

Thermodynamics: An Engineering Approach, 5th Edition
Yunus A. Cengel, Michael A. Boles
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Chapter 7

ENTROPY

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Objectives

- Apply the second law of thermodynamics to processes.
- Define a new property called *entropy* to quantify the second-law effects.
- Establish the *increase of entropy principle*.
- Calculate the entropy changes that take place during processes for pure substances, incompressible substances, and ideal gases.
- Examine a special class of idealized processes, called *isentropic processes*, and develop the property relations for these processes.
- Derive the reversible steady-flow work relations.
- Develop the isentropic efficiencies for various steady-flow devices.
- Introduce and apply the entropy balance to various systems.

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2nd law of thermodynamics leads to expressions that involve inequalities.

An irreversible (i.e., actual) heat engine, for example, is less efficient than a reversible one operating between the same two thermal energy reservoirs.

Likewise, an irreversible refrigerator or a heat pump has a lower coefficient of performance (COP) than a reversible one operating between the same temperature limits.

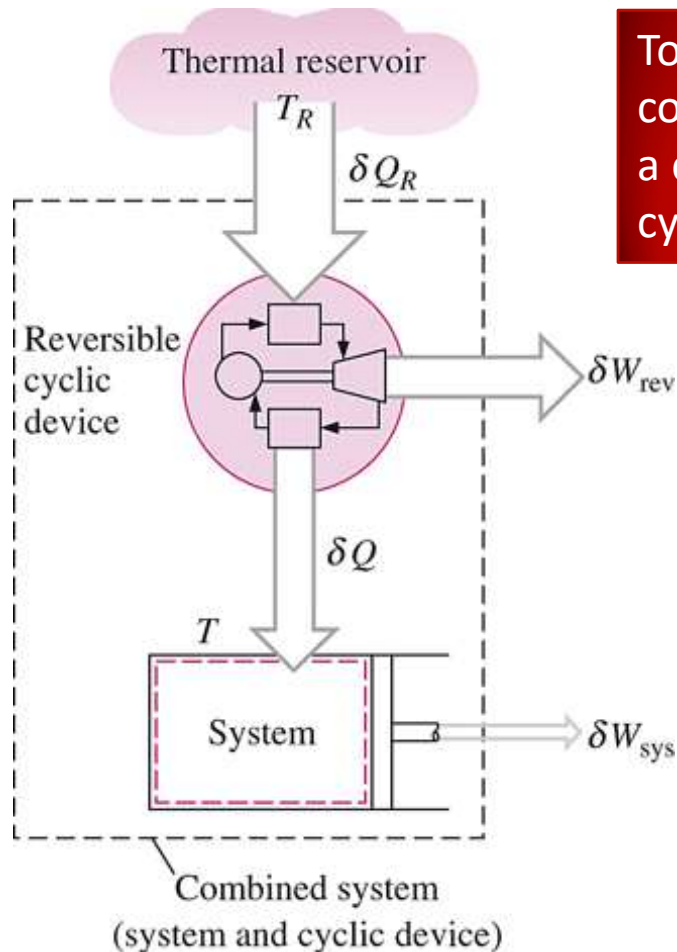
Another important inequality that has major consequences in thermodynamics is the **Clausius inequality**.

It was first stated by the German physicist R. J. E. Clausius (1822–1888), one of the founders of thermodynamics, and is expressed as

$$\oint \frac{\delta Q}{T} \leq 0$$

That is, the cyclic integral of dQ/T is always less than or equal to zero.

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To demonstrate the validity of the Clausius inequality, consider a system connected to a thermal energy reservoir at a constant absolute temperature of T_R through a *reversible* cyclic device.

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\delta W_C = \delta Q_R - dE_C$$

$$\frac{\delta Q_R}{T_R} = \frac{\delta Q}{T}$$

Eliminating dQ_R from the two relations above yields

$$\delta W_C = T_R \frac{\delta Q}{T} - dE_C$$

Then the preceding relation becomes

$$W_C = T_R \oint \frac{\delta Q}{T}$$

Clausius inequality

$$\oint \frac{\delta Q}{T} \leq 0$$

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

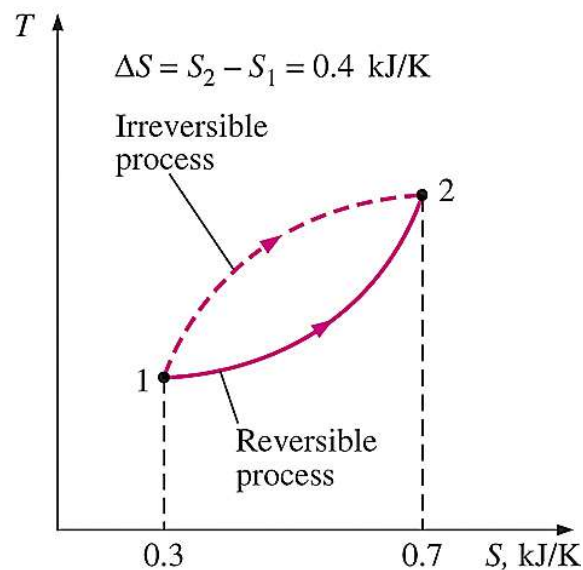
The system considered in the development of the Clausius inequality.

Formal definition of entropy

$$dS = \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

$$\Delta S = S_2 - S_1 = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \quad (\text{kJ/K})$$

The equality in the Clausius inequality holds for totally or just internally reversible cycles and the inequality for the irreversible ones.



Notice that we have actually defined the **change** in entropy instead of entropy itself, just as we defined the change in energy instead of the energy itself when we developed the first-law relation.

The entropy change between two specified states is the same whether the process is reversible or irreversible.

A Special Case: Internally Reversible Isothermal Heat Transfer Processes

$$\Delta S = \int_1^2 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = \int_1^2 \left(\frac{\delta Q}{T_0} \right)_{\text{int rev}} = \frac{1}{T_0} \int_1^2 (\delta Q)_{\text{int rev}}$$

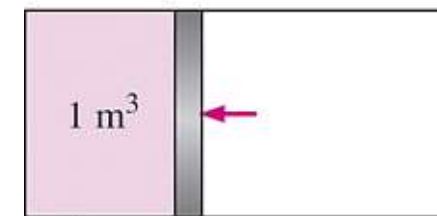
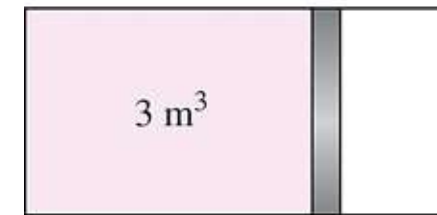
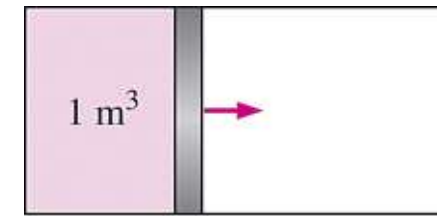
A quantity whose cyclic integral is zero (i.e., a property like volume)

$$\oint \left(\frac{\delta Q}{T} \right)_{\text{int rev}} = 0$$

Entropy is an extensive property of a system.

$$\Delta S = \frac{Q}{T_0}$$

This equation is particularly useful for determining the entropy changes of thermal energy reservoirs.

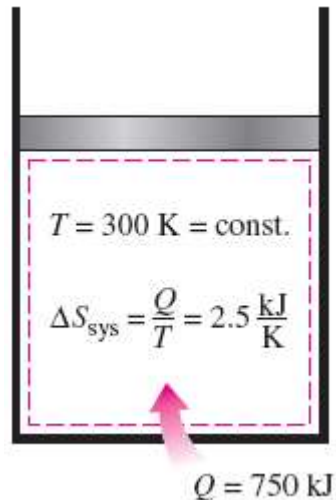


$$\oint dV = \Delta V_{\text{cycle}} = 0$$

The net change in volume (a property) during a cycle is always zero.

EXAMPLE 7-1

A piston–cylinder device contains a liquid–vapor mixture of water at 300 K. During a constant pressure process, 750 kJ of heat is transferred to the water. As a result, part of the liquid in the cylinder vaporizes. Determine the entropy change of the water during this process.



Solution Heat is transferred to a liquid–vapor mixture of water in a piston–cylinder device at constant pressure. The entropy change of water is to be determined.

Assumptions No irreversibilities occur within the system boundaries during the process.

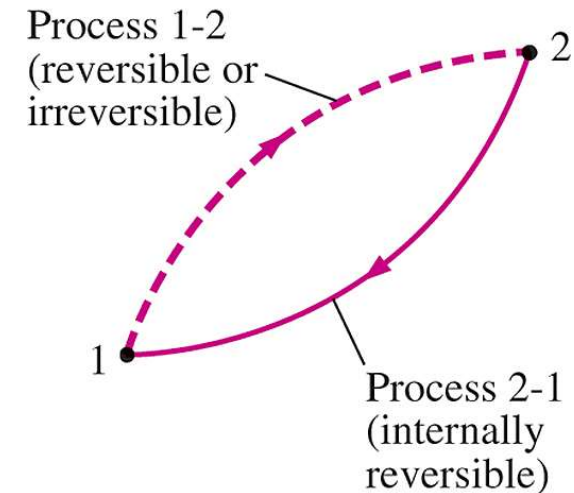
The system undergoes an internally reversible, isothermal process, and thus its entropy change can be determined directly from

$$\Delta S_{\text{sys, isothermal}} = \frac{Q}{T_{\text{sys}}} = \frac{750 \text{ kJ}}{300 \text{ K}} = 2.5 \text{ kJ/K}$$

Discussion Note that the entropy change of the system is positive, as expected, since heat transfer is *to* the system.

THE INCREASE OF ENTROPY PRINCIPLE

Consider a cycle that is made up of two processes: process 1-2, which is arbitrary (reversible or irreversible), and process 2-1, which is internally reversible,



A cycle composed of a reversible and an irreversible process.

$$\oint \frac{\delta Q}{T} \leq 0 \quad \int_1^2 \frac{\delta Q}{T} + \int_2^1 \left(\frac{\delta Q}{T} \right)_{\text{int rev}} \leq 0$$

$$\int_1^2 \frac{\delta Q}{T} + S_1 - S_2 \leq 0 \quad S_2 - S_1 \geq \int_1^2 \frac{\delta Q}{T}$$

The equality holds for an internally reversible process and the inequality for an irreversible process.

$$dS \geq \frac{\delta Q}{T}$$

Some entropy is *generated* or *created* during an irreversible process, and this generation is due entirely to the presence of irreversibilities.

The entropy generation S_{gen} is always a *positive* quantity or zero.
Can the entropy of a system during a process decrease?

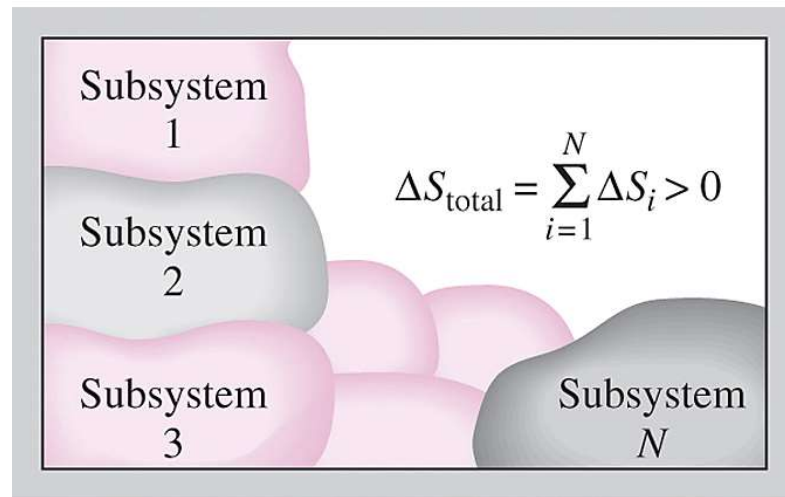
$$\Delta S_{\text{sys}} = S_2 - S_1 = \int_1^2 \frac{\delta Q}{T} + S_{\text{gen}}$$

*the entropy of an isolated system during a process always increases or, in the limiting case of a reversible process, remains constant. In other words, it never decreases. This is known as **the increase of entropy principle**.*

$$\Delta S_{\text{isolated}} \geq 0$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

(Isolated)

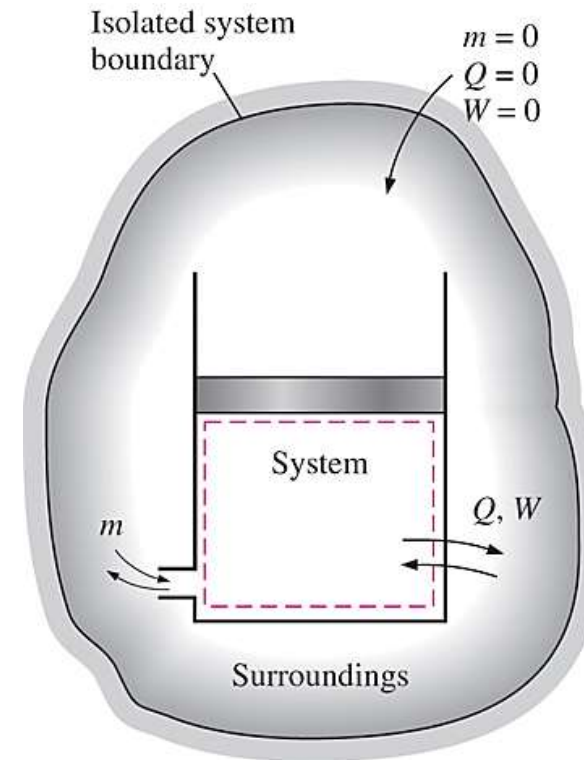


The entropy change of an isolated system is the sum of the entropy changes of its components, and is never less than zero.

$$\Delta S_{\text{isolated}} \geq 0$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} \geq 0$$

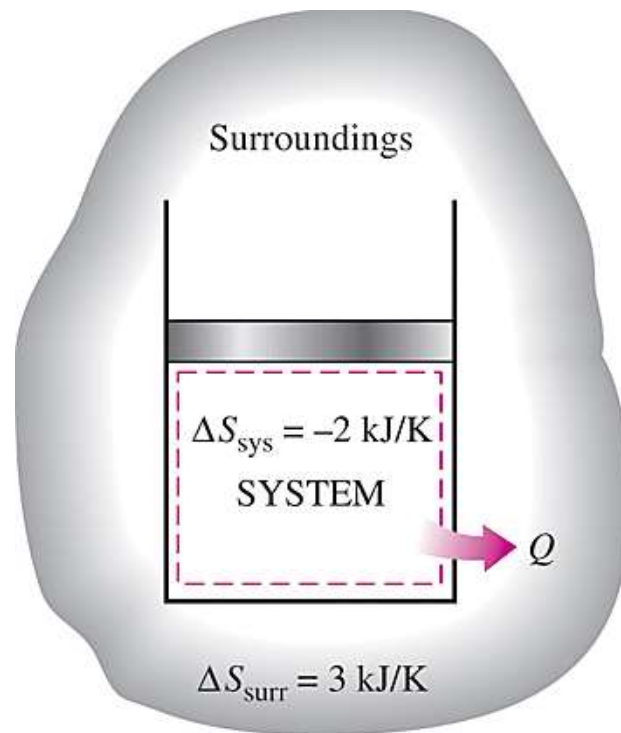
$$S_{\text{gen}} \begin{cases} > 0 & \text{Irreversible process} \\ = 0 & \text{Reversible process} \\ < 0 & \text{Impossible process} \end{cases}$$



A system and its surroundings form an isolated system.

The increase of entropy principle

Some Remarks about Entropy



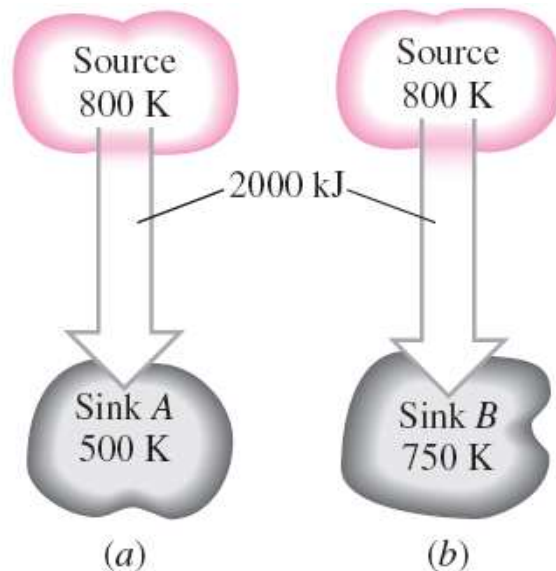
$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{sys}} + \Delta S_{\text{surr}} = 1 \text{ kJ/K}$$

The entropy change of a system can be negative, but the entropy generation cannot.

1. Processes can occur in a *certain* direction only, not in *any* direction. A process must proceed in the direction that complies with the increase of entropy principle, that is, $S_{\text{gen}} \geq 0$. A process that violates this principle is impossible.
2. Entropy is a *nonconserved property*, and there is *no* such thing as the *conservation of entropy principle*. Entropy is conserved during the idealized reversible processes only and increases during *all* actual processes.
3. The performance of engineering systems is degraded by the presence of irreversibilities, and *entropy generation* is a measure of the magnitudes of the irreversibilities during that process. It is also used to establish criteria for the performance of engineering devices.

EXAMPLE 7-2

A heat source at 800 K loses 2000 kJ of heat to a sink at (a) 500 K and (b) 750 K. Determine which heat transfer process is more irreversible.



Solution Heat is transferred from a heat source to two heat sinks at different temperatures. The heat transfer process that is more irreversible is to be determined.

Analysis Both cases involve heat transfer through a finite temperature difference, and therefore both are irreversible. The magnitude of the irreversibility associated with each process can be determined by calculating the total entropy change for each case. The total entropy change for a heat transfer process involving two reservoirs (a source and a sink) is the sum of the entropy changes of each reservoir since the two reservoirs form an adiabatic system.

