Thermodynamics: An Engineering Approach, 5th Edition
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Chapter 9
GAS POWER CYCLES

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Objectives

- Evaluate the performance of gas power cycles for which the working fluid remains a gas throughout the entire cycle.
- Develop simplifying assumptions applicable to gas power cycles.
- Review the operation of reciprocating engines.
- Analyze both closed and open gas power cycles.
- Solve problems based on the Otto, Diesel, Stirling, and Ericsson cycles.
- Solve problems based on the Brayton cycle; the Brayton cycle with regeneration; and the Brayton cycle with intercooling, reheating, and regeneration.
- Analyze jet-propulsion cycles.
- Identify simplifying assumptions for second-law analysis of gas power cycles.
- Perform second-law analysis of gas power cycles.

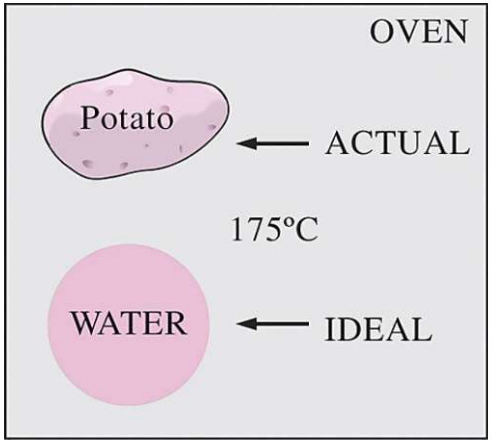
Most power-producing devices operate on cycles.

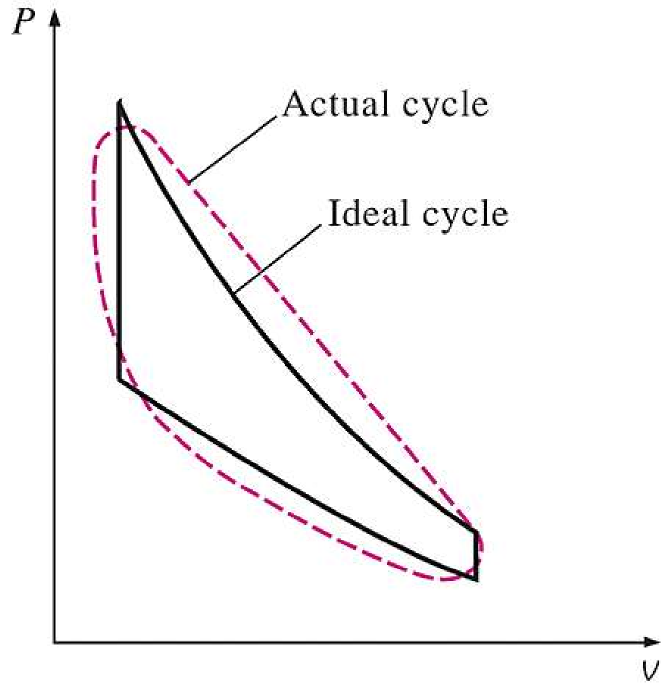
Ideal cycle: A cycle that resembles the actual cycle closely but is made up totally of internally reversible processes is called on**.**

Reversible cycles such as **Carnot cycle** have the highest thermal efficiency of all heat engines operating between the same temperature levels. Unlike ideal cycles, they are totally reversible, and unsuitable as a realistic model.

Thermal efficiency of heat engines

$$\eta_{th} = \frac{W_{net}}{Q_{in}} \quad \text{or} \quad \eta_{th} = \frac{w_{net}}{q_{in}}$$





Modeling is a powerful engineering tool that provides great insight and simplicity at the expense of some loss in accuracy.

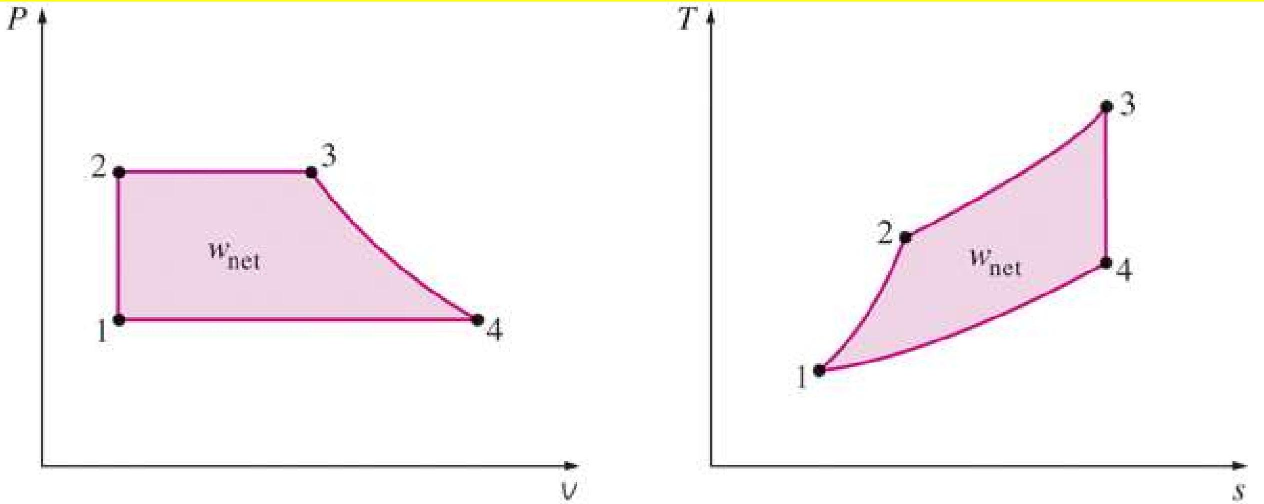
Care should be exercised in the interpretation of the results from ideal cycles.

The analysis of many complex processes can be reduced to a manageable level by utilizing some idealizations.

The idealizations and simplifications in the analysis of power cycles

1. The cycle does **not involve any *friction*.** Therefore, the working fluid does not experience any pressure drop as it flows in pipes or devices such as heat exchangers.
2. All expansion and compression processes take place in a ***quasi-equilibrium*** manner.
3. The pipes connecting the various components of a system are **well insulated**, and *heat transfer* through them is negligible.

On a *T*-*s* diagram, the ratio of the area enclosed by the cyclic curve to the area under the heat-addition process curve represents the thermal efficiency of the cycle. Any modification that increases the ratio of these two areas will also increase the thermal efficiency of the cycle.

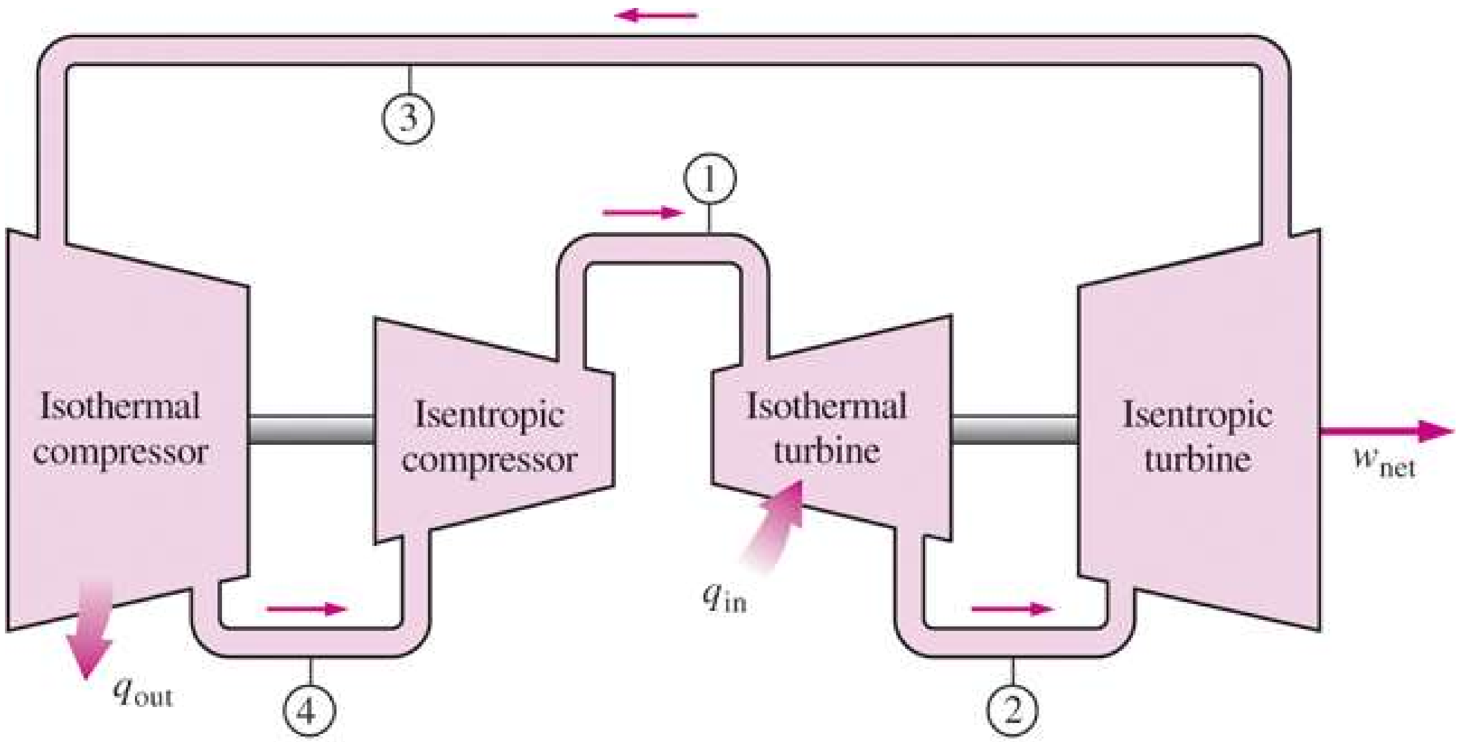


P-*v* and *T*-*s* diagrams, the area enclosed by the process curve represents the net work of the cycle.

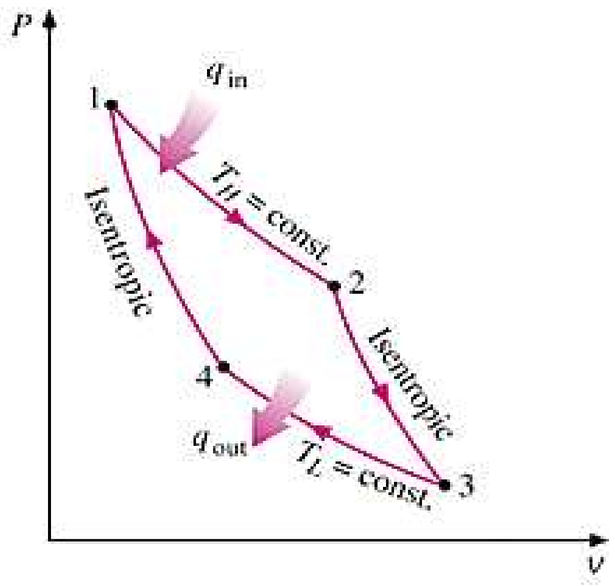
THE CARNOT CYCLE AND ITS VALUE IN

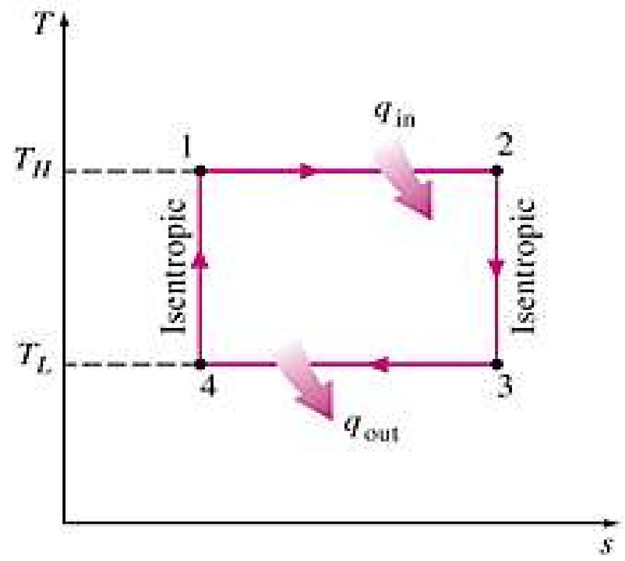
The Carnot cycle is composed of four totally reversible processes: isothermal heat addition, isentropic expansion, isothermal heat rejection, and isentropic compression.

For both ideal and actual cycles: Thermal efficiency increases with an increase in the average temperature at which heat is supplied to the system or with a decrease in the average temperature at which heat is rejected from the system.



A steady-flow Carnot engine.



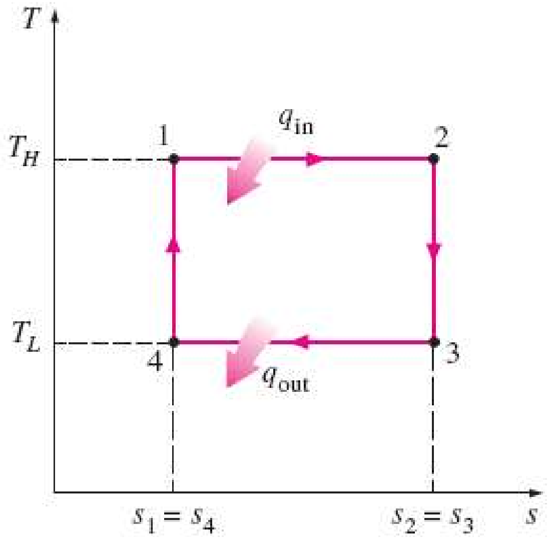


P-v and *T-s* diagrams of a Carnot cycle.

$$\boxed{\eta_{\text{th,Carnot}} = 1 - \frac{T_L}{T_H}}$$

EXAMPLE 9–1

Show that the thermal efficiency of a Carnot cycle operating between the temperature limits of T_H and T_L is solely a function of these two temperatures and is given by



$$\eta_{th,\,Carnot} = 1 - \frac{T_L}{T_H}$$

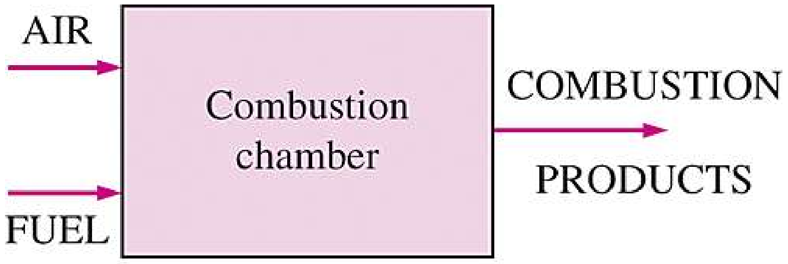
Solution It is to be shown that the efficiency of a Carnot cycle depends on the source and sink temperatures alone.

Analysis All four processes that comprise the Carnot cycle are reversible, and thus the area under each process curve represents the heat transfer for that process. Heat is transferred to the system during process 1-2 and rejected during process 3-4. Therefore, the amount of heat input and heat output for the cycle can be expressed as

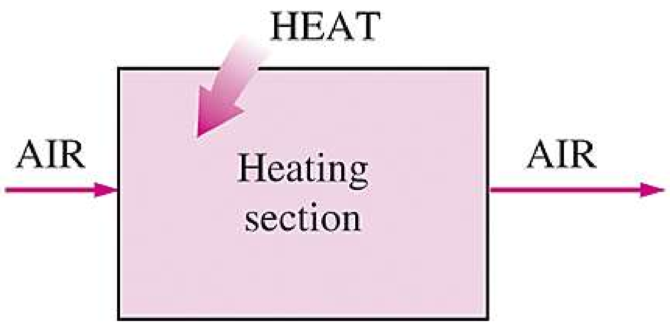
$$q_{in} = T_H(s_2 - s_1) \quad \text{and} \quad q_{out} = T_L(s_3 - s_4) = T_L(s_2 - s_1)$$

$$\eta_{th} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_L(s_2 - s_1)}{T_H(s_2 - s_1)} = 1 - \frac{T_L}{T_H}$$

Discussion Notice that the thermal efficiency of a Carnot cycle is independent of the type of the working fluid used (an ideal gas, steam, etc.) or whether the cycle is executed in a closed or steady-flow system.



(*a*) Actual



(*b*) Ideal

The combustion process is replaced by a heat-addition process in ideal cycles.

Air-standard assumptions:

1. The working fluid is air, which continuously circulates in a closed loop and always behaves as an ideal gas.
2. All the processes that make up the cycle are internally reversible.
3. The combustion process is replaced by a heat-addition process from an external source.
4. The exhaust process is replaced by a heat-rejection process that restores the working fluid to its initial state.

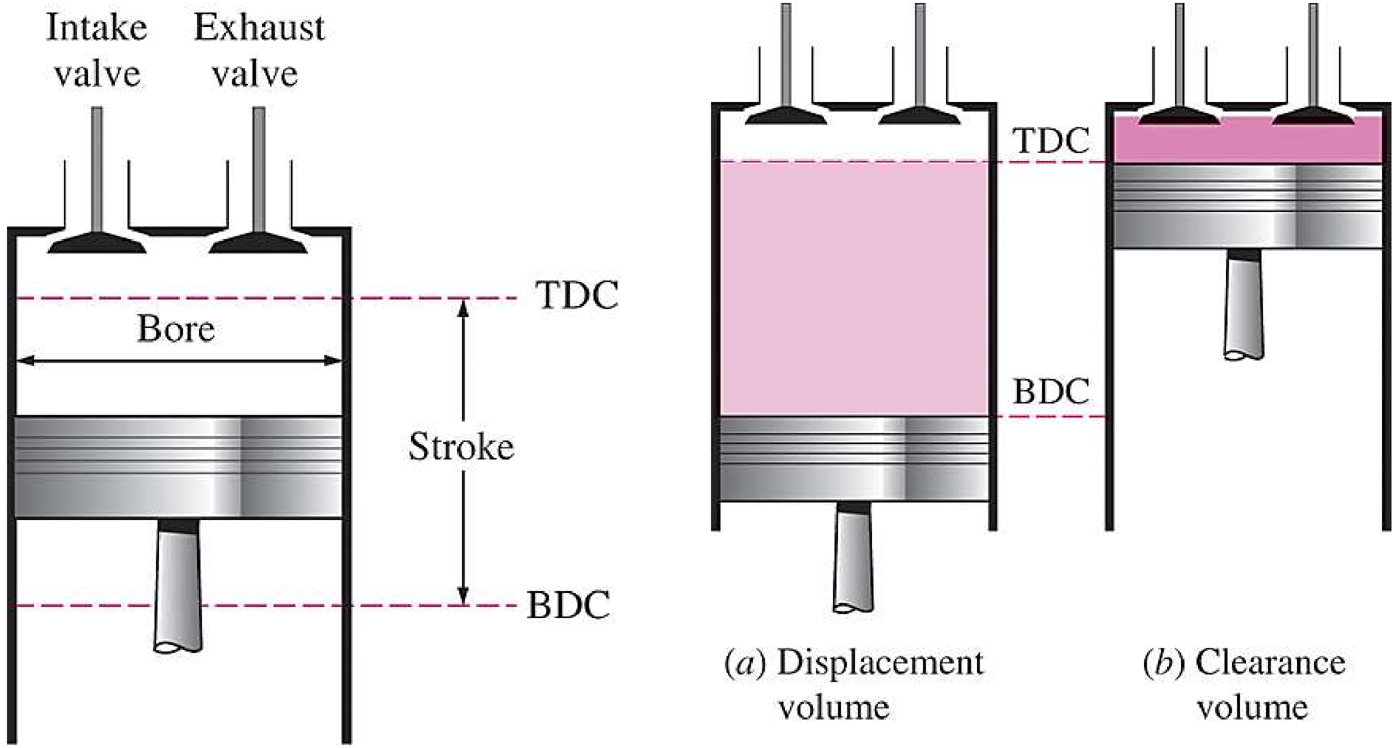
Cold-air-standard assumptions: When the working fluid is considered to be air with constant specific heats at room temperature (25°C).

Air-standard cycle: A cycle for which the air-standard assumptions are applicable.

