Heat and Mass Transfer, 3rd Edition
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CHAPTER 8
INTERNAL FORCED CONVECTION

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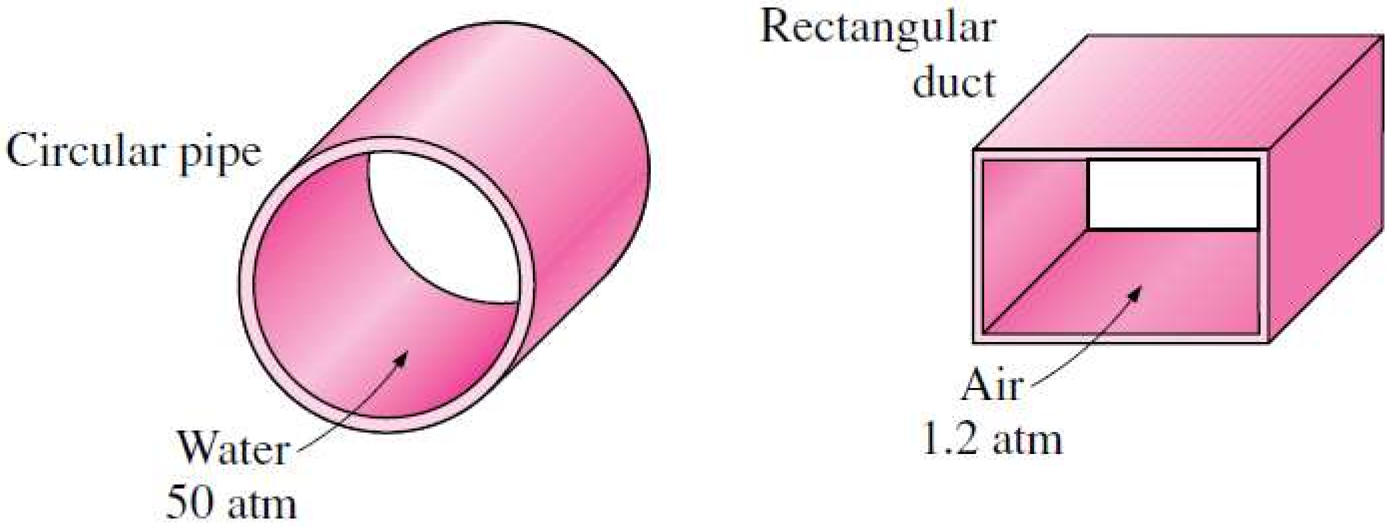
Objectives

- Obtain average velocity from a knowledge of velocity profile, and average temperature from a knowledge of temperature profile in internal flow,
- Have a visual understanding of different flow regions in internal flow, and calculate hydrodynamic and thermal entry lengths
- Analyze heating and cooling of a fluid flowing in a tube under constant surface temperature and constant surface heat flux conditions, and work with the logarithmic mean temperature difference
- Obtain analytic relations for the velocity profile, pressure drop, friction factor, and Nusselt number in fully developed laminar flow, and
- Determine the friction factor and Nusselt number in fully developed turbulent flow using empirical relations, and calculate the heat transfer rate.

INTRODUCTION

- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to *friction*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.
- The pressure drop is then used to determine the *pumping power requirement*.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.

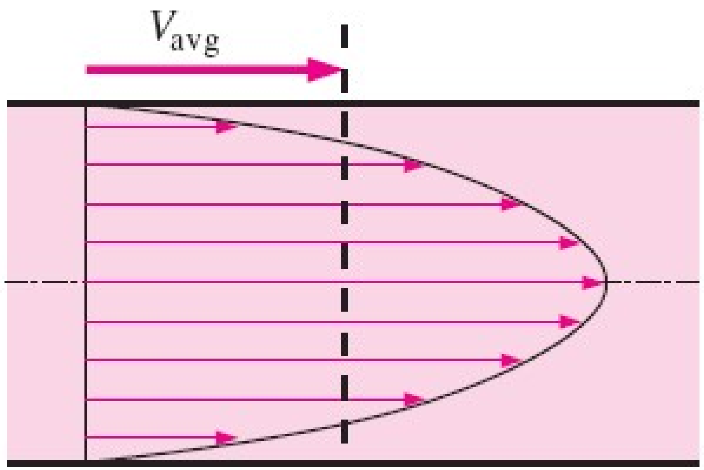
Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

The value of the average velocity V_{avg} at some streamwise cross-section is determined from the requirement that the *conservation of mass* principle be satisfied

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r\, dr$$

The average velocity for incompressible flow in a circular pipe of radius *R*



Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

GENERAL CONSIDERATIONS FOR PIPE FLOW

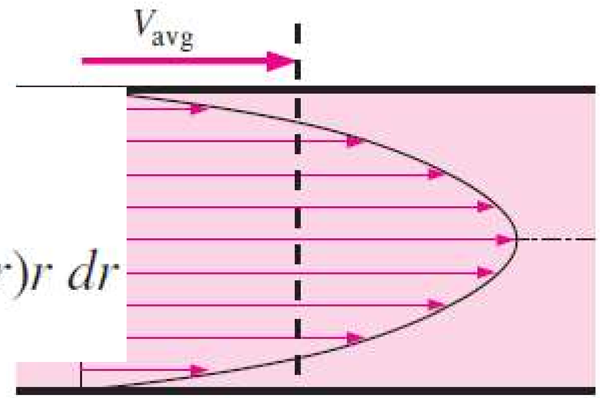
Liquid or gas flow through pipes or ducts is commonly used in practice in heating and cooling applications. The fluid is forced to flow by a fan or pump through a conduit that is sufficiently long to accomplish the desired heat transfer.

Transition from laminar to turbulent flow depends on the Reynolds number as well as the degree of disturbance of the flow by *surface roughness*, *pipe vibrations*, and the *fluctuations in the flow*.

The flow in a pipe is laminar for Re < 2300, fully turbulent for Re > 10,000, and transitional in between.

$$\dot{m} = \rho V_{\text{avg}} A_c = \int \rho u(r)\, dA_c$$

$$V_{\text{avg}} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r\, dr$$

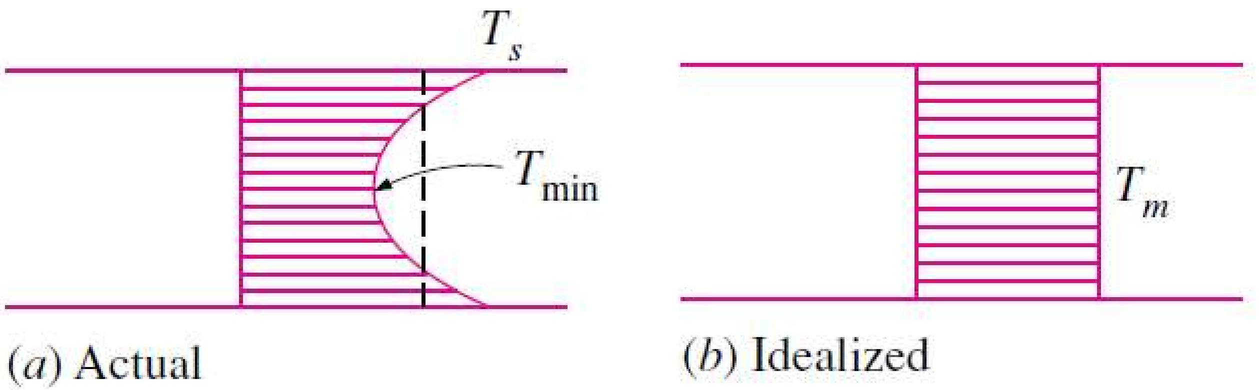


$$\dot{E}_{\text{fluid}} = \dot{m} c_p T_m = \int_{\dot{m}} c_p T(r) \delta \dot{m} = \int_{A_c} \rho c_p T(r) u(r) V dA_c$$

the mean temperature of a fluid with constant density and specific heat flowing in a circular pipe of radius *R*

$$T_m = \frac{\int_{\dot{m}} c_p T(r) \delta \dot{m}}{\dot{m} c_p} = \frac{\int_0^R c_p T(r) \rho u(r) 2\pi r dr}{\rho V_{\text{avg}} (\pi R^2) c_p} = \frac{2}{V_{\text{avg}} R^2} \int_0^R T(r) u(r)\, r dr$$