(a) For the heat transfer process to a sink at 500 K:

$$\Delta S_{\text{source}} = \frac{Q_{\text{source}}}{T_{\text{source}}} = \frac{-2000 \text{ kJ}}{800 \text{ K}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = \frac{Q_{\text{sink}}}{T_{\text{sink}}} = \frac{2000 \text{ kJ}}{500 \text{ K}} = +4.0 \text{ kJ/K}$$

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{source}} + \Delta S_{\text{sink}} = (-2.5 + 4.0) \text{ kJ/K} = +1.5 \text{ kJ/K}$$

Therefore, 1.5 kJ/K of entropy is generated during this process. Noting that both reservoirs have undergone internally reversible processes, the entire entropy generation took place in the partition.

(b) Repeating the calculations in part (a) for a sink temperature of 750 K, we obtain

$$\Delta S_{\text{source}} = -2.5 \text{ kJ/K}$$

$$\Delta S_{\text{sink}} = +2.7 \text{ kJ/K}$$

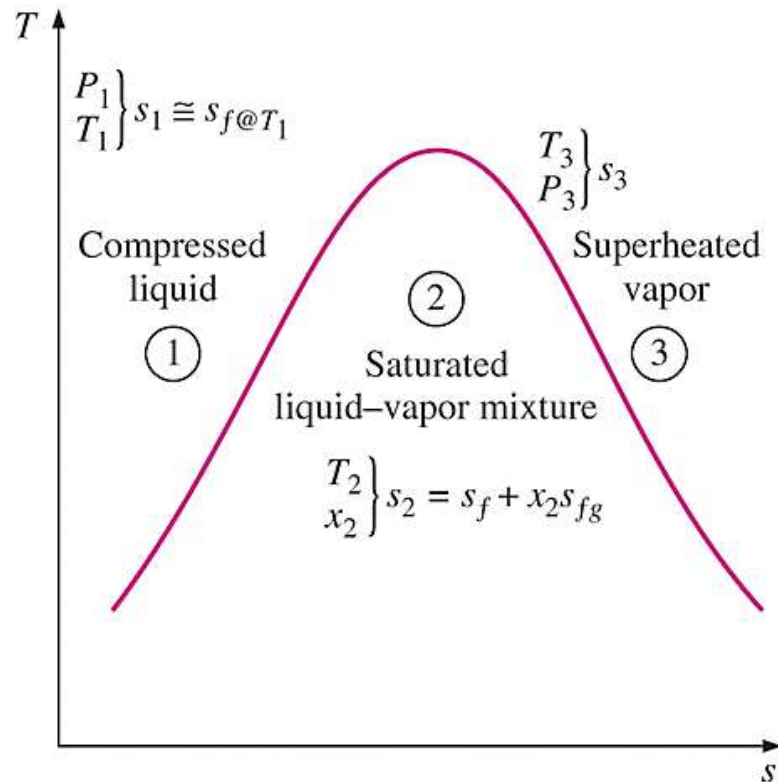
$$S_{\text{gen}} = \Delta S_{\text{total}} = (-2.5 + 2.7) \text{ kJ/K} = \mathbf{+0.2 \text{ kJ/K}}$$

The total entropy change for the process in part (b) is smaller, and therefore it is less irreversible. This is expected since the process in (b) involves a smaller temperature difference and thus a smaller irreversibility.

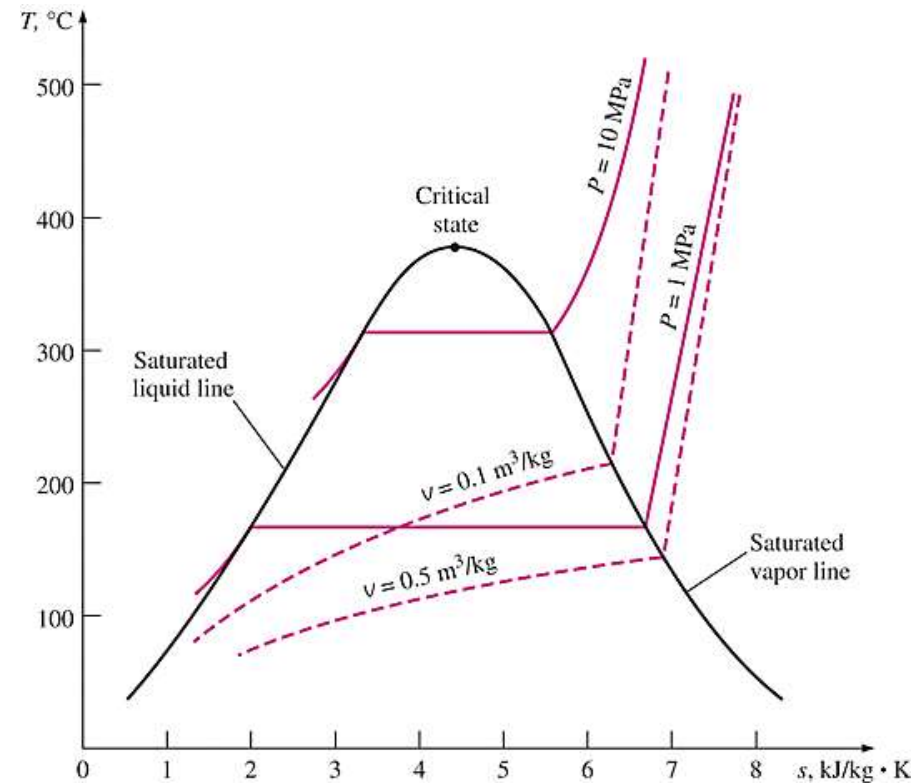
Discussion The irreversibilities associated with both processes could be eliminated by operating a Carnot heat engine between the source and the sink. For this case it can be shown that $\Delta S_{\text{total}} = 0$.

ENTROPY CHANGE OF PURE SUBSTANCES

Entropy is a property, and thus the value of entropy of a system is fixed once the state of the system is fixed.



The entropy of a pure substance is determined from the tables (like other properties).



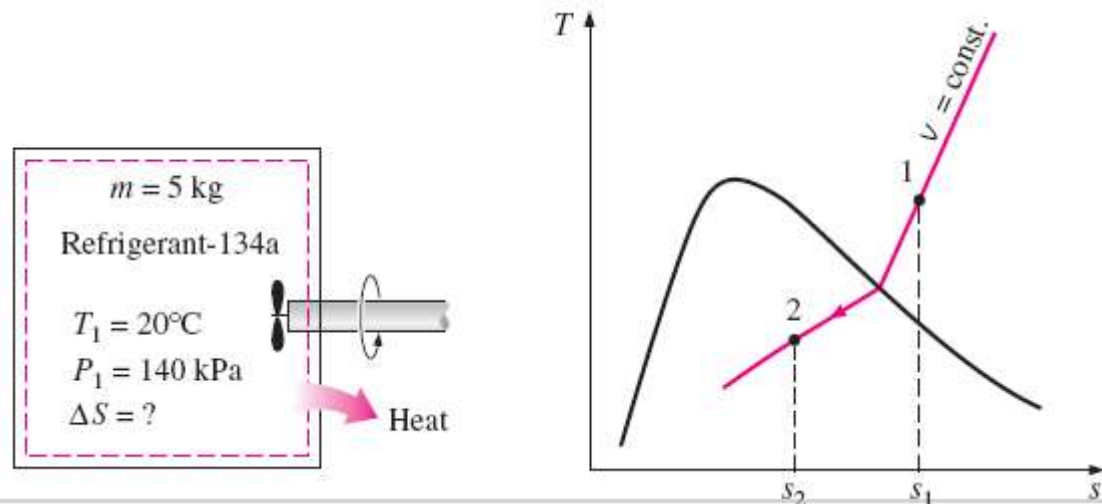
Schematic of the T-s diagram for water.

Entropy change

$$\Delta S = m\Delta s = m(s_2 - s_1) \quad (\text{kJ/K})$$

EXAMPLE 7-3

A rigid tank contains 5 kg of refrigerant-134a initially at 20°C and 140 kPa. The refrigerant is now cooled while being stirred until its pressure drops to 100 kPa. Determine the entropy change of the refrigerant during this process.



Solution The refrigerant in a rigid tank is cooled while being stirred until the pressure drops to a specified value. The entropy change of the refrigerant is to be determined.

Assumptions The volume of the tank is constant and thus $v_2 = v_1$.

$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 140 \text{ kPa} \\ T_1 = 20^\circ\text{C} \end{array} \right\} \quad \begin{array}{l} s_1 = 1.0624 \text{ kJ/kg} \cdot \text{K} \\ v_1 = 0.16544 \text{ m}^3/\text{kg} \end{array}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 100 \text{ kPa} \\ (v_2 = v_1) \end{array} \right\} \quad \begin{array}{l} v_f = 0.0007259 \text{ m}^3/\text{kg} \\ v_g = 0.19254 \text{ m}^3/\text{kg} \end{array}$$

$$x_2 = \frac{v_2 - v_f}{v_{fg}} = \frac{0.16544 - 0.0007259}{0.19254 - 0.0007259} = 0.859$$

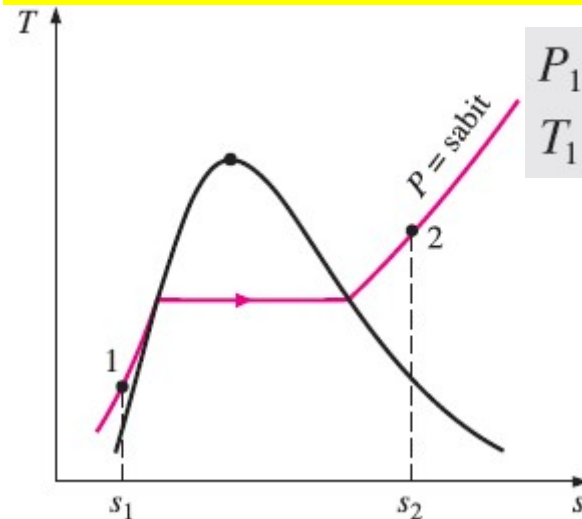
$$s_2 = s_f + x_2 s_{fg} = 0.07188 + (0.859)(0.87995) = 0.8278 \text{ kJ/kg} \cdot \text{K}$$

$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (5 \text{ kg})(0.8278 - 1.0624) \text{ kJ/kg} \cdot \text{K} \\ &= -1.173 \text{ kJ/K} \end{aligned}$$

Discussion The negative sign indicates that the entropy of the system is decreasing during this process. This is not a violation of the second law, however, since it is the *entropy generation* S_{gen} that cannot be negative.

EXAMPLE 7-4

A piston–cylinder device initially contains 1.5 kg of liquid water at 150 kPa and 20°C. The water is now heated at constant pressure by the addition of 4000 kJ of heat. Determine the entropy change of the water during this process.



$$\left. \begin{array}{l} P_1 = 150 \text{ kPa} \\ T_1 = 20^\circ\text{C} \end{array} \right\} \begin{array}{l} s_1 \cong s_{f@20^\circ\text{C}} = 0.2965 \text{ kJ/kg} \cdot \text{K} \\ h_1 \cong h_{f@20^\circ\text{C}} = 83.915 \text{ kJ/kg} \end{array}$$

$$\Delta U + W_b = \Delta H$$

$$Q_g - W_s = \Delta U$$

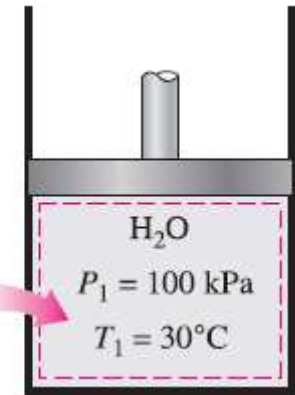
$$Q_g = \Delta H = m(h_2 - h_1)$$

$$4000 \text{ kJ} = (1.5 \text{ kg})(h_2 - 83.915 \text{ kJ/kg})$$

$$h_2 = 2750.6 \text{ kJ/kg}$$

$$\left. \begin{array}{l} P_2 = 150 \text{ kPa} \\ h_2 = 2750.6 \text{ kJ/kg} \end{array} \right\} \begin{array}{l} s_2 = 7.3674 \text{ kJ/kg} \cdot \text{K} \\ \text{(Tablo A-6, doğrusal oranlama)} \end{array}$$

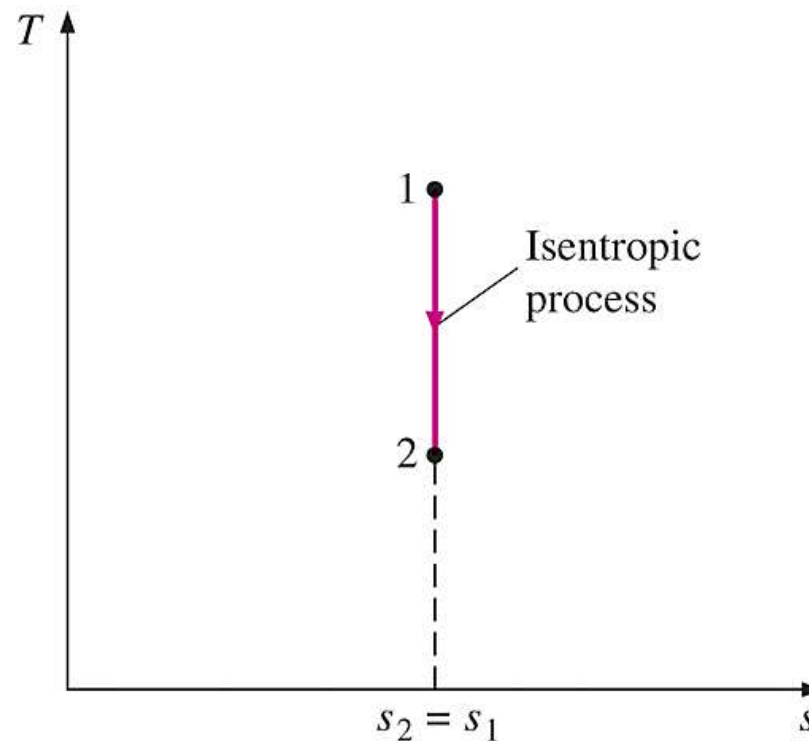
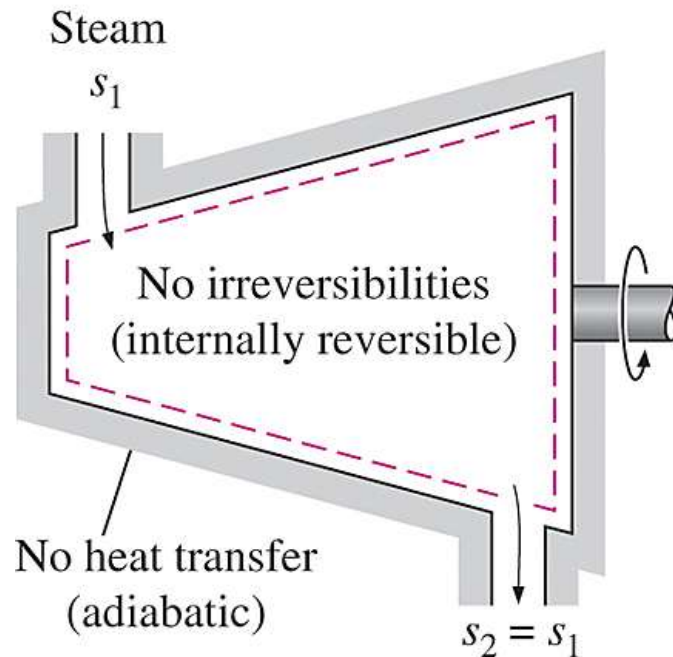
$$\begin{aligned} \Delta S &= m(s_2 - s_1) = (1.5 \text{ kg})(7.3674 - 0.2965) \text{ kJ/kg} \cdot \text{K} \\ &= \mathbf{10.61 \text{ kJ/kg}} \end{aligned}$$



ISENTROPIC PROCESSES

A process during which the entropy remains constant is called an **isentropic process**.

$$\Delta s = 0 \quad \text{or} \quad s_2 = s_1 \quad (\text{kJ/kg} \cdot \text{K})$$



During an internally reversible, adiabatic (isentropic) process, the entropy remains constant.

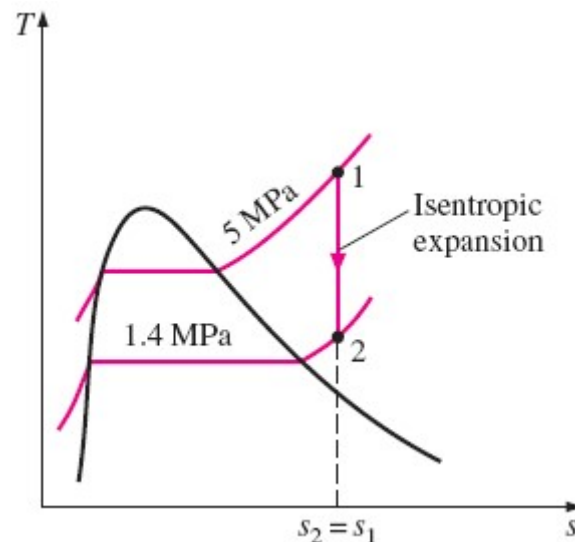
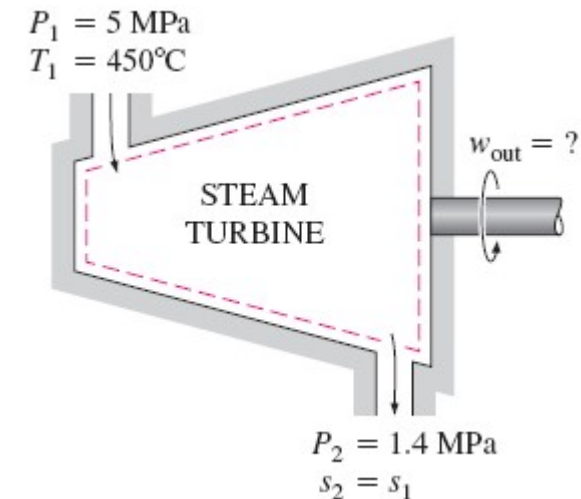
The isentropic process appears as a *vertical* line segment on a T - s diagram.