Compression ratio

$$r = \frac{V_{max}}{V_{min}} = \frac{V_{BDC}}{V_{TDC}}$$

- Spark-ignition (SI) engines
- Compression-ignition (CI) engines

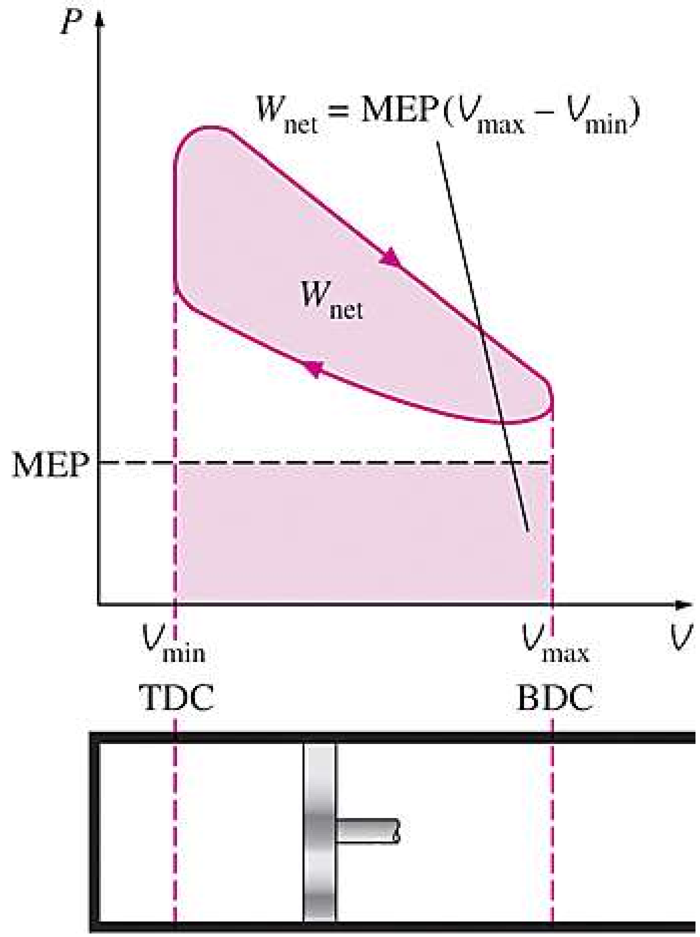


Nomenclature for reciprocating engines.

Mean effective pressure

$$\text{MEP} = \frac{W_{net}}{V_{max} - V_{min}} = \frac{w_{net}}{v_{max} - v_{min}} \quad \text{(kPa)}$$

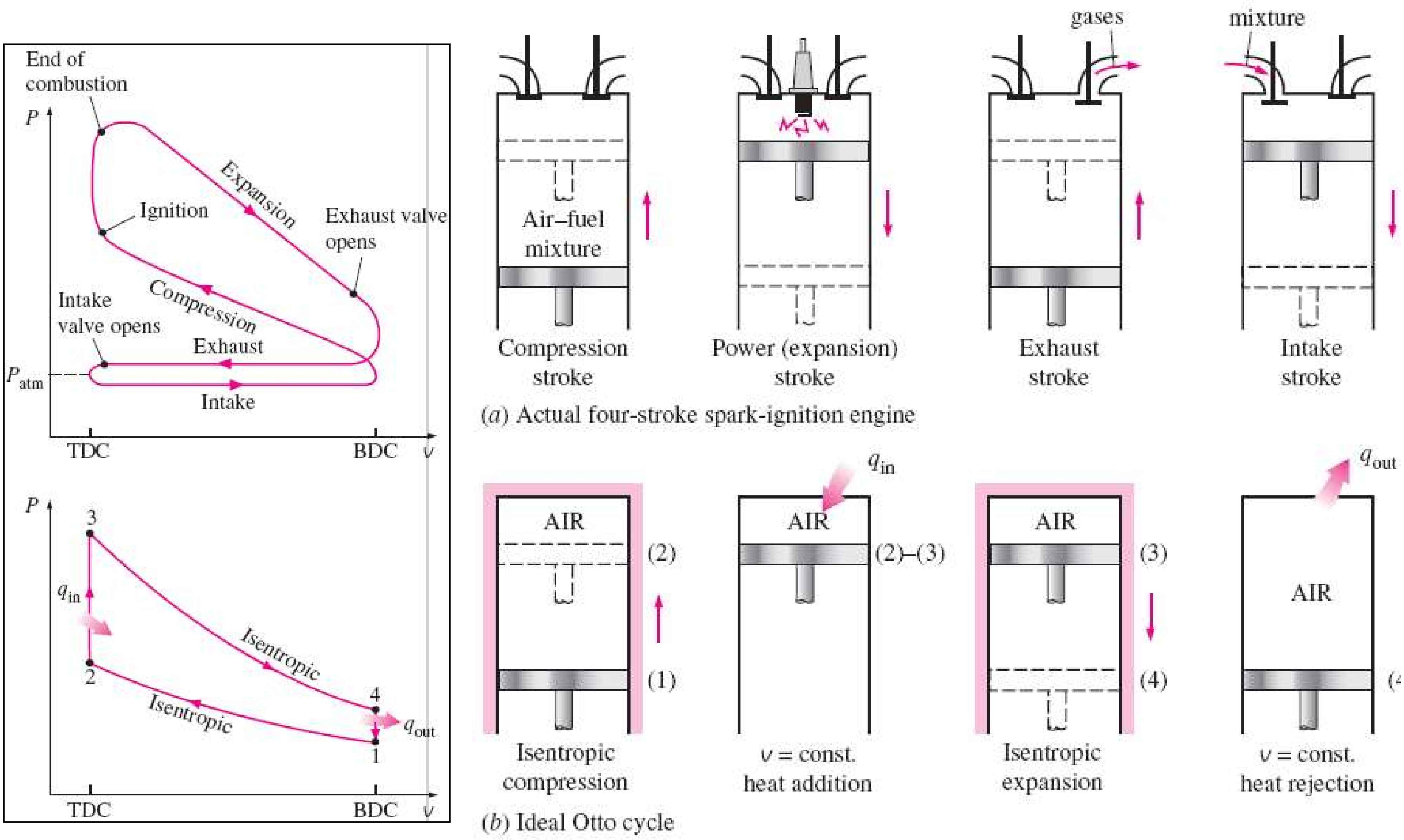
The mean effective pressure can be used as a parameter to compare the performances of reciprocating engines of equal size. The engine with a larger value of MEP will deliver more net work per cycle and thus will perform better.



$$W_{net} = \text{MEP} \times \text{Piston area} \times \text{Stroke}$$
$$= \text{MEP} \times \text{Displacement volume}$$

The net work output of a cycle is equivalent to the product of the mean effective pressure and the displacement volume.

otto cycle: the ideal cycle for spark-ignition engines



Actual and ideal cycles in spark-ignition engines and their *P-v* diagrams.

Four-stroke cycle

1 cycle = 4 stroke = 2 revolution

Two-stroke cycle

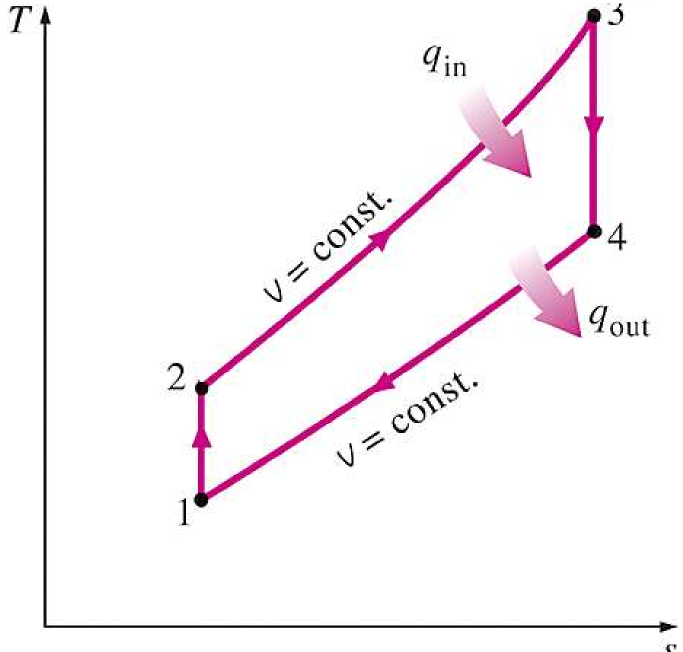
1 cycle = 2 stroke = 1 revolution

1-2 Isentropic compression

2-3 Constant-volume heat addition

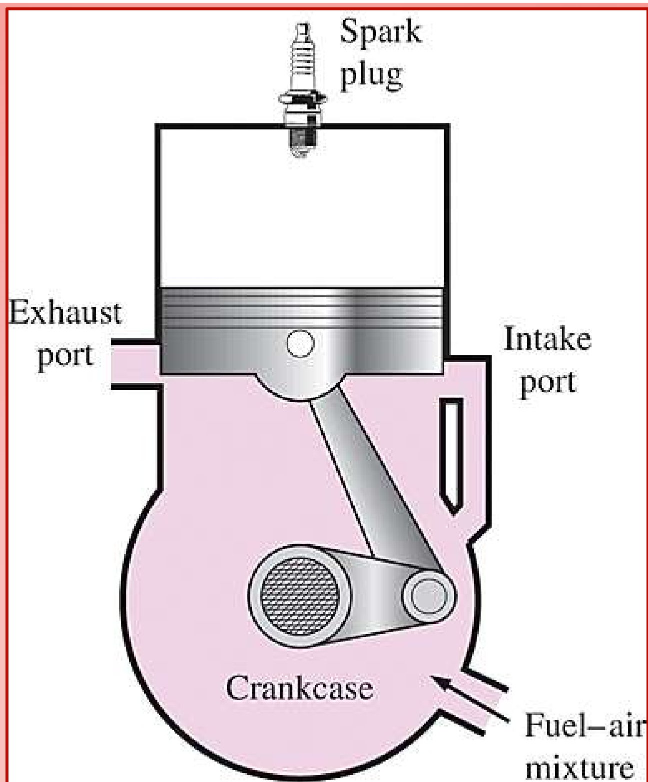
3-4 Isentropic expansion

4-1 Constant-volume heat rejection



T-s diagram of the ideal Otto cycle.

The two-stroke engines are generally less efficient than their four-stroke counterparts but they are relatively simple and inexpensive, and they have high power-to-weight and power-to-volume ratios.



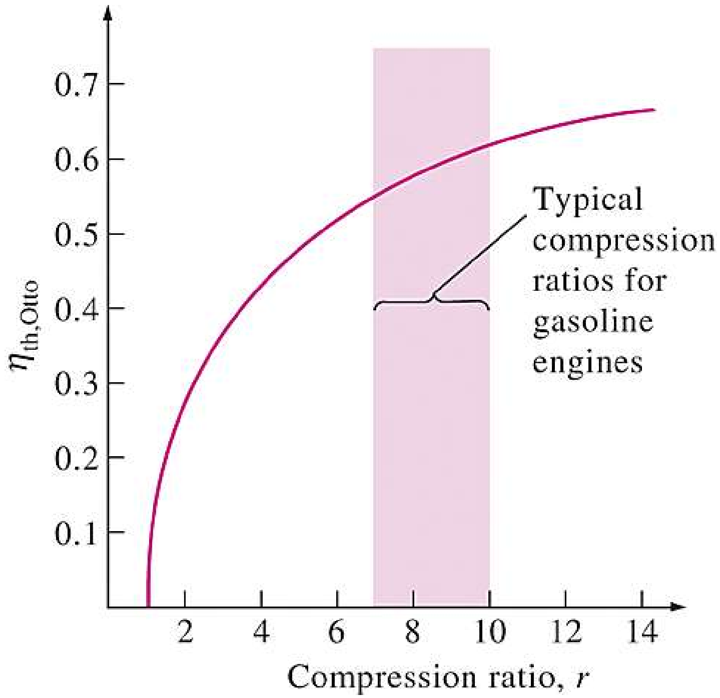
Schematic of a two-stroke reciprocating engine.

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = \Delta u \qquad \text{(kJ/kg)}$$

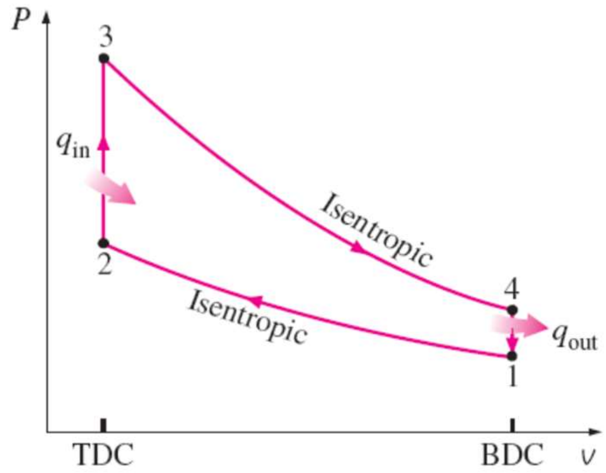
$$q_{in} = u_3 - u_2 = c_v(T_3 - T_2)$$

$$q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\eta_{th,\,Otto} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{T_3 - T_2} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$



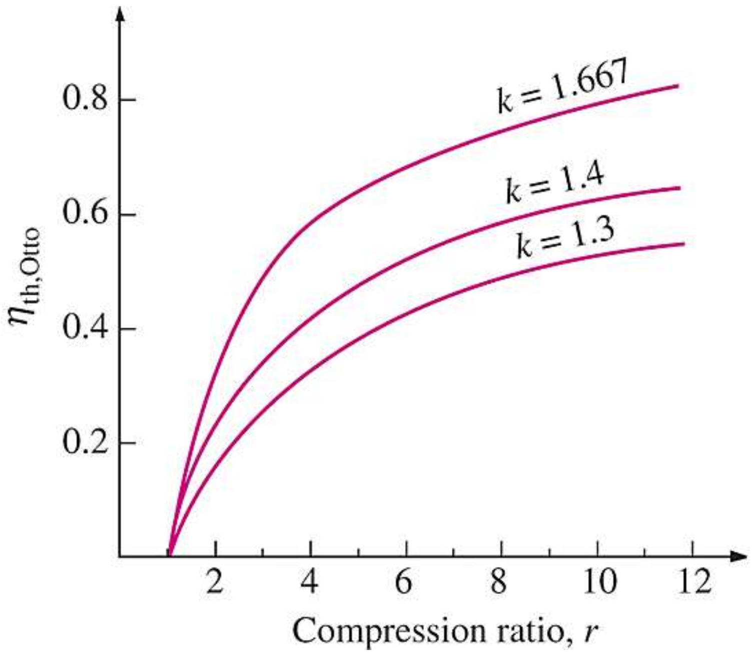
$$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{k-1} = \left(\frac{v_3}{v_4}\right)^{k-1} = \frac{T_4}{T_3}$$



The thermal efficiency of the Otto cycle increases with the specific heat ratio *k* of the working fluid.

$$\eta_{th,\,Otto} = 1 - \frac{1}{r^{k-1}}$$

In SI engines, the compression ratio is limited by auto ignition or engine knock.

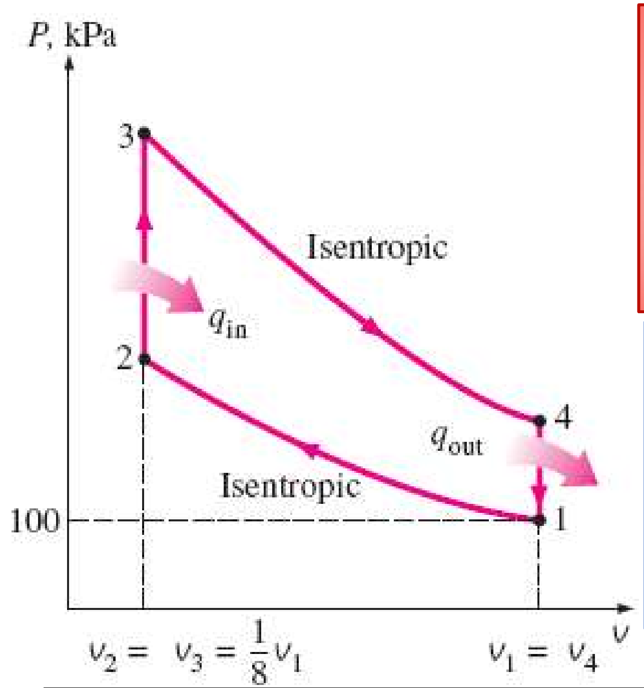


Thermal efficiency of the ideal Otto cycle as a function of compression ratio (k = 1.4).

The working fluid in actual engines contains larger molecules such as carbon dioxide and the specific heat ratio decreases with temperature, which is one of the reasons that the actual cycles have lower thermal efficiencies than the ideal Otto cycle. The thermal efficiencies of actual spark-ignition engines range from **about 25 to 30 %.**

EXAMPLE 9–2

An ideal Otto cycle has a compression ratio of 8. At the beginning of the compression process, air is at 100 kPa and 17°C, and 800 kJ/kg of heat is transferred to air during the constant-volume heat-addition process. Accounting for the variation of specific heats of air with temperature, determine (*a*) the maximum temperature and pressure that occur during the cycle,(*b*) the net work output, (*c*) the thermal efficiency, and (*d*) the mean effective pressure for the cycle.



Solution An ideal Otto cycle with specified compression ratio and heat input is considered.

Assumptions **1** The air-standard assumptions are applicable. **2** Kinetic and potential energy changes are negligible. **3** The variation of specific heats with temperature is to be accounted for.