The fluid properties in internal flow are usually evaluated at the *bulk mean fluid temperature*, which is the arithmetic average of the mean temperatures at the inlet and the exit: $T_b = (T_{m,i} + T_{m,e})/2$



Actual and idealized temperature profiles for flow in a tube

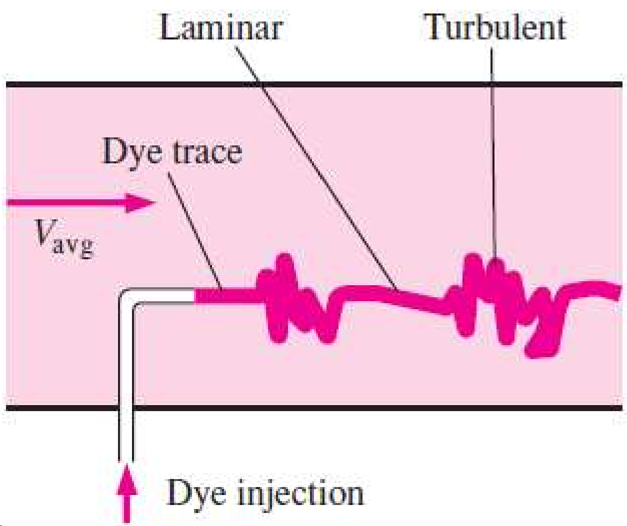
For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter**

$$D_h = \frac{4A_c}{p}$$

For flow in a circular pipe:

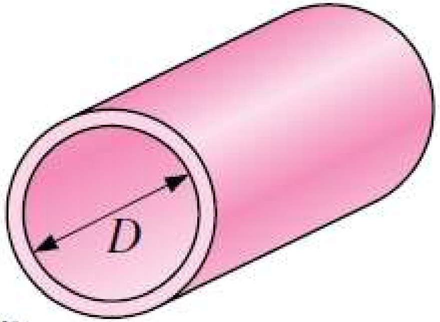
$$\text{laminar for Re} < 2300$$

$$\text{fully turbulent for Re} > 10{,}000$$



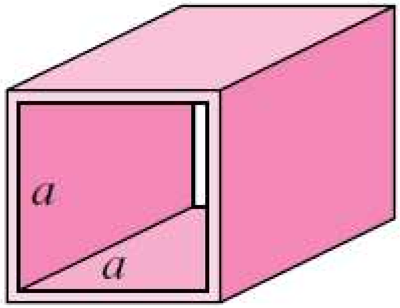
In the transitional flow region of 2300 ≤ Re ≤ 10,000, the flow switches between laminar and turbulent seemingly randomly.

Circular tube:



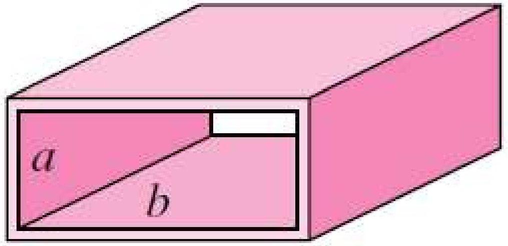
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:



$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

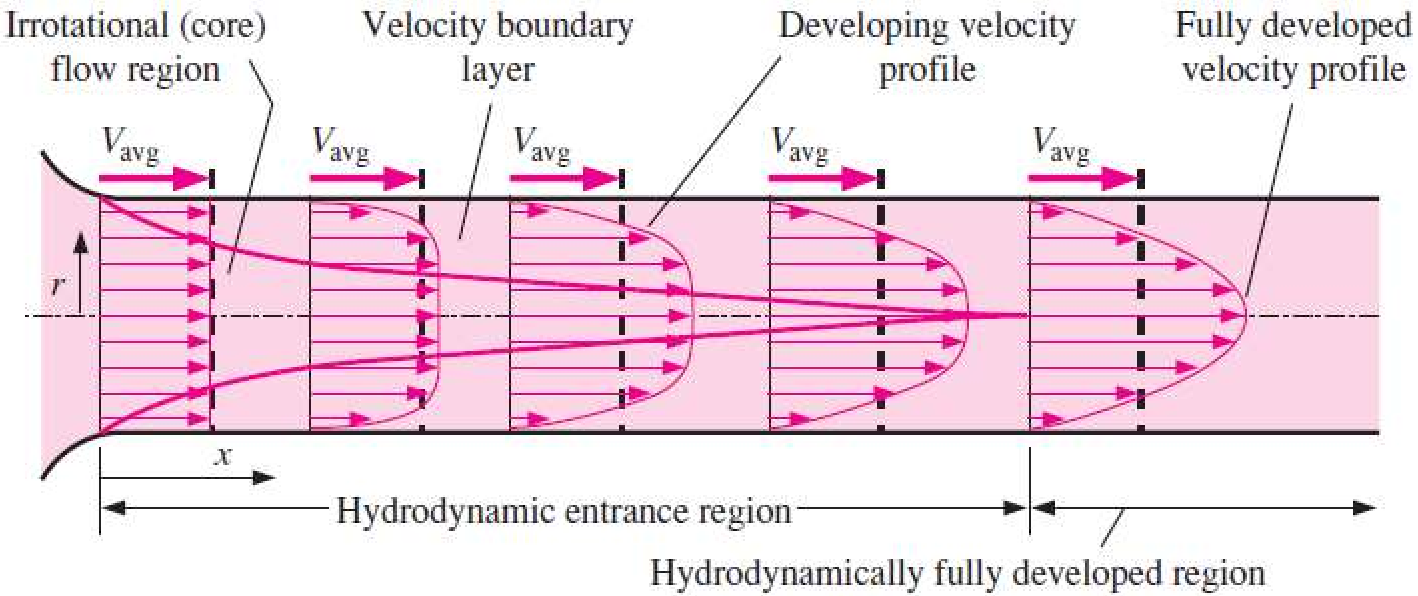
The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

THE ENTRANCE REGION

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.

Boundary layer region: The viscous effects and the velocity changes are significant.

Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

Thermal Entrance Region

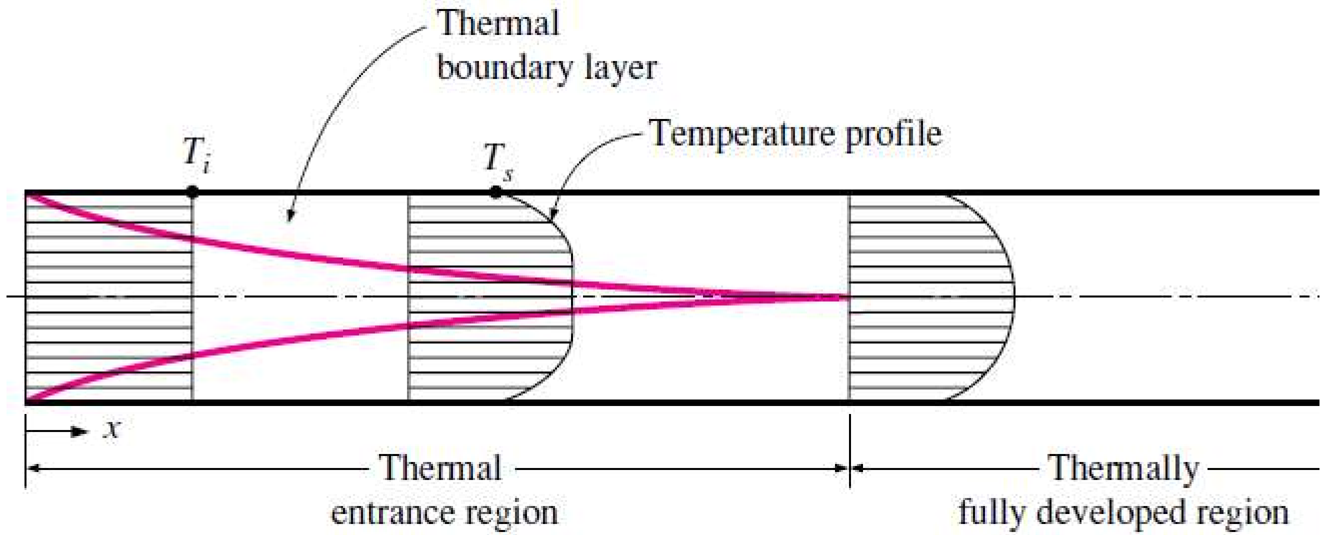
Thermal entrance region: The thermal boundary layer develops and reaches the tube center.