EXAMPLE 7-5

Steam enters an adiabatic turbine at 5 MPa and 450°C and leaves at a pressure of 1.4 MPa. Determine the work output of the turbine per unit mass of steam if the process is reversible.

Solution Steam is expanded in an adiabatic turbine to a specified pressure in a reversible manner. The work output of the turbine is to be determined.

Assumptions 1 This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV}=0$, $\Delta E_{CV}=0$, and $\Delta S_{CV}=0$. 2 The process is reversible. 3 Kinetic and potential energies are negligible. 4 The turbine is adiabatic and thus there is no heat transfer.



$$\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{\text{Rate of net energy transfer by heat, work, and mass}} = \underbrace{\Delta \dot{E}_{system}}_{\text{Rate of change in internal, kinetic, potential, etc., energies}} \overset{0 \text{ (steady)}}{=} 0$$

$$\begin{aligned}\dot{E}_{in} &= \dot{E}_{out} \\ \dot{m}h_1 &= \dot{W}_{out} + \dot{m}h_2 \quad (\text{since } \dot{Q} = 0, \text{ ke} \cong \text{pe} \cong 0) \\ \dot{W}_{out} &= \dot{m}(h_1 - h_2)\end{aligned}$$

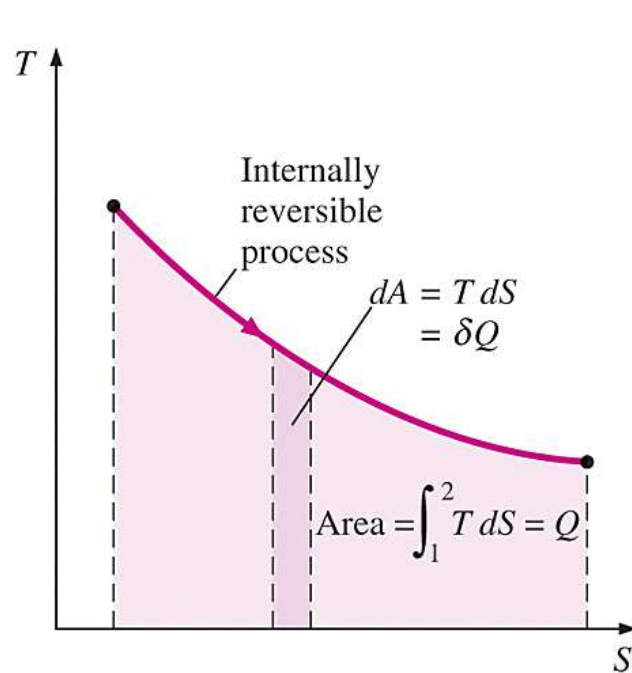
The inlet state is completely specified since two properties are given. But only one property (pressure) is given at the final state, and we need one more property to fix it. The second property comes from the observation that the process is reversible and adiabatic, and thus isentropic. Therefore, $s_2 = s_1$, and

$$\begin{array}{ll} \text{State 1:} & \left. \begin{array}{l} P_1 = 5 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array} \right\} \begin{array}{l} h_1 = 3317.2 \text{ kJ/kg} \\ s_1 = 6.8210 \text{ kJ/kg} \cdot \text{K} \end{array} \\ \\ \text{State 2:} & \left. \begin{array}{l} P_2 = 1.4 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} h_2 = 2967.4 \text{ kJ/kg} \end{array}$$

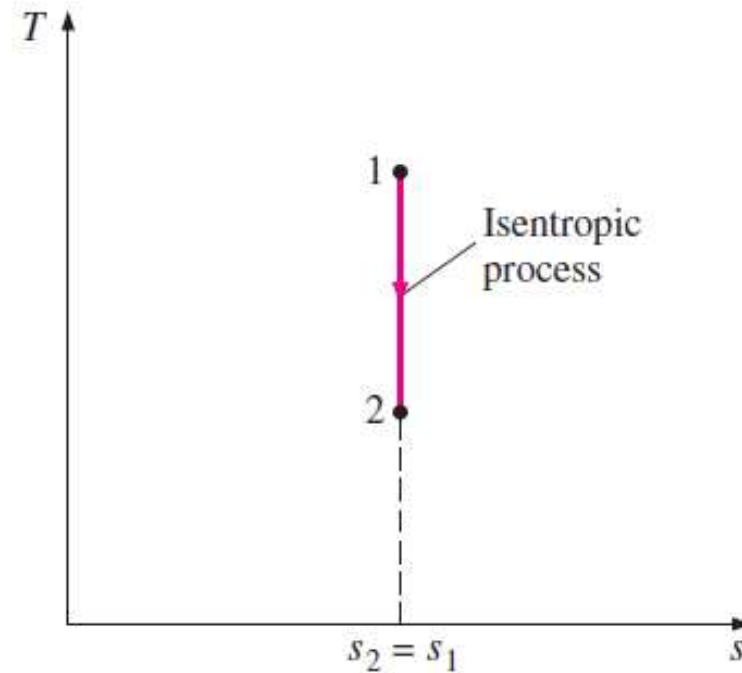
Then the work output of the turbine per unit mass of the steam becomes

$$w_{\text{out}} = h_1 - h_2 = 3317.2 - 2967.4 = \mathbf{349.8 \text{ kJ/kg}}$$

PROPERTY DIAGRAMS INVOLVING ENTROPY



On a T - S diagram, the area under the process curve represents the heat transfer for internally reversible processes.



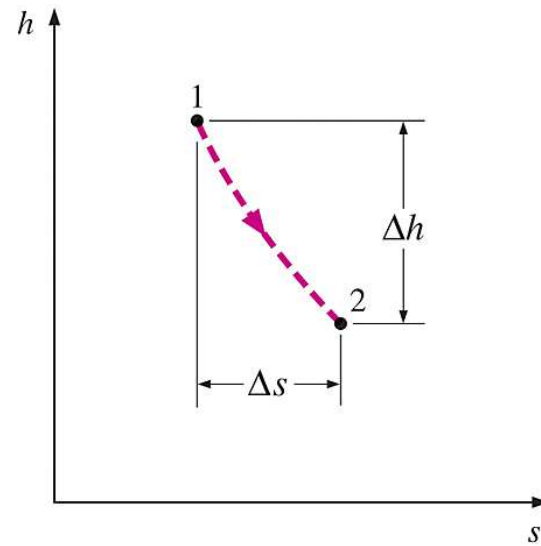
The isentropic process appears as a *vertical* line segment on a T - s diagram.

$$\delta Q_{\text{int rev}} = T dS \quad (\text{kJ})$$

$$\delta q_{\text{int rev}} = T ds \quad (\text{kJ/kg})$$

$$Q_{\text{int rev}} = \int_1^2 T dS \quad (\text{kJ})$$

$$q_{\text{int rev}} = \int_1^2 T ds \quad (\text{kJ/kg})$$



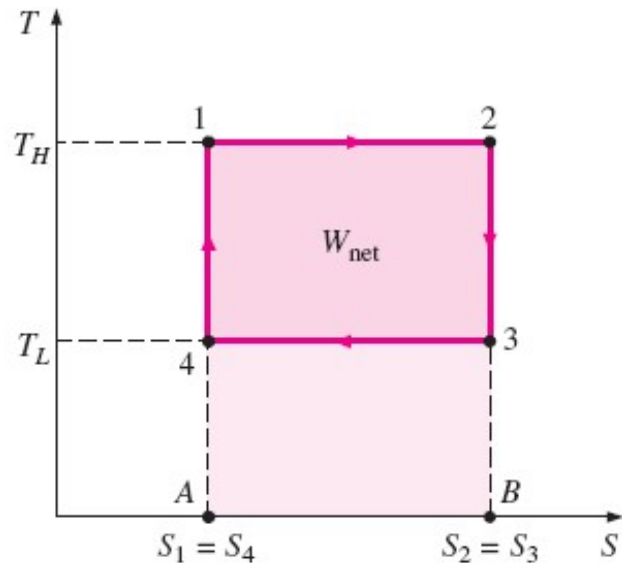
For adiabatic steady-flow devices, the vertical distance Δh on an h - s diagram is a measure of work, and the horizontal distance Δs is a measure of irreversibilities.

$$Q_{\text{int rev}} = T_0 \Delta S \quad (\text{kJ})$$

$$q_{\text{int rev}} = T_0 \Delta s \quad (\text{kJ/kg})$$

EXAMPLE 7-6

Show the Carnot cycle on a T - S diagram and indicate the areas that represent the heat supplied Q_H , heat rejected Q_L , and the net work output $W_{\text{net, out}}$ on this diagram.



Solution The Carnot cycle is to be shown on a T - S diagram, and the areas that represent Q_H , Q_L , and $W_{\text{net, out}}$ are to be indicated.

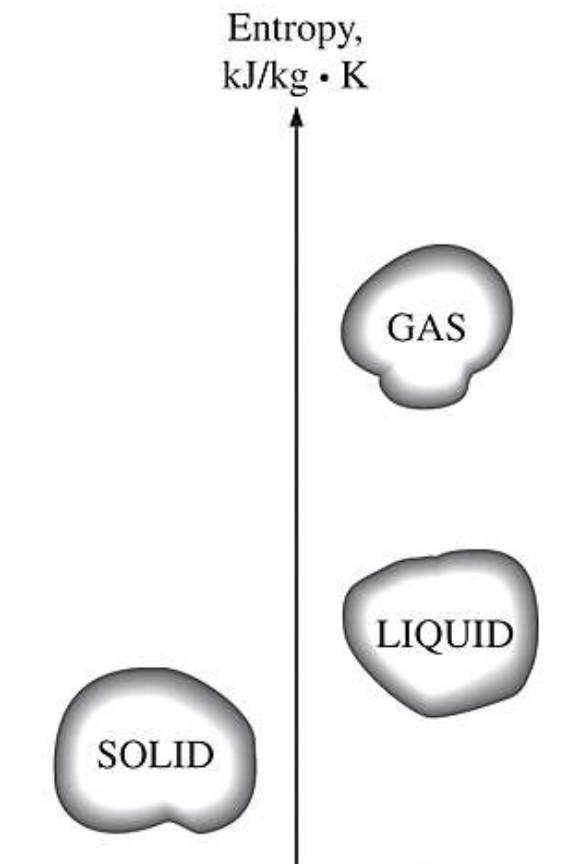
Analysis Recall that the Carnot cycle is made up of two reversible isothermal ($T=\text{constant}$) processes and two isentropic ($s=\text{constant}$) processes. These four processes form a rectangle on a T - S diagram,

On a T - S diagram, the area under the process curve represents the heat transfer for that process. Thus the area $A12B$ represents Q_H , the area $A43B$ represents Q_L , and the difference between these two (the area in color) represents the net work since

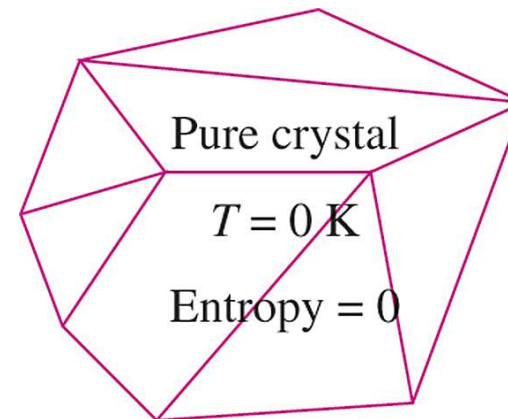
$$W_{\text{net, out}} = Q_H - Q_L$$

Therefore, the area enclosed by the path of a cycle (area 1234) on a T - S diagram represents the net work. Recall that the area enclosed by the path of a cycle also represents the net work on a P - V diagram.

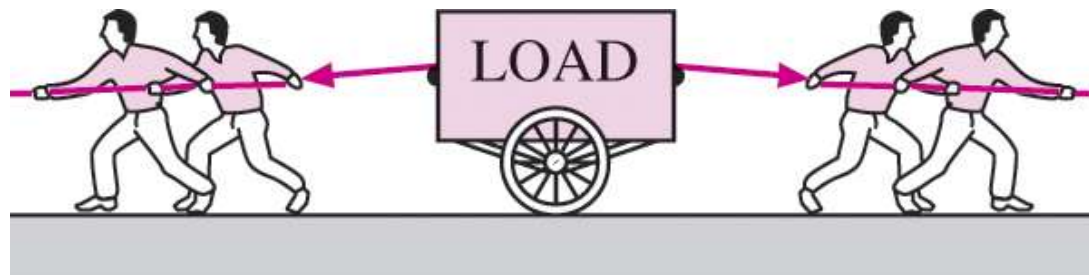
WHAT IS ENTROPY?



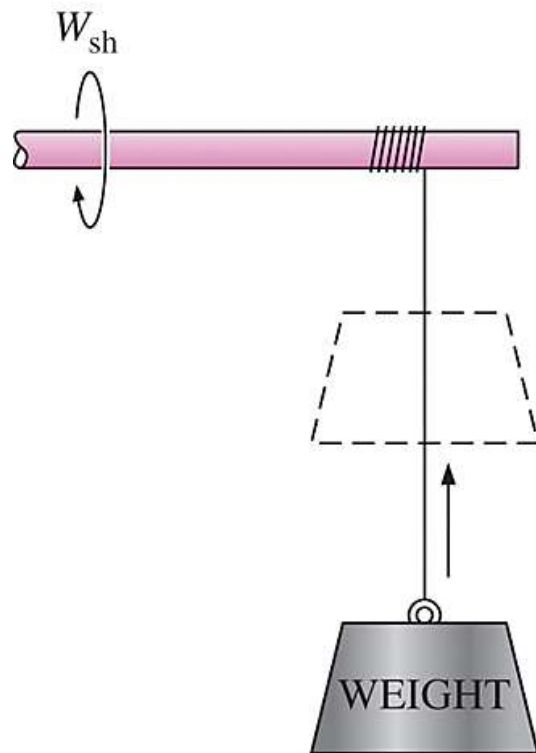
The level of molecular disorder (entropy) of a substance increases as it melts or evaporates.



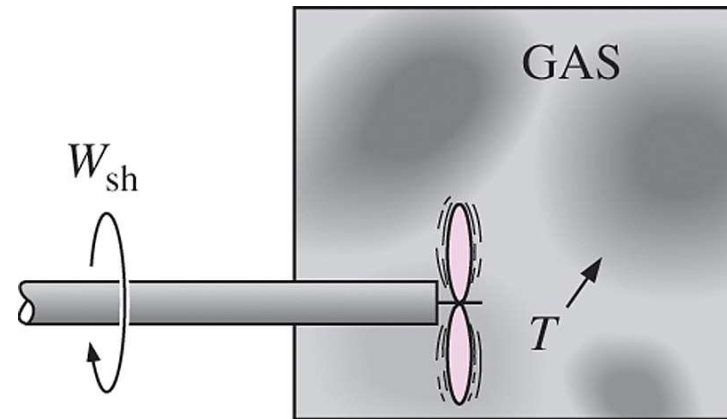
A pure crystalline substance at absolute zero temperature is in perfect order, and its entropy is zero **(the third law of thermodynamics)**.



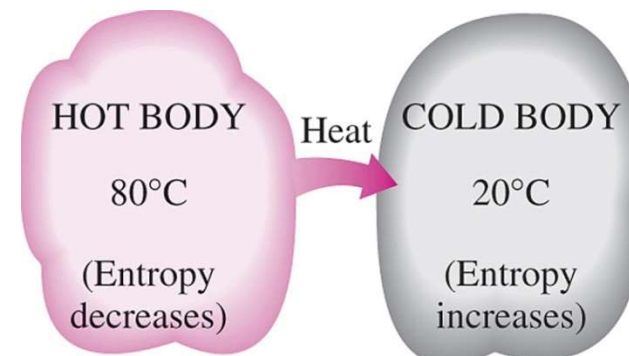
Disorganized energy does not create much useful effect, no matter how large it is.



In the absence of friction, raising a weight by a rotating shaft does not create any disorder (entropy), and thus energy is not degraded during this process.



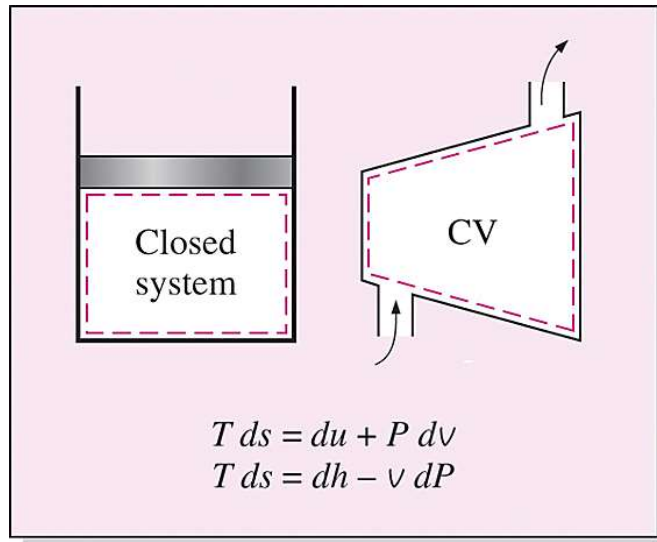
The paddle-wheel work done on a gas increases the level of disorder (entropy) of the gas, and thus energy is degraded during this process.



During a heat transfer process, the net entropy increases. (The increase in the entropy of the cold body more than offsets the decrease in the entropy of the hot body.)