(*a*) The maximum temperature and pressure in an Otto cycle occur at the end of the constant-volume heat-addition process (state 3). To determine the temperature and pressure of air at the end of the isentropic compression process (state 2), using data from Table A–17:

The properties P_r (relative pressure) and v_r (relative specific volume are dimensionless quantities used in the analysis of isentropic processes, should not be confused with the properties pressure and specific volume.

$$T_1 = 290 \text{ K} \rightarrow u_1 = 206.91 \text{ kJ/kg}$$

$$v_{r1} = 676.1$$

Process 1-2 (isentropic compression of an ideal gas):

$$\frac{v_{r2}}{v_{r1}} = \frac{v_2}{v_1} = \frac{1}{r} \rightarrow v_{r2} = \frac{v_{r1}}{r} = \frac{676.1}{8} = 84.51 \rightarrow T_2 = 652.4 \text{ K}$$

$$u_2 = 475.11 \text{ kJ/kg}$$

$$\frac{P_2 v_2}{T_2} = \frac{P_1 v_1}{T_1} \rightarrow P_2 = P_1 \left(\frac{T_2}{T_1}\right)\left(\frac{v_1}{v_2}\right)$$

$$= (100 \text{ kPa})\left(\frac{652.4 \text{ K}}{290 \text{ K}}\right)(8) = 1799.7 \text{ kPa}$$

Process 2-3 (constant-volume heat addition):

$$q_{in} = u_3 - u_2$$

$$800 \text{ kJ/kg} = u_3 - 475.11 \text{ kJ/kg}$$

$$u_3 = 1275.11 \text{ kJ/kg} \rightarrow T_3 = \mathbf{1575.1 \text{ K}}$$

$$v_{r3} = 6.108$$

$$\frac{P_3 v_3}{T_3} = \frac{P_2 v_2}{T_2} \rightarrow P_3 = P_2 \left(\frac{T_3}{T_2}\right)\left(\frac{v_2}{v_3}\right)$$

$$= (1.7997 \text{ MPa})\left(\frac{1575.1 \text{ K}}{652.4 \text{ K}}\right)(1) = \mathbf{4.345 \text{ MPa}}$$

(*b*) The net work output for the cycle is determined either by finding the boundary (*P dV*) work involved in each process by integration and adding them or by finding the net heat transfer that is equivalent to the net work done during the cycle. We take the latter approach. However, first we need to find the internal energy of the air at state 4:

Process 3-4 (isentropic expansion of an ideal gas):

$$\frac{v_{r4}}{v_{r3}} = \frac{v_4}{v_3} = r \rightarrow v_{r4} = r v_{r3} = (8)(6.108) = 48.864 \rightarrow T_4 = 795.6 \text{ K}$$

$$u_4 = 588.74 \text{ kJ/kg}$$

Process 4-1 (constant-volume heat rejection):

$$-q_{\text{out}} = u_1 - u_4 \rightarrow q_{\text{out}} = u_4 - u_1$$

$$q_{\text{out}} = 588.74 - 206.91 = 381.83 \text{ kJ/kg}$$

$$w_{\text{net}} = q_{\text{net}} = q_{\text{in}} - q_{\text{out}} = 800 - 381.83 = \mathbf{418.17\ kJ/kg}$$

(*c*) The thermal efficiency of the cycle is determined from its definition,

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{418.17 \text{ kJ/kg}}{800 \text{ kJ/kg}} = \mathbf{0.523\ or\ 52.3\%}$$

$$\eta_{\text{th, Otto}} = 1 - \frac{1}{r^{k-1}} = 1 - r^{1-k} = 1 - (8)^{1-1.4} = 0.565 \text{ or } 56.5\%$$

(*d*) The mean effective pressure is determined from its definition,

$$\text{MEP} = \frac{w_{\text{net}}}{v_1 - v_2} = \frac{w_{\text{net}}}{v_1 - v_1/r} = \frac{w_{\text{net}}}{v_1(1 - 1/r)}$$

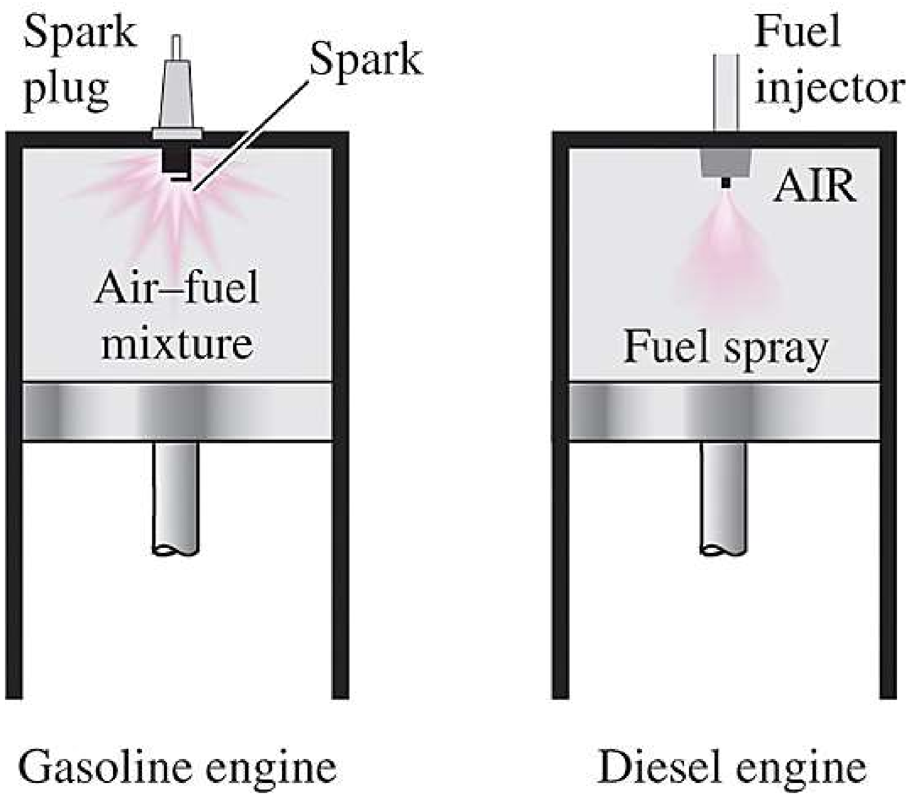
$$v_1 = \frac{RT_1}{P_1} = \frac{(0.287\ \text{kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(290\ \text{K})}{100\ \text{kPa}} = 0.832\ \text{m}^3/\text{kg}$$

$$\text{MEP} = \frac{418.17\ \text{kJ/kg}}{(0.832\ \text{m}^3/\text{kg})(1 - \frac{1}{8})}\left(\frac{1\ \text{kPa} \cdot \text{m}^3}{1\ \text{kJ}}\right) = \mathbf{574.4\ kPa}$$

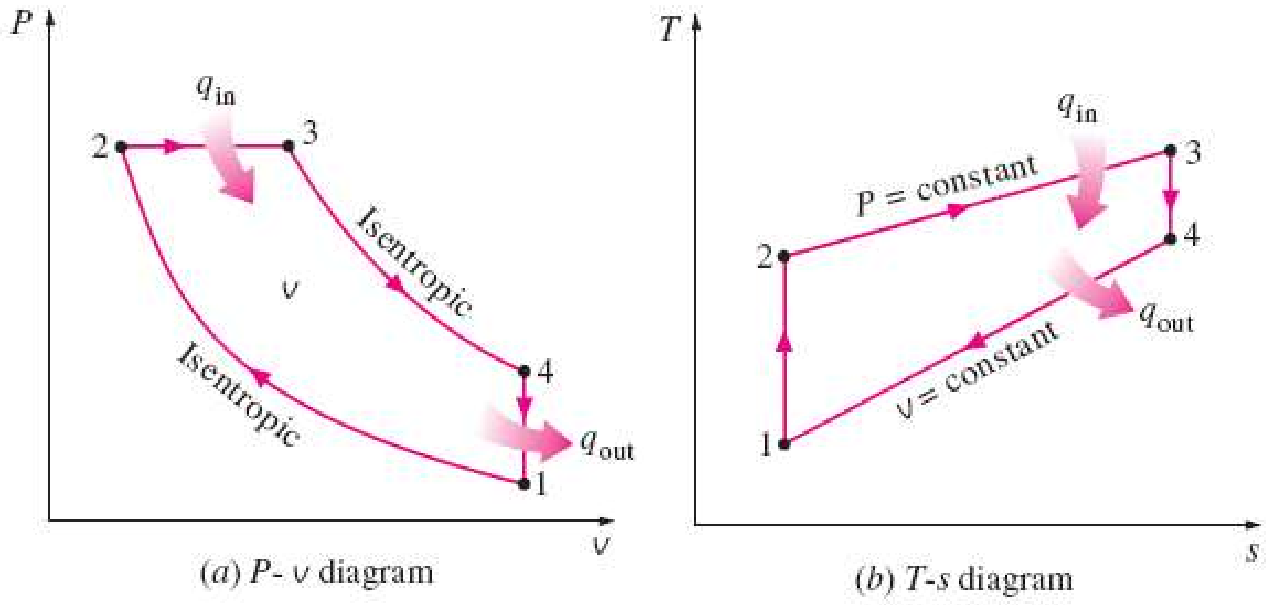
Therefore, a constant pressure of 574.4 kPa during the power stroke would produce the same net work output as the entire cycle.

DIESEL CYCLE: THE IDEAL CYCLE FOR COMPRESSION-IGNITION ENGINES

In diesel engines, only air is compressed during the compression stroke, eliminating the possibility of auto ignition (engine knock). Therefore, diesel engines can be designed to operate at much higher compression ratios than SI engines, typically between **12 and 24**.



In diesel engines, the spark plug is replaced by a fuel injector, and only air is compressed during the compression process.



1-2 isentropic compression
2-3 constant-pressure heat addition
3-4 isentropic expansion
4-1 constant-volume heat rejection.

$$q_{in} - w_{b,out} = u_3 - u_2 \rightarrow q_{in} = P_2(v_3 - v_2) + (u_3 - u_2)$$
$$= h_3 - h_2 = c_p(T_3 - T_2)$$

$$-q_{out} = u_1 - u_4 \rightarrow q_{out} = u_4 - u_1 = c_v(T_4 - T_1)$$

$$\boxed{\eta_{th,\,Diesel} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{T_4 - T_1}{k(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{kT_2(T_3/T_2 - 1)}}$$

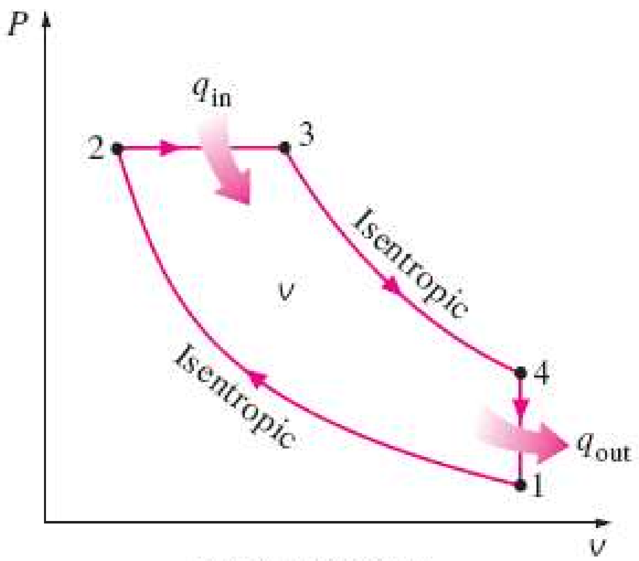
Cutoff ratio

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$

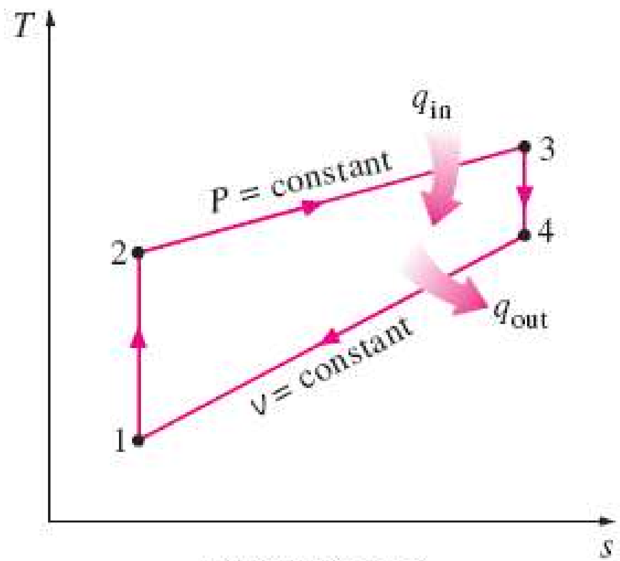
$$\eta_{th,\,Diesel} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

for the same compression ratio

$$\eta_{th,\,Otto} > \eta_{th,\,Diesel}$$



(a) P- v diagram

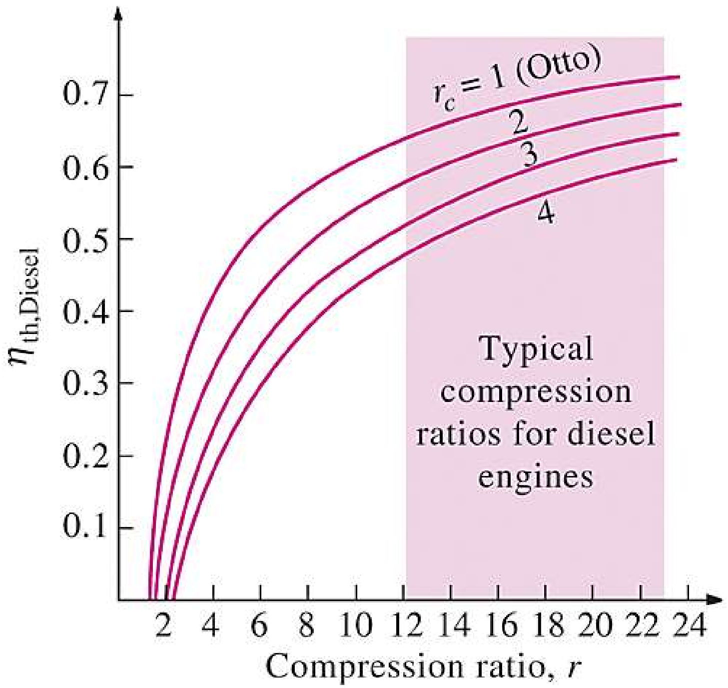


(b) T-s diagram

$$\eta_{\text{th, Diesel}} = 1 - \frac{1}{r^{k-1}}\left[\frac{r_c^k - 1}{k(r_c - 1)}\right]$$

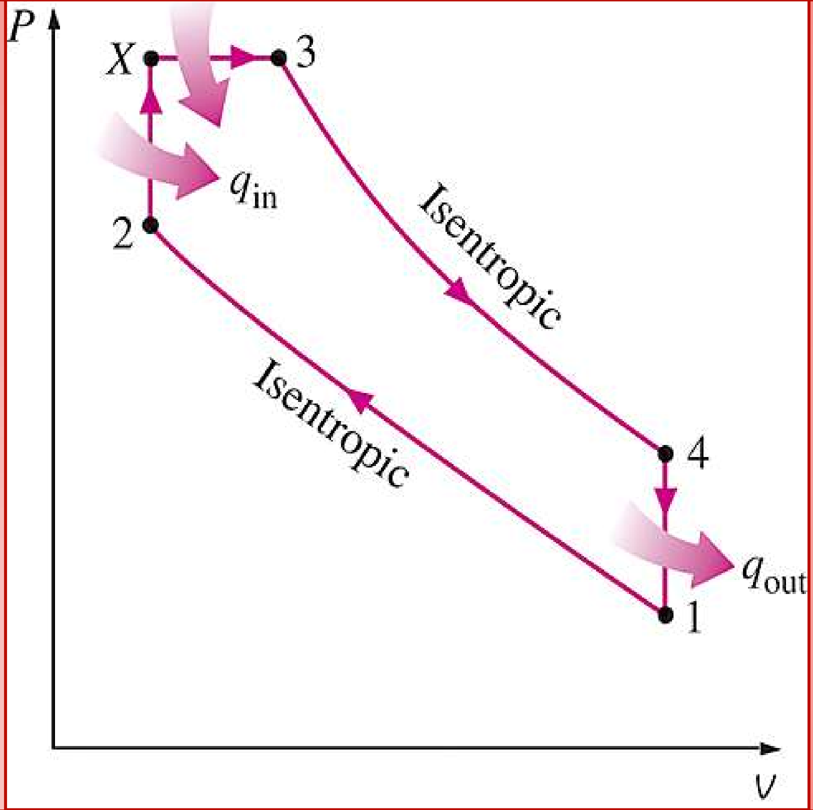
Cutoff ratio

$$r_c = \frac{V_3}{V_2} = \frac{v_3}{v_2}$$



Thermal efficiency of the ideal Diesel cycle as a function of compression and cutoff ratios (k=1.4).

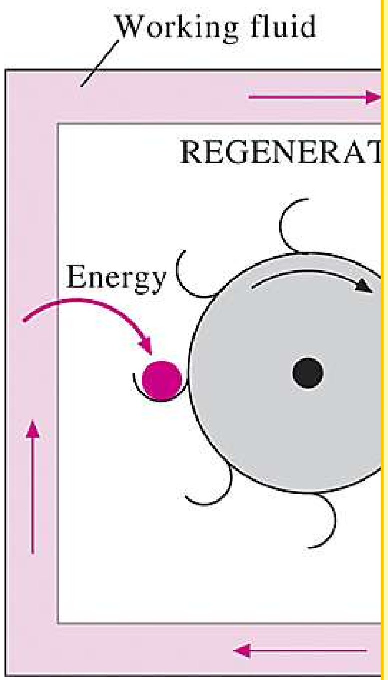
Dual cycle: A more realistic ideal cycle model for modern, high-speed compression ignition engine.



P-v diagram of an ideal dual cycle.

Stirling cycle

- **1-2** *T* = *constant* expansion (heat additi
- **2-3** *v* = *constant* regeneration (internal I ... id to the regenerator)
- **3-4** *T* = *constant* compression (heat reje
- **4-1** *v* = *constant* regeneration (internal I ... or back to the working fluid)



A regenerator is a device that borrows energy from the working fluid during one part of the cycle and pays it back during another part.

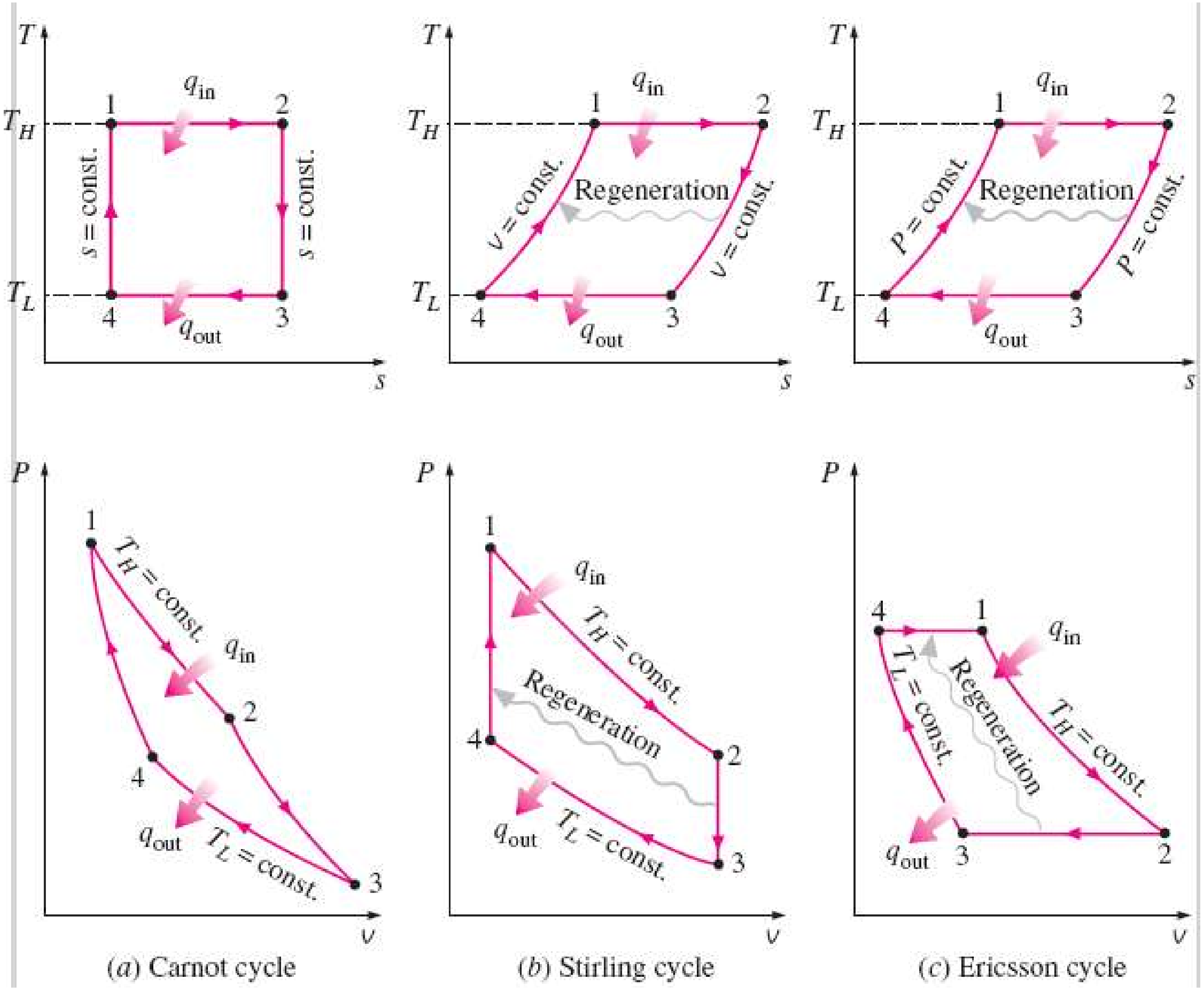
QUESTIONS

Diesel engines operate at higher air-fuel ratios than gasoline engines. Why?

Despite higher power to weight ratios, two-stroke engines are not used in automobiles. Why?

The stationary diesel engines are among the most efficient power producing devices (about 50%). Why?

What is a turbocharger? Why are they mostly used in diesel engines compared to gasoline engines.



