Examples

Chapter 6
MOMENTUM ANALYSIS OF FLOW SYSTEMS

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Newton's Laws and Conservation of Momentum
Sample Questions and Answers

EXAMPLE 6–1

Express Newton's first, second, and third laws.

Newton's first law states that "*a body at rest remains at rest, and a body in motion remains in motion at the same velocity in a straight path when the net force acting on it is zero.*" Therefore, a body tends to preserve its state or inertia. Newton's second law states that "*the acceleration of a body is proportional to the net force acting on it and is inversely proportional to its mass.*" Newton's third law states "*when a body exerts a force on a second body, the second body exerts an equal and opposite force on the first.*"

EXAMPLE 6–2

Is momentum a vector? If so, in what direction does it point?

Since momentum ($m\vec{V}$) is the product of a vector (velocity) and a scalar (mass), momentum must be a vector that points in the same direction as the velocity vector.

EXAMPLE 6–3

Express the conservation of momentum principle. What can you say about the momentum of a body if the net force acting on it is zero?

The conservation of momentum principle is expressed as “the momentum of a system remains constant when the net force acting on it is zero, and thus the momentum of such systems is conserved”. The momentum of a body remains constant if the net force acting on it is zero.

EXAMPLE 6–4

Express Newton’s second law of motion for rotating bodies. What can you say about the angular velocity and angular momentum of a rotating nonrigid body of constant mass if the net torque acting on it is zero?

Newton’s second law of motion, also called the angular momentum equation, is expressed as “*the rate of change of the angular momentum of a body is equal to the net torque acting it.*” For a non-rigid body with zero net torque, the angular momentum remains constant, but the angular velocity changes in accordance with $I\omega = constant$ where I is the moment of inertia of the body.

EXAMPLE 6–5

Consider two rigid bodies having the same mass and angular speed. Do you think these two bodies must have the same angular momentum? Explain.

No. Two rigid bodies having the same mass and angular speed will have different angular momentums unless they also have the same moment of inertia *I*.

Linear Momentum Equation
Sample Questions and Answers

EXAMPLE 6–6

Explain the importance of the Reynolds transport theorem in fluid mechanics, and describe how the linear momentum equation is obtained from it.

The relationship between the time rates of change of an extensive property for a system and for a control volume is expressed by the Reynolds transport theorem, which provides the link between the system and control volume concepts. The linear momentum equation is obtained by setting $b = \vec{V}$ and thus $B = m\vec{V}$ in the Reynolds transport theorem.

EXAMPLE 6–7

Describe body forces and surface forces, and explain how the net force acting on control volume is determined. Is fluid weight a body force or surface force? How about pressure?

The forces acting on the control volume consist of **body forces** that act throughout the entire body of the control volume (such as gravity, electric, and magnetic forces) and **surface forces** that act on the control surface (such as the pressure forces and reaction forces at points of contact). The net force acting on a control volume is the sum of all body and surface forces. Fluid weight is a body force, and pressure is a surface force (acting per unit area).

EXAMPLE 6–8

How do surface forces arise in the momentum analysis of a control volume? How can we minimize the number of surface forces exposed during analysis?

All of these surface forces arise as the control volume is isolated from its surroundings for analysis, and the effect of any detached object is accounted for by a force at that location. We can minimize the number of surface forces exposed by choosing the control volume such that the forces that we are not interested in remain internal, and thus they do not complicate the analysis. A well-chosen control volume exposes only the forces that are to be determined (such as reaction forces) and a minimum number of other forces.

EXAMPLE 6–9

What is the importance of the momentum-flux correction factor in the momentum analysis of slow systems? For which type of flow is it significant and must it be considered in analysis: laminar flow, turbulent flow, or jet flow?

The momentum-flux correction factor β enables us to express the momentum flux in terms of the mass flow rate and mean flow velocity as

$\int_{A_c} \rho\vec{V}\left(\vec{V}\cdot\vec{n}\right)dA_c = \beta\dot{m}\vec{V}_{avg}$. The value of β is unity for uniform flow, such as a jet flow, nearly unity for turbulent flow (between 1.01 and 1.04), but about 1.3 for laminar flow. So it should be considered in laminar flow.