THE $T ds$ RELATIONS

$$\delta Q_{\text{int rev}} - \delta W_{\text{int rev, out}} = dU$$



The $T ds$ relations are valid for both reversible and irreversible processes and for both closed and open systems.

the second $T ds$ equation

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

$$\delta Q_{\text{int rev}} = T dS$$

$$\delta W_{\text{int rev, out}} = P dV$$

$$T dS = dU + P dV \quad (\text{kJ})$$

$$T ds = du + P dv \quad (\text{kJ/kg})$$

the first $T ds$, or Gibbs equation

$$h = u + Pv$$

$$\left. \begin{array}{l} dh = du + P dv + v dP \\ T ds = du + P dv \end{array} \right\} T ds = dh - v dP$$

Differential changes in entropy in terms of other properties

ENTROPY CHANGE OF LIQUIDS AND SOLIDS

Liquids and solids can be approximated as *incompressible substances* since their specific volumes remain nearly constant during a process.

$$ds = \frac{du}{T} + \frac{P dv}{T}$$

$$dv \cong 0 \text{ for liquids and solids}$$

$$ds = \frac{du}{T} = \frac{c dT}{T}$$

$$\text{since } c_p = c_v = c \text{ and } du = c dT$$

$$\text{Liquids, solids: } s_2 - s_1 = \int_1^2 c(T) \frac{dT}{T} \cong c_{av} \ln \frac{T_2}{T_1} \quad (\text{kJ/kg} \cdot \text{K})$$

For an isentropic process of an incompressible substance

$$s_2 - s_1 = c_{av} \ln \frac{T_2}{T_1} = 0 \quad \rightarrow \quad T_2 = T_1$$

EXAMPLE 7-7

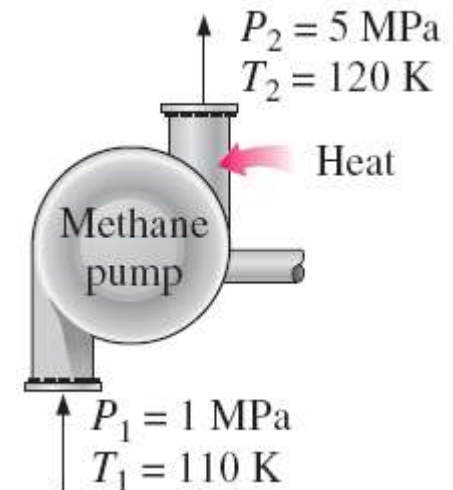
Liquid methane is commonly used in various cryogenic applications. The critical temperature of methane is 191 K (or -82°C), and thus methane must be maintained below 191 K to keep it in liquid phase. The properties of liquid methane at various temperatures and pressures are given in Table 7–1. Determine the entropy change of liquid methane as it undergoes a process from 110 K and 1 MPa to 120 K and 5 MPa (a) using actual data for methane and (b) approximating liquid methane as an incompressible substance. What is the error involved in the latter case?

Solution Liquid methane undergoes a process between two specified states. The entropy change of methane is to be determined by using compressed liquid data and by assuming methane to be incompressible, and the results are to be compared.

(a) We consider a unit mass of liquid methane. The entropies of the methane at the initial and final states are

State 1:	$\left. \begin{array}{l} P_1 = 1 \text{ MPa} \\ T_1 = 110 \text{ K} \end{array} \right\}$	$\begin{array}{l} s_1 = 4.875 \text{ kJ/kg} \cdot \text{K} \\ c_{p1} = 3.471 \text{ kJ/kg} \cdot \text{K} \end{array}$
State 2:	$\left. \begin{array}{l} P_2 = 5 \text{ MPa} \\ T_2 = 120 \text{ K} \end{array} \right\}$	$\begin{array}{l} s_2 = 5.145 \text{ kJ/kg} \cdot \text{K} \\ c_{p2} = 3.486 \text{ kJ/kg} \cdot \text{K} \end{array}$

$$\Delta s = s_2 - s_1 = 5.145 - 4.875 = \mathbf{0.270 \text{ kJ/kg} \cdot \text{K}}$$



(b) Approximating liquid methane as an incompressible substance, its entropy change is determined to be

$$\Delta s = c_{av} \ln \frac{T_2}{T_1} = (3.4785 \text{ kJ/kg} \cdot \text{K}) \ln \frac{120 \text{ K}}{110 \text{ K}} = \mathbf{0.303 \text{ kJ/kg} \cdot \text{K}}$$

$$c_{p, av} = \frac{c_{p1} + c_{p2}}{2} = \frac{3.471 + 3.486}{2} = 3.4785 \text{ kJ/kg} \cdot \text{K}$$

Therefore, the error involved in approximating liquid methane as an incompressible substance is

$$\text{Error} = \frac{|\Delta s_{\text{actual}} - \Delta s_{\text{ideal}}|}{\Delta s_{\text{actual}}} = \frac{|0.270 - 0.303|}{0.270} = \mathbf{0.122 \text{ (or 12.2\%)}}$$

Discussion This result is not surprising since the density of liquid methane changes during this process from 425.8 to 415.2 kg/m³ (about 3 %), which makes us question the validity of the incompressible substance assumption. Still, this assumption enables us to obtain reasonably accurate results with less effort, which proves to be very convenient in the absence of compressed liquid data.

THE ENTROPY CHANGE OF IDEAL GASES

From the first $T ds$ relation

$$ds = \frac{du}{T} + \frac{P dv}{T} \quad \begin{aligned} du &= c_v dT \\ P &= RT/v \end{aligned}$$

$$ds = c_v \frac{dT}{T} + R \frac{dv}{v}$$

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

From the second $T ds$ relation

$$ds = \frac{dh}{T} - \frac{v dP}{T}$$

$$dh = c_p dT \quad v = RT/P$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} P v &= RT \\ du &= C_v dT \\ dh &= C_p dT \end{aligned}$$

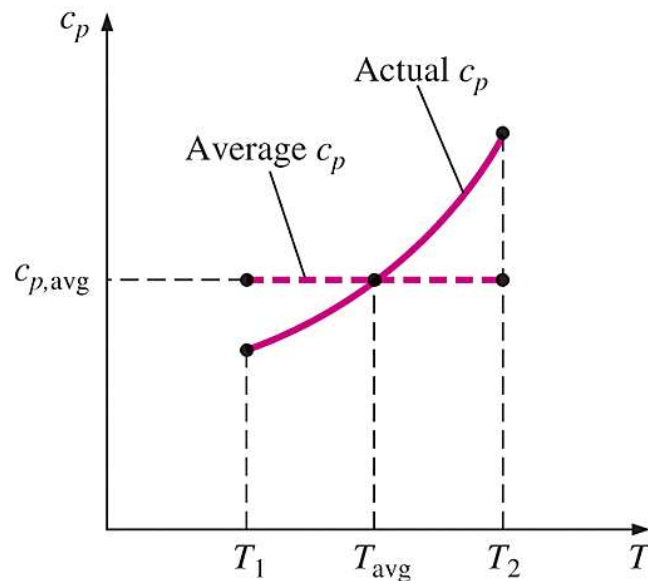
Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = \int_1^2 c_v(T) \frac{dT}{T} + R \ln \frac{v_2}{v_1}$$

$$s_2 - s_1 = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad (\text{kJ/kg} \cdot \text{K})$$

$$s_2 - s_1 = \int_1^2 c_p(T) \frac{dT}{T} - R \ln \frac{P_2}{P_1}$$

$$s_2 - s_1 = c_{p,av} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$



Entropy change of an ideal gas on a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{v,av} \ln \frac{T_2}{T_1} + R_u \ln \frac{v_2}{v_1} \quad (\text{kJ/mol} \cdot \text{K})$$

$$\bar{s}_2 - \bar{s}_1 = \bar{c}_{p,av} \ln \frac{T_2}{T_1} - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/mol} \cdot \text{K})$$

Under the constant-specific-heat assumption, the specific heat is assumed to be constant at some average value.

Variable Specific Heats (Exact Analysis)

We choose absolute zero as the reference temperature and define a function s° as

$$s^\circ = \int_0^T c_p(T) \frac{dT}{T}$$

$$\int_1^2 c_p(T) \frac{dT}{T} = s_2^\circ - s_1^\circ$$

On a unit-mass basis

$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \quad (\text{kJ/kg} \cdot \text{K})$$

On a unit-mole basis

$$\bar{s}_2 - \bar{s}_1 = \bar{s}_2^\circ - \bar{s}_1^\circ - R_u \ln \frac{P_2}{P_1} \quad (\text{kJ/kmol} \cdot \text{K})$$

T, K	$s^\circ(T), \text{kJ/kg} \cdot \text{K}$
.	.
.	.
.	.
300	1.70203
310	1.73498
320	1.76690
.	.
.	.
.	.

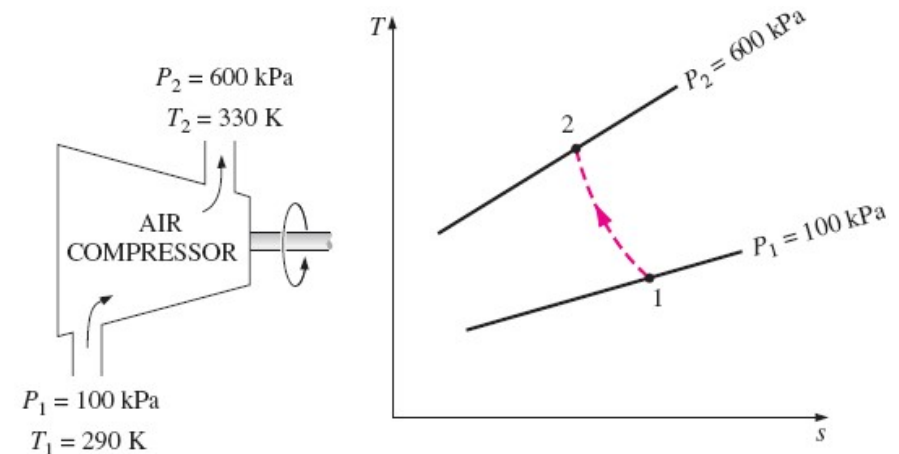
The entropy of an ideal gas depends on both T and P . The function s represents only the temperature-dependent part of entropy.

EXAMPLE 7-9

Air is compressed from an initial state of 100 kPa and 17°C to a final state of 600 kPa and 57°C. Determine the entropy change of air during this compression process by using (a) property values from the air table and (b) average specific heats.

Solution Air is compressed between two specified states. The entropy change of air is to be determined by using tabulated property values and also by using average specific heats.

Assumptions Air is an ideal gas since it is at a high temperature and low pressure relative to its critical-point values. Therefore, entropy change relations developed under the ideal-gas assumption are applicable.



(a) The properties of air are given in the air table (Table A–21). Reading s values at given temperatures and substituting, we find

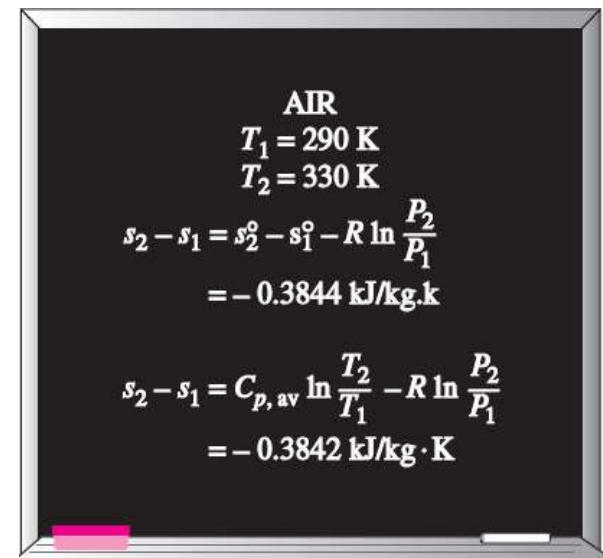
$$s_2 - s_1 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1}$$

$$\begin{aligned} &= [(1.79783 - 1.66802) \text{ kJ/kg} \cdot \text{K}] - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\ &= \mathbf{-0.3844 \text{ kJ/kg} \cdot \text{K}} \end{aligned}$$

(b) The entropy change of air during this process can also be determined approximately from Eq. 7–34 by using a c_p value at the average temperature of 37°C (Table A–2b) and treating it as a constant:

$$\begin{aligned}s_2 - s_1 &= c_{p, \text{av}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\&= (1.006 \text{ kJ/kg} \cdot \text{K}) \ln \frac{330 \text{ K}}{290 \text{ K}} - (0.287 \text{ kJ/kg} \cdot \text{K}) \ln \frac{600 \text{ kPa}}{100 \text{ kPa}} \\&= \mathbf{-0.3842 \text{ kJ/kg} \cdot \text{K}}\end{aligned}$$

Discussion The two results above are almost identical since the change in temperature during this process is relatively small. When the temperature change is large, however, they may differ significantly. For those cases, Eq. 7–39 should be used instead of Eq. 7–34 since it accounts for the variation of specific heats with temperature.



AIR
 $T_1 = 290 \text{ K}$
 $T_2 = 330 \text{ K}$

$$\begin{aligned}s_2 - s_1 &= s_2^o - s_1^o - R \ln \frac{P_2}{P_1} \\&= -0.3844 \text{ kJ/kg.k}\end{aligned}$$
$$\begin{aligned}s_2 - s_1 &= c_{p, \text{av}} \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} \\&= -0.3842 \text{ kJ/kg} \cdot \text{K}\end{aligned}$$

Isentropic Processes of Ideal Gases

Constant Specific Heats (Approximate Analysis)

$$s_2 - s_1 = c_{v,av} \ln \frac{T_2}{T_1} + R \ln \frac{v_2}{v_1} \quad (\text{kJ/kg} \cdot \text{K})$$

Setting this eq. equal to zero, we get

$$\ln \frac{T_2}{T_1} = -\frac{R}{c_v} \ln \frac{v_1}{v_2} \quad \ln \frac{T_2}{T_1} = \ln \left(\frac{v_1}{v_2} \right)^{R/c_v}$$

$$R = c_p - c_v, \quad k = c_p/c_v \quad R/c_v = k - 1$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k}$$

$$\left(\frac{P_2}{P_1} \right)_{s=\text{const.}} = \left(\frac{v_1}{v_2} \right)^k$$

$$\begin{aligned} TV^{k-1} &= \text{constant} \\ TP^{(1-k)/k} &= \text{constant} \quad (\text{ideal gas}) \\ PV^k &= \text{constant} \end{aligned}$$

$$\left(\frac{T_2}{T_1} \right)_{s=\text{const.}} = \left(\frac{P_2}{P_1} \right)^{(k-1)/k} = \left(\frac{v_1}{v_2} \right)^{k-1}$$

VALID FOR
*ideal gas
*isentropic process
*constant specific heats

The isentropic relations of ideal gases are valid for the isentropic processes of ideal gases only.

Isentropic Processes of Ideal Gases

Variable Specific Heats (Exact Analysis)

$$0 = s_2^\circ - s_1^\circ - R \ln \frac{P_2}{P_1} \rightarrow s_2^\circ - s_1^\circ + R \ln \frac{P_2}{P_1}$$

Relative Pressure and Relative Specific Volume

$$\frac{P_2}{P_1} = \exp \frac{s_2^\circ - s_1^\circ}{R}$$

$\exp(s^\circ/R)$ is the relative pressure P_r .

$$\frac{P_2}{P_1} = \frac{\exp(s_2^\circ/R)}{\exp(s_1^\circ/R)}$$

$$\left(\frac{P_2}{P_1} \right)_{s=\text{const.}} = \frac{P_{r2}}{P_{r1}}$$

$$\frac{P_1 v_1}{T_1} = \frac{P_2 v_2}{T_2} \rightarrow \frac{v_2}{v_1} = \frac{T_2}{T_1} \frac{P_1}{P_2} = \frac{T_2}{T_1} \frac{P_{r1}}{P_{r2}} = \frac{T_2/P_{r2}}{T_1/P_{r1}}$$

$$\left(\frac{v_2}{v_1} \right)_{s=\text{const.}} = \frac{v_{r2}}{v_{r1}}$$

T/P_r is the relative specific volume v_r .

Process: isentropic

Given: P_1 , T_1 , and P_2

Find: T_2

T	P_r
.	.
.	.
.	.
T_2	← read $P_{r2} = \frac{P_2}{P_1} P_{r1}$
.	.
.	.
.	.
T_1	→ read P_{r1}
.	.
.	.
.	.

The use of P_r data for calculating the final temperature during an isentropic process.

Process: isentropic

Given: v_1 , T_1 , and v_2

Find: T_2

T	v_r
.	.
.	.
.	.
T_2	← read $v_{r2} = \frac{v_2}{v_1} v_{r1}$
.	.
.	.
.	.
T_1	→ read v_{r1}
.	.
.	.
.	.

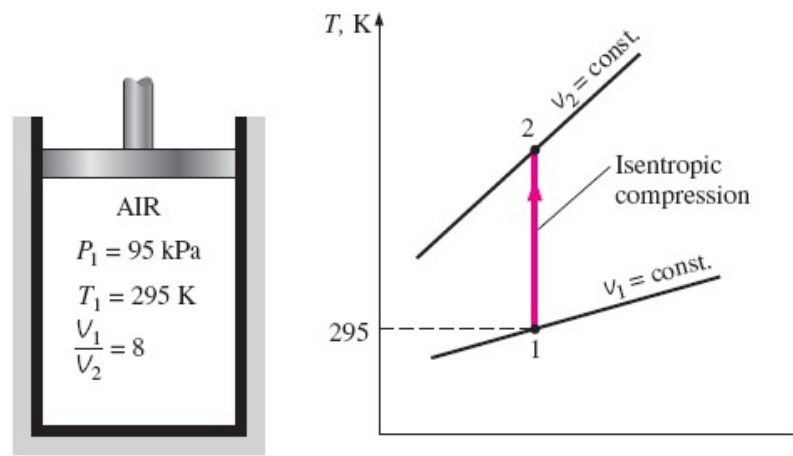
The use of v_r data for calculating the final temperature during an isentropic process

EXAMPLE 7-10

Air is compressed in a car engine from 22°C and 95 kPa in a reversible and adiabatic manner. If the compression ratio V_1/V_2 of this piston–cylinder device is 8, determine the final temperature of the air.

Solution Air is compressed in a car engine isentropically. For a given compression ratio, the final air temperature is to be determined.

Assumptions At specified conditions, air can be treated as an ideal gas. Therefore, the isentropic relations developed earlier for ideal gases are applicable.