Thermal entry length: The length of this region.

Thermally developing flow: Flow in the thermal entrance region. This is the region where the temperature profile develops.

Thermally fully developed region: The region beyond the thermal entrance region in which the dimensionless temperature profile remains unchanged.

Fully developed flow: The flow is both hydrodynamically and thermally developed.



The development of the thermal boundary layer in a tube.

Hydrodynamically fully developed:

$$\frac{\partial u(r, x)}{\partial x} = 0 \quad \longrightarrow \quad u = u(r)$$

Thermally fully developed:

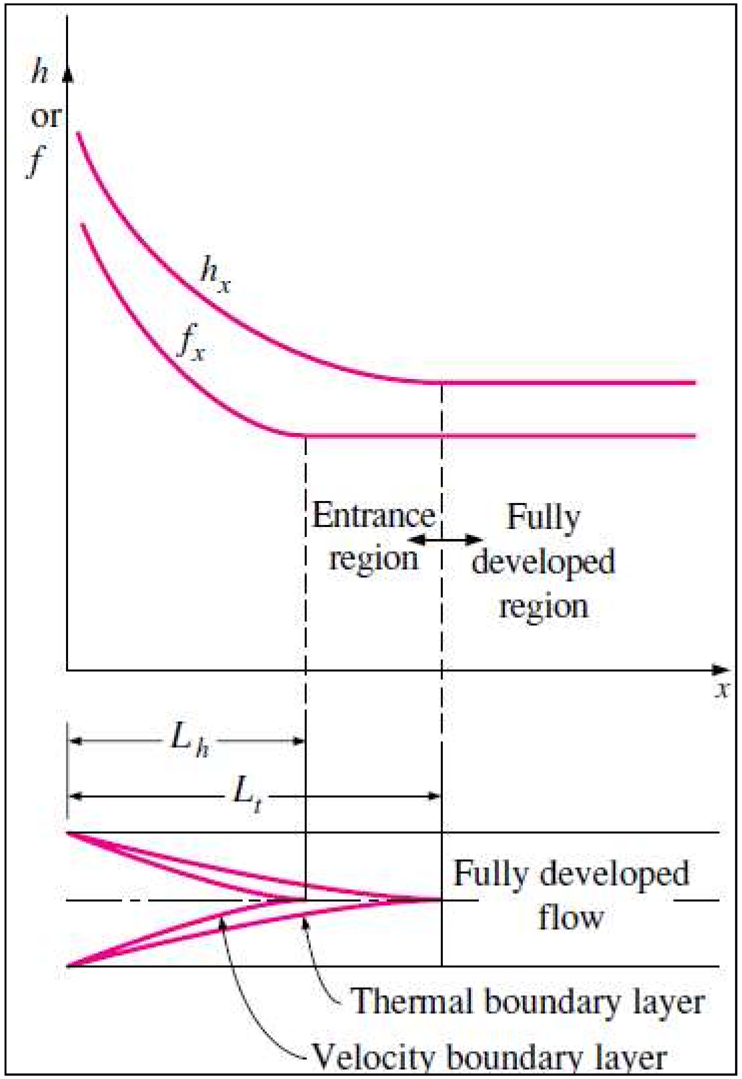
$$\frac{\partial}{\partial x}\left[\frac{T_s(x) - T(r, x)}{T_s(x) - T_m(x)}\right] = 0$$

$$\dot{q}_s = h_x(T_s - T_m) = k \left.\frac{\partial T}{\partial r}\right|_{r=R} \quad \longrightarrow \quad h_x = \frac{k(\partial T/\partial r)|_{r=R}}{T_s - T_m}$$

In the thermally fully developed region of a tube, the local convection coefficient is constant

Therefore, both *the friction and convection coefficients remain constant in the fully developed region of a tube.*

The pressure drop and heat flux are *higher* in the entrance regions of a tube, and the effect of the entrance region is always to *increase* the average friction factor and heat transfer coefficient for the entire tube.



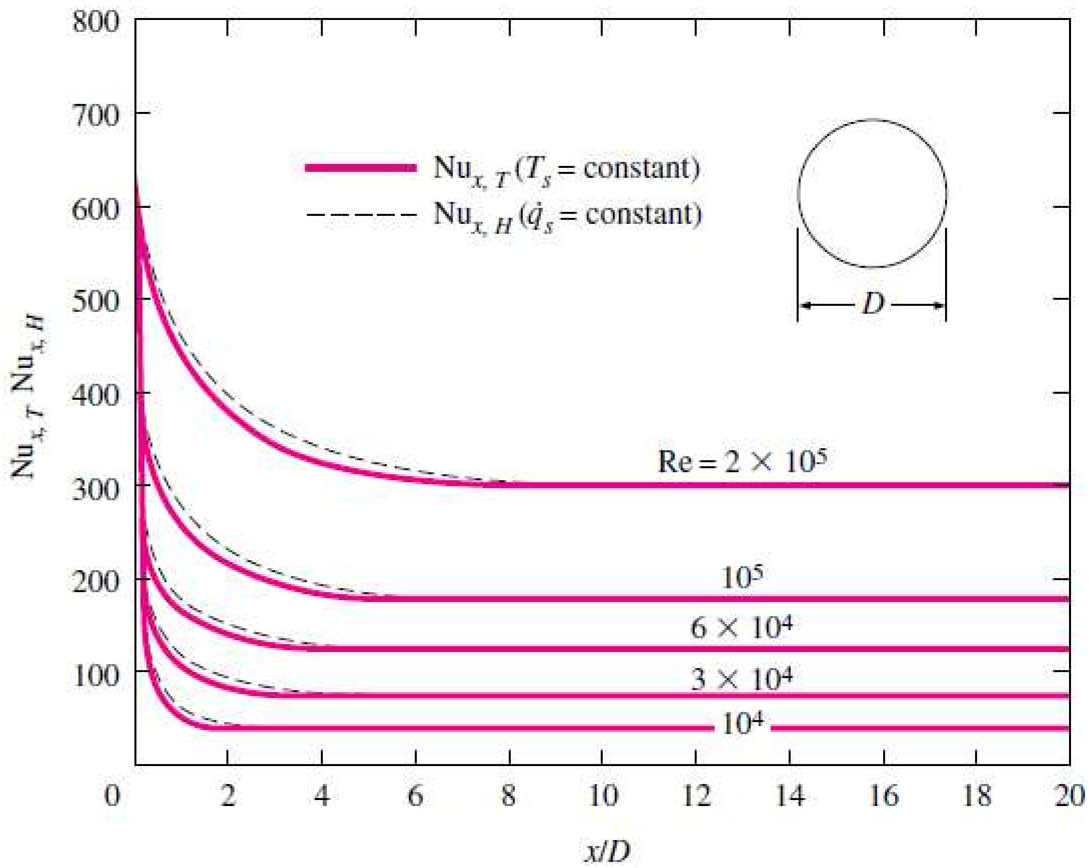
Variation of the friction factor and the convection heat transfer coefficient in the flow direction for flow in a tube (Pr>1).

Entry Lengths

$$L_{h,\text{ laminar}} \approx 0.05 \text{ Re } D$$
$$L_{t,\text{ laminar}} \approx 0.05 \text{ Re Pr } D = \text{Pr } L_{h,\text{ laminar}}$$

$$L_{h,\text{ turbulent}} = 1.359 D \text{ Re}^{1/4}$$

$$L_{h,\text{ turbulent}} \approx L_{t,\text{ turbulent}} \approx 10D$$



- The Nusselt numbers and thus *h* values are much higher in the entrance region.
- The Nusselt number reaches a constant value at a distance of less than 10 diameters, and thus the flow can be assumed to be fully developed for *x* > 10*D.*

- The Nusselt numbers for the uniform surface temperature and uniform surface heat flux conditions are identical in the fully developed regions, and nearly identical in the entrance regions.

