c*) The required mass flow rate for a 5-MW power output is

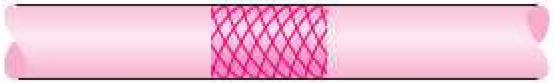
$$\dot{m} = \frac{\dot{W}_{out}}{w_{out}} = \frac{5000\ \text{kJ/s}}{872.48\ \text{kJ/kg}} = \mathbf{5.73\ kg/s}$$

Discussion Two observations; First, the change in potential energy is insignificant in comparison to the changes in enthalpy and kinetic energy. Second, as a result of low pressure and thus high specific volume, the steam velocity at the turbine exit can be very high.

Throttling valves



(*a*) An adjustable valve



(*b*) A porous plug

(*c*) A capillary tube

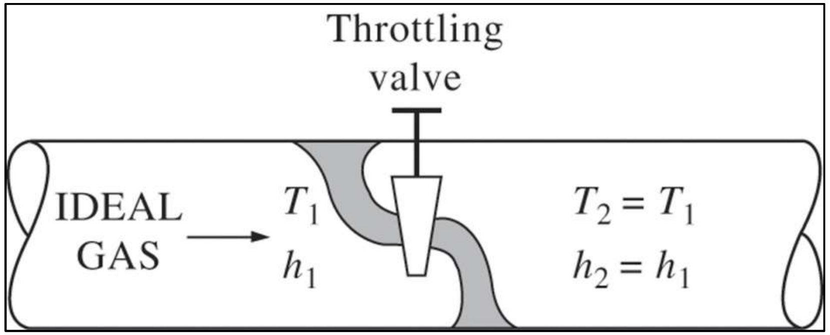
Throttling valves are *any kind of flow-restricting devices* that cause a significant pressure drop in the fluid.

What is the difference between a turbine and a throttling valve?

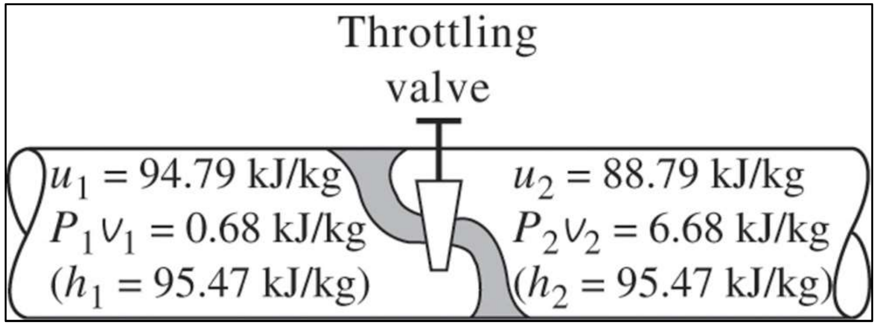
The pressure drop in the fluid is often accompanied by a *large drop in temperature*, and for that reason throttling devices are commonly used in refrigeration and air-conditioning applications.

$$h_2 \cong h_1 \qquad \text{(kJ/kg)} \qquad u_1 + P_1 v_1 = u_2 + P_2 v_2$$

$$\text{Internal energy} + \text{Flow energy} = \text{Constant}$$



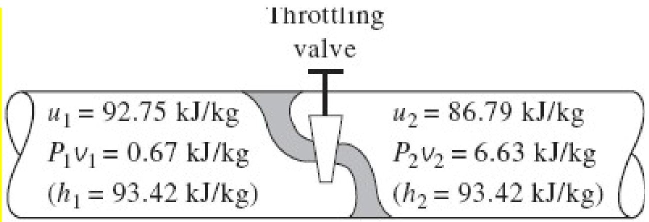
The temperature of an ideal gas does not change during a throttling (h = cons.) process since $h = h(T)$.



During a throttling process, the enthalpy of a fluid remains constant. But internal and flow energies may be converted to each other.

EXAMPLE 5–8

Refrigerant-134a enters the capillary tube of a refrigerator as saturated liquid at 0.8 MPa and is throttled to a pressure of 0.12 MPa. Determine the quality of the refrigerant at the final state and the temperature drop during this process.



Solution Refrigerant-134a that enters a capillary tube as saturated liquid is throttled to a specified pressure.