EXAMPLE 6–10

Write the momentum equation for steady one-dimensional flow for the case of no external forces and explain the physical significance of its terms.

The momentum equation for steady one-dimensional flow for the case of no external forces is

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V}$$

where the left hand side is the net force acting on the control volume, and first term on the right hand side is the incoming momentum flux and the second term is the outgoing momentum flux by mass.

EXAMPLE 6–11

In the application of the momentum equation, explain why we can usually disregard the atmospheric pressure and work with gage pressures only.

In the application of the momentum equation, we can disregard the atmospheric pressure and work with gage pressures only since the atmospheric pressure acts in all directions, and its effect cancels out in every direction.

EXAMPLE 6–12

Two firefighters are fighting a fire with identical water hoses and nozzles, except that one is holding the hose straight so that the water leaves the nozzle in the same direction it comes, while the other holds it backward so that the water makes a U-turn before being discharged. Which firefighter will experience a greater reaction force?

The fireman who holds the hose backwards so that the water makes a U-turn before being discharged will experience a greater reaction force since the numerical values of momentum fluxes across the nozzle are added in this case instead of being subtracted.

EXAMPLE 6–13

A rocket in space (no friction or resistance to motion) can expel gases relative to itself at some high velocity V. Is V the upper limit to the rocket's ultimate velocity?

No, V is not the upper limit to the rocket's ultimate velocity. Without friction the rocket velocity will continue to increase as more gas outlets the nozzle.

EXAMPLE 6–14

A rocket in space (no friction or resistance to motion) can expel gases relative to itself at some high velocity V. Is V the upper limit to the rocket's ultimate velocity?

No, V is not the upper limit to the rocket's ultimate velocity. Without friction the rocket velocity will continue to increase as more gas outlets the nozzle.

EXAMPLE 6–14

Describe in terms of momentum and airflow why a helicopter hovers.

A helicopter hovers because the strong downdraft of air, caused by the overhead propeller blades, manifests a momentum in the air stream. This momentum must be countered by the helicopter lift force.

EXAMPLE 6–15

Does it take more, equal, or less power for a helicopter to hover at the top of a high mountain than it does at sea level? Explain.

As the air density decreases, it requires more energy for a helicopter to hover, because more air must be forced into the downdraft by the helicopter blades to provide the same lift force. Therefore, it takes more power for a helicopter to hover on the top of a high mountain than it does at sea level.

EXAMPLE 6–16

In a given location, would a helicopter require more energy in summer or winter to achieve a specified performance? Explain.

In winter the air is generally colder, and thus denser. Therefore, less air must be driven by the blades to provide the same helicopter lift, requiring less power.

EXAMPLE 6–17

A horizontal water jet from a nozzle of constant exit cross section impinges normally on a stationary vertical flat plate. A certain force F is required to hold the plate against the water stream. If the water velocity is doubled, will the necessary holding force also be doubled? Explain.

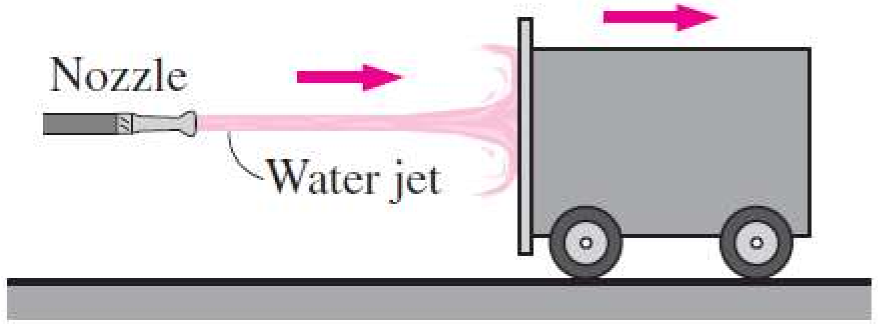
The force required to hold the plate against the horizontal water stream will increase by a factor of 4 when the velocity is doubled since

$$F = \dot{m}V = (\rho A V)V = \rho A V^2$$

and thus the force is proportional to the square of the velocity.

EXAMPLE 6–18

constant-velocity horizontal water jet from a stationary nozzle impinges normally on a vertical flat plate that is held in a nearly frictionless track. As the water jet hits the plate, it begins to move due to the water force. Will the acceleration of the plate remain constant or change? Explain.