GENERAL THERMAL ANALYSIS

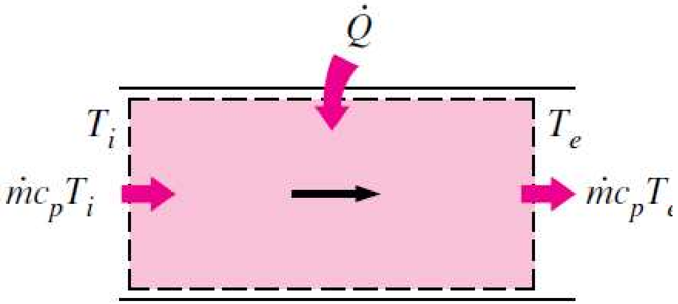
Rate of heat transfer

$$\dot{Q} = \dot{m} c_p (T_e - T_i) \qquad \text{(W)}$$

Surface heat flux

$$\dot{q}_s = h_x (T_s - T_m) \qquad (\text{W/m}^2)$$

h_x the *local* heat transfer coefficient



Energy balance:

$$\dot{Q} = \dot{m} c_p (T_e - T_i)$$

The heat transfer to a fluid flowing in a tube is equal to the increase in the energy of the fluid.

The thermal conditions at the surface can be approximated to be

(T_s= const) (q_s = const)

The constant surface temperature condition is realized when a phase change process such as boiling or condensation occurs at the outer surface of a tube.

The constant surface heat flux condition is realized when the tube is subjected to radiation or electric resistance heating uniformly from all directions.

We may have either T_s=constant or q_s= constant at the surface of a tube, but not both.

Constant Surface Heat Flux (q_s = constant)

Rate of heat transfer:

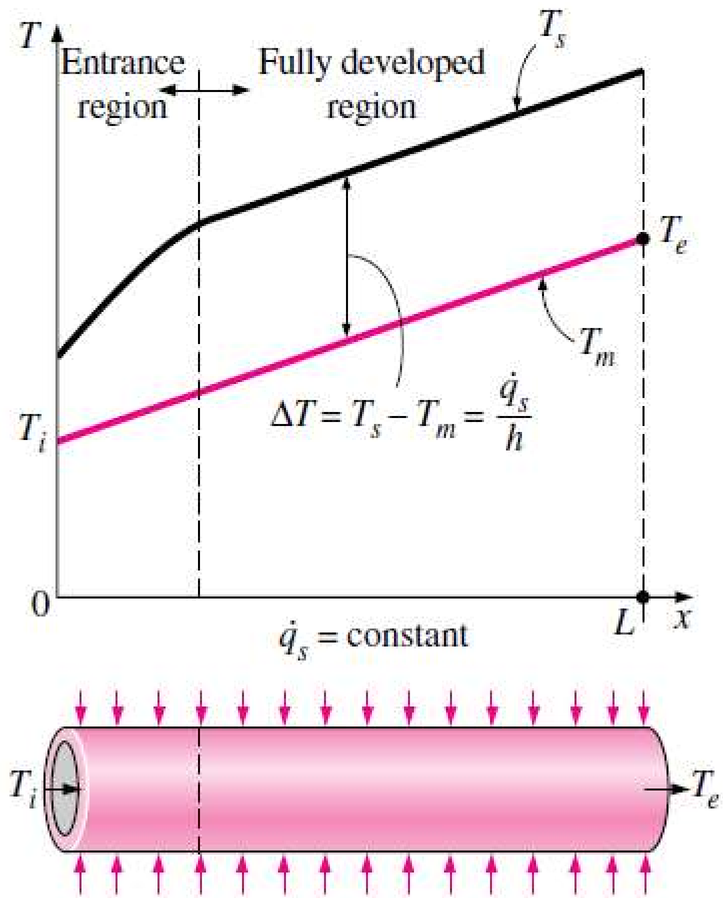
$$\dot{Q} = \dot{q}_s A_s = \dot{m} c_p (T_e - T_i) \qquad \text{(W)}$$

Mean fluid temperature at the tube exit:

$$T_e = T_i + \frac{\dot{q}_s A_s}{\dot{m} c_p}$$

Surface temperature:

$$\dot{q}_s = h(T_s - T_m) \longrightarrow T_s = T_m + \frac{\dot{q}_s}{h}$$



Variation of the *tube surface* and the *mean fluid* temperatures along the tube for the case of constant surface heat flux.

$$\dot{m}\, c_p\, dT_m = \dot{q}_s(p dx) \longrightarrow \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = \text{constant}$$

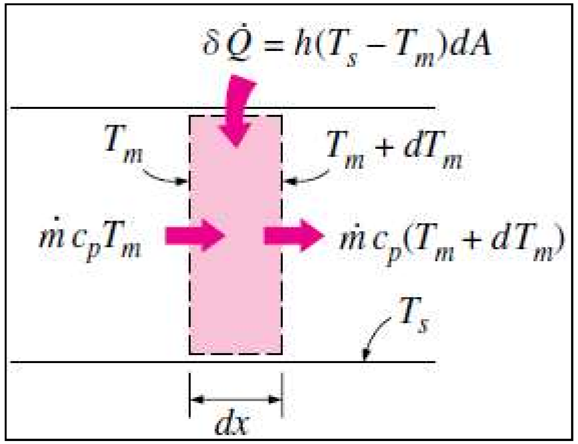
$$\frac{dT_m}{dx} = \frac{dT_s}{dx}$$

$$\frac{\partial}{\partial x}\left(\frac{T_s - T}{T_s - T_m}\right) = 0 \longrightarrow \frac{1}{T_s - T_m}\left(\frac{\partial T_s}{\partial x} - \frac{\partial T}{\partial x}\right) = 0$$

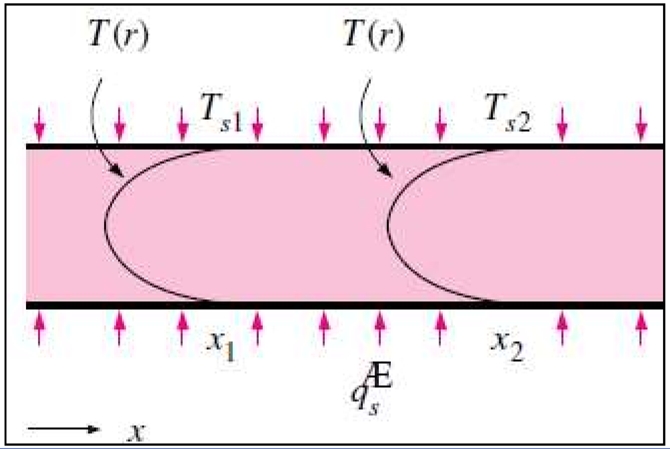
$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx}$$

$$\boxed{\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{\dot{q}_s p}{\dot{m} c_p} = \text{constant}}$$

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{avg} c_p R} = \text{constant}$$



Energy interactions for a differential control volume in a tube.



The shape of the temperature profile remains unchanged in the fully developed region of a tube subjected to constant surface heat flux.

Constant Surface Temperature (T_s = constant)

Rate of heat transfer to or from a fluid flowing in a tube

$$\dot{Q} = hA_s\Delta T_{avg} = hA_s(T_s - T_m)_{avg} \qquad \text{(W)}$$

Two suitable ways of expressing ΔT_{avg}

- arithmetic mean temperature difference
- logarithmic mean temperature difference

Arithmetic mean temperature difference

$$\Delta T_{avg} \approx \Delta T_{am} = \frac{\Delta T_i + \Delta T_e}{2} = \frac{(T_s - T_i) + (T_s - T_e)}{2} = T_s - \frac{T_i + T_e}{2}$$

$$= T_s - T_b$$

Bulk mean fluid temperature: $T_b = (T_i + T_e)/2$

By using arithmetic mean temperature difference, we assume that the mean fluid temperature varies linearly along the tube, which is hardly ever the case when T_s = constant.

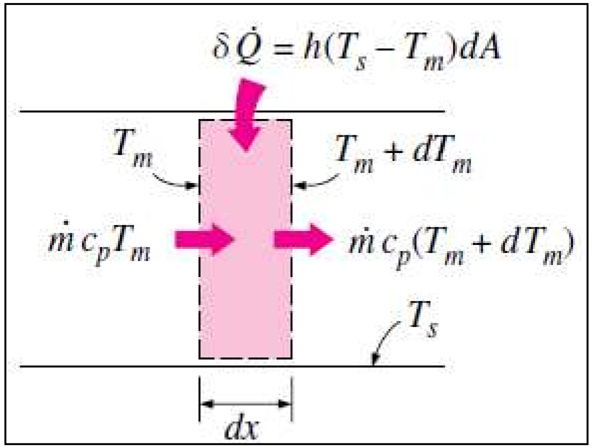
This simple approximation often gives acceptable results, but not always.