Assumptions Heat transfer and Kinetic energy change of the refrigerant is negligible.

$$\text{At inlet: } \left.\begin{array}{l} P_1 = 0.8 \text{ MPa} \\ \text{sat. liquid} \end{array}\right\} \quad \begin{array}{l} T_1 = T_{\text{sat @ 0.8 MPa}} = 31.31°\text{C} \\ h_1 = h_{f \text{ @ 0.8 MPa}} = 95.47 \text{ kJ/kg} \end{array}$$

$$\text{At exit: } \begin{array}{l} P_2 = 0.12 \text{ MPa} \\ (h_2 = h_1) \end{array} \longrightarrow \begin{array}{l} h_f = 22.49 \text{ kJ/kg} \quad T_{\text{sat}} = -22.32°\text{C} \\ h_g = 236.97 \text{ kJ/kg} \end{array}$$

The quality at this state is

$$x_2 = \frac{h_2 - h_f}{h_{fg}} = \frac{95.47 - 22.49}{236.97 - 22.49} = \mathbf{0.340}$$

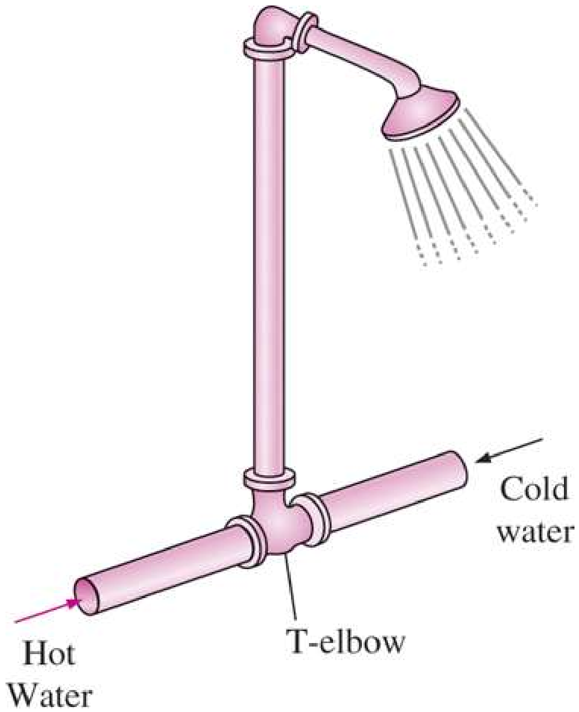
Since the exit state is a saturated mixture at 0.12 MPa, the exit temperature must be the saturation temperature at this pressure, which is 22.32°C.

$$\Delta T = T_2 - T_1 = (-22.32 - 31.31)°\text{C} = \mathbf{-53.63°C}$$

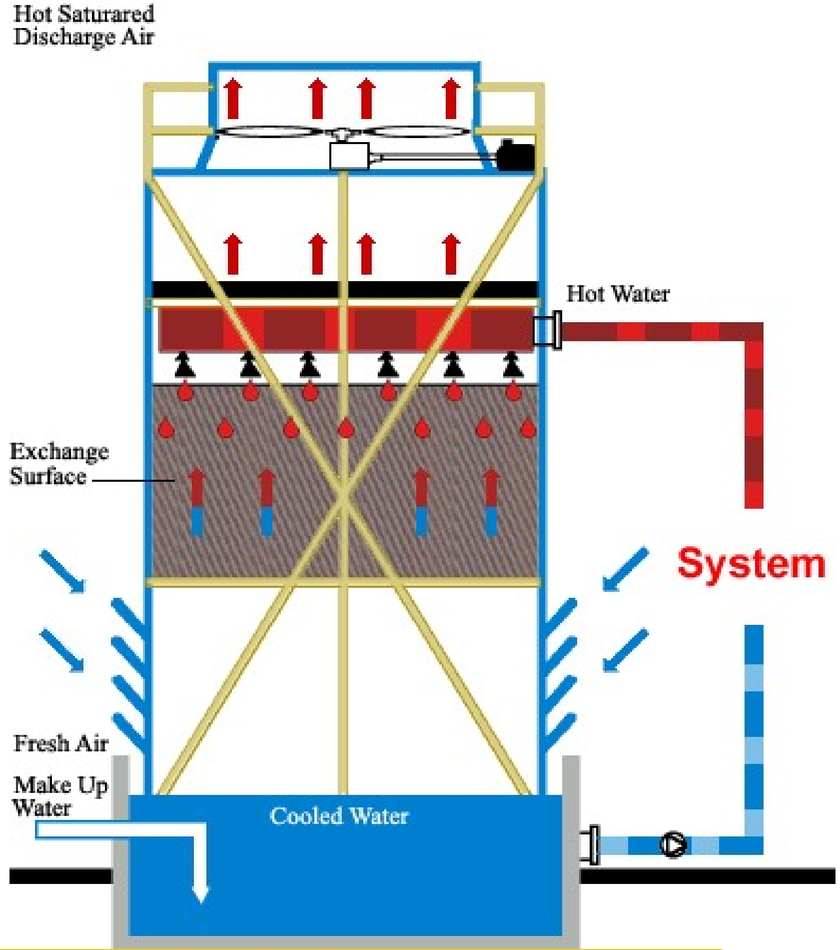
Discussion 34.0 % of the refrigerant vaporizes during this throttling process, and the energy needed to vaporize this refrigerant is absorbed from the refrigerant itself.