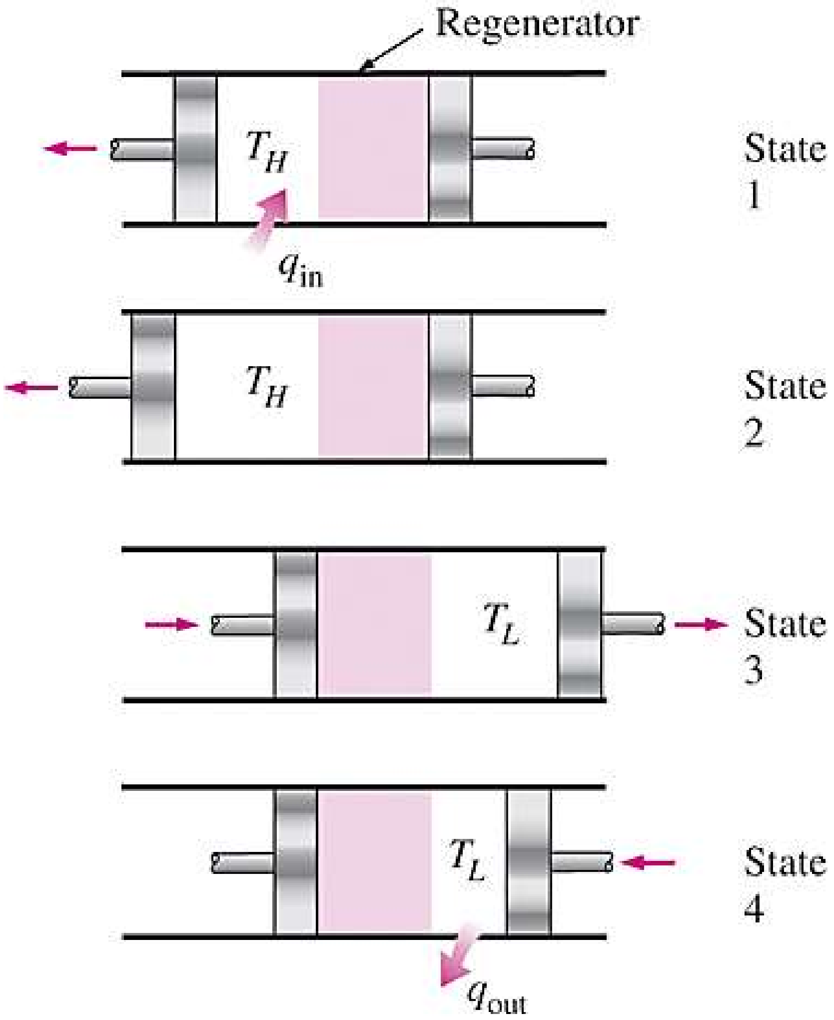
T-s and *P-v* diagrams of Carnot, Stirling, and Ericsson cycles.

The Stirling and Ericsson cycles give a message: ***Regeneration can increase efficiency.***

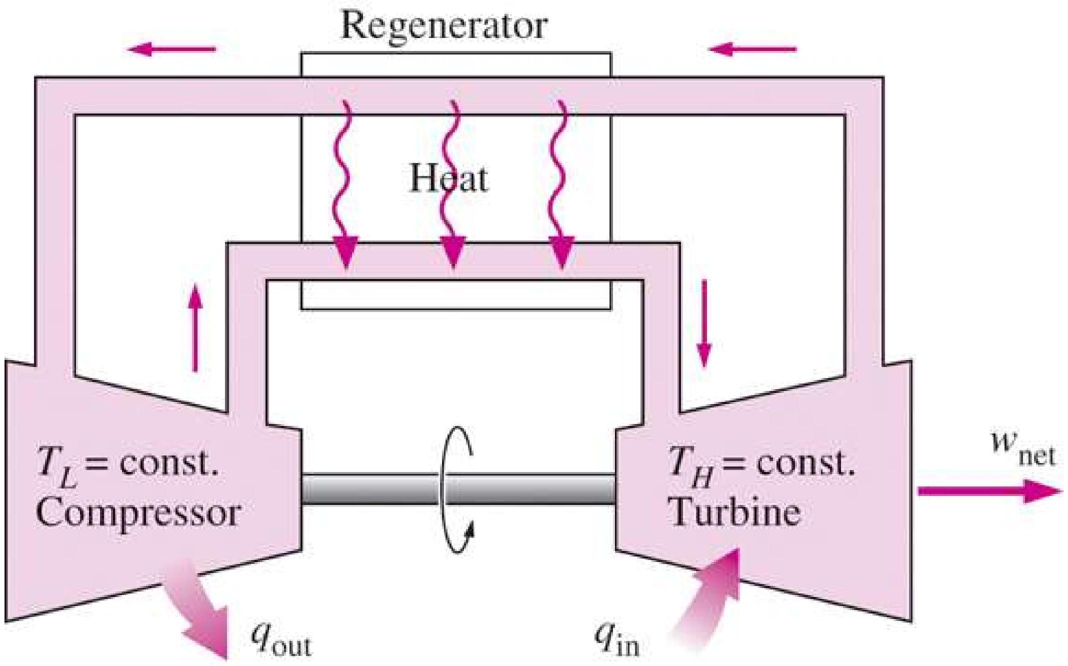
Both the Stirling and Ericsson cycles are totally reversible, as is the Carnot cycle, and thus:

$$\eta_{\text{th, Stirling}} = \eta_{\text{th, Ericsson}} = \eta_{\text{th, Carnot}} = 1 - \frac{T_L}{T_H}$$

The Ericsson cycle is very much like the Stirling cycle, except that the two constant-volume processes are replaced by two constant-pressure processes.



The execution of the Stirling cycle.



A steady-flow Ericsson engine.

Stirling and Ericsson cycles are difficult to achieve in practice because they involve heat transfer through a differential temperature difference in all components including the regenerator.

This would require providing infinitely large surface areas for heat transfer or allowing an infinitely long time for the process. Neither is practical. In reality, all heat transfer processes will take place through a finite temperature difference, the regenerator will not have an efficiency of 100 percent, and the pressure losses in the regenerator will be considerable.

More research and development are needed before these engines can compete with the gasoline or diesel engines. Both the Stirling and the Ericsson engines are *external combustion* engines.

Despite the physical limitations and impracticalities associated with them, both the Stirling and Ericsson cycles give a strong message to design engineers: *Regeneration can increase efficiency.* It is no coincidence that modern gas-turbine and steam power plants make extensive use of regeneration.

External combustion offers several advantages.

- ✓ A variety of fuels can be used as a source of thermal energy.
- ✓ There is more time for combustion, and thus the combustion process is more complete, which means less air pollution and more energy extraction from the fuel.
- ✓ These engines operate on closed cycles, and thus a working fluid that has the most desirable characteristics can be utilized as the working fluid.
- ✓ Hydrogen and helium are two gases commonly employed in these engines.
- ✓ In fact, the Brayton cycle with intercooling, reheating, and regeneration, which is utilized in large gas-turbine power plants and discussed later in this chapter, closely resembles the Ericsson cycle.

EXAMPLE 9-4

Using an ideal gas as the working fluid, show that the thermal efficiency of an Ericsson cycle is identical to the efficiency of a Carnot cycle operating between the same temperature limits.

Solution It is to be shown that the thermal efficiencies of Carnot and Ericsson cycles are identical.

Analysis Heat is transferred to the working fluid isothermally from an external source at temperature *TH* during process 1-2, and it is rejected again isothermally to an external sink at temperature *TL* during process 3-4.

$$q = T\,\Delta s$$

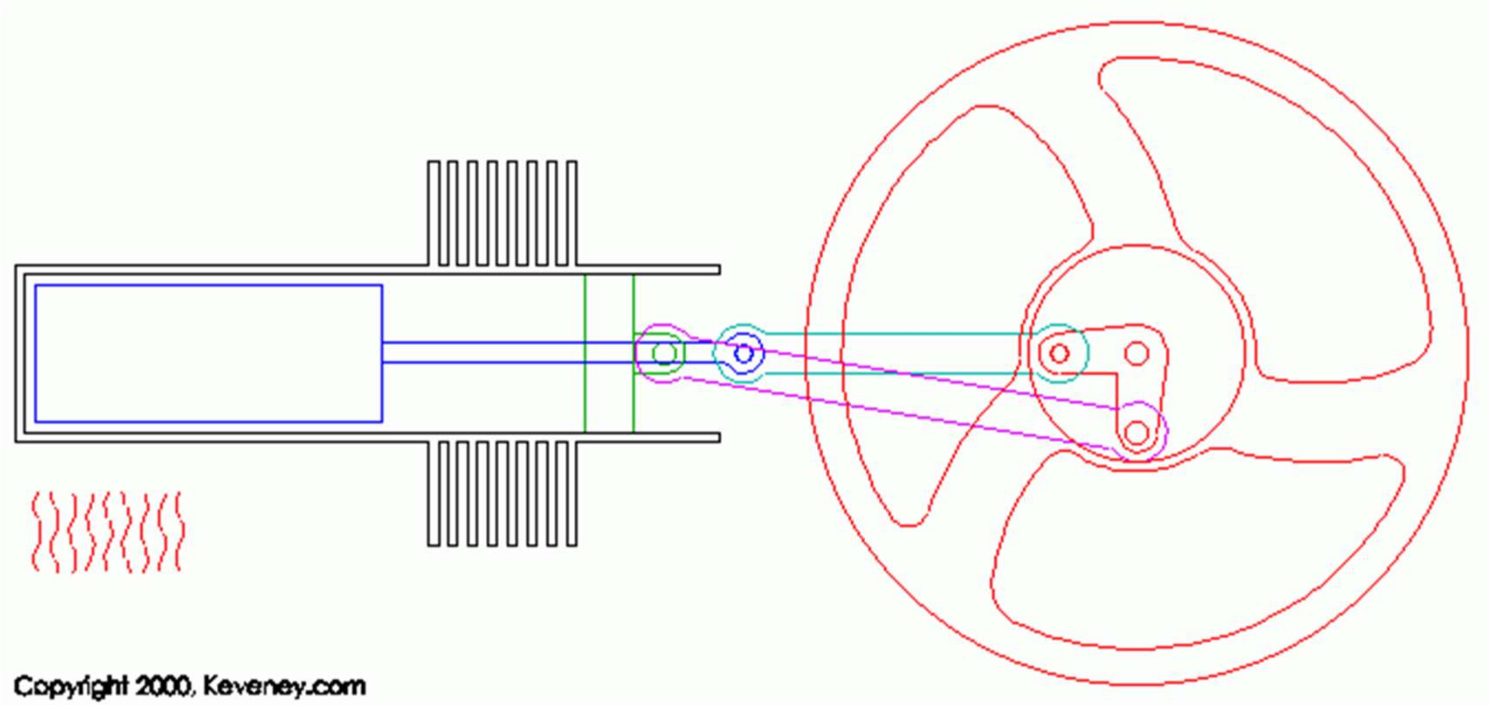
$$\Delta s = c_p \ln \frac{T_e}{T_i}^{\nearrow 0} - R \ln \frac{P_e}{P_i} = -R \ln \frac{P_e}{P_i}$$

$$q_{\text{in}} = T_H(s_2 - s_1) = T_H\left(-R \ln \frac{P_2}{P_1}\right) = RT_H \ln \frac{P_1}{P_2}$$

$$q_{\text{out}} = T_L(s_4 - s_3) = -T_L\left(-R \ln \frac{P_4}{P_3}\right) = RT_L \ln \frac{P_4}{P_3}$$

since $P_1 = P_4$ and $P_3 = P_2$.

$$\eta_{\text{th, Ericsson}} = 1 - \frac{q_{\text{out}}}{q_{\text{in}}} = 1 - \frac{RT_L \ln(P_4/P_3)}{RT_H \ln(P_1/P_2)} = 1 - \frac{T_L}{T_H}$$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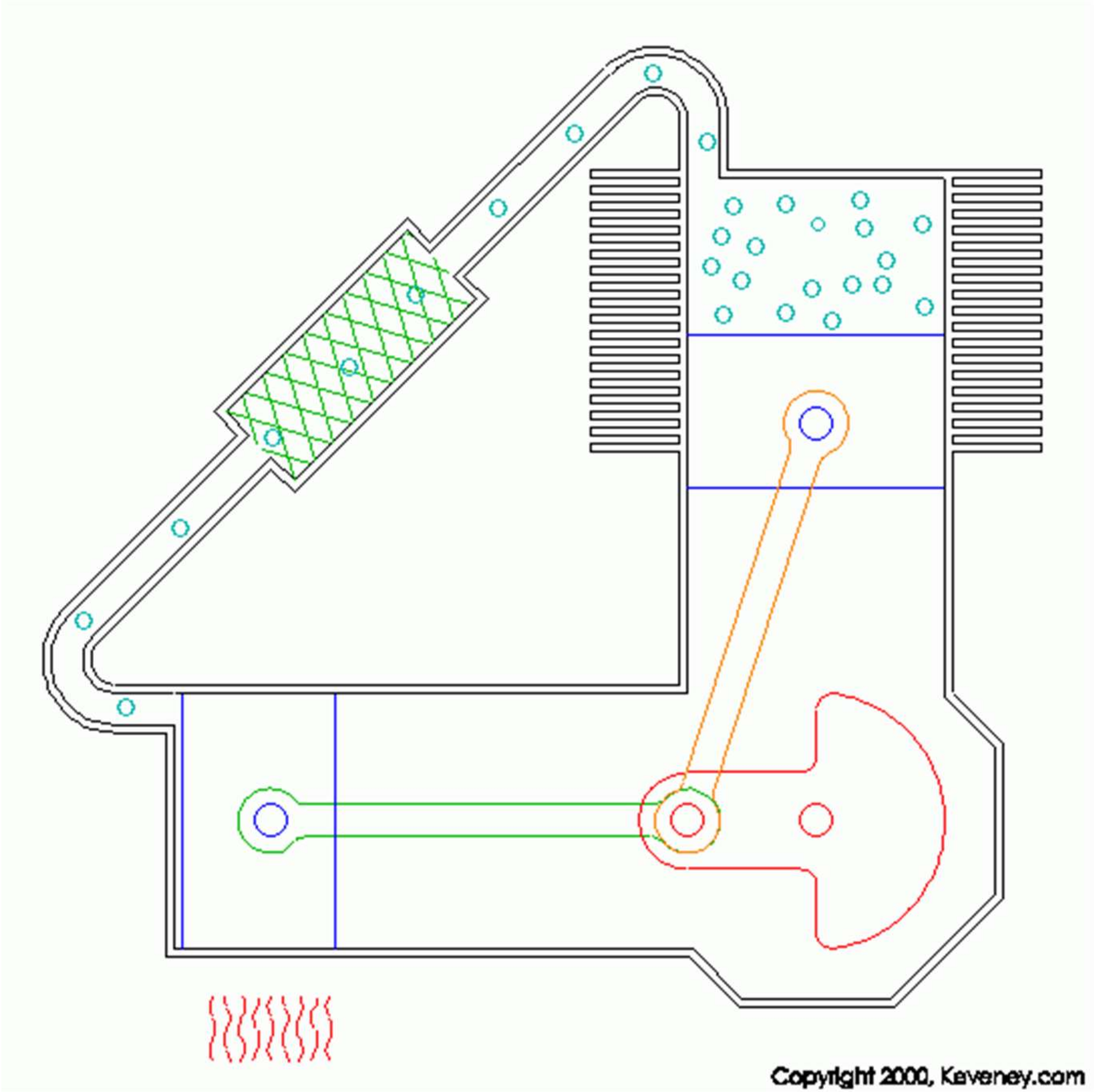
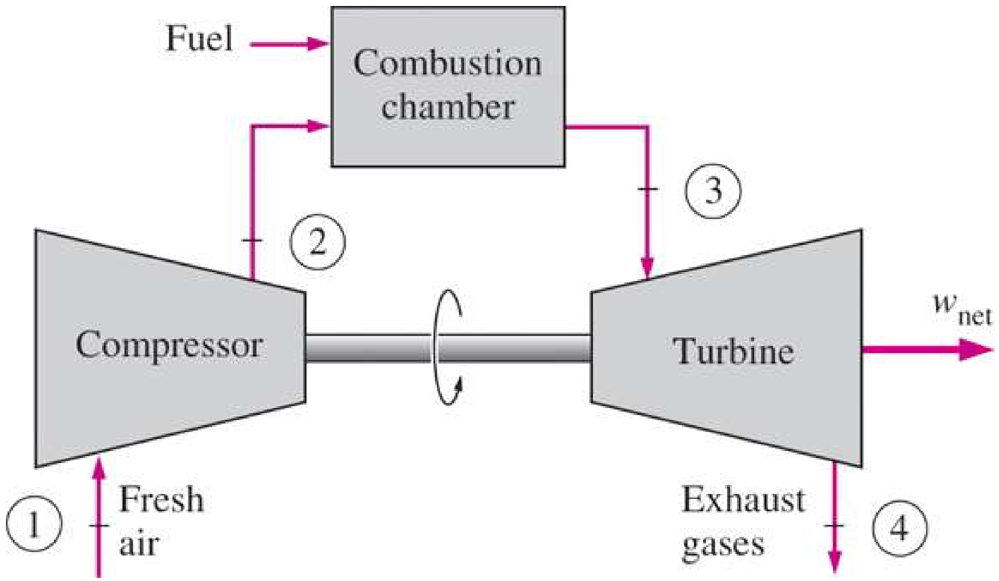
Rejenerative Stirling Cycles

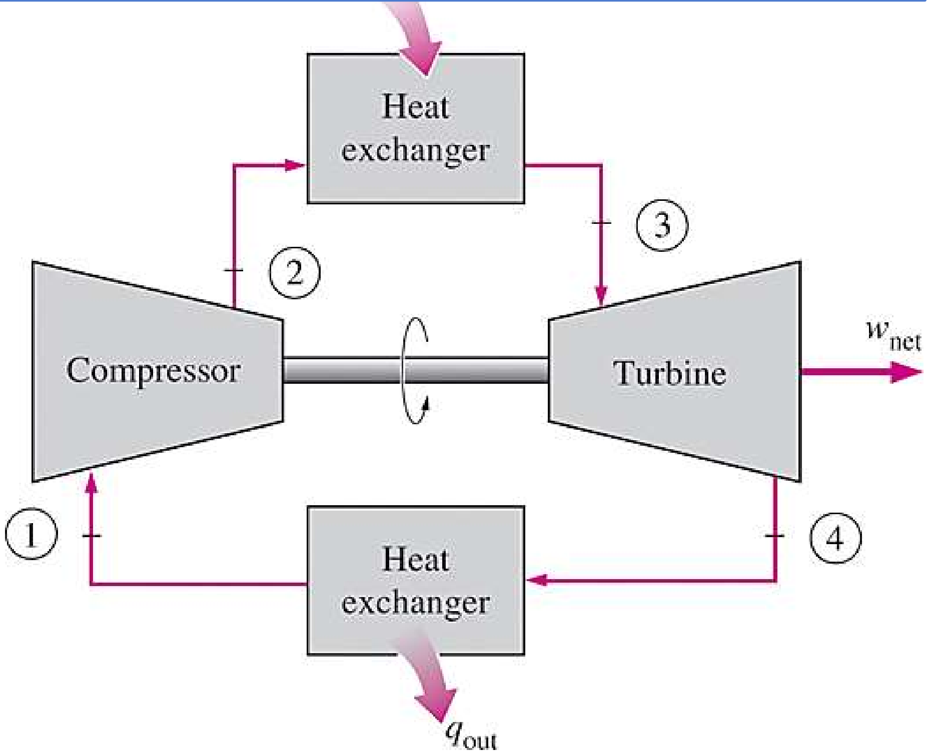


The combustion process is replaced by a constant-pressure heat-addition process from an external source, and the exhaust process is replaced by a constant-pressure heat-rejection process to the ambient air.

1-2 Isentropic compression (in a compressor)
2-3 Constant-pressure heat addition
3-4 Isentropic expansion (in a turbine)
4-1 Constant-pressure heat rejection



An open-cycle gas-turbine engine.