For closed systems: $\frac{V_2}{V_1} = \frac{v_2}{v_1}$ At $T_1 = 295 \text{ K}$: $v_{r1} = 647.9$

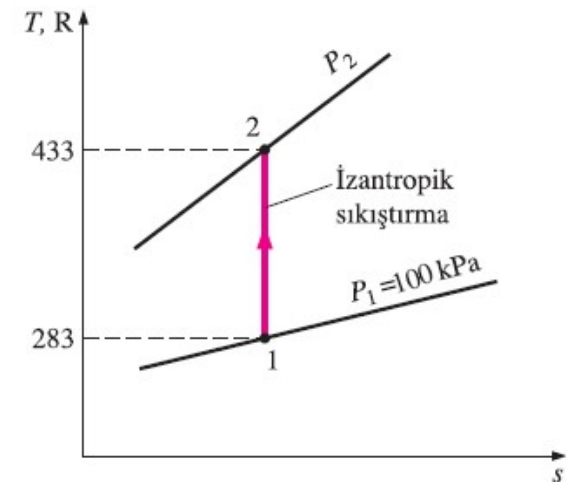
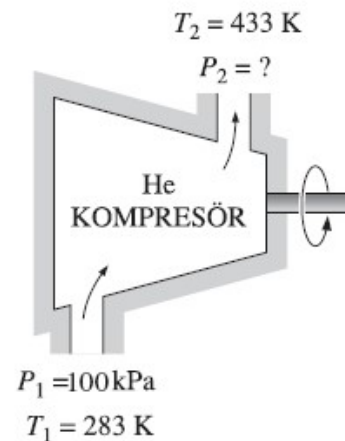
$$v_{r2} = v_{r1} \left(\frac{v_2}{v_1} \right) = (647.9) \left(\frac{1}{8} \right) = 80.99 \rightarrow T_2 = \mathbf{662.7 \text{ K}}$$

Therefore, the temperature of air will increase by 367.7°C during this process.

EXAMPLE 7-11

Helium gas is compressed in an adiabatic compressor from an initial state of 100 kPa and 10°C to a final temperature of 160°C in a reversible manner. Determine the exit pressure of helium.

Assumptions At specified conditions helium can be treated as an ideal gas since it is at a high temperature relative to its critical-point value of 450F. Therefore, the isentropic relations developed earlier for ideal gases are applicable.



$$P_2 = P_1 \left(\frac{T_2}{T_1} \right)^{k/(k-1)} = (100 \text{ kPa}) \left(\frac{780 \text{ R}}{510 \text{ R}} \right)^{1.667/0.667} = \mathbf{289 \text{ kPa}}$$

REVERSIBLE STEADY-FLOW WORK

The work done during a process depends on the path followed as well as on the properties at the end states. Recall that reversible (quasi-equilibrium) moving boundary work associated with closed systems is expressed in terms of the fluid properties as

$$\left. \begin{array}{l} \delta q_{\text{rev}} = T ds \\ T ds = dh - v dP \end{array} \right\} \delta q_{\text{rev}} = dh - v dP$$

$$-\delta w_{\text{rev}} = v dP + d\text{ke} + d\text{pe}$$

$$w_{\text{rev}} = - \int_1^2 v dP - \Delta\text{ke} - \Delta\text{pe} \quad (\text{kJ/kg})$$

When kinetic and potential energies are negligible

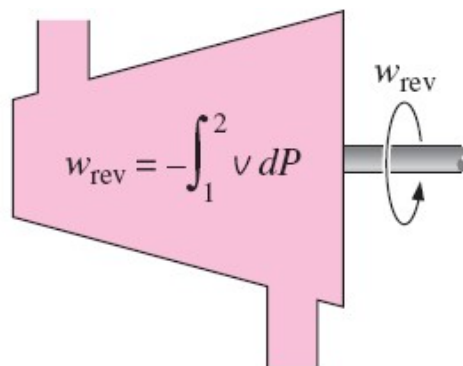
$$w_{\text{rev}} = - \int_1^2 v dP \quad (\text{kJ/kg})$$

$$w_{\text{rev, in}} = \int_1^2 v dP + \Delta\text{ke} + \Delta\text{pe}$$

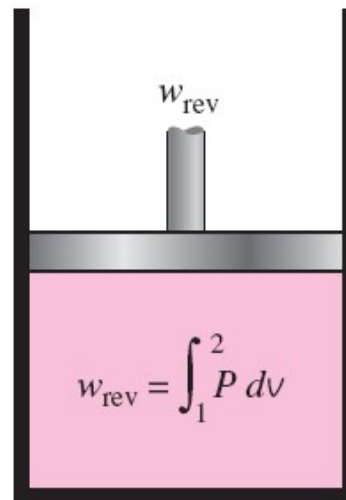
$$w_{\text{rev}} = -v(P_2 - P_1) - \Delta\text{ke} - \Delta\text{pe} \quad (\text{kJ/kg})$$

For the steady flow of a liquid through a device that involves no work interactions (such as a pipe section), the work term is zero (**Bernoulli equation**):

$$v(P_2 - P_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1) = 0$$

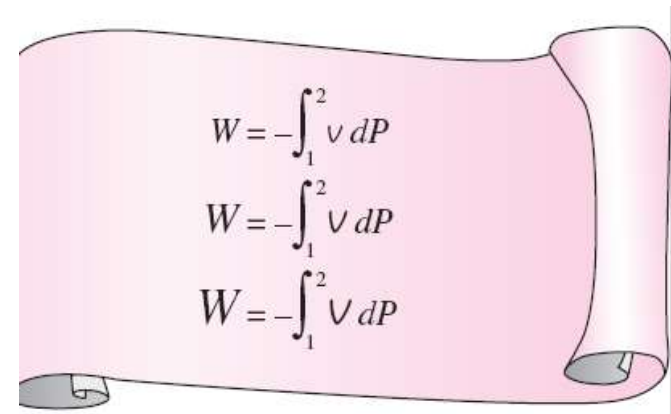


(a) Steady-flow system



(b) Closed system

Reversible work relations for steady-flow and closed systems.



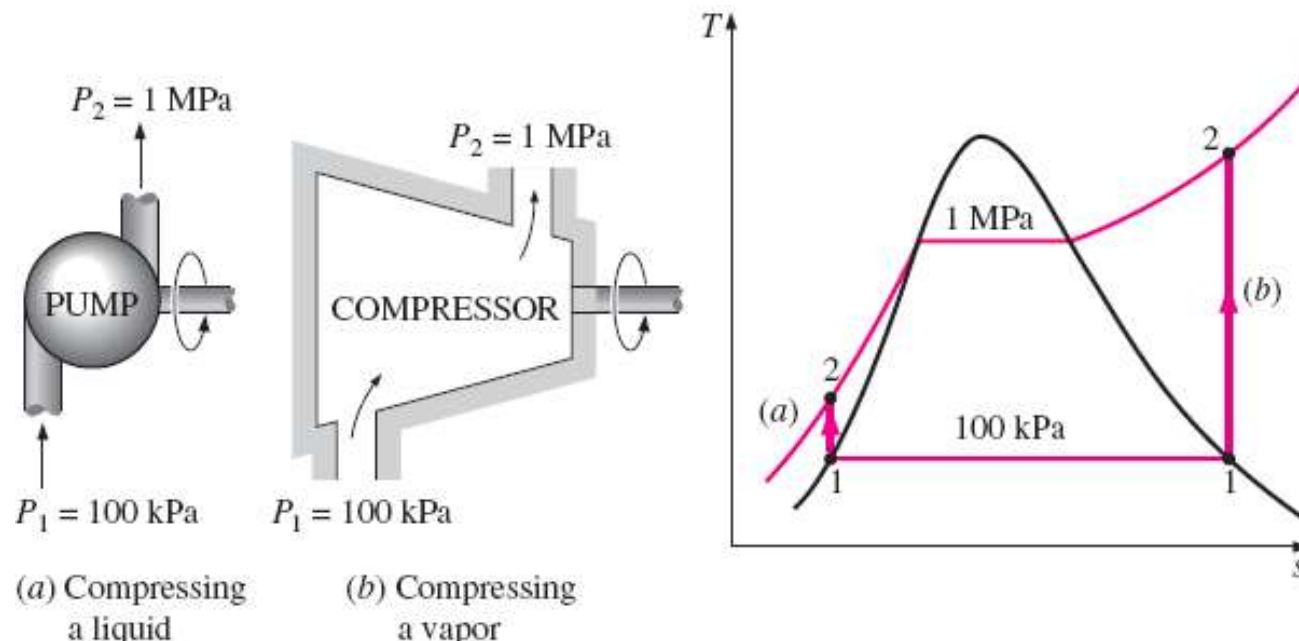
The larger the specific volume, the greater the work produced (or consumed) by a steady-flow device.

EXAMPLE 7-12

Determine the compressor work input required to compress steam isentropically from 100 kPa to 1 MPa, assuming that the steam exists as (a) saturated liquid and (b) saturated vapor at the inlet state.

Solution Steam is to be compressed from a given pressure to a specified pressure isentropically. The work input is to be determined for the cases of steam being a saturated liquid and saturated vapor at the inlet.

Assumptions 1 Steady operating conditions exist. 2 Kinetic and potential energy changes are negligible. 3 The process is given to be isentropic.



(a) In this case, steam is a saturated liquid initially, and its specific volume is

$$v_1 = v_{f@100 \text{ kPa}} = 0.001043 \text{ m}^3/\text{kg}$$

$$\begin{aligned} w_{\text{rev, in}} &= \int_1^2 v \, dP \cong v_1(P_2 - P_1) \\ &= (0.001043 \text{ m}^3/\text{kg})[(1000 - 100) \text{ kPa}] \left(\frac{1 \text{ kJ}}{1 \text{ kPa} \cdot \text{m}^3} \right) \\ &= \mathbf{0.94 \text{ kJ/kg}} \end{aligned}$$

(b) This time, steam is a saturated vapor initially and remains a vapor during the entire compression process. Since the specific volume of a gas changes considerably during a compression process, we need to know how v varies with P to perform the integration in Eq. 7-53. This relation, in general, is not readily available. But for an isentropic process, it is easily obtained from the

$$\left. \begin{aligned} T \, ds &= dh - v \, dP \quad (\text{Eq. 7-24}) \\ ds &= 0 \quad (\text{isentropic process}) \end{aligned} \right\} v \, dP = dh$$

$$w_{\text{rev, in}} = \int_1^2 v \, dP = \int_1^2 dh = h_2 - h_1$$

$$\begin{array}{ll} \text{State 1:} & \left. \begin{array}{l} P_1 = 100 \text{ kPa} \\ \text{(sat. vapor)} \end{array} \right\} \begin{array}{l} h_1 = 2675.0 \text{ kJ/kg} \\ s_1 = 7.3589 \text{ kJ/kg} \cdot \text{K} \end{array} \\ \\ \text{State 2:} & \left. \begin{array}{l} P_2 = 1 \text{ MPa} \\ s_2 = s_1 \end{array} \right\} h_2 = 3194.5 \text{ kJ/kg} \end{array}$$

$$w_{\text{rev, in}} = (3194.5 - 2675.0) \text{ kJ/kg} = \mathbf{519.5 \text{ kJ/kg}}$$

Discussion Note that compressing steam in the vapor form would require over 500 times more work than compressing it in the liquid form between the same pressure limits.

Proof that Steady-Flow Devices Deliver the Most and Consume the Least Work when the Process Is Reversible

Taking heat input and work output positive:

$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe$$

Actual

$$\delta q_{\text{rev}} - \delta w_{\text{rev}} = dh + dke + dpe$$

Reversible

$$\delta q_{\text{act}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta w_{\text{rev}}$$

$$\delta w_{\text{rev}} - \delta w_{\text{act}} = \delta q_{\text{rev}} - \delta q_{\text{act}}$$

$$\delta q_{\text{rev}} = T ds \quad ds \geq \frac{\delta q_{\text{act}}}{T}$$

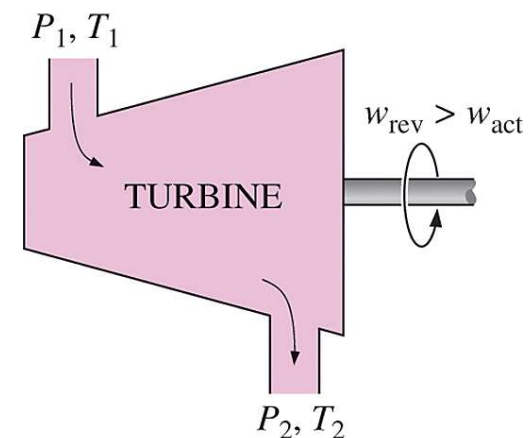
$$\delta q_{\text{act}} - \delta w_{\text{act}} = dh + dke + dpe$$

$$\frac{\delta w_{\text{rev}} - \delta w_{\text{act}}}{T} = ds - \frac{\delta q_{\text{act}}}{T} \geq 0$$

$$\delta w_{\text{rev}} \geq \delta w_{\text{act}}$$

$$w_{\text{rev}} \geq w_{\text{act}}$$

A reversible turbine delivers more work than an irreversible one if both operate between the same end states.



Work-producing devices such as turbines deliver more work, and work-consuming devices such as pumps and compressors require less work when they operate reversibly.

MINIMIZING THE COMPRESSOR WORK

When kinetic and potential energies are negligible

$$w_{\text{rev, in}} = \int_1^2 v \, dP$$

Isentropic ($Pv^k = \text{constant}$):

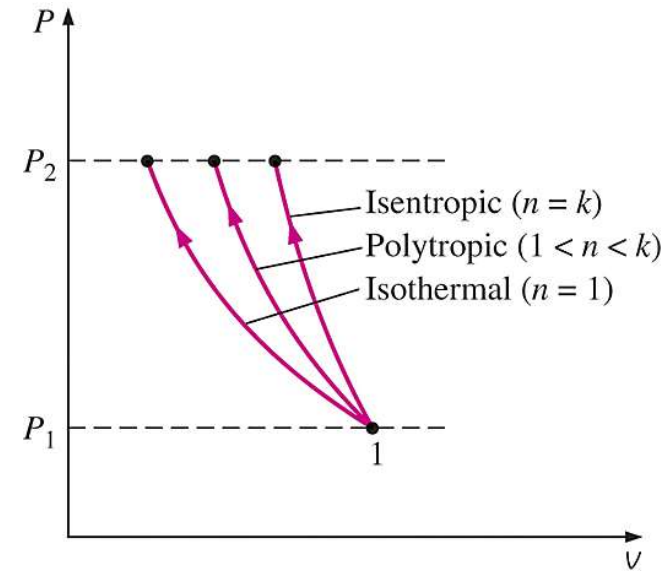
$$w_{\text{comp, in}} = \frac{kR(T_2 - T_1)}{k - 1} = \frac{kRT_1}{k - 1} \left[\left(\frac{P_2}{P_1} \right)^{(k-1)/k} - 1 \right]$$

Polytropic ($Pv^n = \text{constant}$):

$$w_{\text{comp, in}} = \frac{nR(T_2 - T_1)}{n - 1} = \frac{nRT_1}{n - 1} \left[\left(\frac{P_2}{P_1} \right)^{(n-1)/n} - 1 \right]$$

Isothermal ($Pv = \text{constant}$):

$$w_{\text{comp, in}} = RT \ln \frac{P_2}{P_1}$$

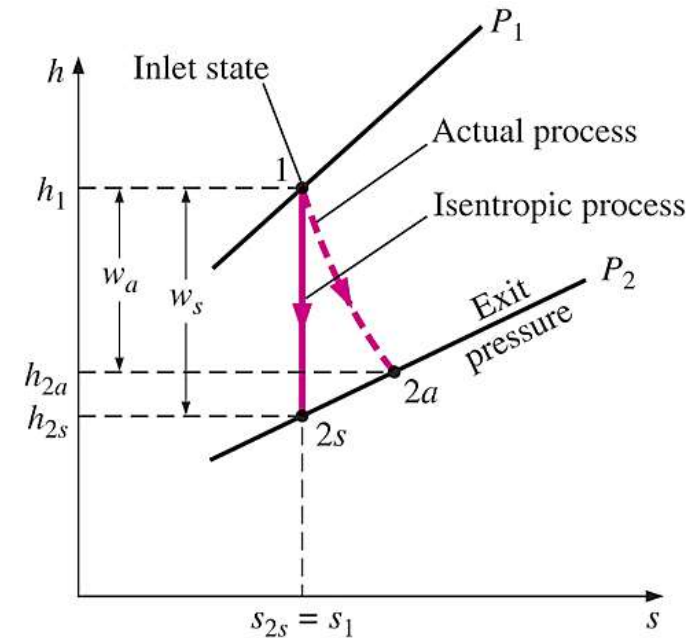
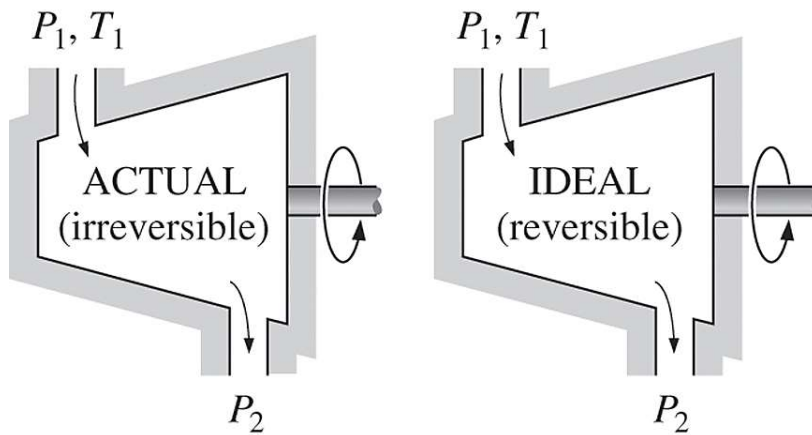


P - v diagrams of isentropic, polytropic, and isothermal compression processes between the same pressure limits.