Mixing chambers

In engineering applications, the section where the mixing process takes place is commonly referred to as a **mixing chamber**.



The T-elbow of an ordinary shower serves as the mixing chamber for the hot- and the cold-water streams.



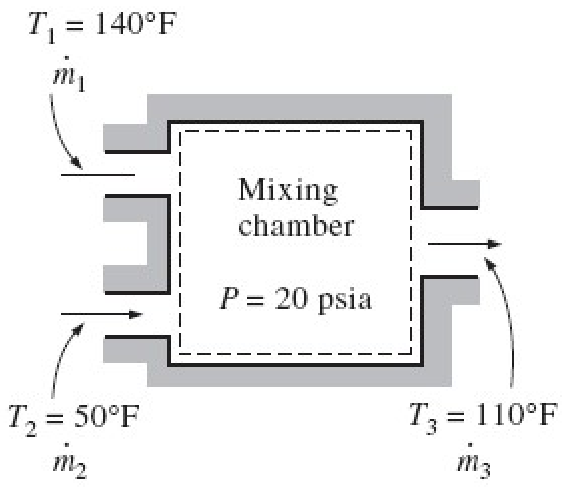
Energy balance for the adiabatic mixing chamber in the figure is:

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3$$

$$(\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

EXAMPLE 5-8

Consider an ordinary shower where hot water at 140°F is mixed with cold water at 50°F. If it is desired that a steady stream of warm water at 110°F be supplied, determine the ratio of the mass flow rates of the hot to cold water. Assume the heat losses from the mixing chamber to be negligible and the mixing to take place at a pressure of 20 psia.



$$\textit{Mass balance:} \qquad \dot{m}_{in} - \dot{m}_{out} = \Delta \dot{m}_{system}^{\nearrow 0 \text{ (steady)}} = 0$$

$$\dot{m}_{in} = \dot{m}_{out} \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (\text{since } \dot{Q} \cong 0, \dot{W} = 0, \text{ke} \cong \text{pe} \cong 0)$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = (\dot{m}_1 + \dot{m}_2) h_3 \qquad y h_1 + h_2 = (y + 1) h_3$$

where $y = \dot{m}_1 / \dot{m}_2$ is the desired mass flow rate ratio.

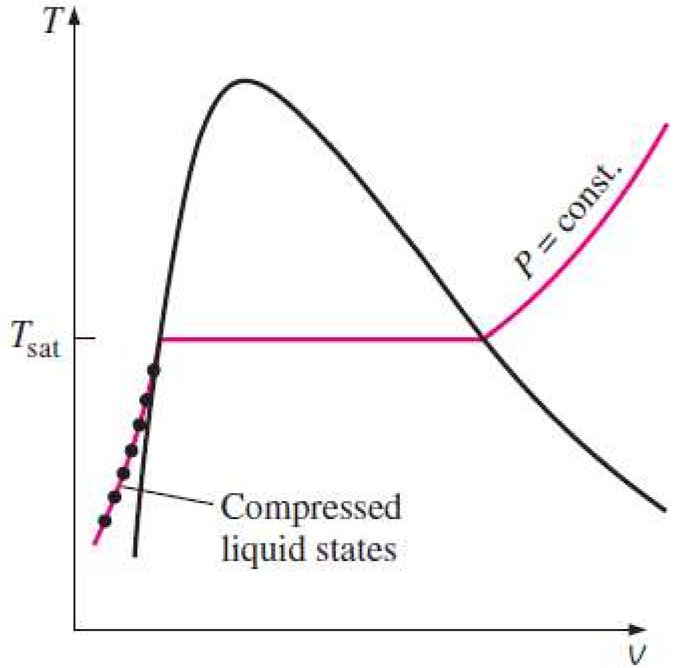
$$h_1 \cong h_{f\,@\,140°\mathrm{F}} = 107.96 \text{ Btu/lbm}$$