A closed-cycle gas-turbine engine.

$$(q_{in} - q_{out}) + (w_{in} - w_{out}) = h_{exit} - h_{inlet}$$

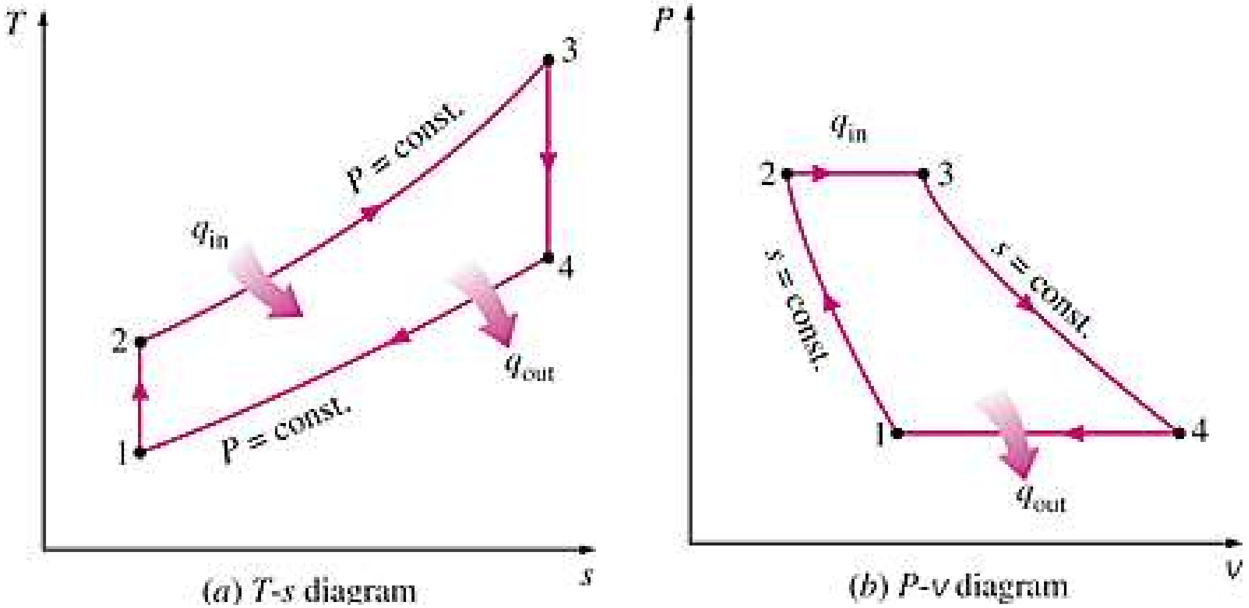
$$q_{in} = h_3 - h_2 = c_p(T_3 - T_2)$$

$$q_{out} = h_4 - h_1 = c_p(T_4 - T_1)$$

Thermal efficiency

$$\eta_{th,\,Brayton} = \frac{w_{net}}{q_{in}} = 1 - \frac{q_{out}}{q_{in}} = 1 - \frac{c_p(T_4 - T_1)}{c_p(T_3 - T_2)} = 1 - \frac{T_1(T_4/T_1 - 1)}{T_2(T_3/T_2 - 1)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{(k-1)/k} = \left(\frac{P_3}{P_4}\right)^{(k-1)/k} = \frac{T_3}{T_4}$$



T-s and *P-v* diagrams for the ideal Brayton cycle.

Pressure ratio

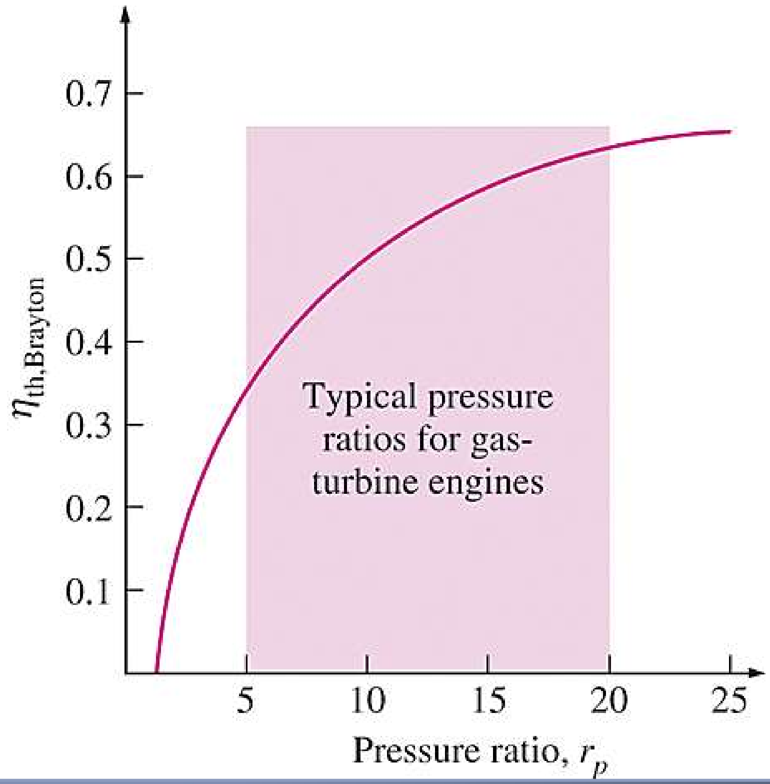
$$r_p = \frac{P_2}{P_1}$$

$$\eta_{th,\,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}}$$

Pressure ratio

$$r_p = \frac{P_2}{P_1}$$

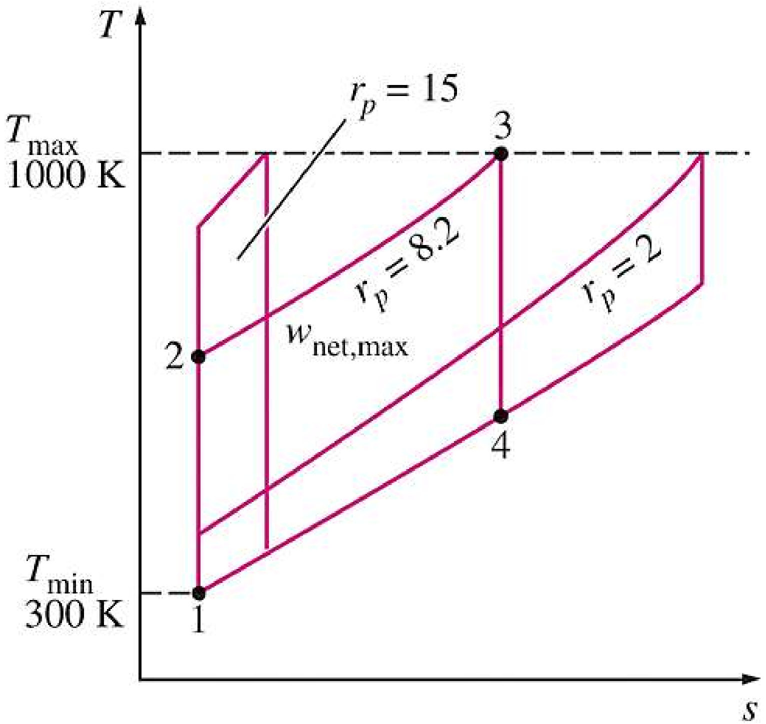
$$\boxed{\eta_{\text{th, Brayton}} = 1 - \frac{1}{r_p^{(k-1)/k}}}$$



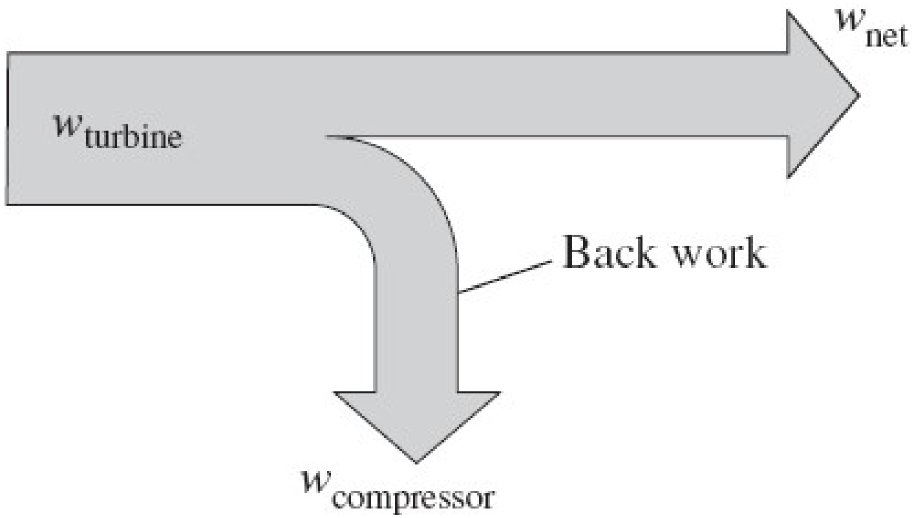
Thermal efficiency of the ideal Brayton cycle as a function of the pressure ratio.

The two major application areas of gas-turbine engines are *aircraft propulsion* and *electric power generation*.

The highest temperature in the cycle is limited by the maximum temperature that the turbine blades can withstand. This also limits the pressure ratios that can be used in the cycle. The air in gas turbines supplies the necessary oxidant for the combustion of the fuel, and it serves as a coolant to keep the temperature of various components within safe limits. An air–fuel ratio of 50 or above is not uncommon.



For fixed values of T_{min} and T_{max}, the net work of the Brayton cycle first increases with the pressure ratio, then reaches a maximum at $r_p = (T_{max}/T_{min})^{k/[2(k-1)]}$, and finally decreases.



The fraction of the turbine work used to drive the compressor is called the back work ratio.

1. Increasing the turbine inlet (or firing) temperatures
2. Increasing the efficiencies of turbomachinery components (turbines compressors)
3. Adding modifications to the basic cycle (intercooling, regeneration or recuperation, and reheating).

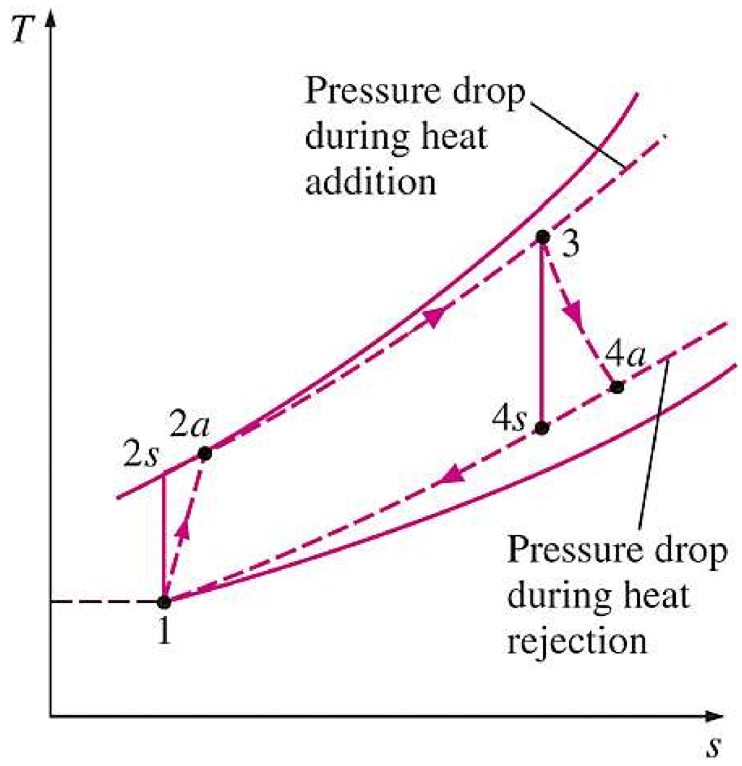
Deviation of Actual Gas-Turbine Cycles from Idealized Ones

Reasons: Irreversibilities in turbine and compressors, pressure drops, heat losses

Isentropic efficiencies of the compressor and turbine

$$\eta_C = \frac{w_s}{w_a} \cong \frac{h_{2s} - h_1}{h_{2a} - h_1}$$

$$\eta_T = \frac{w_a}{w_s} \cong \frac{h_3 - h_{4a}}{h_3 - h_{4s}}$$



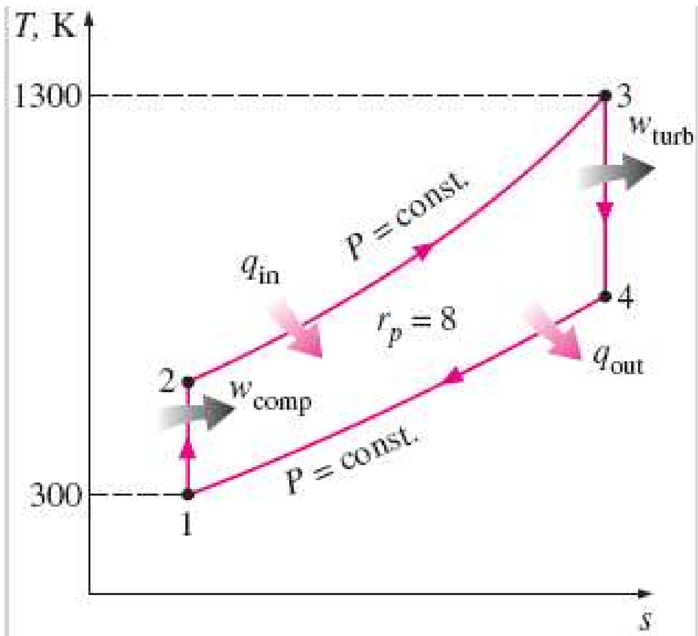
The deviation of an actual gas-turbine cycle from the ideal Brayton cycle as a result of irreversibilities.

EXAMPLE 9–5

A gas-turbine power plant operating on an ideal Brayton cycle has a pressure ratio of 8. The gas temperature is 300 K at the compressor inlet and 1300 K at the turbine inlet. Utilizing the air-standard assumptions, determine (*a*) the gas temperature at the exits of the compressor and the turbine, (*b*) the back work ratio, and (*c*) the thermal efficiency.

Solution A power plant operating on the ideal Brayton cycle with a specified pressure ratio is considered. The compressor and turbine exit temperatures, back work ratio, and the thermal efficiency are to be determined.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible. **4** The variation of specific heats with temperature is to be considered.



(*a*) The air temperatures at the compressor and turbine exits are determined from isentropic relations.

Process 1-2 (isentropic compression of an ideal gas):

$$T_1 = 300\text{ K} \rightarrow h_1 = 300.19\text{ kJ/kg}$$
$$P_{r1} = 1.386$$

at compressor exit

$$P_{r2} = \frac{P_2}{P_1} P_{r1} = (8)(1.386) = 11.09 \rightarrow T_2 = \mathbf{540\text{ K}}$$

$$h_2 = 544.35\text{ kJ/kg}$$

Process 3-4 (isentropic expansion of an ideal gas):

$$T_3 = 1300\text{ K} \rightarrow h_3 = 1395.97\text{ kJ/kg} \qquad P_{r3} = 330.9$$

at turbine exit

$$P_{r4} = \frac{P_4}{P_3} P_{r3} = \left(\frac{1}{8}\right)(330.9) = 41.36 \rightarrow T_4 = \mathbf{770\ K} \qquad h_4 = 789.11\text{ kJ/kg}$$

(*b*) To find the back work ratio, we need to find the work input to the compressor and the work output of the turbine:

$$w_{\text{comp, in}} = h_2 - h_1 = 544.35 - 300.19 = 244.16\text{ kJ/kg}$$

$$w_{\text{turb, out}} = h_3 - h_4 = 1395.97 - 789.11 = 606.86\text{ kJ/kg}$$

$$\text{Back work ratio } r_{\text{bw}} = \frac{w_{\text{comp, in}}}{w_{\text{turb, out}}} = \frac{244.16\text{ kJ/kg}}{606.86\text{ kJ/kg}} = \mathbf{0.402}$$

(*c*) The thermal efficiency of the cycle is the ratio of the net power output to the total heat input:

$$q_{\text{in}} = h_3 - h_2 = 1395.97 - 544.35 = 851.62\text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 606.86 - 244.16 = 362.7\text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{362.7\text{ kJ/kg}}{851.62\text{ kJ/kg}} = \mathbf{0.426\ or\ 42.6\%}$$

The thermal efficiency could also be determined from

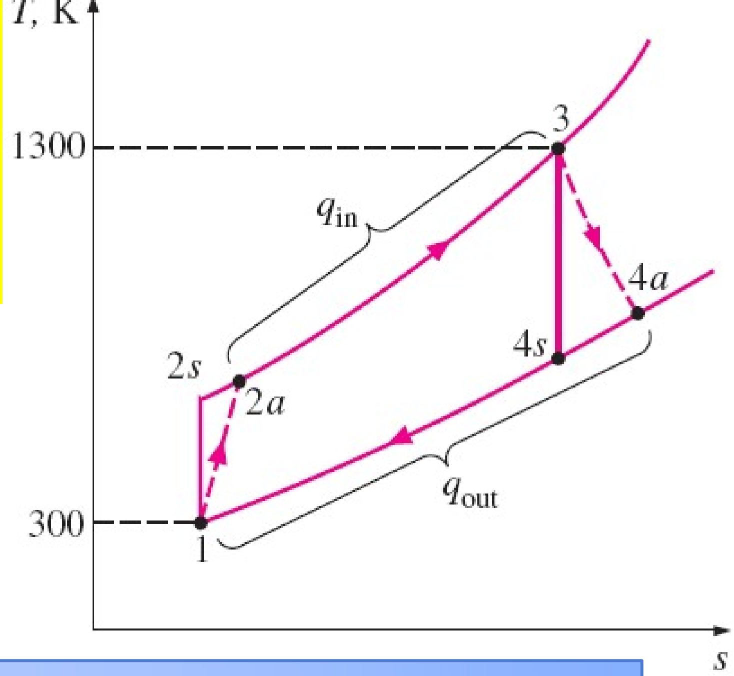
$$\eta_{th} = 1 - \frac{q_{out}}{q_{in}}$$

$$q_{out} = h_4 - h_1 = 789.11 - 300.19 = 488.92 \text{ kJ/kg}$$

$$\eta_{th,\,Brayton} = 1 - \frac{1}{r_p^{(k-1)/k}} = 1 - \frac{1}{8^{(1.4-1)/1.4}} = 0.448$$

EXAMPLE 9–6

Assuming a compressor efficiency of 80 percent and a turbine efficiency of 85 percent, determine (*a*) the back work ratio, (*b*) the thermal efficiency, and (*c*) the turbine exit temperature of the gas-turbine cycle discussed in Example 9–5.



Solution The Brayton cycle discussed in Example 9–5 is reconsidered. For specified turbine and compressor efficiencies, the back work ratio, the thermal efficiency, and the turbine exit temperature are to be determined.

(*a*) The actual compressor work and turbine work are determined by using the definitions of compressor and turbine efficiencies,

$$\textit{Compressor:} \qquad w_{\text{comp, in}} = \frac{w_s}{\eta_C} = \frac{244.16 \text{ kJ/kg}}{0.80} = 305.20 \text{ kJ/kg}$$

$$\textit{Turbine:} \qquad w_{\text{turb, out}} = \eta_T w_s = (0.85)(606.86 \text{ kJ/kg}) = 515.83 \text{ kJ/kg}$$

$$r_{\text{bw}} = \frac{w_{\text{comp, in}}}{w_{\text{turb, out}}} = \frac{305.20 \text{ kJ/kg}}{515.83 \text{ kJ/kg}} = \mathbf{0.592}$$

That is, the compressor is now consuming 59.2 % of the work produced by the turbine (up from 40.2 %). This increase is due to the irreversibilities that occur within the compressor and the turbine.