The adiabatic compression ($Pv^k = \text{constant}$) requires the maximum work and the isothermal compression ($T = \text{constant}$) requires the minimum.

ISENTROPIC EFFICIENCIES OF STEADY-FLOW DEVICES

The isentropic process involves no irreversibilities and serves as the ideal process for **adiabatic devices**.



The $h-s$ diagram for the actual and isentropic processes of an adiabatic turbine.

Isentropic Efficiency of Turbines

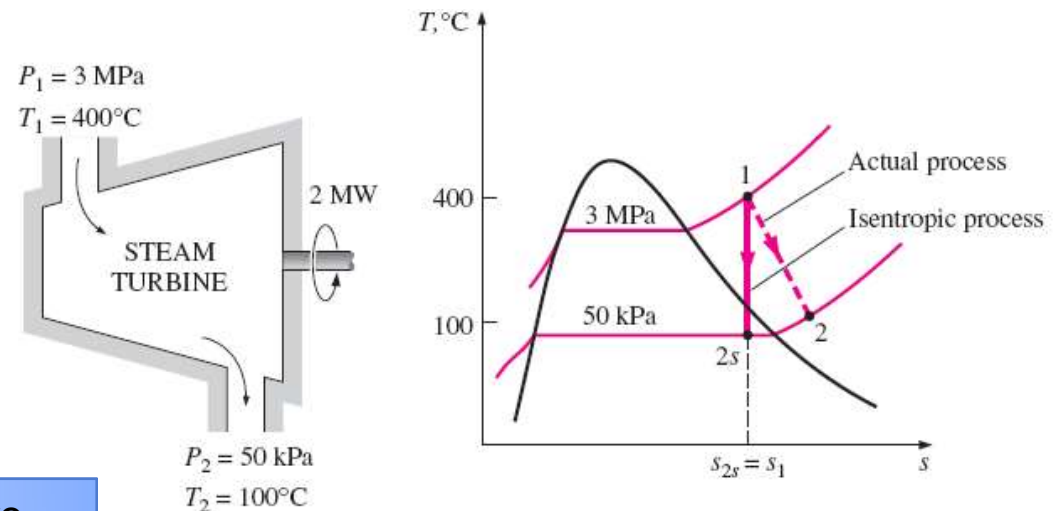
$$\eta_T = \frac{\text{Actual turbine work}}{\text{Isentropic turbine work}} = \frac{w_a}{w_s} \quad \eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$

EXAMPLE 7-14

Steam enters an adiabatic turbine steadily at 3 MPa and 400°C and leaves at 50 kPa and 100°C. If the power output of the turbine is 2 MW, determine (a) the isentropic efficiency of the turbine and (b) the mass flow rate of the steam flowing through the turbine.

Solution Steam flows steadily between specified inlet and exit states. For a specified power output, the isentropic efficiency and the mass flow rate are to be determined.

Assumptions 1 Steady operating conditions exist. 2 The changes in kinetic and potential energies are negligible. 3 The turbine is adiabatic.



(a) The enthalpies at various states are

State 1:	$P_1 = 3 \text{ MPa}$	$\left. \begin{array}{l} h_1 = 3231.7 \text{ kJ/kg} \\ s_1 = 6.9235 \text{ kJ/kg} \cdot \text{K} \end{array} \right\}$
	$T_1 = 400^\circ\text{C}$	
State 2a:	$P_{2a} = 50 \text{ kPa}$	$\left. \begin{array}{l} h_{2a} = 2682.4 \text{ kJ/kg} \end{array} \right\}$
	$T_{2a} = 100^\circ\text{C}$	

The exit enthalpy of the steam for the isentropic process h_{2s} is determined from the requirement that the entropy of the steam remain constant ($s_{2s} = s_1$):

$$\text{State } 2s: \quad \begin{array}{l} P_{2s} = 50 \text{ kPa} \\ (s_{2s} = s_1) \end{array} \rightarrow \begin{array}{l} s_f = 1.0912 \text{ kJ/kg} \cdot \text{K} \\ s_g = 7.5931 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$x_{2s} = \frac{s_{2s} - s_f}{s_{fg}} = \frac{6.9235 - 1.0912}{6.5019} = 0.897$$

$$h_{2s} = h_f + x_{2s}h_{fg} = 340.54 + 0.897(2304.7) = 2407.9 \text{ kJ/kg}$$

$$\eta_T \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{3231.7 - 2682.4}{3231.7 - 2407.9} = \mathbf{0.667, \text{ or } 66.7\%}$$

(b) The mass flow rate of steam through this turbine is determined from the energy balance for steady-flow systems:

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 = \dot{W}_{a, \text{out}} + \dot{m}h_{2a}$$

$$\dot{W}_{a, \text{out}} = \dot{m}(h_1 - h_{2a})$$

$$2 \text{ MW} \left(\frac{1000 \text{ kJ/s}}{1 \text{ MW}} \right) = \dot{m}(3231.7 - 2682.4) \text{ kJ/kg}$$

$$\dot{m} = \mathbf{3.64 \text{ kg/s}}$$

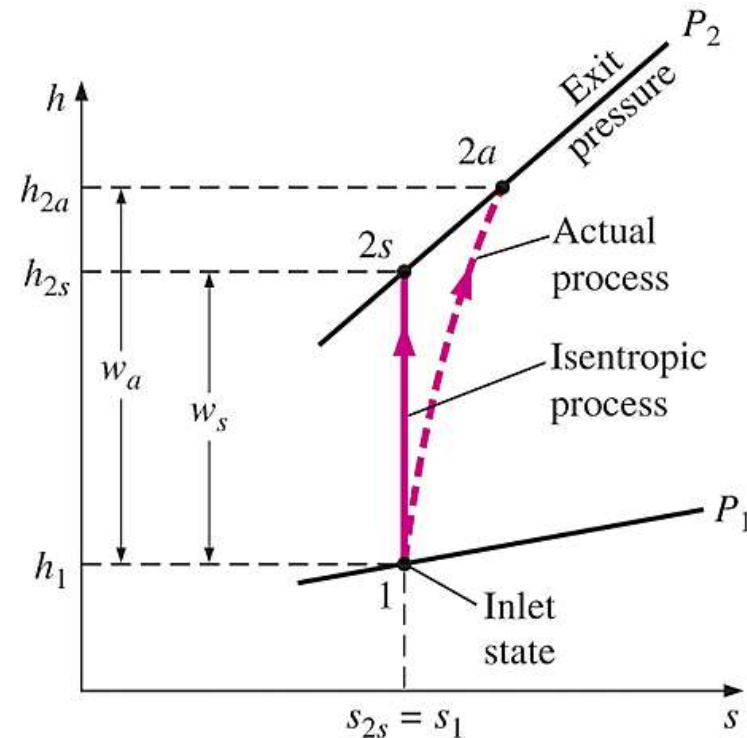
Isentropic Efficiencies of Compressors and Pumps

$$\eta_c = \frac{\text{Isentropic compressor work}}{\text{Actual compressor work}} = \frac{w_s}{w_a}$$

$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \quad \text{When kinetic and potential energies are negligible}$$

$$\eta_P = \frac{w_s}{w_a} = \frac{v(P_2 - P_1)}{h_{2a} - h_1} \quad \text{For a pump}$$

$$\eta_c = \frac{w_t}{w_a} \quad \text{Isothermal efficiency}$$



The h - s diagram of the actual and isentropic processes of an adiabatic compressor.

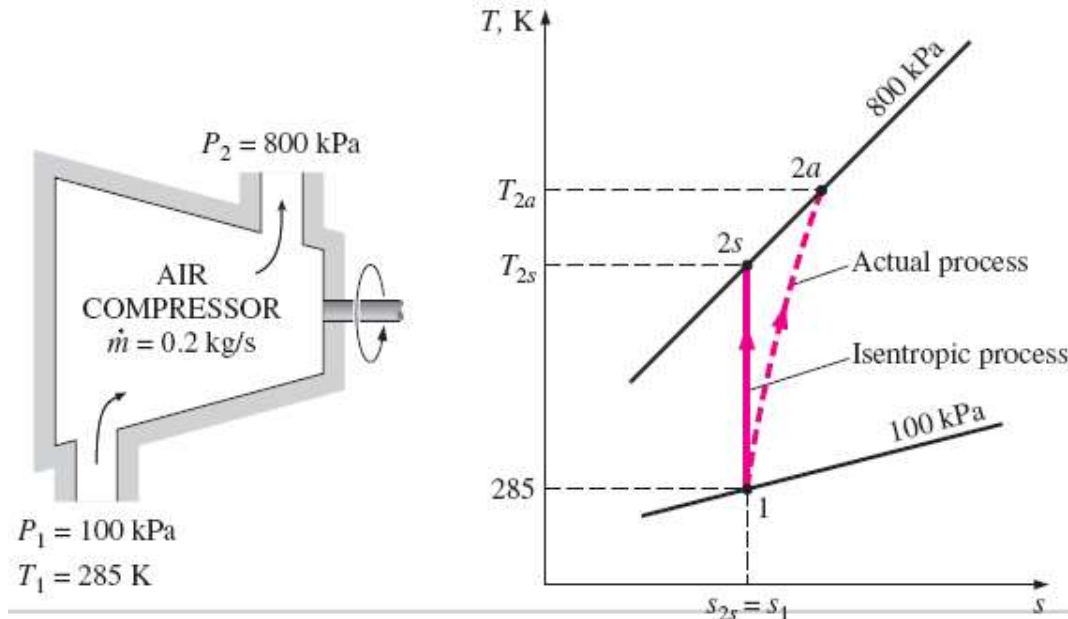
EXAMPLE 7-15

Air is compressed by an adiabatic compressor from 100 kPa and 12°C to a pressure of 800 kPa at a steady rate of 0.2 kg/s. If the isentropic efficiency of the compressor is 80 percent, determine (a) the exit temperature of air and (b) the required power input to the compressor.

Solution Air is compressed to a specified pressure at a specified rate. For a given isentropic efficiency, the exit temperature and the power input are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The changes in kinetic and potential energies are negligible. 4 The compressor is adiabatic.

(a) We know only one property (pressure) at the exit state, and we need to know one more to fix the state and thus determine the exit temperature. The property that can be determined with minimal effort in this case is h_{2a} since the isentropic efficiency of the compressor is given. At the compressor inlet,



$$T_1 = 285 \text{ K} \rightarrow h_1 = 285.14 \text{ kJ/kg}$$

$$(P_{r1} = 1.1584)$$

$$P_{r2} = P_{r1} \left(\frac{P_2}{P_1} \right) = 1.1584 \left(\frac{800 \text{ kPa}}{100 \text{ kPa}} \right) = 9.2672$$

$$P_{r2} = 9.2672 \rightarrow h_{2s} = 517.05 \text{ kJ/kg}$$

$$\eta_c \cong \frac{h_{2s} - h_1}{h_{2a} - h_1} \rightarrow 0.80 = \frac{(517.05 - 285.14) \text{ kJ/kg}}{(h_{2a} - 285.14) \text{ kJ/kg}}$$

$$h_{2a} = 575.03 \text{ kJ/kg} \rightarrow T_{2a} = \mathbf{569.5 \text{ K}}$$

(b) The required power input to the compressor is determined from the energy balance for steady-flow devices,

$$\dot{E}_{\text{in}} = \dot{E}_{\text{out}}$$

$$\dot{m}h_1 + \dot{W}_{a, \text{in}} = \dot{m}h_{2a}$$

$$\dot{W}_{a, \text{in}} = \dot{m}(h_{2a} - h_1)$$

$$= (0.2 \text{ kg/s})[(575.03 - 285.14) \text{ kJ/kg}] = \mathbf{58.0 \text{ kW}}$$

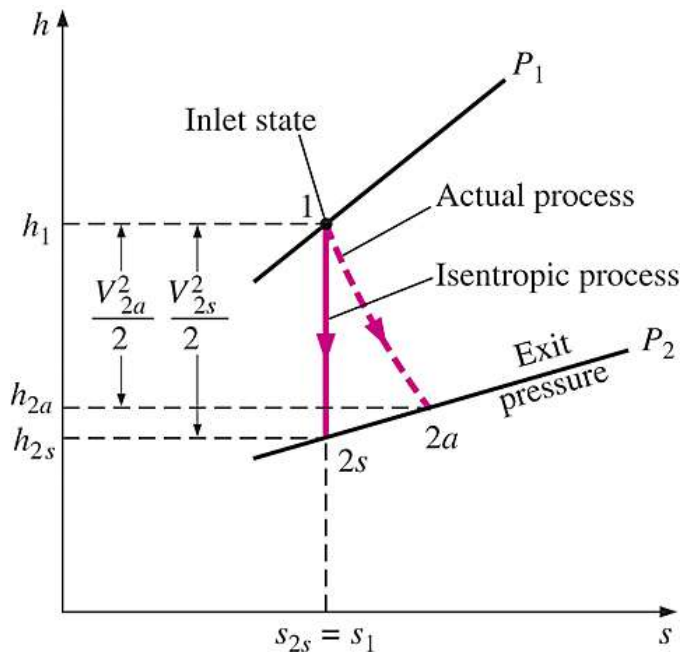
Discussion Notice that in determining the power input to the compressor, we used h_{2a} instead of h_{2s} since h_{2a} is the actual enthalpy of the air as it exits the compressor. The quantity h_{2s} is a hypothetical enthalpy value that the air would have if the process were isentropic.

Isentropic Efficiency of Nozzles

Nozzles are essentially adiabatic devices and are used to accelerate a fluid. Therefore, the isentropic process serves as a suitable model for nozzles.

If the inlet velocity of the fluid is small relative to the exit velocity, the energy balance is

$$h_1 = h_{2a} + \frac{V_{2a}^2}{2}$$

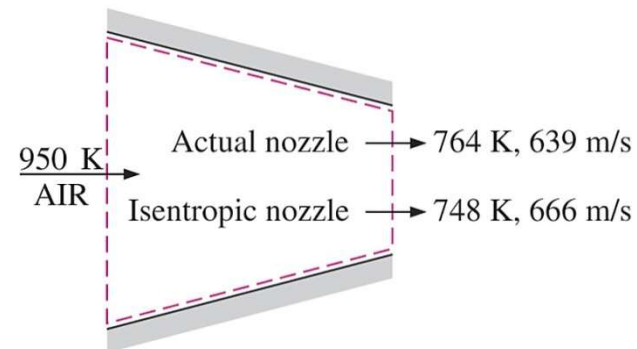


The $h-s$ diagram of the actual and isentropic processes of an adiabatic nozzle.

$$\eta_N = \frac{\text{Actual KE at nozzle exit}}{\text{Isentropic KE at nozzle exit}} = \frac{V_{2a}^2}{V_{2s}^2}$$

Then,

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}}$$



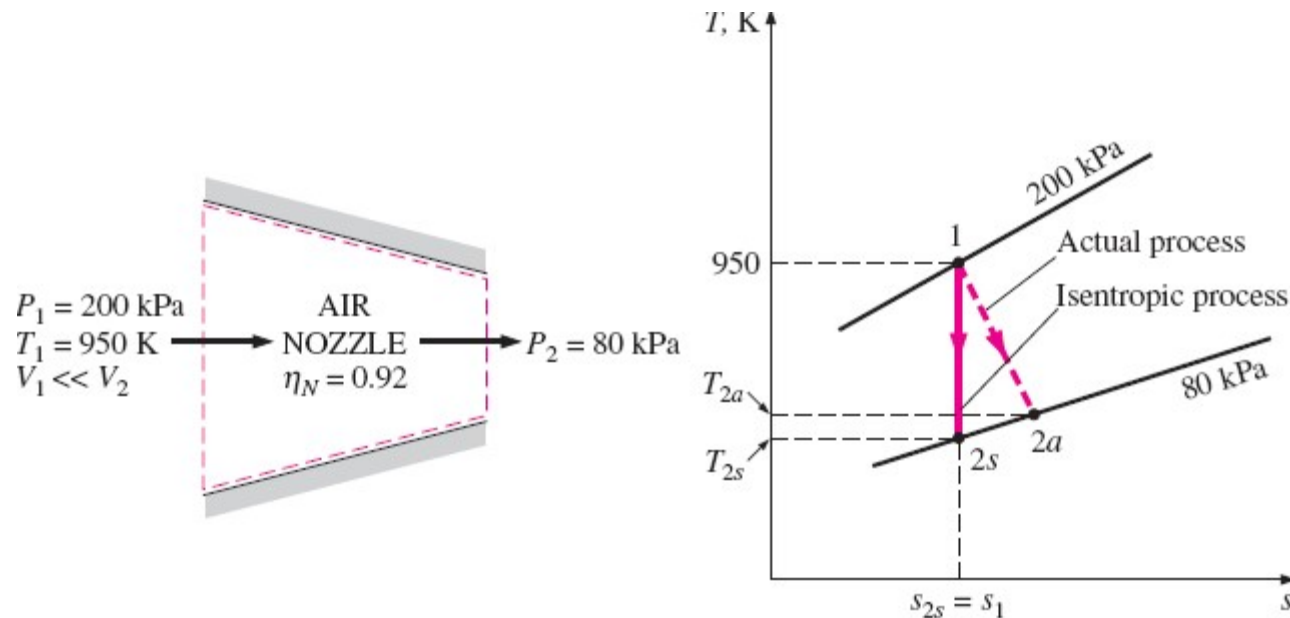
A substance leaves actual nozzles at a higher temperature (thus a lower velocity) as a result of friction.

EXAMPLE 7-16

Air at 200 kPa and 950 K enters an adiabatic nozzle at low velocity and is discharged at a pressure of 80 kPa. If the isentropic efficiency of the nozzle is 92 percent, determine (a) the maximum possible exit velocity, (b) the exit temperature, and (c) the actual velocity of the air. Assume constant specific heats for air.

Solution The acceleration of air in a nozzle is considered. For specified exit pressure and isentropic efficiency, the maximum and actual exit velocities and the exit temperature are to be determined.

Assumptions 1 Steady operating conditions exist. 2 Air is an ideal gas. 3 The inlet kinetic energy is negligible. 4 The nozzle is adiabatic.