$$h_2 \cong h_{f\,@\,50°\mathrm{F}} = 18.07 \text{ Btu/lbm}$$

$$h_3 \cong h_{f\,@\,110°\mathrm{F}} = 78.02 \text{ Btu/lbm}$$

$$y = \frac{h_3 - h_2}{h_1 - h_3} = \frac{78.02 - 18.07}{107.99 - 78.02} = \mathbf{2.0}$$

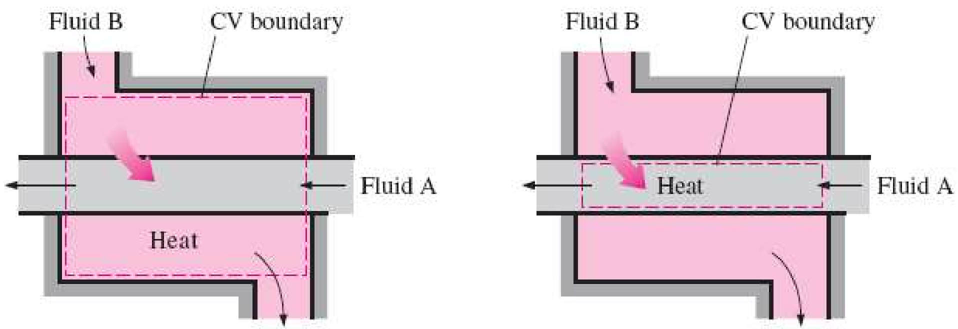
Discussion Note that the mass flow rate of the hot water must be twice the mass flow rate of the cold water for the mixture to leave at 110°F.



A substance exists as a compressed liquid at temperatures below the saturation temperatures at the given pressure.

Heat exchangers

Heat exchangers are devices where two moving fluid streams exchange heat without mixing. Heat exchangers are widely used in various industries, and they come in various designs.

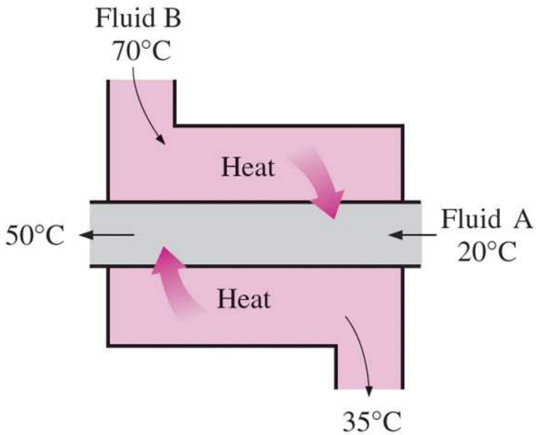


(*a*) System: Entire heat exchanger ($Q_{CV} = 0$)

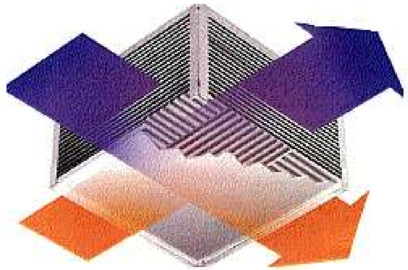
(*b*) System: Fluid A ($Q_{CV} \neq 0$)

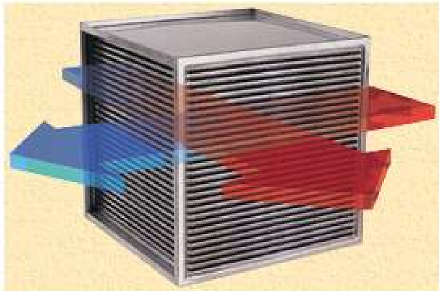
The heat transfer associated with a heat exchanger may be zero or nonzero depending on how the control volume is selected.