(*b*) In this case, air will leave the compressor at a higher temperature and enthalpy, which are determined to be

$$w_{\text{comp, in}} = h_{2a} - h_1 \rightarrow h_{2a} = h_1 + w_{\text{comp, in}}$$
$$= 300.19 + 305.20$$
$$= 605.39 \text{ kJ/kg} \quad (\text{and } T_{2a} = 598 \text{ K})$$

$$q_{\text{in}} = h_3 - h_{2a} = 1395.97 - 605.39 = 790.58 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{out}} - w_{\text{in}} = 515.83 - 305.20 = 210.63 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{210.63 \text{ kJ/kg}}{790.58 \text{ kJ/kg}} = \mathbf{0.266 \text{ or } 26.6\%}$$

(*c*) The air temperature at the turbine exit is determined from an energy balance on the turbine:

$$w_{\text{turb, out}} = h_3 - h_{4a} \rightarrow h_{4a} = h_3 - w_{\text{turb, out}}$$
$$= 1395.97 - 515.83$$
$$= 880.14 \text{ kJ/kg}$$

Then, from Table A–17,

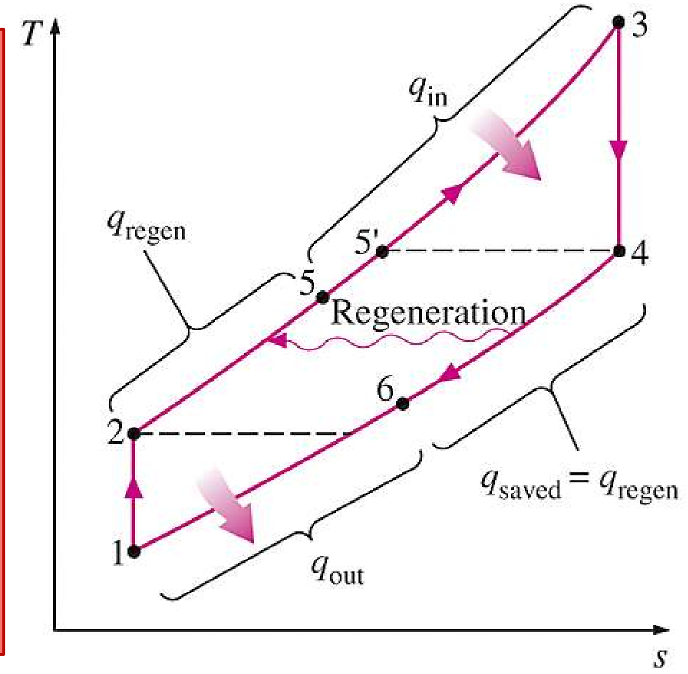
$$T_{4a} = \mathbf{853 \text{ K}}$$

This value is considerably higher than the air temperature at the compressor exit (T_{2a}=598 K), which suggests the use of regeneration to reduce fuel cost.

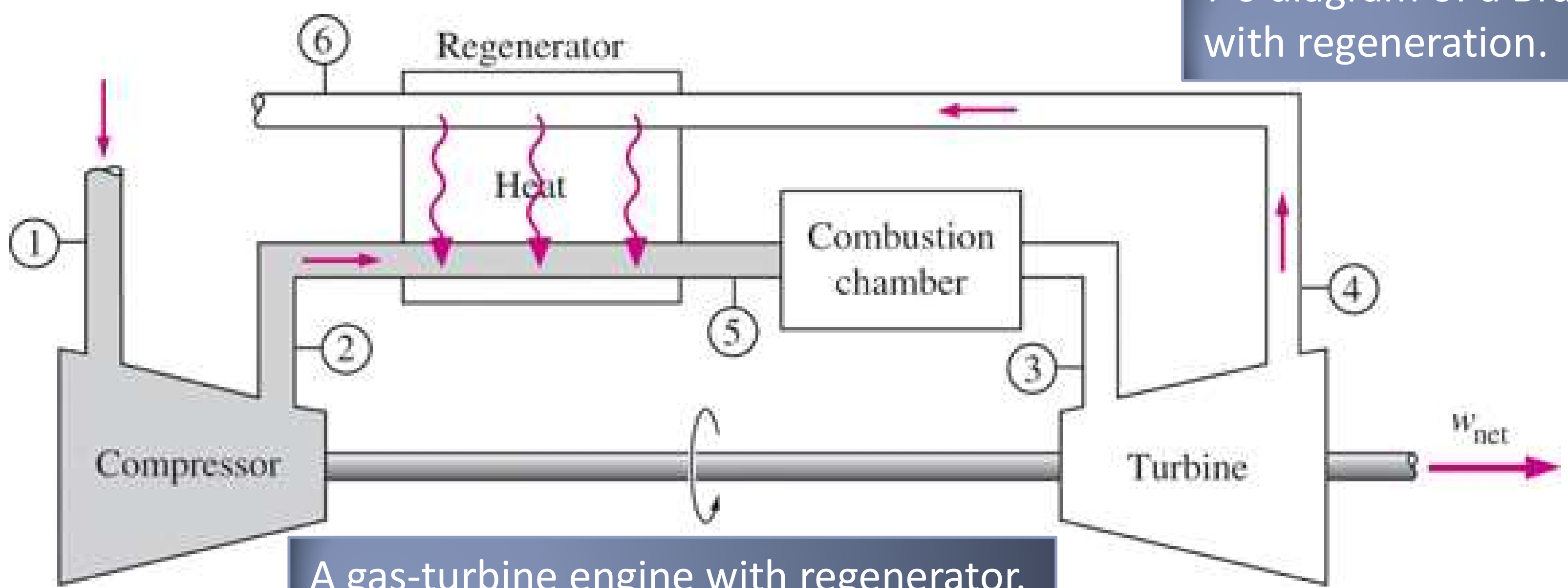
In gas-turbine engines, the temperature of the exhaust gas leaving the turbine is often considerably higher than the temperature of the air leaving the compressor.

Therefore, the high-pressure air leaving the compressor can be heated by the hot exhaust gases in a counter-flow heat exchanger (a *regenerator* or a *recuperator*).

The thermal efficiency of the Brayton cycle increases as a result of regeneration since less fuel is used for the same work output.



T-s diagram of a Brayton cycle with regeneration.



A gas-turbine engine with regenerator.

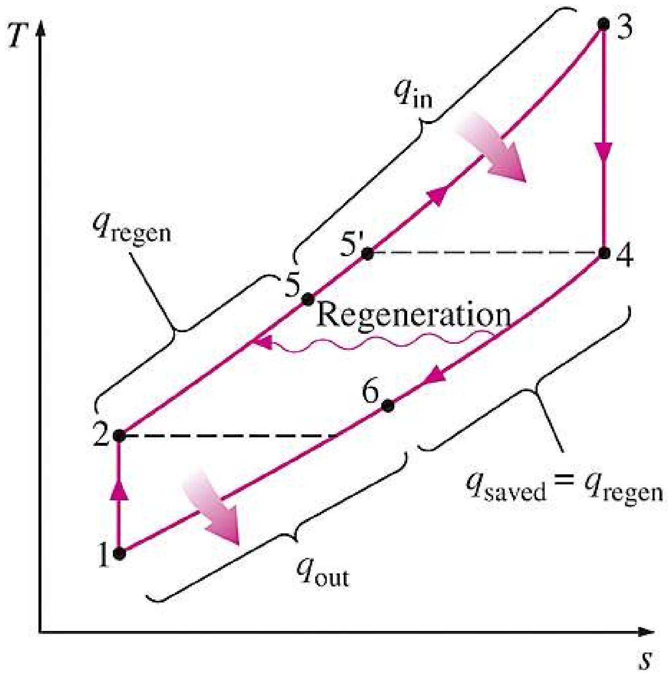
Effectiveness of regenerator

The thermal efficiency depends on the ratio of the minimum to maximum temperatures as well as the pressure ratio.

Regeneration is most effective at lower pressure ratios and low minimum-to-maximum temperature ratios.

$$q_{\text{regen, max}} = h_{5'} - h_2 = h_4 - h_2$$

$$q_{\text{regen, act}} = h_5 - h_2$$



T-s diagram of a Brayton cycle with regeneration.

Effectiveness of regenerator

$$\epsilon = \frac{q_{\text{regen, act}}}{q_{\text{regen, max}}} = \frac{h_5 - h_2}{h_4 - h_2}$$

Effectiveness under cold-air standard assumptions

$$\epsilon \cong \frac{T_5 - T_2}{T_4 - T_2}$$

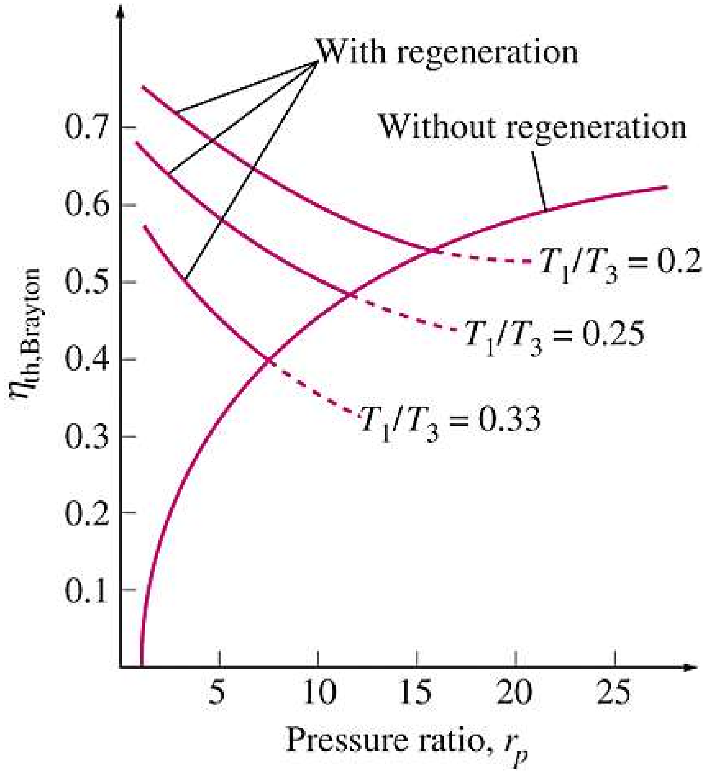
$$\eta_{\text{th, regen}} = 1 - \left(\frac{T_1}{T_3}\right)(r_p)^{(k-1)/k}$$

Under cold-air standard assumptions

Pressure ratio

$$r_p = \frac{P_2}{P_1}$$

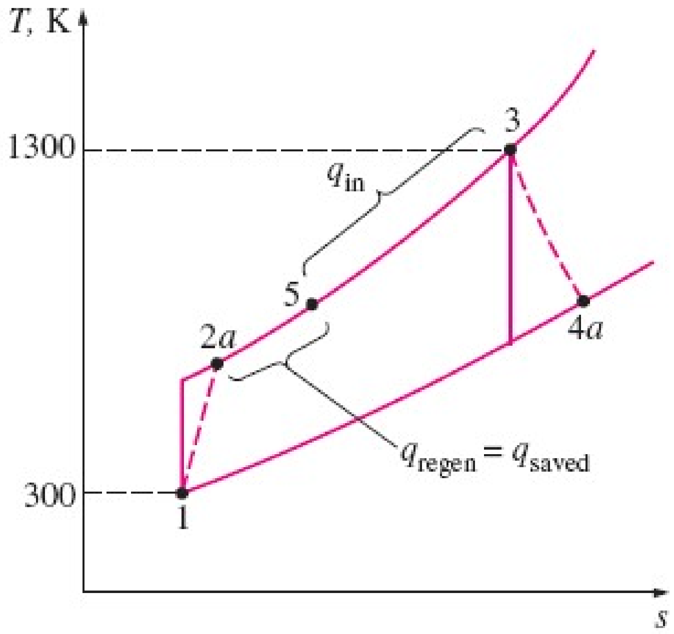
$$\eta_{\text{th, regen}} = 1 - \left(\frac{T_1}{T_3}\right)(r_p)^{(k-1)/k}$$



Thermal efficiency of the ideal Brayton cycle with and without regeneration.

EXAMPLE 9–7

Determine the thermal efficiency of the gas-turbine power plant described in Example 9–6 if a regenerator having an effectiveness of 80 percent is installed.



Solution The gas-turbine power plant discussed in Example 9–6 is equipped with a regenerator. For a specified effectiveness, the thermal efficiency of the cycle is to be determined.

Analysis We first determine the enthalpy of the air at the exit of the regenerator, using the definition of effectiveness:

$$\epsilon = \frac{h_5 - h_{2a}}{h_{4a} - h_{2a}}$$

$$0.80 = \frac{(h_5 - 605.39)\ \text{kJ/kg}}{(880.14 - 605.39)\ \text{kJ/kg}} \rightarrow h_5 = 825.19\ \text{kJ/kg}$$

$$q_{in} = h_3 - h_5 = (1395.97 - 825.19)\ \text{kJ/kg} = 570.78\ \text{kJ/kg}$$

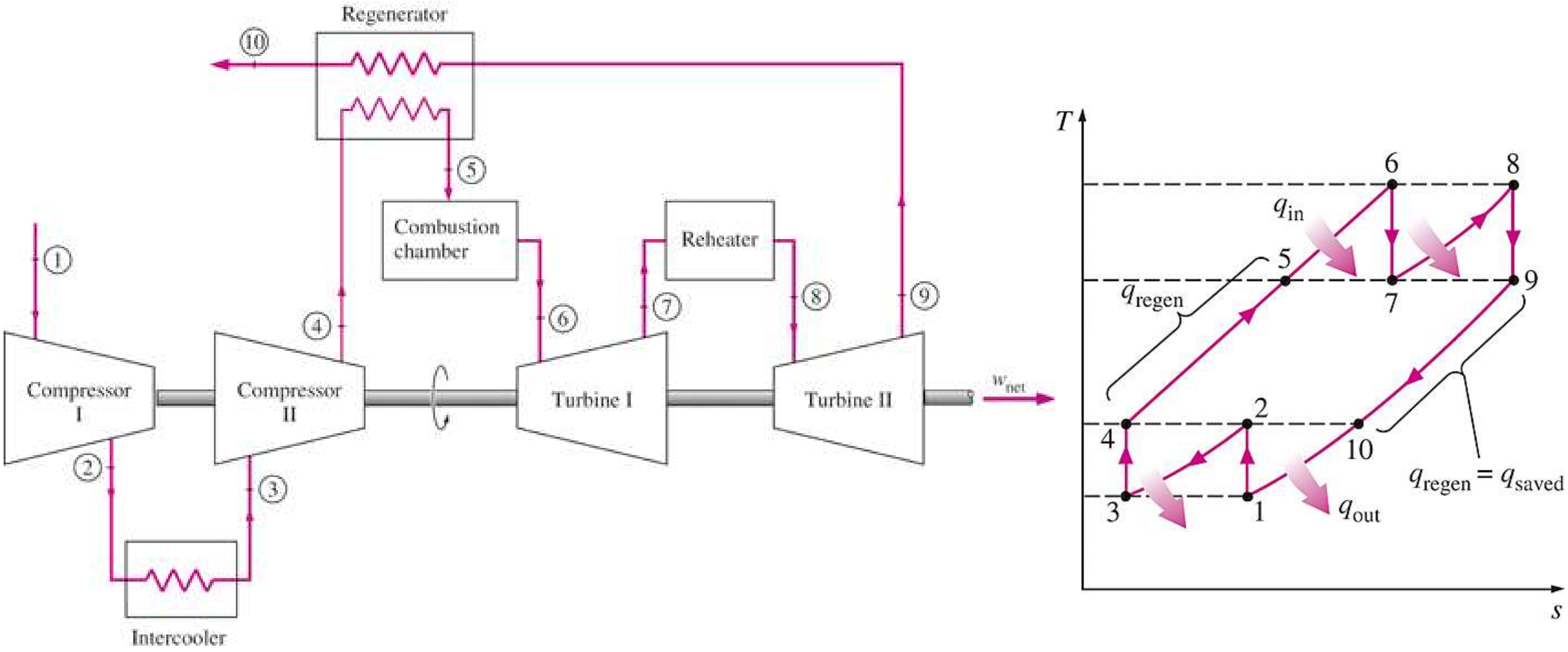
$$\eta_{th} = \frac{w_{net}}{q_{in}} = \frac{210.63\ \text{kJ/kg}}{570.78\ \text{kJ/kg}} = \mathbf{0.369\ or\ 36.9\%}$$

Discussion Note that the thermal efficiency of the power plant has gone up from 26.6 to 36.9 percent as a result of installing a regenerator that helps to recuperate some of the thermal energy of the exhaust gases.

THE BRAYTON CYCLE WITH INTERCOOLING, REHEATING, AND REGENERATION

For minimizing work input to compressor and maximizing work output from turbine:

$$\frac{P_2}{P_1} = \frac{P_4}{P_3} \quad \text{and} \quad \frac{P_6}{P_7} = \frac{P_8}{P_9}$$

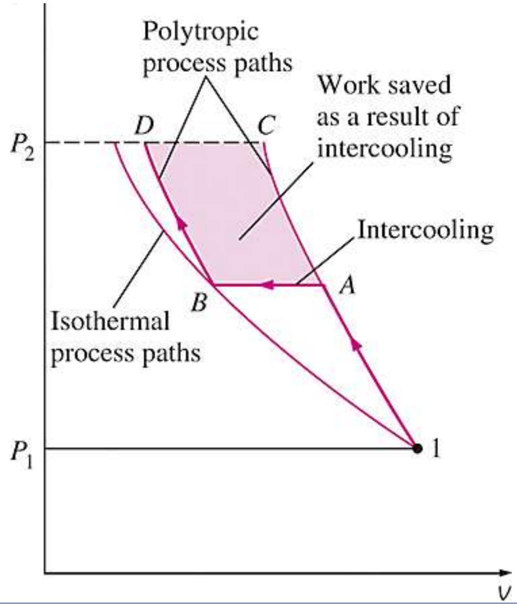


A gas-turbine engine with two-stage compression with intercooling, two-stage expansion with reheating, and regeneration and its *T-s* diagram.

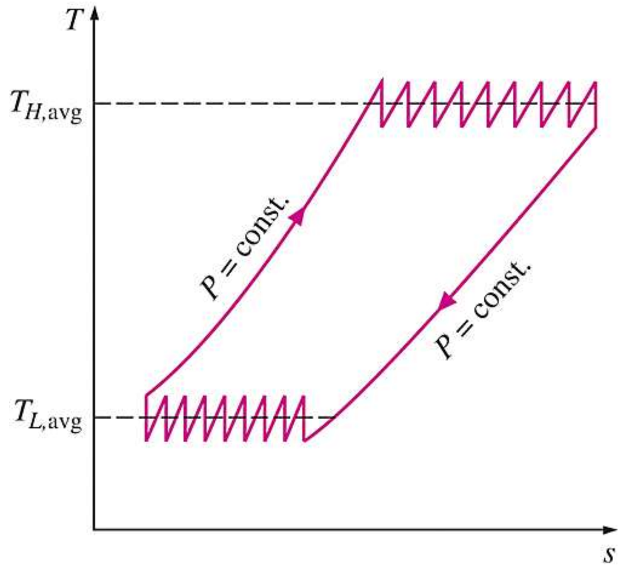
Multistage compression with intercooling: The work required to compress a gas between two specified pressures can be decreased by carrying out the compression process in stages and cooling the gas in between. This keeps the specific volume as low as possible.

Multistage expansion with reheating keeps the specific volume of the working fluid as high as possible during an expansion process, thus maximizing work output.

Intercooling and reheating always decreases the thermal efficiency unless they are accompanied by regeneration. **Why?**



Comparison of work inputs to a single-stage compressor (1*AC*) and a two-stage compressor with intercooling (1*ABD*).



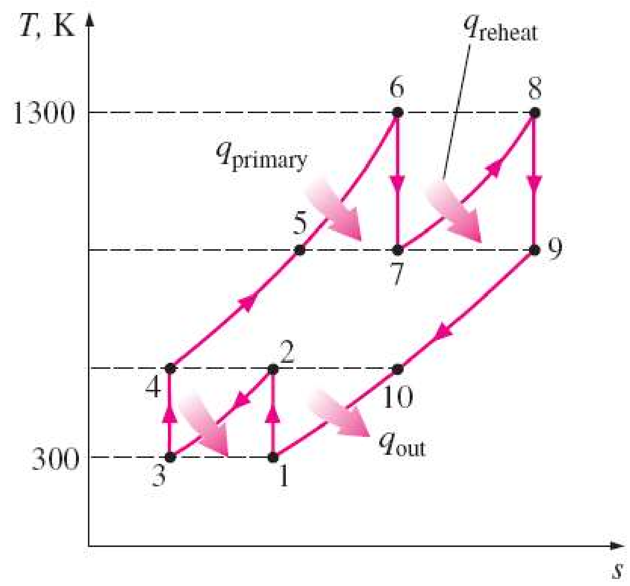
Compression and expansion stages increases, the gas-turbine cycle with intercooling, reheating, and regeneration approaches the Ericsson cycle.

EXAMPLE 9-8

An ideal gas-turbine cycle with two stages of compression and two stages of expansion has an overall pressure ratio of 8. Air enters each stage of the compressor at 300 K and each stage of the turbine at 1300 K. Determine the back work ratio and the thermal efficiency of this gas-turbine cycle, assuming (*a*) no regenerators and (*b*) an ideal regenerator with 100 percent effectiveness. Compare the results with those obtained in Example 9–5.