(a) the maximum possible exit velocity

$$\frac{T_{2s}}{T_1} = \left(\frac{P_{2s}}{P_1} \right)^{(k-1)/k}$$

$$T_{2s} = T_1 \left(\frac{P_{2s}}{P_1} \right)^{(k-1)/k} = (950 \text{ K}) \left(\frac{80 \text{ kPa}}{200 \text{ kPa}} \right)^{0.354/1.354} = 748 \text{ K}$$

$$e_{\text{in}} = e_{\text{out}}$$
$$h_1 + \frac{V_1^2}{2} = h_{2s} + \frac{V_{2s}^2}{2}$$

$$V_{2s} = \sqrt{2(h_1 - h_{2s})} = \sqrt{2c_{p, \text{av}}(T_1 - T_{2s})}$$
$$= \sqrt{2(1.099 \text{ kJ/kg} \cdot \text{K})[(950 - 748) \text{ K}] \left(\frac{1000 \text{ m}^2/\text{s}^2}{1 \text{ kJ/kg}} \right)}$$
$$= \mathbf{666 \text{ m/s}}$$

(b) The actual exit temperature of the air will be higher than the isentropic exit temperature evaluated above, and it is determined from

$$\eta_N \cong \frac{h_1 - h_{2a}}{h_1 - h_{2s}} = \frac{c_{p, \text{av}}(T_1 - T_{2a})}{c_{p, \text{av}}(T_1 - T_{2s})}$$

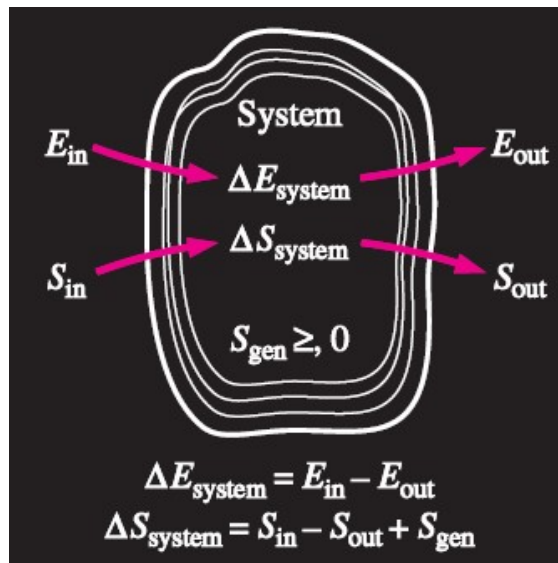
$$0.92 = \frac{950 - T_{2a}}{950 - 748} \rightarrow T_{2a} = \mathbf{764 \text{ K}}$$

(c) The actual exit velocity of air can be determined from the definition of isentropic efficiency of a nozzle,

$$\eta_N = \frac{V_{2a}^2}{V_{2s}^2} \rightarrow V_{2a} = \sqrt{\eta_N V_{2s}^2} = \mathbf{639 \text{ m/s}}$$

ENTROPY BALANCE

The property **entropy** is a measure of molecular disorder or randomness of a system, and the second law of thermodynamics states that entropy can be created but it cannot be destroyed.



The entropy change of a system during a process is equal to the net entropy transfer through the system boundary and the entropy generated within the system.

$$\left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{entering} \end{array} \right) - \left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{leaving} \end{array} \right) + \left(\begin{array}{c} \text{Total} \\ \text{entropy} \\ \text{generated} \end{array} \right) = \left(\begin{array}{c} \text{Change in the} \\ \text{total entropy} \\ \text{of the system} \end{array} \right)$$

Energy and entropy balances for a system.

$$S_{in} - S_{out} + S_{gen} = \Delta S_{system}$$

Entropy Change of a System, ΔS_{system}

Entropy change = Entropy at final state – Entropy at initial state

$$\Delta S_{\text{system}} = S_{\text{final}} - S_{\text{initial}} = S_2 - S_1$$

When the properties of the system are not uniform

the entropy change of steady-flow devices such as nozzles, compressors, turbines, pumps, and heat exchangers is zero during steady operation.

When the properties of the system are not uniform, the entropy of the system can be determined by integration from

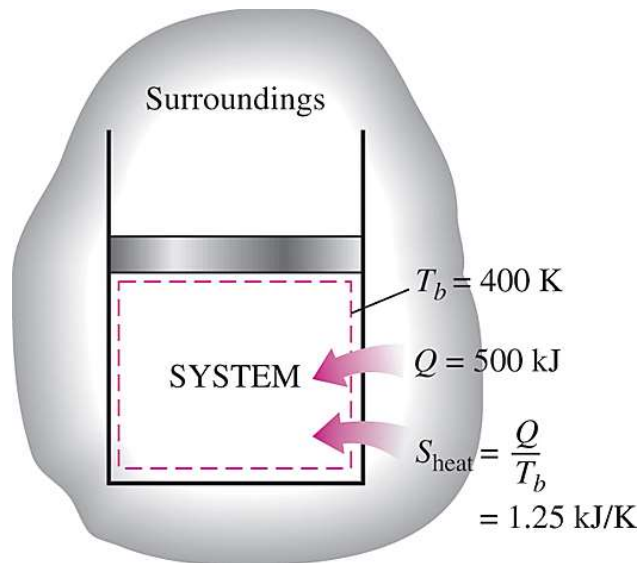
$$S_{\text{system}} = \int s \, \delta m = \int_V s \rho \, dV$$

Mechanisms of Entropy Transfer, S_{in} and S_{out}

1 Heat Transfer

Entropy transfer by heat transfer:

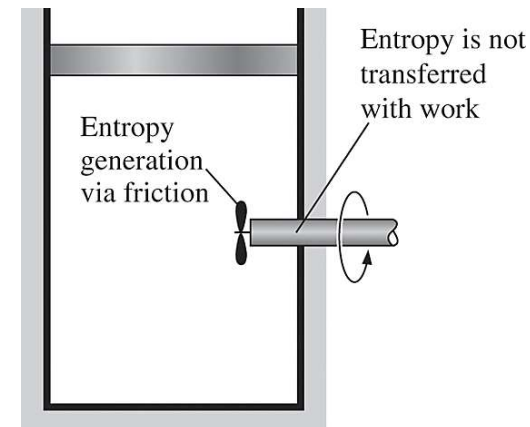
$$S_{\text{heat}} = \frac{Q}{T} \quad (T = \text{constant}) \quad S_{\text{heat}} = \int_1^2 \frac{\delta Q}{T} \cong \sum \frac{Q_k}{T_k}$$



Heat transfer is always accompanied by entropy transfer in the amount of Q/T , where T is the boundary temperature.

Entropy transfer by work:

$$S_{\text{work}} = 0$$



No entropy accompanies work as it crosses the system boundary. But entropy may be generated within the system as work is dissipated into a less useful form of energy.

Mechanisms of Entropy Transfer, S_{in} and S_{out}

2 Mass Flow

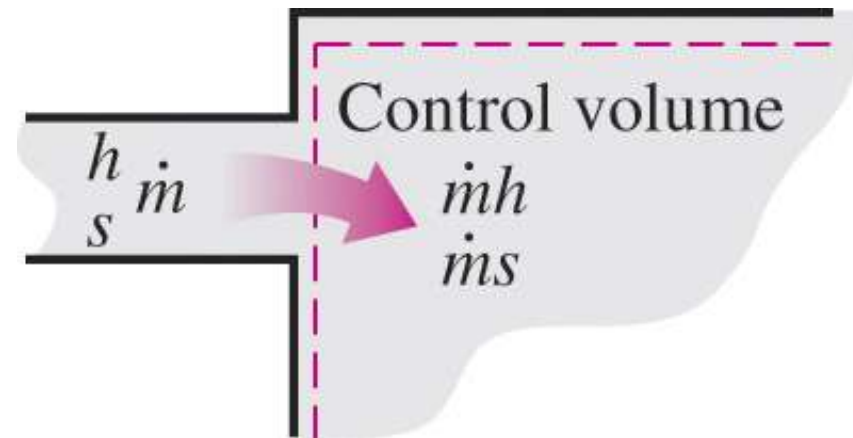
Entropy transfer by mass:

$$S_{\text{mass}} = ms$$

When the properties of the mass change during the process

$$\dot{S}_{\text{mass}} = \int_{A_c} s \rho V_n dA_c$$

$$S_{\text{mass}} = \int s \delta m = \int_{\Delta t} \dot{S}_{\text{mass}} dt$$



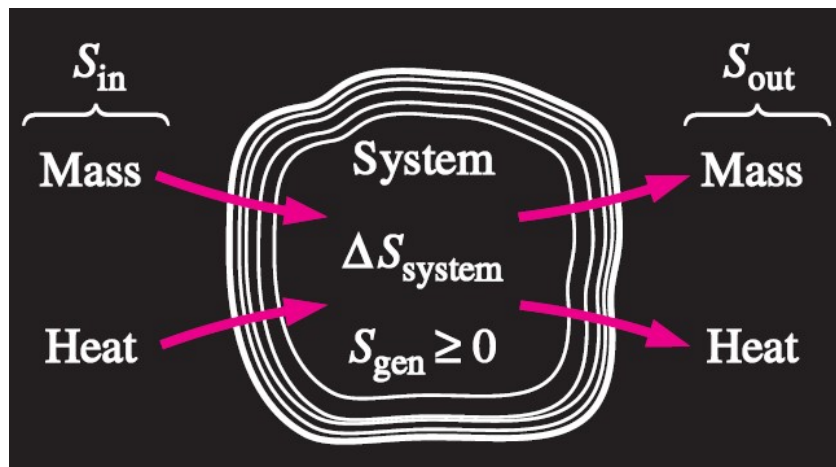
Mass contains entropy as well as energy, and thus mass flow into or out of system is always accompanied by energy and entropy transfer.

Entropy Generation, S_{gen}

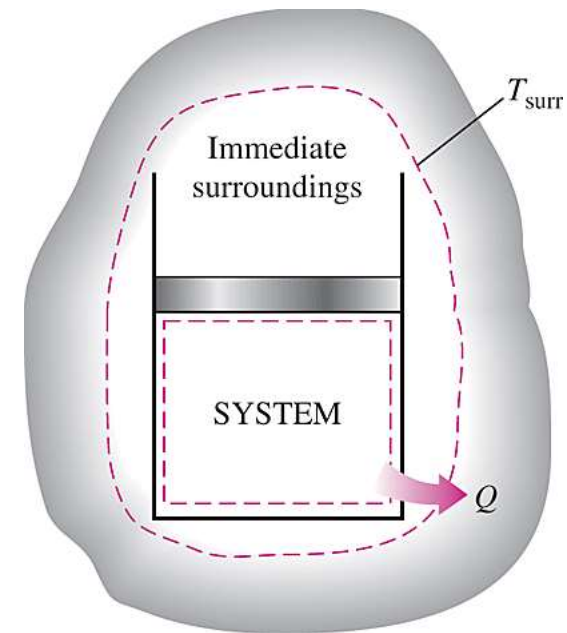
$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}} \quad (\text{kJ/K})$$

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \quad (\text{kW/K})$$

$$(s_{\text{in}} - s_{\text{out}}) + s_{\text{gen}} = \Delta s_{\text{system}} \quad (\text{kJ/kg} \cdot \text{K})$$



Mechanisms of entropy transfer for a general system.



Entropy generation outside system boundaries can be accounted for by writing an entropy balance on an extended system that includes the system and its immediate surroundings.

Closed Systems

$$\text{Closed system:} \quad \sum \frac{Q_k}{T_k} + S_{\text{gen}} = \Delta S_{\text{system}} = S_2 - S_1 \quad (\text{kJ/K})$$

The entropy change of a closed system during a process is equal to the sum of the net entropy transferred through the system boundary by heat transfer and the entropy generated within the system boundaries.

$$\text{Adiabatic closed system:} \quad S_{\text{gen}} = \Delta S_{\text{adiabatic system}}$$

$$\text{System + Surroundings:} \quad S_{\text{gen}} = \sum \Delta S = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}}$$

$$\Delta S_{\text{system}} = m(s_2 - s_1) \quad \Delta S_{\text{surr}} = Q_{\text{surr}}/T_{\text{surr}}$$

Control Volumes

$$\sum \frac{Q_k}{T_k} + \sum m_i s_i - \sum m_e s_e + S_{\text{gen}} = (S_2 - S_1)_{\text{CV}}$$

$$\sum \frac{\dot{Q}_k}{T_k} + \sum \dot{m}_i s_i - \sum \dot{m}_e s_e + \dot{S}_{\text{gen}} = \Delta \dot{S}_{\text{CV}}$$

Steady-flow:

$$\dot{S}_{\text{gen}} = \sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \sum \frac{\dot{Q}_k}{T_k}$$

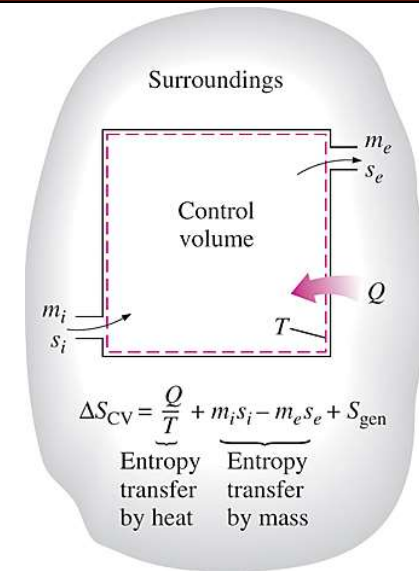
Steady-flow, single-stream:

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i) - \sum \frac{\dot{Q}_k}{T_k}$$

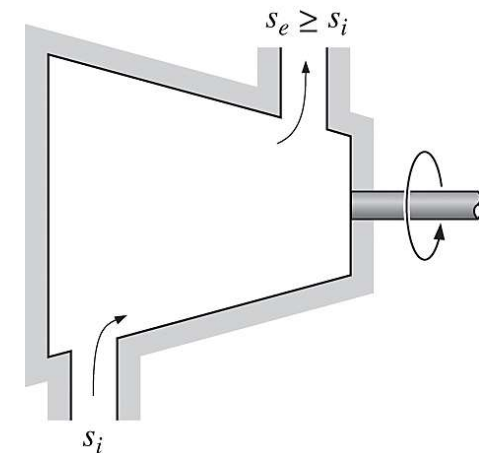
Steady-flow, single-stream, adiabatic:

$$\dot{S}_{\text{gen}} = \dot{m}(s_e - s_i)$$

The entropy of a substance always increases (or remains constant in the case of a reversible process) as it flows through a single-stream, adiabatic, steady-flow device.



The entropy of a control volume changes as a result of mass flow as well as heat transfer.

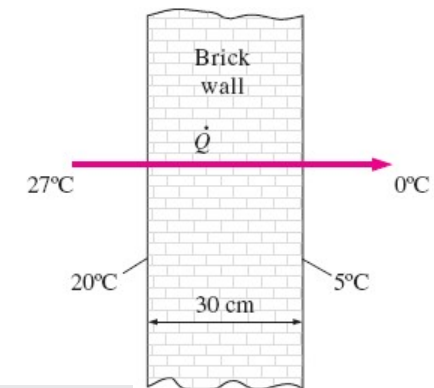


EXAMPLE 7-17

Consider steady heat transfer through a 5-m 7-m brick wall of a house of thickness 30 cm. On a day when the temperature of the outdoors is 0°C, the house is maintained at 27°C. The temperatures of the inner and outer surfaces of the brick wall are measured to be 20°C and 5°C, respectively, and the rate of heat transfer through the wall is 1035 W. Determine the rate of entropy generation in the wall, and the rate of total entropy generation associated with this heat transfer process..

Solution Steady heat transfer through a wall is considered. For specified heat transfer rate, wall temperatures, and environment temperatures, the entropy generation rate within the wall and the total entropy generation rate are to be determined.

Assumptions 1 The process is steady, and thus the rate of heat transfer through the wall is constant. 2 Heat transfer through the wall is 1-D.



$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \xrightarrow{0}$$

$$\left(\frac{\dot{Q}}{T} \right)_{\text{in}} - \left(\frac{\dot{Q}}{T} \right)_{\text{out}} + \dot{S}_{\text{gen}} = 0$$

$$\frac{1035 \text{ W}}{293 \text{ K}} - \frac{1035 \text{ W}}{278 \text{ K}} + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen, wall}} = \mathbf{0.191 \text{ W/K}}$$

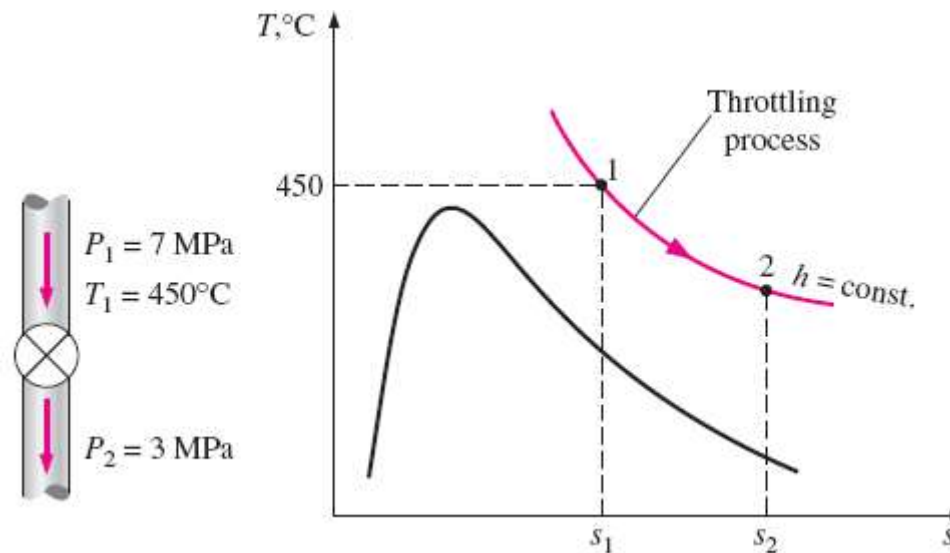
$$\frac{1035 \text{ W}}{300 \text{ K}} - \frac{1035 \text{ W}}{273 \text{ K}} + \dot{S}_{\text{gen, total}} = 0 \rightarrow \dot{S}_{\text{gen, total}} = \mathbf{0.341 \text{ W/K}}$$

EXAMPLE 7-18

Steam at 7 MPa and 450°C is throttled in a valve to a pressure of 3 MPa during a steady-flow process. Determine the entropy generated during this process and check if the increase of entropy principle is satisfied.