Therefore, we need a better way to evaluate ΔT_{avg}.

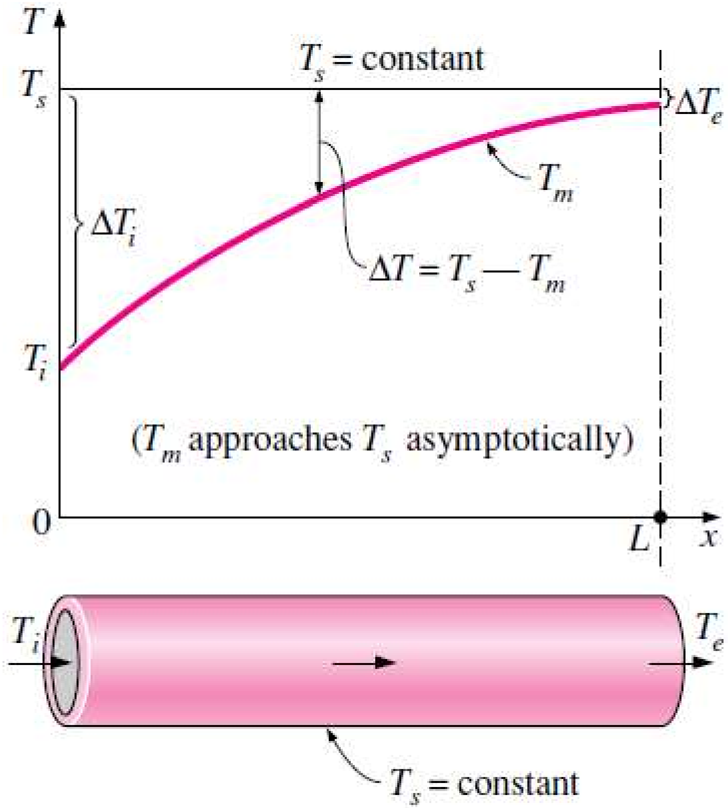
$$\dot{m} c_p dT_m = h(T_s - T_m) dA_s$$

$$dA_s = p dx \qquad\qquad dT_m = -d(T_s - T_m)$$

$$\frac{d(T_s - T_m)}{T_s - T_m} = -\frac{hp}{\dot{m} c_p} dx$$



Energy interactions for a differential control volume in a tube.



The variation of the *mean fluid* temperature along the tube for the case of constant temperature.

Integrating from $x = 0$ (tube inlet, $T_m = T_i$) to $x = L$ (tube exit, $T_m = T_e$)

$$\ln \frac{T_s - T_e}{T_s - T_i} = -\frac{hA_s}{\dot{m} c_p}$$

$$T_e = T_s - (T_s - T_i) \exp(-hA_s / \dot{m} c_p)$$

$$\dot{Q} = hA_s\Delta T_{\ln}$$

logarithmic mean temperature difference

$$\Delta T_{\ln} = \frac{T_i - T_e}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

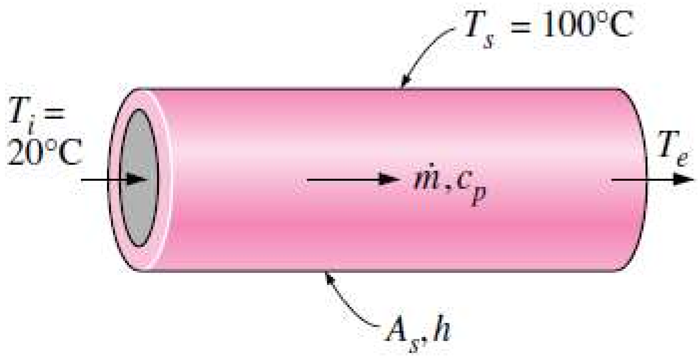
NTU: *Number of transfer units.* A measure of the effectiveness of the heat transfer systems.

For NTU = 5, $T_e = T_s$, and the limit for heat transfer is reached.

A small value of NTU indicates more opportunities for heat transfer.

$\Delta T_{\ln}$ is an *exact* representation of the *average temperature difference* between the fluid and the surface.

When ΔT_e differs from ΔT_i by no more than 40%, the error in using the arithmetic mean temperature difference is less than 1 %.



NTU = $hA_s / \dot{m}c_p$	T_e, °C
0.01	20.8
0.05	23.9
0.10	27.6
0.50	51.5
1.00	70.6
5.00	99.5
10.00	100.0

An NTU greater than 5 indicates that the fluid flowing in a tube will reach the surface temperature at the exit regardless of the inlet temperature.

LAMINAR FLOW IN TUBES

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

$$\frac{dP}{dx} = -\frac{2\tau_w}{R} \qquad u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\, dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\, dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$

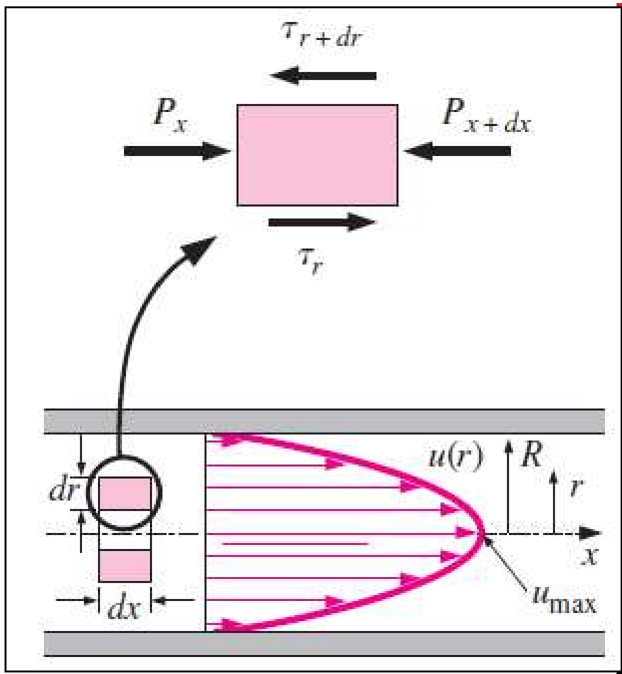
$$u_{max} = 2V_{avg}$$

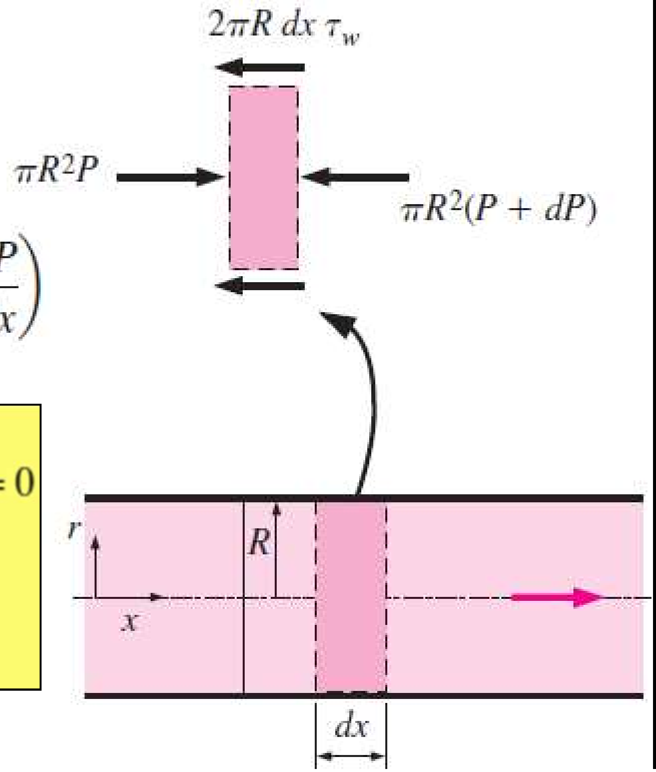
Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\, dx\, \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$





Pressure Drop

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

Laminar flow:

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

Pressure loss:

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

Darcy friction factor

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

Circular tube, laminar:

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

head loss

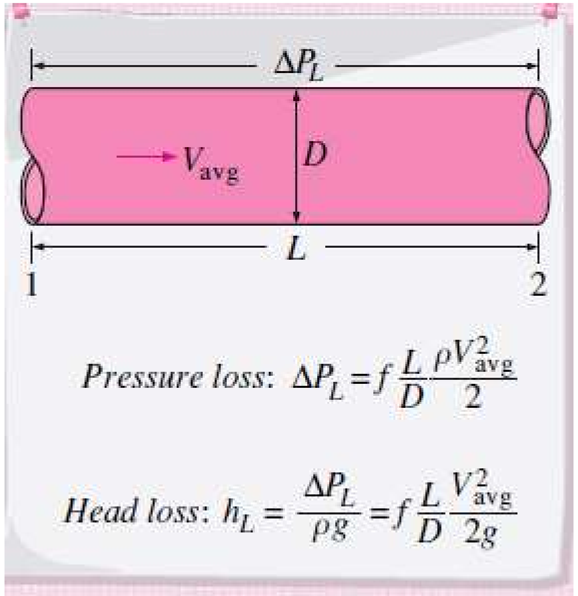
$$\boxed{h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}}$$

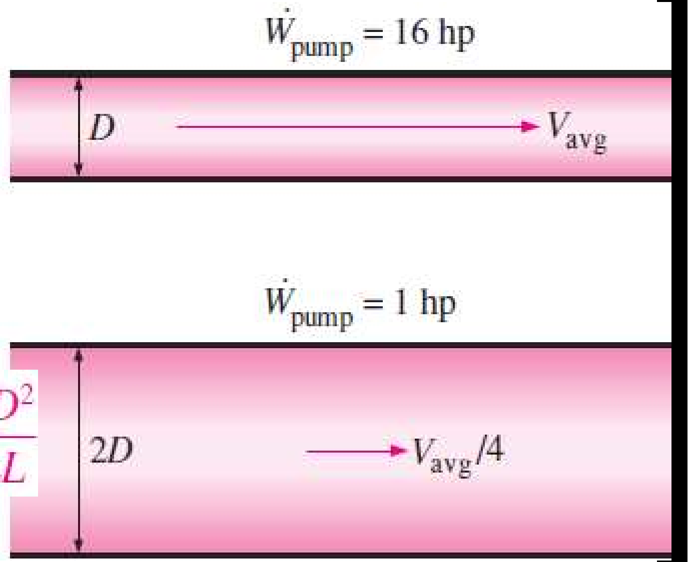
$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

Horizontal tube:

$$V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

$$\dot{V} = V_{avg}A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\,\pi D^4}{128\mu L}$$





Temperature Profile and the Nusselt Number

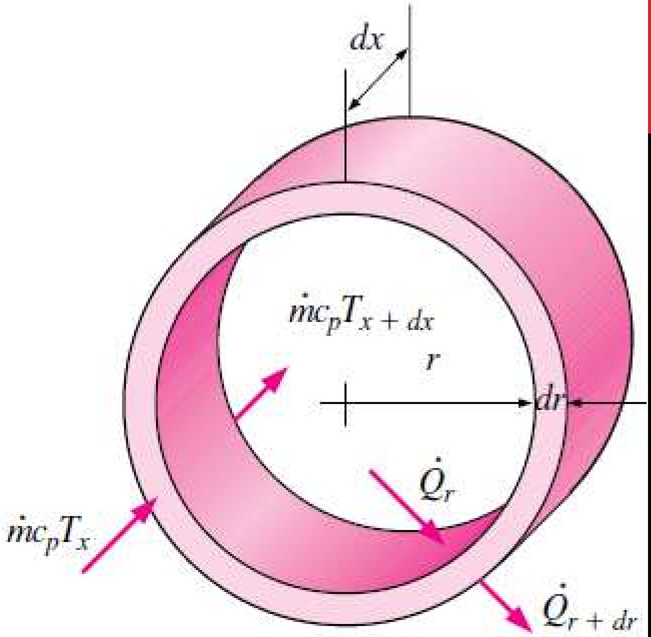
$$\dot{m}c_pT_x - \dot{m}c_pT_{x+dx} + \dot{Q}_r - \dot{Q}_{r+dr} = 0$$

$$\rho c_p u \frac{T_{x+dx} - T_x}{dx} = -\frac{1}{2\pi r dx}\frac{\dot{Q}_{r+dr} - \dot{Q}_r}{dr}$$

$$u\frac{\partial T}{\partial x} = -\frac{1}{2\rho c_p \pi r dx}\frac{\partial \dot{Q}}{\partial r}$$

$$\frac{\partial \dot{Q}}{\partial r} = \frac{\partial}{\partial r}\left(-k2\pi r dx\frac{\partial T}{\partial r}\right) = -2\pi k dx \frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)$$

$$\boxed{u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{dr}\left(r\frac{\partial T}{\partial r}\right)}$$



The differential volume element used in the derivation of energy balance relation.

The rate of net energy transfer to the control volume by mass flow is equal to the net rate of heat conduction in the radial direction.

Constant Surface Heat Flux

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{dT_m}{dx} = \frac{2\dot{q}_s}{\rho V_{\text{avg}} c_p R} = \text{constant}$$

$$\frac{4\dot{q}_s}{kR}\left(1 - \frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \qquad T = \frac{\dot{q}_s}{kR}\left(r^2 - \frac{r^4}{4R^2}\right) + C_1 r + C_2$$

Applying the boundary conditions $\partial T/\partial x = 0$ at $r = 0$ (because of symmetry) and $T=T_s$ at $r = R$

$$T = T_s - \frac{\dot{q}_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{r^4}{4R^4}\right) \qquad T_m = T_s - \frac{11}{24}\frac{\dot{q}_s R}{k}$$

$$\dot{q}_s = h(T_s - T_m) \qquad h = \frac{24}{11}\frac{k}{R} = \frac{48}{11}\frac{k}{D} = 4.36\frac{k}{D}$$

Circular tube, laminar ($\dot{q}_s$ =constant):

$$\boxed{\text{Nu} = \frac{hD}{k} = 4.36}$$

Therefore, for fully developed laminar flow in a circular tube subjected to constant surface heat flux, the Nusselt number is a constant.

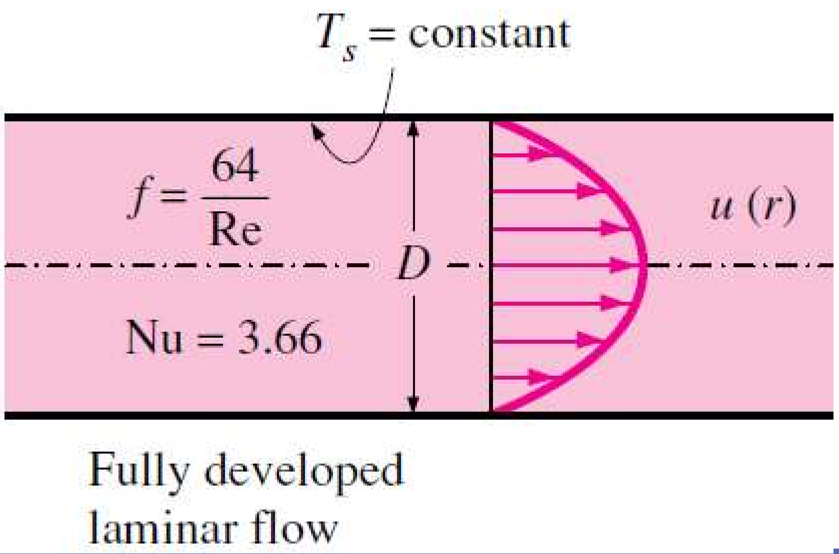
There is no dependence on the Reynolds or the Prandtl numbers.

Constant Surface Temperature

$$\text{Circular tube, laminar } (T_s = \text{constant}): \qquad \text{Nu} = \frac{hD}{k} = 3.66$$

The thermal conductivity *k* for use in the Nu relations should be evaluated at the bulk mean fluid temperature.

For laminar flow, the effect of *surface roughness* on the friction factor and the heat transfer coefficient is negligible.



In laminar flow in a tube with constant surface temperature, both the *friction factor* and the *heat transfer coefficient* remain constant in the fully developed region.