Mass and energy balances for the adiabatic heat exchanger in the figure is:



A heat exchanger can be as simple as two concentric pipes.





$$\dot{E}_{in} = \dot{E}_{out}$$

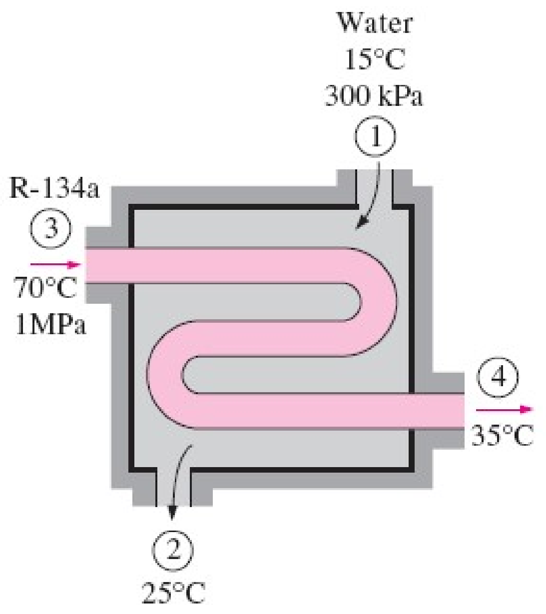
$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_2 h_2 + \dot{m}_4 h_4$$

EXAMPLE 5–10

Refrigerant-134a is to be cooled by water in a condenser. The refrigerant enters the condenser with a mass flow rate of 6 kg/min at 1 MPa and 70°C and leaves at 35°C. The cooling water enters at 300 kPa and 15°C and leaves at 25°C. Neglecting any pressure drops, determine (*a*) the mass flow rate of the cooling water required and (*b*) the heat transfer rate from the refrigerant to water.



Solution Refrigerant-134a is cooled by water in a condenser. The mass flow rate of the cooling water and the rate of heat transfer from the refrigerant to the water are to be determined.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV}=0$ and $\Delta E_{CV}=0$. **2** The kinetic and potential energies are negligible, $\Delta ke \cong \Delta pe \cong 0$. **3** Heat losses from the system are negligible and thus $\dot{Q} \cong 0$. **4** There is no work interaction.

(*a*) Under the stated assumptions and observations, the mass and energy balances for this steady-flow system can be expressed in the rate form as follows

Mass balance: $\dot{m}_{in} = \dot{m}_{out}$

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_w$$

$$\dot{m}_3 = \dot{m}_4 = \dot{m}_R$$

$$\dot{m}_1 h_1 + \dot{m}_3 h_3 = \dot{m}_3 h_3 + \dot{m}_4 h_4 \qquad (\text{since } \dot{Q} \cong 0,\ \dot{W} = 0,\ \text{ke} \cong \text{pe} \cong 0)$$

$$\dot{m}_w (h_1 - h_2) = \dot{m}_R (h_4 - h_3)$$

$$h_1 \cong h_{f @ 15^\circ C} = 62.982 \text{ kJ/kg}$$

$$h_2 \cong h_{f @ 25^\circ C} = 104.83 \text{ kJ/kg}$$

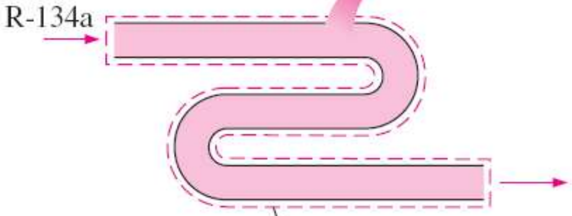
$$\left.\begin{array}{l} P_3 = 1 \text{ MPa} \\ T_3 = 70^\circ\text{C} \end{array}\right\} \quad h_3 = 303.85 \text{ kJ/kg}$$

$$\left.\begin{array}{l} P_4 = 1 \text{ MPa} \\ T_4 = 35^\circ\text{C} \end{array}\right\} \quad h_4 \cong h_{f @ 35^\circ C} = 100.87 \text{ kJ/kg}$$

$$\dot{m}_w (62.982 - 104.83) \text{ kJ/kg} = (6 \text{ kg/min})[(100.87 - 303.85) \text{ kJ/kg}]$$

$$\dot{m}_w = \mathbf{29.10 \text{ kg/min}}$$

(*b*) the heat transfer rate from the refrigerant to water.