Solution Steam is throttled to a specified pressure. The entropy generated during this process is to be determined, and the validity of the increase of entropy principle is to be shown.

Assumptions **1** This is a steady-flow process since there is no change with time at any point and thus $\Delta m_{CV}=0$, $\Delta E_{CV}=0$, and $\Delta S_{CV}=0$. **2** Heat transfer to or from the valve is negligible. **3** The kinetic and potential energy changes are negligible, $\Delta ke = \Delta pe = 0$.



$$\text{State 1:} \quad \left. \begin{array}{l} P_1 = 7 \text{ MPa} \\ T_1 = 450^\circ\text{C} \end{array} \right\} \quad \begin{array}{l} h_1 = 3288.3 \text{ kJ/kg} \\ s_1 = 6.6353 \text{ kJ/kg} \cdot \text{K} \end{array}$$

$$\text{State 2:} \quad \left. \begin{array}{l} P_2 = 3 \text{ MPa} \\ h_2 = h_1 \end{array} \right\} \quad s_2 = 7.0046 \text{ kJ/kg} \cdot \text{K}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$

$$s_{\text{gen}} = s_2 - s_1 = 7.0046 - 6.6353 = \mathbf{0.3693 \text{ kJ/kg} \cdot \text{K}}$$

This is the amount of entropy generated per unit mass of steam as it is throttled from the inlet state to the final pressure, and it is caused by unrestrained expansion. The increase of entropy principle is obviously satisfied during this process since the entropy generation is positive.

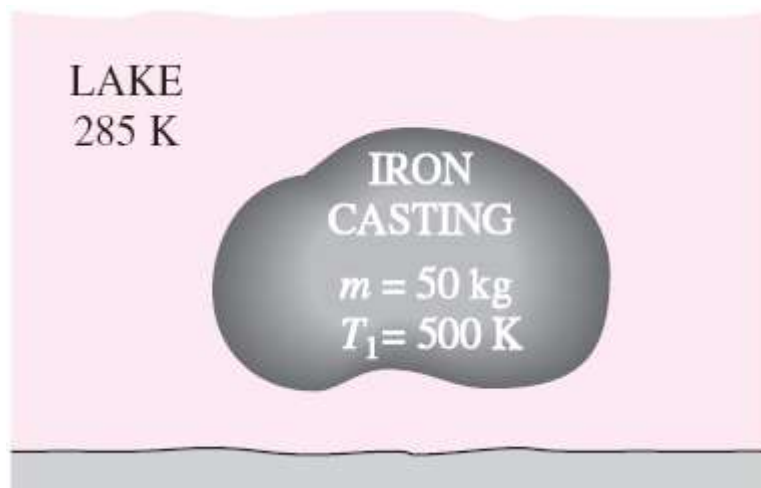
EXAMPLE 7-19

A 50-kg block of iron casting at 500 K is thrown into a large lake that is at a temperature of 285 K. The iron block eventually reaches thermal equilibrium with the lake water. Assuming an average specific heat of 0.45 kJ/kg · K for the iron, determine (a) the entropy change of the iron block, (b) the entropy change of the lake water, and (c) the entropy generated during this process.

Solution A hot iron block is thrown into a lake, and cools to the lake temperature. The entropy changes of the iron and of the lake as well as the entropy generated during this process are to be determined.

Assumptions **1** Both the water and the iron block are incompressible substances. **2** Constant specific heats can be used for the water and the iron. **3** The kinetic and potential energy changes of the iron are negligible, $\Delta KE = \Delta PE = 0$ and thus $\Delta E = \Delta U$. **4** There are no work interactions.

(a) Approximating the iron block as an incompressible substance, its entropy change can be determined from



$$\begin{aligned}\Delta S_{\text{iron}} &= m(s_2 - s_1) = mc_{\text{av}} \ln \frac{T_2}{T_1} \\ &= (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K}) \ln \frac{285 \text{ K}}{500 \text{ K}} \\ &= -12.65 \text{ kJ/K}\end{aligned}$$

(b) the entropy change of the lake water

$$-Q_{\text{out}} = \Delta U = mc_{\text{av}}(T_2 - T_1)$$

$$Q_{\text{out}} = mc_{\text{av}}(T_1 - T_2) = (50 \text{ kg})(0.45 \text{ kJ/kg} \cdot \text{K})(500 - 285) \text{ K} = 4838 \text{ kJ}$$

$$\Delta S_{\text{lake}} = \frac{Q_{\text{lake}}}{T_{\text{lake}}} = \frac{+4838 \text{ kJ}}{285 \text{ K}} = \mathbf{16.97 \text{ kJ/K}}$$

(c) the entropy generated during this process.

$$\underbrace{S_{\text{in}} - S_{\text{out}}}_{\text{Net entropy transfer by heat and mass}} + \underbrace{S_{\text{gen}}}_{\text{Entropy generation}} = \underbrace{\Delta S_{\text{system}}}_{\text{Change in entropy}}$$

$$-\frac{Q_{\text{out}}}{T_b} + S_{\text{gen}} = \Delta S_{\text{system}}$$

$$S_{\text{gen}} = \frac{Q_{\text{out}}}{T_b} + \Delta S_{\text{system}} = \frac{4838 \text{ kJ}}{285 \text{ K}} - (12.65 \text{ kJ/K}) = \mathbf{4.32 \text{ kJ/K}}$$

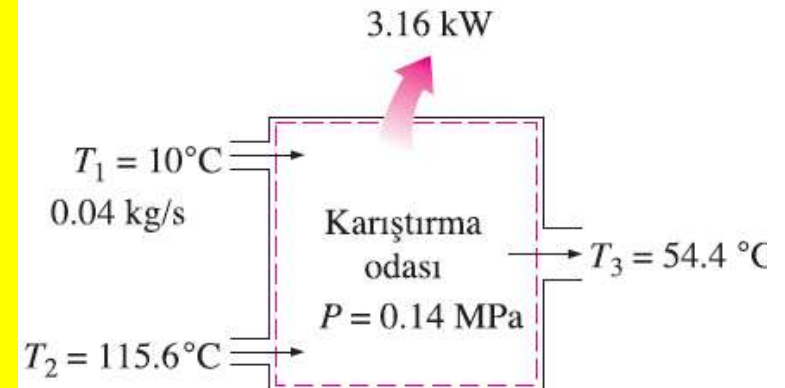
Discussion The entropy generated can also be determined by taking the iron block and the entire lake as the system, which is an isolated system, and applying an entropy balance. An isolated system involves no heat or entropy transfer, and thus the entropy generation in this case becomes equal to the total entropy change,

$$S_{\text{gen}} = \Delta S_{\text{total}} = \Delta S_{\text{system}} + \Delta S_{\text{lake}} = -12.65 + 16.97 = 4.32 \text{ kJ/K}$$

which is the same result obtained above.

EXAMPLE 7-20

Water at 200 kPa and 10°C enters a mixing chamber at a rate of 150 kg/min where it is mixed steadily with steam entering at 200 kPa and 150°C. The mixture leaves the chamber at 200 kPa and 70°C, and heat is lost to the surrounding air at 20°C at a rate of 190 kJ/min. Neglecting the changes in kinetic and potential energies, determine the rate of entropy generation during this process.



Kütle dengesi: $\dot{m}_g - \dot{m}_ç = \frac{dm_{\text{sistem}}}{dt} \overset{0 \text{ (sürekli)}}{=} 0 \rightarrow \dot{m}_1 + \dot{m}_2 = \dot{m}_3$

Enerji dengesi: $\underbrace{\dot{E}_g - \dot{E}_ç}_{\text{Birim zamanda ısı, iş ve kütle ile gerçekleşen enerji geçişi}} = \underbrace{\frac{dE_{\text{sistem}}}{dt}}_{\text{Birim zamanda sistemin iç, kinetik, potansiyel vb. enerjilerindeki değişim}} \overset{0 \text{ (sürekli)}}{=} 0$

$$\dot{E}_g = \dot{E}_ç$$

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 + \dot{Q}_ç$$

$$(\dot{W} = 0, \text{ ke} \cong \text{pe} \cong 0 \text{ olduğundan})$$

$$\dot{Q}_ç = \dot{m}_1 h_1 + \dot{m}_2 h_2 - (\dot{m}_1 + \dot{m}_2) h_3$$

$$\text{Hal 1:} \quad \left. \begin{array}{l} P_1 = 200 \text{ kPa} \\ T_1 = 10^\circ \text{C} \end{array} \right\} \quad \begin{array}{l} h_1 = h_{f@10^\circ \text{C}} = 42.022 \text{ kJ/kg} \\ s_1 = s_{f@10^\circ \text{C}} = 0.1511 \text{ kJ/kg.K} \end{array}$$

$$\text{Hal 2:} \quad \left. \begin{array}{l} P_2 = 200 \text{ kPa} \\ T_2 = 150^\circ \text{C} \end{array} \right\} \quad \begin{array}{l} h_2 = 2769.1 \text{ kJ/kg} \\ s_2 = 7.2810 \text{ kJ/kg.K} \end{array}$$

$$\text{Hal 3:} \quad \left. \begin{array}{l} P_3 = 200 \text{ kPa} \\ T_3 = 70^\circ \text{C} \end{array} \right\} \quad \begin{array}{l} h_3 = h_{f@70^\circ \text{C}} = 293.07 \text{ kJ/kg} \\ s_3 = s_{f@70^\circ \text{C}} = 0.9551 \text{ kJ/kg.K} \end{array}$$

$$190 \text{ kJ/dk} = [150 \times 42.022 + \dot{m}_2 \times 2769.1 - (150 + \dot{m}_2) \times 293.07] \text{ kJ/dk}$$

$$\dot{m}_2 = 15.29 \text{ kJ/dk}$$

$$\dot{m}_1 s_1 + \dot{m}_2 s_2 - \dot{m}_3 s_3 - \frac{\dot{Q}_\xi}{T_b} + \dot{S}_{\text{üretim}} = 0$$

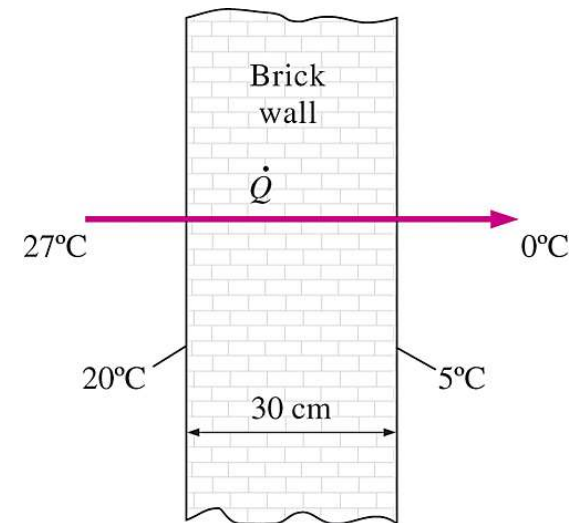
$$\begin{aligned} \dot{S}_{\text{üretim}} &= \dot{m}_3 s_3 - \dot{m}_1 s_1 - \dot{m}_2 s_2 + \frac{\dot{Q}_\xi}{T_b} \\ &= (165.29 \times 0.9551 - 150 \times 0.1511 - 15.29 \times 7.2810) \text{ kJ/dk} \cdot \text{K} \\ &\quad + \frac{190 \text{ kJ/dk}}{293 \text{ K}} \\ &= \mathbf{24.53 \text{ kJ/dk} \cdot \text{K}} \end{aligned}$$

EXAMPLES

Entropy balance for heat transfer through a wall

$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \xrightarrow{0}$$

$$\left(\frac{\dot{Q}}{T}\right)_{\text{in}} - \left(\frac{\dot{Q}}{T}\right)_{\text{out}} + \dot{S}_{\text{gen}} = 0$$

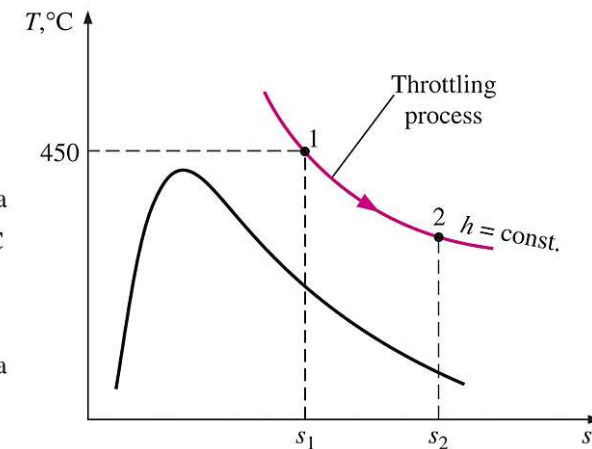
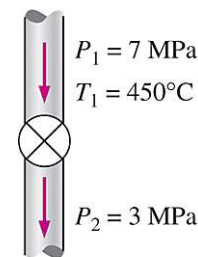


Entropy balance for a throttling process

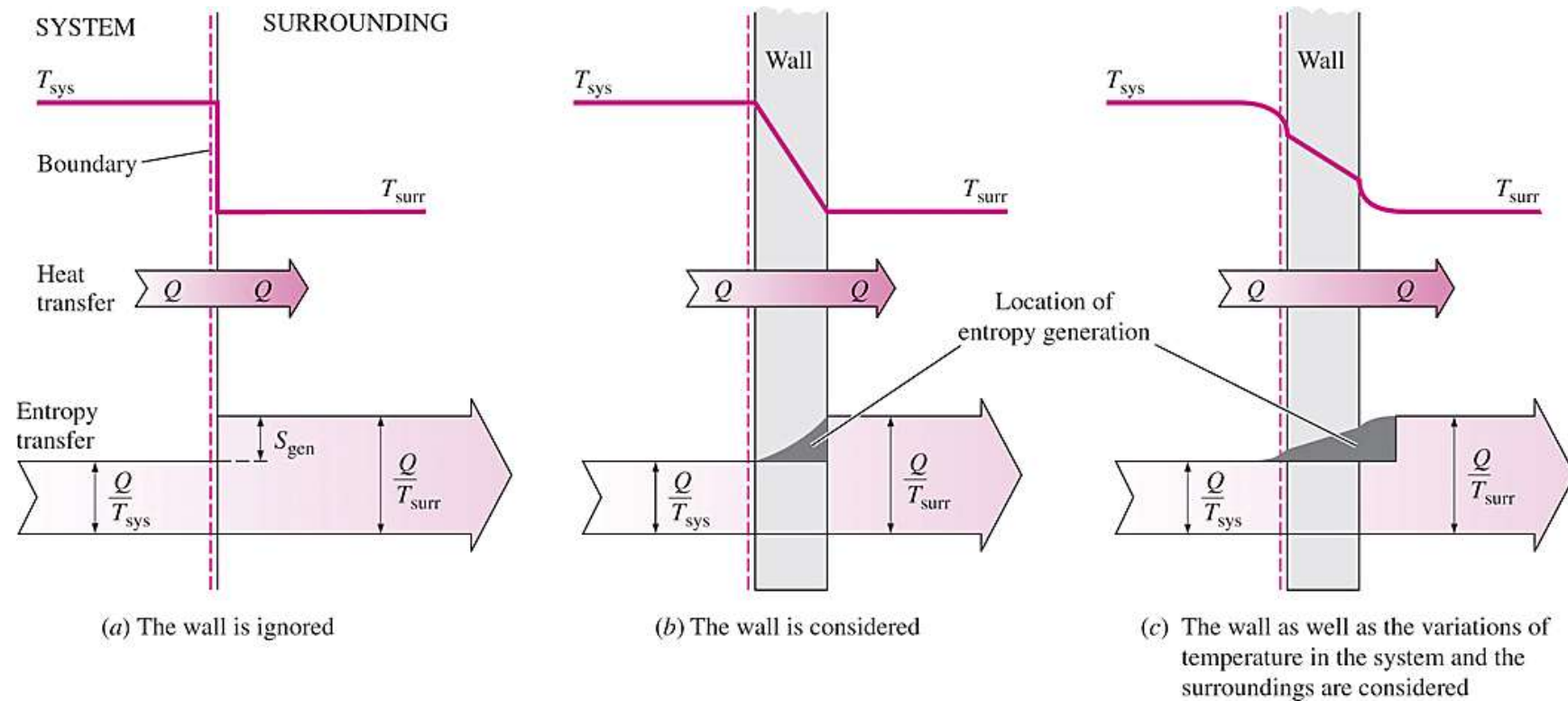
$$\underbrace{\dot{S}_{\text{in}} - \dot{S}_{\text{out}}}_{\text{Rate of net entropy transfer by heat and mass}} + \underbrace{\dot{S}_{\text{gen}}}_{\text{Rate of entropy generation}} = \underbrace{\Delta \dot{S}_{\text{system}}}_{\text{Rate of change in entropy}} \xrightarrow{0 \text{ (steady)}}$$

$$\dot{m}s_1 - \dot{m}s_2 + \dot{S}_{\text{gen}} = 0$$

$$\dot{S}_{\text{gen}} = \dot{m}(s_2 - s_1)$$



Entropy generation associated with a heat transfer process



Graphical representation of entropy generation during a heat transfer process through a finite temperature difference.

SUMMARY

- Entropy
- The Increase of entropy principle
- Some remarks about entropy
- Entropy change of pure substances
- Isentropic processes
- Property diagrams involving entropy
- What is entropy?
- The $T ds$ relations
- Entropy change of liquids and solids
- The entropy change of ideal gases
- Reversible steady-flow work
- Minimizing the compressor work
- Isentropic efficiencies of steady-flow devices
- Entropy balance