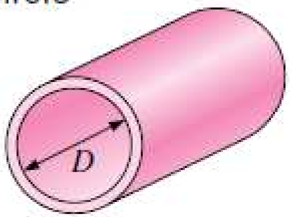
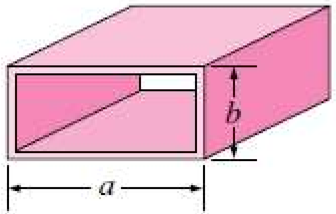
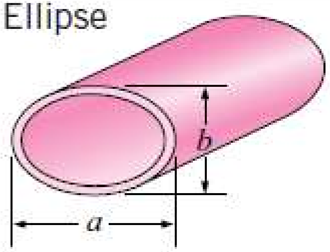
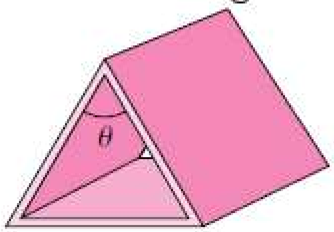
Laminar Flow in Noncircular Tubes

Nusselt number relations are given in the table for *fully developed laminar flow* in tubes of various cross sections.

The Reynolds and Nusselt numbers for flow in these tubes are based on the hydraulic diameter $D_h = 4A_c/p$,

Once the Nusselt number is available, the convection heat transfer coefficient is determined from $h = k\text{Nu}/D_h$.

Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\mathrm{Re} = V_{avg}D_h/\nu$, and $\mathrm{Nu} = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Facto f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Isosceles Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re

Developing Laminar Flow in the Entrance Region

For a circular tube of length *L* subjected to constant surface temperature, the average Nusselt number for the *thermal entrance region*:

Entry region, laminar:

$$\mathrm{Nu} = 3.66 + \frac{0.065\,(D/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.04[(D/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$

The average Nusselt number is larger at the entrance region, and it approaches asymptotically to the fully developed value of 3.66 as $L \to \infty$.

When the difference between the surface and the fluid temperatures is large, it may be necessary to account for the variation of viscosity with temperature:

$$\mathrm{Nu} = 1.86\left(\frac{\mathrm{Re}\,\mathrm{Pr}\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

All properties are evaluated at the bulk mean fluid temperature, except for μ_s, which is evaluated at the surface temperature.

The average Nusselt number for the thermal entrance region of flow between *isothermal parallel plates* of length *L* is

$$\mathrm{Re} \leq 2800$$

Entry region, laminar:

$$\mathrm{Nu} = 7.54 + \frac{0.03\,(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}}{1 + 0.016[(D_h/L)\,\mathrm{Re}\,\mathrm{Pr}]^{2/3}}$$

TURBULENT FLOW IN TUBES

First Petukhov equation

Smooth tubes: $f = (0.790 \ln \mathrm{Re} - 1.64)^{-2}$

$3000 < \mathrm{Re} < 5 \times 10^6$

Chilton–Colburn analogy

$\mathrm{Nu} = 0.125 f \,\mathrm{Re}\mathrm{Pr}^{1/3}$

Colburn equation

$$f = 0.184\ \mathrm{Re}^{-0.2}$$

$$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{1/3} \qquad \left(\begin{array}{l} 0.7 \le \mathrm{Pr} \le 160 \\ \mathrm{Re} > 10{,}000 \end{array}\right)$$

$\mathrm{Nu} = 0.023\ \mathrm{Re}^{0.8}\ \mathrm{Pr}^{n}$ *Dittus–Boelter equation*

$n = 0.4$ for *heating* and 0.3 for *cooling*

When the variation in properties is large due to a large temperature difference

$$\mathrm{Nu} = 0.027\ \mathrm{Re}^{0.8}\mathrm{Pr}^{1/3}\left(\frac{\mu}{\mu_s}\right)^{0.14} \qquad \left(\begin{array}{c} 0.7 \le \mathrm{Pr} \le 17{,}600 \\ \mathrm{Re} \ge 10{,}000 \end{array}\right)$$

All properties are evaluated at T_b except μ_s, which is evaluated at T_s.

Second Petukhov equation

$$\mathrm{Nu} = \frac{(f/8)\,\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \left(\begin{array}{l} 0.5 \le \mathrm{Pr} \le 2000 \\ 10^4 < \mathrm{Re} < 5 \times 10^6 \end{array}\right)$$

Gnielinski relation

$$\mathrm{Nu} = \frac{(f/8)(\mathrm{Re} - 1000)\,\mathrm{Pr}}{1 + 12.7(f/8)^{0.5}(\mathrm{Pr}^{2/3} - 1)} \qquad \left(\begin{array}{l} 0.5 \le \mathrm{Pr} \le 2000 \\ 3 \times 10^3 < \mathrm{Re} < 5 \times 10^6 \end{array}\right)$$

For liquid metals ($0.004 < \mathrm{Pr} < 0.01$), the following relations are recommended by Sleicher and Rouse for $10^4 < \mathrm{Re} < 10^6$:

Liquid metals, T_s = constant: $\quad \mathrm{Nu} = 4.8 + 0.0156\,\mathrm{Re}^{0.85}\,\mathrm{Pr}_s^{0.93}$

Liquid metals, $\dot{q}_s$ = constant: $\quad \mathrm{Nu} = 6.3 + 0.0167\,\mathrm{Re}^{0.85}\,\mathrm{Pr}_s^{0.93}$

In turbulent flow, wall roughness increases the heat transfer coefficient *h* by a factor of 2 or more. The convection heat transfer coefficient for rough tubes can be calculated approximately from *Gnielinski relation* or *Chilton–Colburn analogy* by using the friction factor determined from the *Moody chart* or the *Colebrook equation*.

The relations above are not very sensitive to the *thermal conditions* at the tube surfaces and can be used for both T_s = constant and q_s = constant.

The Moody Chart

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness ε/D.

Explicit Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

*Smooth surface. All values are for Re = 10^6 and are calculated from Eq. 8–74.

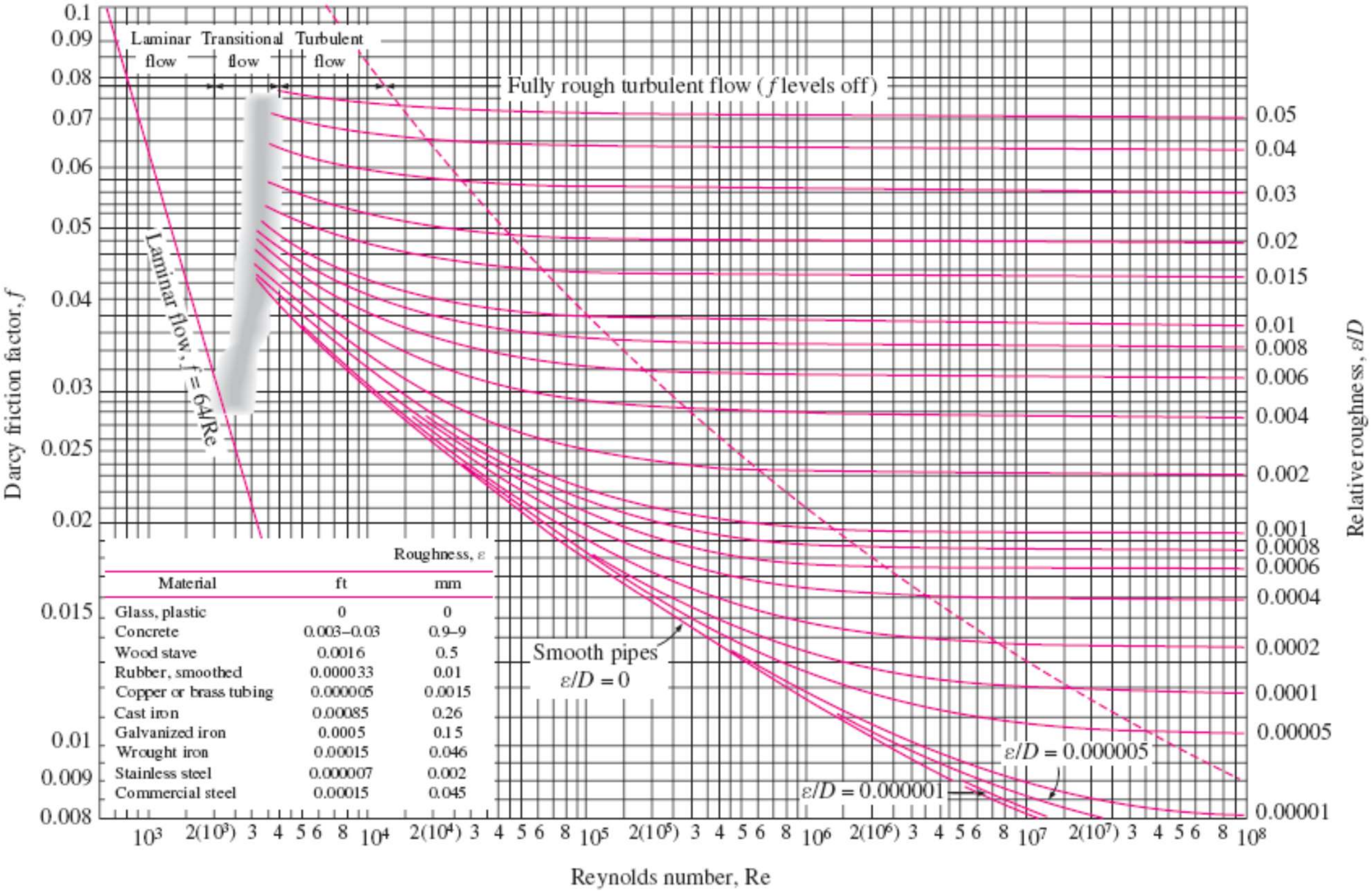
The friction factor is minimum for a smooth pipe and increases with roughness.