The acceleration will not be constant since the force is not constant. The impulse force exerted by water on the plate is $F = \dot{m}V = (\rho AV)V = \rho AV^2$, where V is the relative velocity between the water and the plate, which is moving. The plate acceleration will be **a = F/m**. But as the plate begins to move, V decreases, so the acceleration must also decrease.

EXAMPLE 6–19

A horizontal water jet of constant velocity V from a stationary nozzle impinges normally on a vertical flat plate that is held in a nearly frictionless track. As the water jet hits the plate, it begins to move due to the water force. What is the highest velocity the plate can attain? Explain.

The maximum velocity possible for the plate is the velocity of the water jet. As long as the plate is moving slower than the jet, the water will exert a force on the plate, which will cause it to accelerate, until terminal jet velocity is reached.

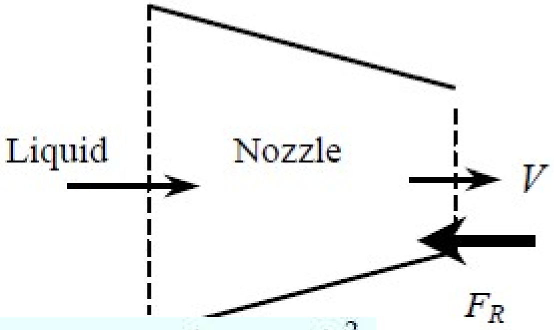
EXAMPLE 6–20

Show that the force exerted by a liquid jet on a stationary nozzle as it leaves with a velocity *V* is proportional to V^2 or, alternatively, to $\dot{m}^2$.

Assumptions **1** The flow is steady and incompressible. **2** The nozzle is given to be stationary. **3** The nozzle involves a 90° turn and thus the incoming and outgoing flow streams are normal to each other. **4** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero.

Analysis We take the nozzle as the control volume, and the flow direction at the outlet as the *x* axis. Note that the nozzle makes a 90° turn, and thus it does not contribute to any pressure force or momentum flux term at the inlet in the *x* direction. Noting that $\dot{m} = \rho AV$ where A is the nozzle outlet area and *V* is the average nozzle outlet velocity, the momentum equation for steady one-dimensional flow in the *x* direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = \beta \dot{m}_{out} V_{out} = \beta \dot{m} V$$



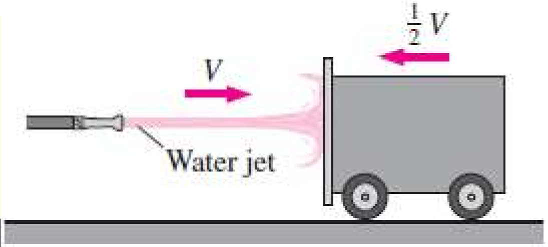
where F_{Rx} is the reaction force on the nozzle due to liquid jet at the nozzle outlet. Then,

$$\dot{m} = \rho AV \quad \rightarrow \quad F_{Rx} = \beta \dot{m} V = \beta \rho AVV = \beta \rho AV^2 \quad \text{or} \quad F_{Rx} = \beta \dot{m} V = \beta \dot{m} \frac{\dot{m}}{\rho A} = \beta \frac{\dot{m}^2}{\rho A}$$

Therefore, the force exerted by a liquid jet of velocity V on this stationary nozzle is proportional to V^2, or alternatively, to $\dot{m}^2$.

EXAMPLE 6–21

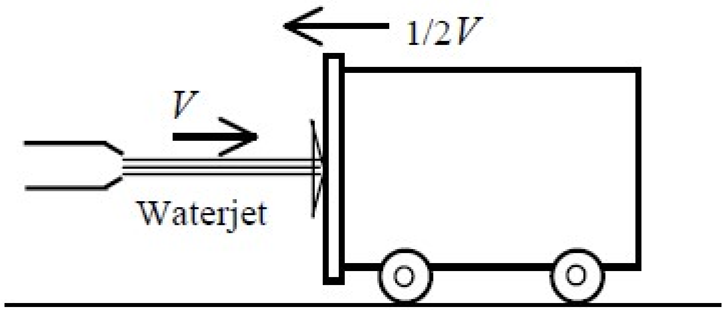
A horizontal water jet of constant velocity *V* impinges normally on a vertical flat plate and splashes off the sides in the vertical plane. The plate is moving toward the oncoming water jet with velocity *½V* If a force *F* is required to maintain the plate stationary, how much force is required to move the plate toward the water jet?