Solution An ideal gas-turbine cycle with two stages of compression and two stages of expansion is considered. The back work ratio and the thermal efficiency of the cycle are to be determined for the cases of no regeneration and maximum regeneration.

Assumptions **1** Steady operating conditions exist. **2** The air-standard assumptions are applicable. **3** Kinetic and potential energy changes are negligible.



$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt{8} = 2.83 \quad \text{and} \quad \frac{P_6}{P_7} = \frac{P_8}{P_9} = \sqrt{8} = 2.83$$

At inlets: $T_1 = T_3, \quad h_1 = h_3 \quad \text{and} \quad T_6 = T_8, \quad h_6 = h_8$

At exits: $T_2 = T_4, \quad h_2 = h_4 \quad \text{and} \quad T_7 = T_9, \quad h_7 = h_9$

(*a*) In the absence of any regeneration, the back work ratio and the thermal efficiency are determined by using data from Table A–17 as follows:

$$T_1 = 300 \text{ K} \rightarrow h_1 = 300.19 \text{ kJ/kg} \qquad P_{r1} = 1.386$$

$$P_{r2} = \frac{P_2}{P_1} P_{r1} = \sqrt{8}(1.386) = 3.92 \rightarrow T_2 = 403.3 \text{ K} \qquad h_2 = 404.33 \text{ kJ/kg}$$

$$T_6 = 1300 \text{ K} \rightarrow h_6 = 1395.97 \text{ kJ/kg} \qquad P_{r6} = 330.9$$

$$P_{r7} = \frac{P_7}{P_6} P_{r6} = \frac{1}{\sqrt{8}}(330.9) = 117.0 \rightarrow T_7 = 1006.4 \text{ K} \qquad h_7 = 1053.35 \text{ kJ/kg}$$

$$w_{\text{comp, in}} = 2(w_{\text{comp, in, I}}) = 2(h_2 - h_1) = 2(404.33 - 300.19) = 208.28 \text{ kJ/kg}$$

$$w_{\text{turb, out}} = 2(w_{\text{turb, out, I}}) = 2(h_6 - h_7) = 2(1395.97 - 1053.35) = 685.24 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{turb, out}} - w_{\text{comp, in}} = 685.24 - 208.28 = 476.96 \text{ kJ/kg}$$

$$\begin{aligned} q_{\text{in}} &= q_{\text{primary}} + q_{\text{reheat}} = (h_6 - h_4) + (h_8 - h_7) \\ &= (1395.97 - 404.33) + (1395.97 - 1053.35) = 1334.26 \text{ kJ/kg} \end{aligned}$$

$$r_{\text{bw}} = \frac{w_{\text{comp, in}}}{w_{\text{turb, out}}} = \frac{208.28 \text{ kJ/kg}}{685.24 \text{ kJ/kg}} = \mathbf{0.304 \text{ or } 30.4\%}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{476.96 \text{ kJ/kg}}{1334.26 \text{ kJ/kg}} = \mathbf{0.357 \text{ or } 35.7\%}$$

A comparison of these results with those obtained in Example 9–5 reveals that multistage compression with intercooling and multistage expansion with reheating improve the back work ratio (it drops from 40.2 to 30.4 %) but hurt the thermal efficiency (it drops from 42.6 to 35.7 %).

(*b*) an ideal regenerator with 100 percent effectiveness.

The heat input and the thermal efficiency in this case are

$$\begin{aligned} q_{\text{in}} &= q_{\text{primary}} + q_{\text{reheat}} = (h_6 - h_5) + (h_8 - h_7) \\ &= (1395.97 - 1053.35) + (1395.97 - 1053.35) = 685.24 \text{ kJ/kg} \end{aligned}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{476.96 \text{ kJ/kg}}{685.24 \text{ kJ/kg}} = \mathbf{0.696 \text{ or } 69.6\%}$$

That is, the thermal efficiency almost doubles as a result of regeneration compared to the no-regeneration case. The overall effect of two-stage compression and expansion with intercooling, reheating, and regeneration on the thermal efficiency is an increase of 63 %. and the thermal efficiency will approach.

$$\eta_{\text{th, Ericsson}} = \eta_{\text{th, Carnot}} = 1 - \frac{T_L}{T_H} = 1 - \frac{300 \text{ K}}{1300 \text{ K}} = 0.769$$

Adding a second stage increases the thermal efficiency from 42.6 to 69.6 %, an increase of 27 % points. This is a significant increase in efficiency. Adding more stages, however can increase the efficiency an additional 7.3 % points at most, and usually cannot be justified economically.

IDEAL JET-PROPULSION CYCLES

Gas-turbine engines are widely used to power aircraft because they are light and compact and have a high power-to-weight ratio.

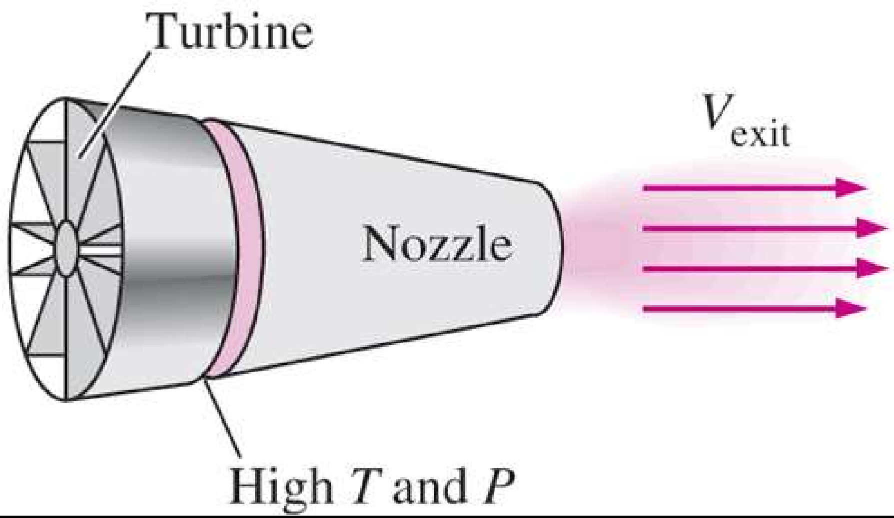
Aircraft gas turbines operate on an open cycle called a **jet-propulsion cycle**.

The ideal jet-propulsion cycle differs from the simple ideal Brayton cycle in that the gases are not expanded to the ambient pressure in the turbine. Instead, they are expanded to a pressure such that the power produced by the turbine is just sufficient to drive the compressor and the auxiliary equipment.

The net work output of a jet-propulsion cycle is zero. The gases that exit the turbine at a relatively high pressure are subsequently accelerated in a nozzle to provide the thrust to propel the aircraft.

Aircraft are propelled by accelerating a fluid in the opposite direction to motion.

In jet engines, the high-temperature and high-pressure gases leaving the turbine are accelerated in a nozzle to provide thrust.



Thrust (propulsive force)

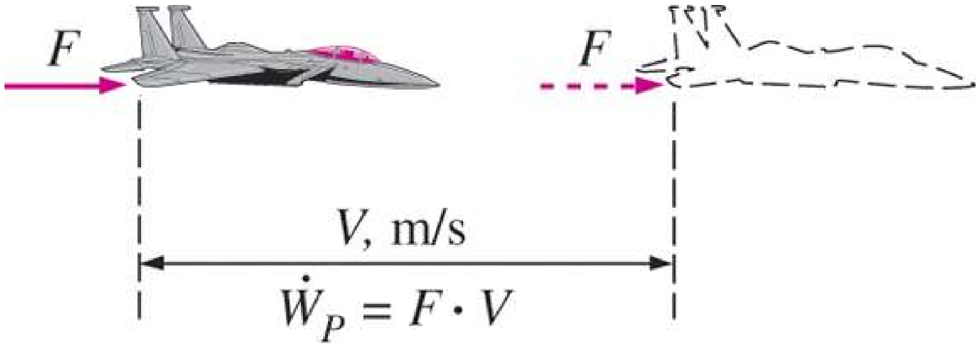
$$F = (\dot{m}V)_{exit} - (\dot{m}V)_{inlet} = \dot{m}(V_{exit} - V_{inlet}) \qquad \text{(N)}$$

Propulsive power

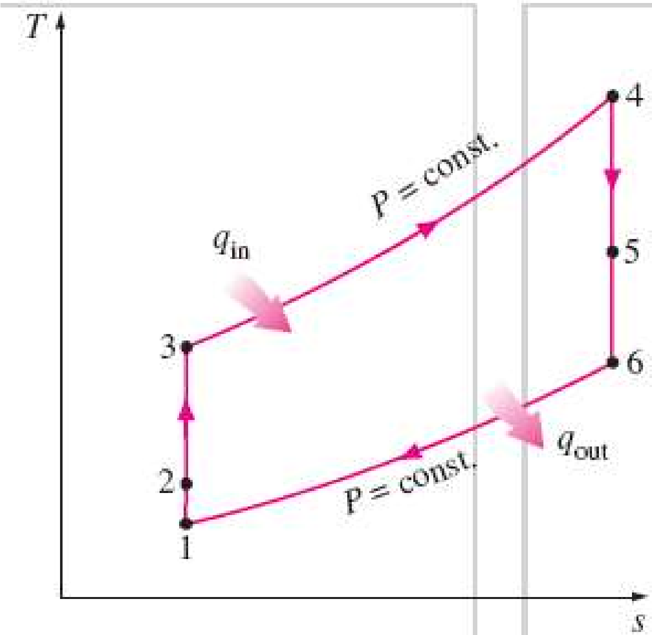
$$\dot{W}_P = (F)V_{aircraft} = \dot{m}(V_{exit} - V_{inlet})V_{aircraft} \qquad \text{(kW)}$$

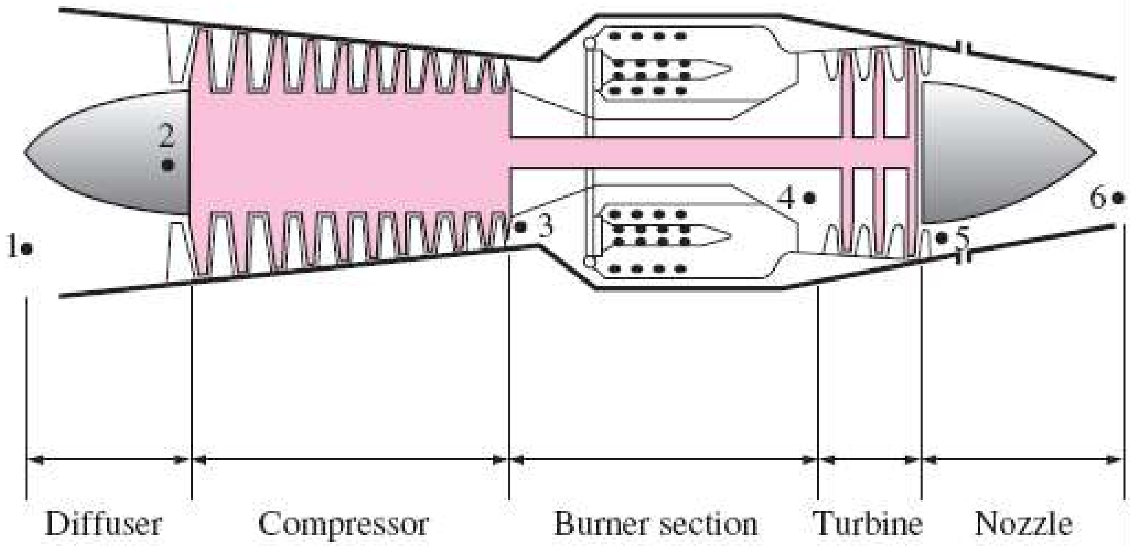
Propulsive efficiency

$$\eta_P = \frac{\text{Propulsive power}}{\text{Energy input rate}} = \frac{\dot{W}_P}{\dot{Q}_{in}}$$



Propulsive power is the thrust acting on the aircraft through a distance per unit time.





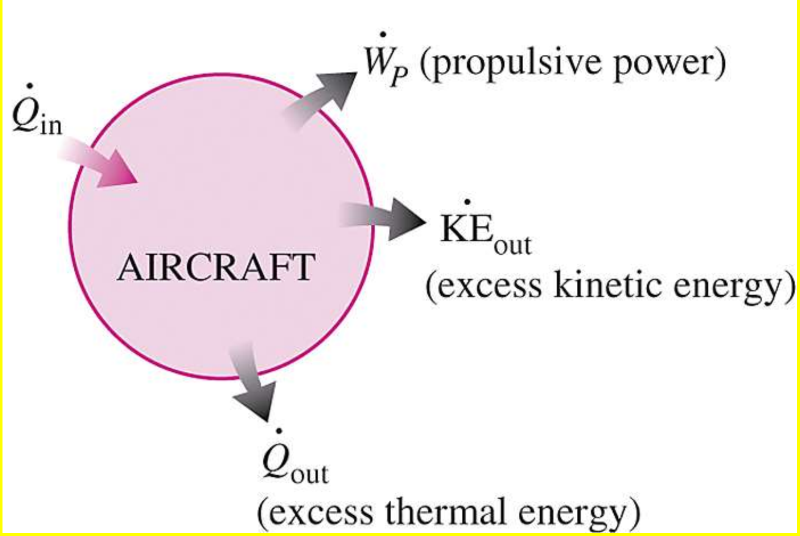
Basic components of a turbojet engine and the *T-s* diagram for the ideal turbojet cycle.

Modifications to Turbojet Engines

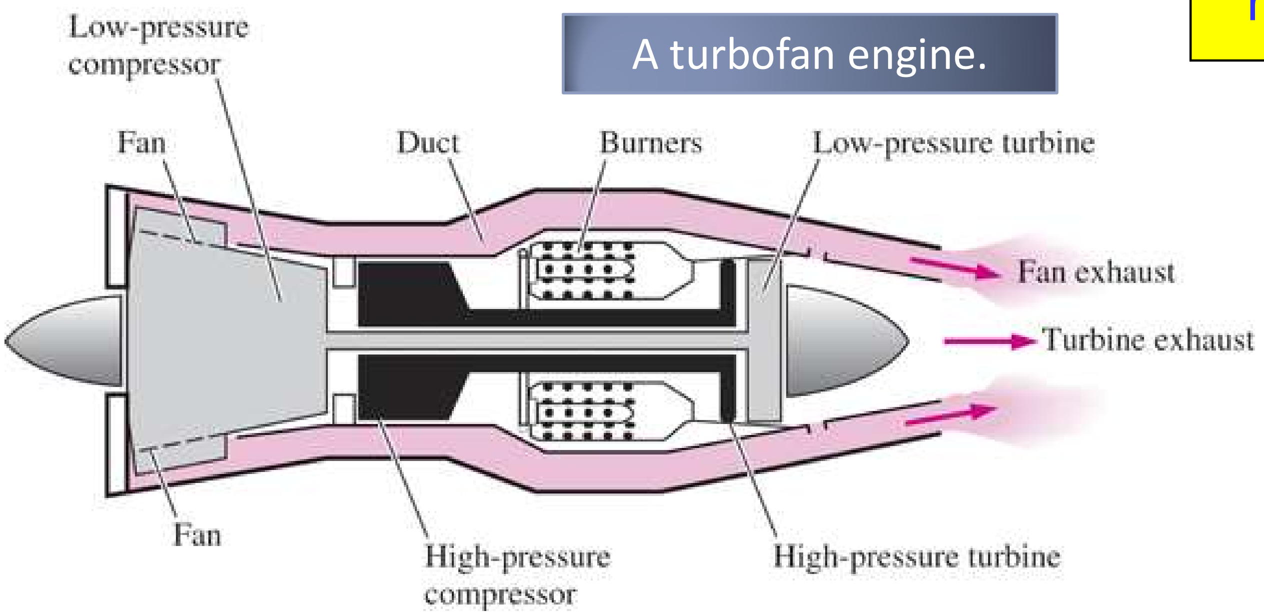
The first airplanes built were all propeller-driven, with propellers powered by engines essentially identical to automobile engines.

Both propeller-driven engines and jet-propulsion-driven engines have their own strengths and limitations, and several attempts have been made to combine the desirable characteristics of both in one engine.

Two such modifications are the ***propjet engine*** and the ***turbofan engine***.

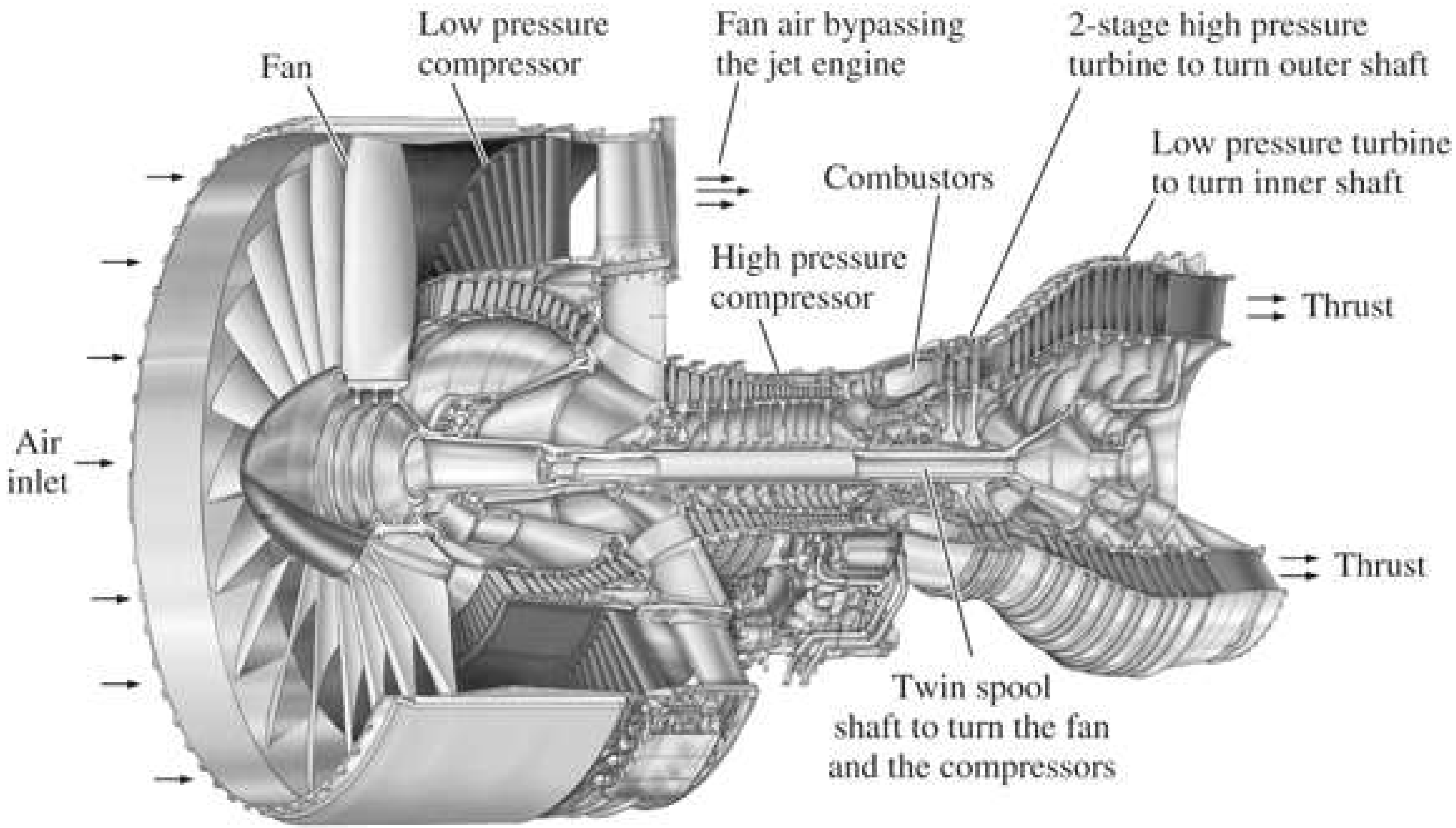


Energy supplied to an aircraft (from the burning of a fuel) manifests itself in various forms.