Colebrook equation (for smooth and rough pipes)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right)$$

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045



The Moody Chart

Developing Turbulent Flow in the Entrance Region

The entry lengths for turbulent flow are typically short, often just 10 tube diameters long, and thus the Nusselt number determined for fully developed turbulent flow can be used approximately for the entire tube.

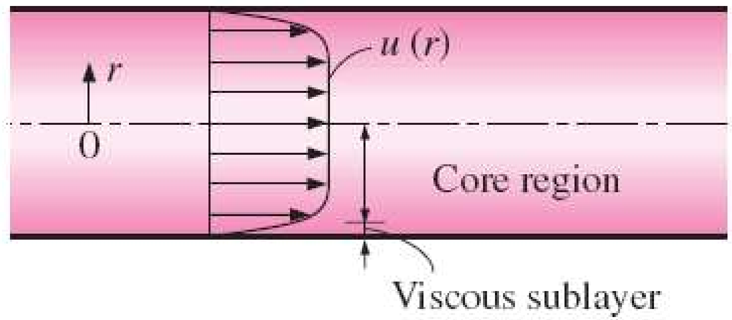
This simple approach gives reasonable results for pressure drop and heat transfer for long tubes and conservative results for short ones.

Correlations for the friction and heat transfer coefficients for the entrance regions are available in the literature for better accuracy.

Turbulent Flow in Noncircular Tubes

Pressure drop and heat transfer characteristics of turbulent flow in tubes are dominated by the very thin viscous sublayer next to the wall surface, and the shape of the core region is not of much significance.

The turbulent flow relations given above for circular tubes can also be used for noncircular tubes with reasonable accuracy by replacing the diameter D in the evaluation of the Reynolds number by the hydraulic diameter $D_h = 4A_c/p$.



In turbulent flow, the velocity profile is nearly a straight line in the core region, and any significant velocity gradients occur in the viscous sublayer.

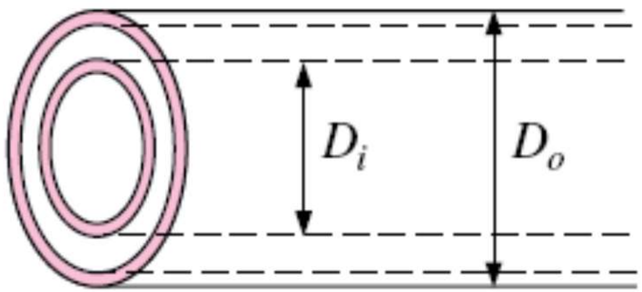
Flow through Tube Annulus

The hydraulic diameter of annulus

$$D_h = \frac{4A_c}{p} = \frac{4\pi(D_o^2 - D_i^2)/4}{\pi(D_o + D_i)} = D_o - D_i$$

For laminar flow, the convection coefficients for the inner and the outer surfaces are determined from

$$\mathrm{Nu}_i = \frac{h_i D_h}{k} \quad \text{and} \quad \mathrm{Nu}_o = \frac{h_o D_h}{k}$$



Tube surfaces are often *roughened, corrugated,* or *finned* in order to *enhance* convection heat transfer.

Nusselt number for fully developed laminar flow in an annulus with one surface isothermal and the other adiabatic (Kays and Perkins, 1972)

D_i/D_o	Nu_i	Nu_o
0	—	3.66
0.05	17.46	4.06
0.10	11.56	4.11
0.25	7.37	4.23
0.50	5.74	4.43
1.00	4.86	4.86

For fully developed turbulent flow, h_i and h_o are approximately equal to each other, and the tube annulus can be treated as a noncircular duct with a hydraulic diameter of $D_h = D_o - D_i$.

The Nusselt number can be determined from a suitable turbulent flow relation such as the Gnielinski equation. To improve the accuracy, Nusselt number can be multiplied by the following correction factors when one of the tube walls is adiabatic and heat transfer is through the other wall:

$$F_i = 0.86 \left(\frac{D_i}{D_o}\right)^{-0.16} \qquad \text{(outer wall adiabatic)}$$

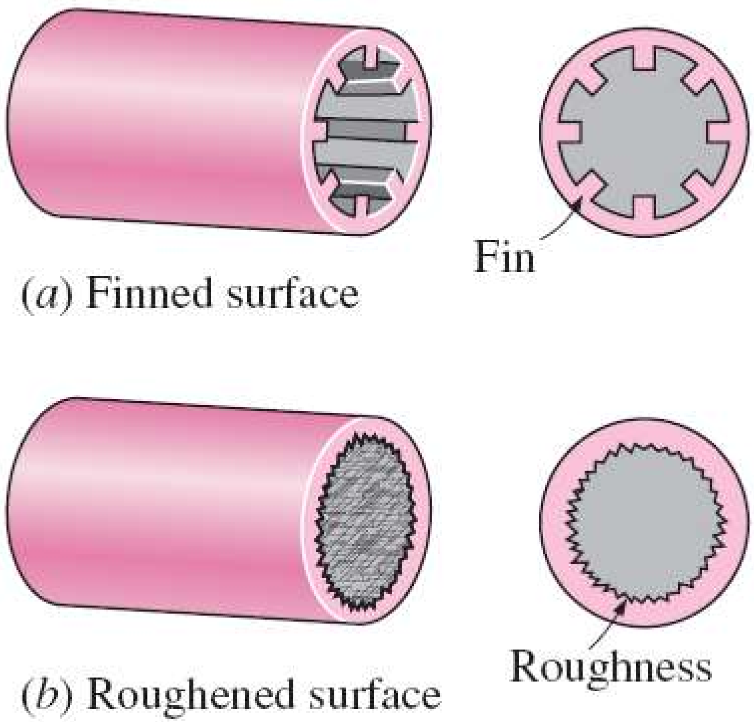
$$F_o = 0.86 \left(\frac{D_i}{D_o}\right)^{-0.16} \qquad \text{(inner wall adiabatic)}$$

Heat Transfer Enhancement

Tubes with rough surfaces have much higher heat transfer coefficients than tubes with smooth surfaces.

Heat transfer in turbulent flow in a tube has been increased by as much as 400 percent by roughening the surface. Roughening the surface, of course, also increases the friction factor and thus the power requirement for the pump or the fan.

The convection heat transfer coefficient can also be increased by inducing pulsating flow by pulse generators, by inducing swirl by inserting a twisted tape into the tube, or by inducing secondary flows by coiling the tube.



(*a*) Finned surface

(*b*) Roughened surface

Tube surfaces are often *roughened, corrugated,* or *finned* in order to *enhance* convection heat transfer.

SUMMARY

General Considerations for Pipe Flow

- Thermal Entrance Region, Entry Lengths

General Thermal Analysis

- Constant Surface Heat Flux
- Constant Surface Temperature

Laminar Flow in Tubes

- Constant Surface Heat Flux, Constant Surface Temperature
- Laminar Flow in Noncircular Tubes, Developing Laminar Flow in the Entrance Region

Turbulent Flow in Tubes

- Developing Turbulent Flow in the Entrance Region,
- Turbulent Flow in Noncircular Tubes
- Flow through Tube Annulus, Heat Transfer Enhancement