$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{Q}_{w,\text{ in}} + \dot{m}_w h_1 = \dot{m}_w h_2$$

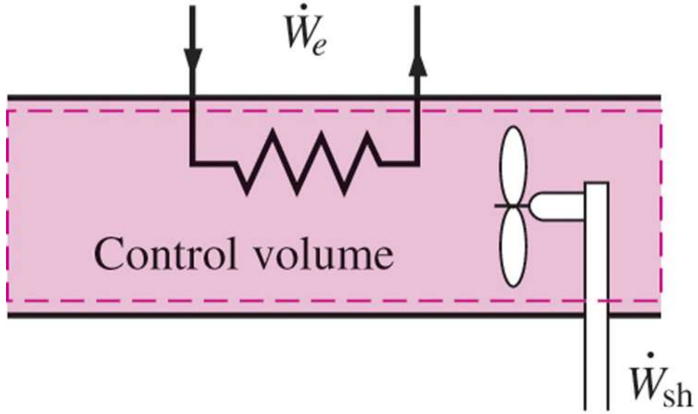
$$\dot{Q}_{w,\text{ in}} + \dot{m}_w (h_2 - h_1) = (29.10 \text{ kg/min})[(104.83 - 62.982) \text{ kJ/kg}]$$

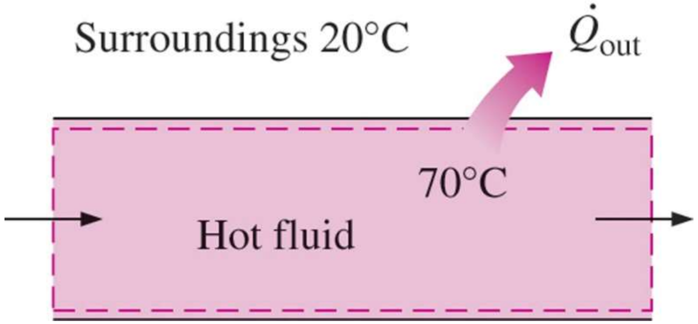
$$= \mathbf{1218 \text{ kJ/min}}$$

Discussion Had we chosen the volume occupied by the refrigerant as the control volume, we would have obtained the same result for $Q_{R,out}$ since the heat gained by the water is equal to the heat lost by the refrigerant.

Pipe and duct flow

The transport of liquids or gases in pipes and ducts is of great importance in many engineering applications. Flow through a pipe or a duct usually satisfies the steady-flow conditions.





Pipe or duct flow may involve more than one form of work at the same time.

Heat losses from a hot fluid flowing through an uninsulated pipe or duct to the cooler environment may be very significant.

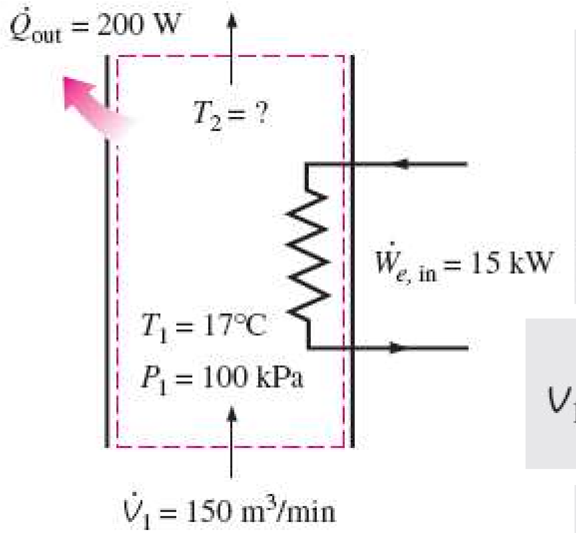
$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,\,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2$$

$$\dot{W}_{e,\,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$

EXAMPLE 5–11

The electric heating systems used in many houses consist of a simple duct with resistance wires. Air is heated as it flows over resistance wires. Consider a 15-kW electric heating system. Air enters the heating section at 100 kPa and 17°C with a volume flow rate of 150 m³/min. If heat is lost from the air in the duct to the surroundings at a rate of 200 W, determine the exit temperature of air.