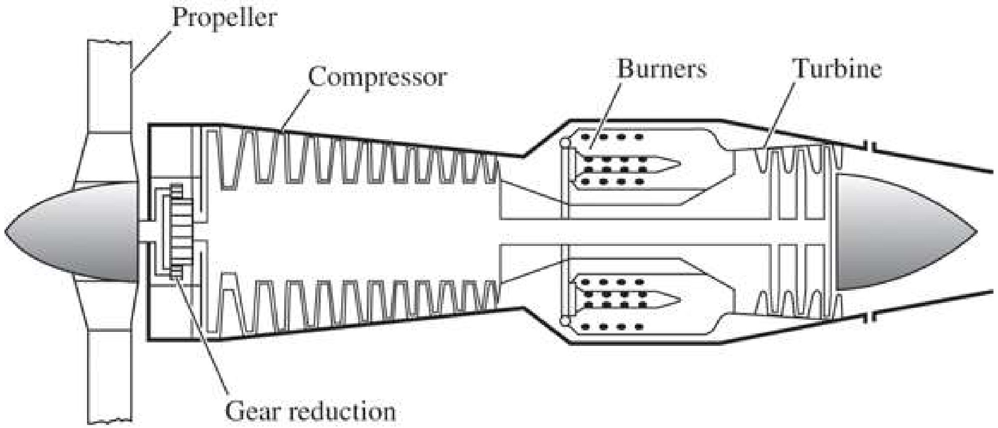
A turbofan engine.

The most widely used engine in aircraft propulsion is the **turbofan** (or *fanjet*) engine wherein a large fan driven by the turbine forces a considerable amount of air through a duct (cowl) surrounding the engine.

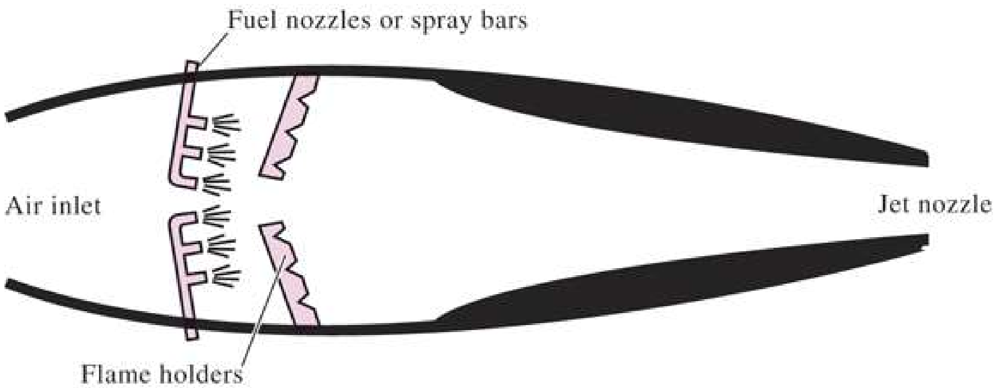


A modern jet engine used to power Boeing 777 aircraft. This is a Pratt & Whitney PW4084 turbofan capable of producing 374 kN of thrust. It is 4.87 m long, has a 2.84 m diameter fan, and it weighs 6800 kg.

Various engine types:
Turbofan, Propjet, Ramjet, Sacramjet, Rocket



A turboprop engine.



A ramjet engine.

Exergy destruction for a closed system

$$X_{\text{dest}} = T_0 S_{\text{gen}} = T_0(\Delta S_{\text{sys}} - S_{\text{in}} + S_{\text{out}})$$
$$= T_0 \left[(S_2 - S_1)_{\text{sys}} - \frac{Q_{\text{in}}}{T_{b,\,\text{in}}} + \frac{Q_{\text{out}}}{T_{b,\,\text{out}}} \right] \qquad (\text{kJ})$$

For a steady-flow system

$$\dot{X}_{\text{dest}} = T_0 \dot{S}_{\text{gen}} = T_0(\dot{S}_{\text{out}} - \dot{S}_{\text{in}}) = T_0 \left(\sum \dot{m}_e s_e - \sum \dot{m}_i s_i - \frac{\dot{Q}_{\text{in}}}{T_{b,\,\text{in}}} + \frac{\dot{Q}_{\text{out}}}{T_{b,\,\text{out}}} \right)$$

Steady-flow, one-inlet, one-exit

$$X_{\text{dest}} = T_0 s_{\text{gen}} = T_0 \left(s_e - s_i - \frac{q_{\text{in}}}{T_{b,\,\text{in}}} + \frac{q_{\text{out}}}{T_{b,\,\text{out}}} \right) \qquad (\text{kJ/kg})$$

Exergy destruction of a cycle

$$x_{dest} = T_0\left(\sum \frac{q_{out}}{T_{b,\,out}} - \sum \frac{q_{in}}{T_{b,\,in}}\right) \qquad \text{(kJ/kg)}$$

For a cycle with heat transfer only with a source and a sink

$$x_{dest} = T_0\left(\frac{q_{out}}{T_L} - \frac{q_{in}}{T_H}\right) \qquad \text{(kJ/kg)}$$

Closed system exergy

$$\phi = (u - u_0) - T_0(s - s_0) + P_0(v - v_0) + \frac{V^2}{2} + gz$$

Stream exergy

$$\psi = (h - h_0) - T_0(s - s_0) + \frac{V^2}{2} + gz \qquad \text{(kJ/kg)}$$

A second-law analysis of these cycles reveals where the largest irreversibilities occur and where to start improvements.

SUMMARY

- Basic considerations in the analysis of power cycles
- The Carnot cycle and its value in engineering
- Air-standard asssumptions
- An overview of reciprocating engines
- Otto cycle: The ideal cycle for spark-ignition engines
- Diesel cycle: The ideal cycle for compression-ignition engines
- Stirling and Ericsson cycles
- Brayton cycle: The ideal cycle for gas-turbine engines
- The Brayton cycle with regeneration
- The Brayton cycle with intercooling, reheating, and regeneration
- Ideal jet-propulsion cycles
- Second-law analysis of gas power cycles

The world's largest turbine, with an output of 340 MW that a new combined cycle power plant achieves a record-breaking efficiency of more than 60 % when it goes into operation in 2011.

After assembly at Siemens' gas turbine plant in Berlin, the world's largest gas turbine hits the road. The turbine arrives on a flat bed trailer at its destination

EXAMPLE 9–16

A gas-turbine power plant operates on the simple Brayton cycle between the pressure limits of 100 and 1200kPa. The working fluid is air, which enters the compressor at 30°C at a rate of 150 m3/min and leaves the turbine at 500°C. Using variable specific heats for air and assuming a compressor isentropic efficiency of 82 percent and a turbine isentropic efficiency of 88 percent, determine (*a) the net power* output, (*b) the back work ratio*, *and (c) the thermal efficiency*.

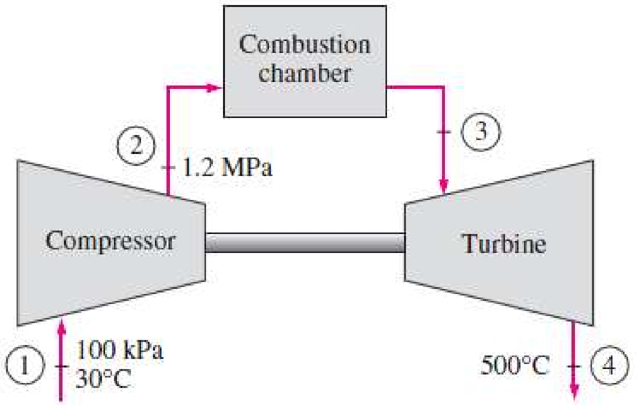


FIGURE P9–84

***Assumptions* 1** *The air-standard assumptions are applicable.* **2** *Kinetic and potential energy changes are* negligible. **3** Air is an ideal gas with variable specific heats.

Properties *The gas constant of air is R = 0.287 kJ/kg·K (Table A-1).*

Analysis (a) *For this problem, we use the* properties from EES software. Remember that for an ideal gas, enthalpy is a function of temperature only whereas entropy is a function of both temperature and pressure.

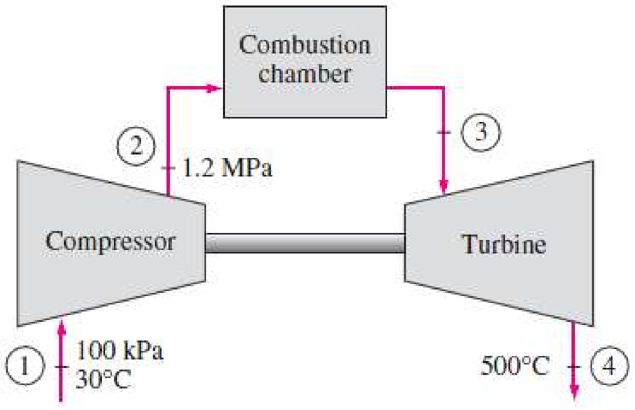


FIGURE P9–84

Process 1–2: compression.

$$T_1 = 30°\text{C} \longrightarrow h_1 = 303.60 \text{ kJ/kg}$$

$$\left.\begin{array}{l} T_1 = 30°\text{C} \\ P_1 = 100 \text{ kPa} \end{array}\right\} s_1 = 5.7159 \text{ kJ/kg} \cdot \text{K}$$

$$\left.\begin{array}{l} P_2 = 1200 \text{ kPa} \\ s_2 = s_1 = 5.7159 \text{ kJ/kg.K} \end{array}\right\} h_{2s} = 617.37 \text{ kJ/kg}$$

$$\eta_C = \frac{h_{2s} - h_1}{h_2 - h_1} \longrightarrow 0.82 = \frac{617.37 - 303.60}{h_2 - 303.60} \longrightarrow h_2 = 686.24 \text{ kJ/kg}$$

Process 3–4: expansion.

$$T_4 = 500°\text{C} \longrightarrow h_4 = 792.62 \text{ kJ/kg}$$

$$\eta_T = \frac{h_3 - h_4}{h_3 - h_{4s}} \longrightarrow 0.88 = \frac{h_3 - 792.62}{h_3 - h_{4s}}$$

We cannot find the enthalpy at state 3 directly. However, using the following lines in EES together with the isentropic efficiency relation, we find h_3 = 1404.7 kJ/kg, T_3 = 1034°C, s_3 = 6.5699 kJ/kg.K. The solution by hand would require a trial and error approach.

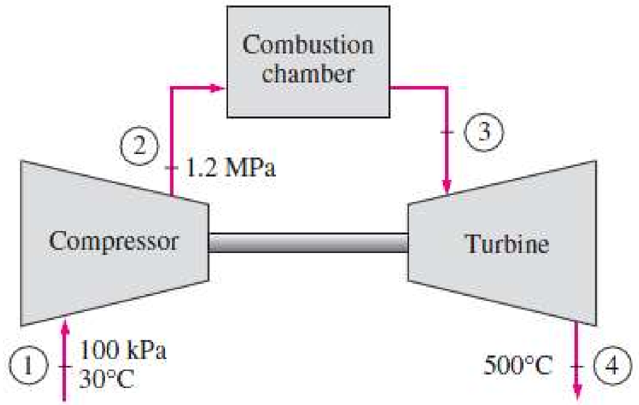


FIGURE P9–84

```
h_3=enthalpy(Air, T=T_3)
s_3=entropy(Air, T=T_3, P=P_2)
h_4s=enthalpy(Air, P=P_1, s=s_3)
```

The mass flow rate is determined from

$$\dot{m} = \frac{P_1 \dot{V}_1}{RT_1} = \frac{(100\,\text{kPa})(150/60\,\text{m}^3/\text{s})}{(0.287\,\text{kPa}\cdot\text{m}^3/\text{kg}\cdot\text{K})(30+273\,\text{K})} = 2.875\,\text{kg/s}$$

The net power output is

$$\dot{W}_{\text{C,in}} = \dot{m}(h_2 - h_1) = (2.875\,\text{kg/s})(686.24 - 303.60)\text{kJ/kg} = 1100\,\text{kW}$$

$$\dot{W}_{\text{T,out}} = \dot{m}(h_3 - h_4) = (2.875\,\text{kg/s})(1404.7 - 792.62)\text{kJ/kg} = 1759\,\text{kW}$$

$$\dot{W}_{\text{net}} = \dot{W}_{\text{T,out}} - \dot{W}_{\text{C,in}} = 1759 - 1100 = \mathbf{659\,kW}$$

(b) The back work ratio is

$$r_{bw} = \frac{\dot{W}_{C,in}}{\dot{W}_{T,out}} = \frac{1100\ \text{kW}}{1759\ \text{kW}} = \mathbf{0.625}$$

(c) The rate of heat input and the thermal efficiency are

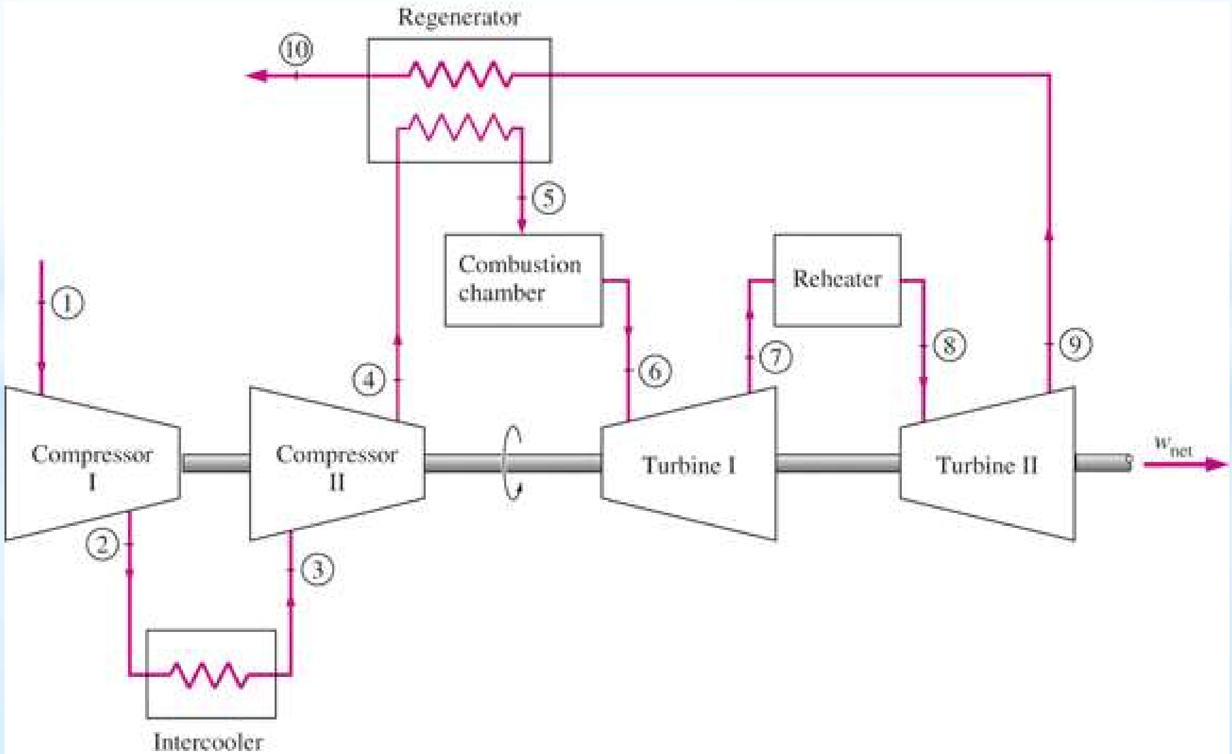
$$\dot{Q}_{in} = \dot{m}(h_3 - h_2) = (2.875\ \text{kg/s})(1404.7 - 686.24)\text{kJ/kg} = 2065\ \text{kW}$$

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{659\ \text{kW}}{2065\ \text{kW}} = \mathbf{0.319}$$

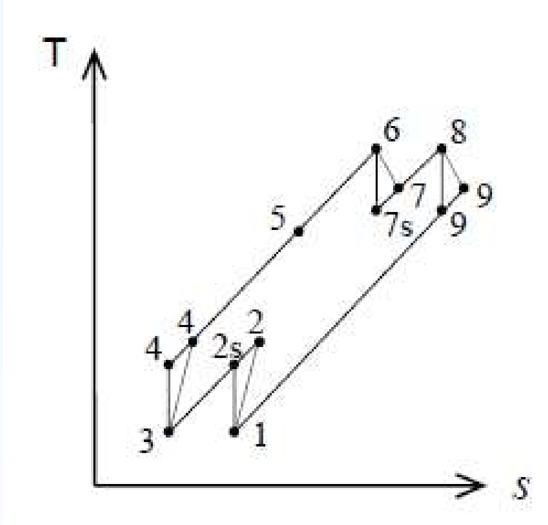
EXAMPLE 9-25

A gas-turbine engine with regeneration operates with two stages of compression and two stages of expansion. The pressure ratio across each stage of the compressor and turbine is 3.5. The air enters each stage of the compressor at 300 K and each stage of the turbine at 1200 K. The compressor and turbine efficiencies are 78 and 86 percent, respectively, and the effectiveness of the regenerator is 72 percent. Determine the back work ratio and the thermal efficiency of the cycle, assuming constant specific heats for air at room temperature.

***Assumptions* 1** *The air-standard assumptions are applicable.*
2 *Kinetic and potential energy changes are* negligible.
3 Air is an ideal gas with constant specific heats.



***Analysis** The work inputs of each stage of compressor are identical, so are the work outputs of each stage* of the turbine.



$$T_{4s} = T_{2s} = T_1\left(\frac{P_2}{P_1}\right)^{(k-1)/k} = (300\text{ K})(3.5)^{0.4/1.4} = 429.1\text{ K}$$

$$\eta_C = \frac{h_{2s}-h_1}{h_2-h_1} = \frac{c_p(T_{2s}-T_1)}{c_p(T_2-T_1)} \longrightarrow T_4 = T_2 = T_1 + (T_{2s}-T_1)/\eta_C$$
$$= 300 + (429.1-300)/(0.78)$$
$$= 465.5\text{ K}$$

$$T_{9s} = T_{7s} = T_6\left(\frac{P_7}{P_6}\right)^{(k-1)/k} = (1200\text{ K})\left(\frac{1}{3.5}\right)^{0.4/1.4} = 838.9\text{ K}$$

$$\eta_T = \frac{h_6-h_7}{h_6-h_{7s}} = \frac{c_p(T_6-T_7)}{c_p(T_6-T_{7s})} \longrightarrow T_9 = T_7 = T_6 - \eta_T(T_6-T_{7s})$$
$$= 1200 - (0.86)(1200-838.9)$$
$$= 889.5\text{ K}$$

$$\varepsilon = \frac{h_5 - h_4}{h_9 - h_4} = \frac{c_p(T_5 - T_4)}{c_p(T_9 - T_4)} \longrightarrow T_5 = T_4 + \varepsilon(T_9 - T_4)$$
$$= 465.5 + (0.72)(889.5 - 465.5)$$
$$= 770.8 \text{ K}$$

$$w_{\text{C,in}} = 2(h_2 - h_1) = 2c_p(T_2 - T_1) = 2(1.005 \text{ kJ/kg}\cdot\text{K})(465.5 - 300)\text{K} = 332.7 \text{ kJ/kg}$$

$$w_{\text{T,out}} = 2(h_6 - h_7) = 2c_p(T_6 - T_7) = 2(1.005 \text{ kJ/kg}\cdot\text{K})(1200 - 889.5)\text{K} = 624.1 \text{ kJ/kg}$$

Thus, $r_{\text{bw}} = \dfrac{w_{\text{C,in}}}{w_{\text{T,out}}} = \dfrac{332.7 \text{ kJ/kg}}{624.1 \text{ kJ/kg}} = \mathbf{53.3\%}$

$$q_{\text{in}} = (h_6 - h_5) + (h_8 - h_7) = c_p[(T_6 - T_5) + (T_8 - T_7)]$$
$$= (1.005 \text{ kJ/kg}\cdot\text{K})[(1200 - 770.8) + (1200 - 889.5)]\text{K} = 743.4 \text{ kJ/kg}$$

$$w_{\text{net}} = w_{\text{T,out}} - w_{\text{C,in}} = 624.1 - 332.7 = 291.4 \text{ kJ/kg}$$

$$\eta_{\text{th}} = \frac{w_{\text{net}}}{q_{\text{in}}} = \frac{291.4 \text{ kJ/kg}}{743.4 \text{ kJ/kg}} = \mathbf{39.2\%}$$