$$\dot{E}_{in} = \dot{E}_{out}$$

$$\dot{W}_{e,\,in} + \dot{m}h_1 = \dot{Q}_{out} + \dot{m}h_2 \qquad (\text{since } \Delta\text{ke} \cong \Delta\text{pe} \cong 0)$$

$$\dot{W}_{e,\,in} - \dot{Q}_{out} = \dot{m}c_p(T_2 - T_1)$$

$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(290\ \text{K})}{100\ \text{kPa}} = 0.832\ \text{m}^3/\text{kg}$$

$$\dot{m} = \frac{\dot{V}_1}{v_1} = \frac{150\ \text{m}^3/\text{min}}{0.832\ \text{m}^3/\text{kg}}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 3.0\ \text{kg/s}$$

$$(15\ \text{kJ/s}) - (0.2\ \text{kJ/s}) = (3\ \text{kg/s})(1.005\ \text{kJ/kg}\cdot{}^\circ\text{C})(T_2 - 17)^\circ\text{C}$$

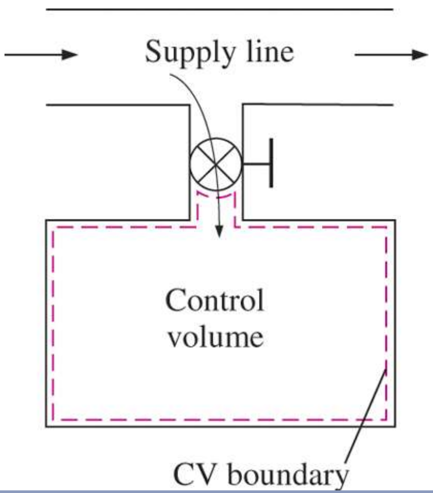
$$T_2 = \mathbf{21.9^\circ C}$$

Discussion Note that heat loss from the duct reduces the exit temperature of air.

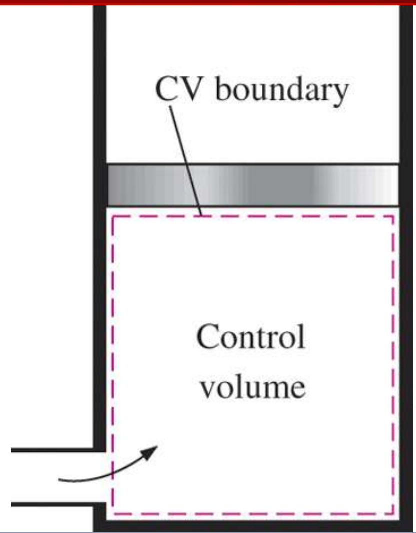
Many processes of interest, however, involve *changes* within the control volume with time. Such processes are called *unsteady-flow,* or *transient-flow,* processes.

Most unsteady-flow processes can be represented reasonably well by the *uniform-flow process*.

Uniform-flow process: The fluid flow at any inlet or exit is uniform and steady, and thus the fluid properties do not change with time or position over the cross section of an inlet or exit. If they do, they are averaged and treated as constants for the entire process.



Charging of a rigid tank from a supply line is an unsteady-flow process since it involves changes within the control volume.



The shape and size of a control volume may change during an unsteady-flow process.

Mass balance

Mass balance: $m_{in} - m_{out} = \Delta m_{system}$

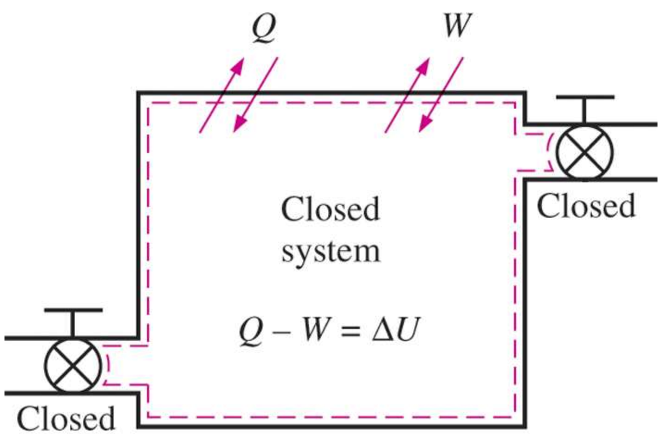
$$\Delta m_{system} = m_{final} - m_{initial}$$

$$\sum m_i - \sum m_e = (m_2 - m_1)_{system}$$

Energy balance

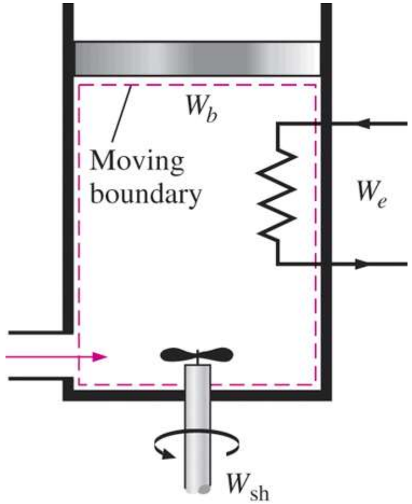
$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\substack{\text{Net energy transfer} \\ \text{by heat, work, and mass}}} = \underbrace{\Delta \dot{E}_{system}}_{\substack{\text{Change in internal, kinetic,} \\ \text{potential, etc., energies}}} \quad (\text{kJ})$$

$$\left(Q_{in} + W_{in} + \sum m_i\theta_i\right) - \left(Q_{out} + W_{out} + \sum m_e\theta_e\right) = (m_2e_2 - m_1e_1)_{system}$$



$$\theta = h + \text{ke} + \text{pe}$$

$$e = u + \text{ke} + \text{pe}$$

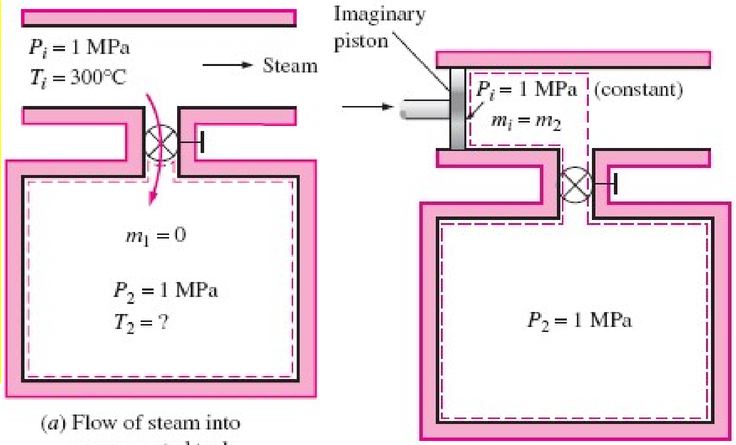


The energy equation of a uniform-flow system reduces to that of a closed system when all the inlets and exits are closed.

A uniform-flow system may involve electrical, shaft, and boundary work all at once.

EXAMPLE 5–12

A rigid, insulated tank that is initially evacuated is connected through a valve to a supply line that carries steam at 1 MPa and 300°C. Now the valve is opened, and steam is allowed to flow slowly into the tank until the pressure reaches 1 MPa, at which point the valve is closed. Determine the final temperature of the steam in the tank.



(a) Flow of steam into an evacuated tank

(b) The closed-system equivalence

$$\text{Mass balance:} \quad m_i - m_e = \Delta m_{\text{system}} \quad \rightarrow \quad m_i = m_2 - \overset{\nearrow 0}{m_1} = m_2$$

$$m_i h_i = m_2 u_2 \qquad (\text{since } W = Q = 0,\ \text{ke} \cong \text{pe} \cong 0,\ m_1 = 0) \qquad u_2 = h_i$$

$$\left.\begin{aligned} P_i &= 1 \text{ MPa} \\ T_i &= 300°\text{C} \end{aligned}\right\} \quad h_i = 3051.6 \text{ kJ/kg} \qquad \left.\begin{aligned} P_2 &= 1 \text{ MPa} \\ u_2 &= 3051.6 \text{ kJ/kg} \end{aligned}\right\} \quad T_2 = \mathbf{456.1°C}$$

Discussion Note that the temperature of the steam in the tank has increased by 156.1°C. This result may be surprising at first, and you may be wondering where the energy to raise the temperature of the steam came from. The answer lies in the enthalpy term *h u Pv*. Part of the energy represented by enthalpy is the flow energy *Pv*, and this flow energy is converted to sensible internal energy once the flow ceases to exist in the control volume, and it shows up as an increase in temperature

SUMMARY

- Conservation of mass
  - Mass and volume flow rates
  - Mass balance for a steady-flow process
  - Mass balance for incompressible flow
- Flow work and the energy of a flowing fluid
  - Energy transport by mass
- Energy analysis of steady-flow systems
- Some steady-flow engineering devices
  - Nozzles and Diffusers
  - Turbines and Compressors
  - Throttling valves
  - Mixing chambers and Heat exchangers
  - Pipe and Duct flow
- Energy analysis of unsteady-flow processes