Assumptions **1** The flow is steady and incompressible. **2** The plate is vertical and the jet is normal to plate. **3** The pressure on both sides of the plate is atmospheric pressure (and thus its effect cancels out). **4** Fiction during motion is negligible. **5** There is no acceleration of the plate. **6** The water splashes off the sides of the plate in a plane normal to the jet. **7** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the plate as the control volume. The relative velocity between the plate and the jet is V when the plate is stationary, and *1.5V* when the plate is moving with a velocity *½V* towards the plate. Then the momentum equation for steady one-dimensional flow in the horizontal direction reduces to

$$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V} \quad \rightarrow \quad -F_R = -\dot{m}_i V_i \quad \rightarrow \quad F_R = \dot{m}_i V_i$$



Stationary plate: $(V_i = V$ and $\dot{m}_i = \rho A V_i = \rho A V) \rightarrow F_R = \rho A V^2 = F$

Moving plate: $(V_i = 1.5V$ and $\dot{m}_i = \rho A V_i = \rho A(1.5V)) \rightarrow F_R = \rho A(1.5V)^2 = 2.25\rho A V^2 = 2.25F$

Therefore, the force required to hold the plate stationary against the oncoming water jet becomes 2.25 times when the jet velocity becomes 1.5 times.

Discussion Note that when the plate is stationary, V is also the jet velocity. But if the plate moves toward the stream with velocity ½V, then the relative velocity is 1.5V, and the amount of mass striking the plate (and falling off its sides) per unit time also increases by 50%.

EXAMPLE 6–22

A 90° elbow is used to direct water flow at a rate of *25 kg/s* in a horizontal pipe upward. The diameter of the entire elbow is 10 cm. The elbow discharges water into the atmosphere, and thus the pressure at the exit is the local atmospheric pressure. The elevation difference between the centers of the exit and the inlet of the elbow is 35 cm. The weight of the elbow and the water in it is considered to be

Assumptions **1** The flow is steady, frictionless, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis (a) We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$ 30kg/s Noting that $\dot{m} = \rho A V$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\ \text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \;\rightarrow\; P_1-P_2=\rho g(z_2-z_1) \;\rightarrow\; P_{1,\,gage}=\rho g(z_2-z_1)$$

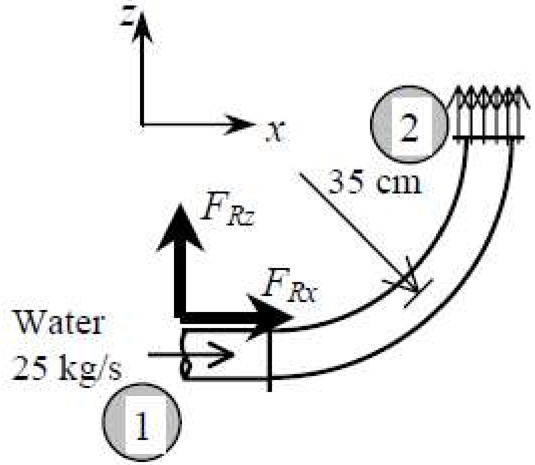
Substituting,

$$P_{1,\,gage}=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.35\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=3.434\ \text{kN/m}^2=\mathbf{3.434\ kPa}$$

(b) The momentum equation for steady one-dimensional flow $\sum \vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the x and y axes become

$$F_{Rx}+P_{1,\,gage}A_1=0-\beta\dot{m}(+V_1)=-\beta\dot{m}V$$
$$F_{Rz}=\beta\dot{m}(+V_2)=\beta\dot{m}V$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,



$$F_{Rx} = -\beta \dot{m} V - P_{1,\,gage} A_1$$

$$= -1.03(25\ \mathrm{kg/s})(3.18\ \mathrm{m/s})\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg \cdot m/s^2}}\right) - (3434\ \mathrm{N/m^2})[\pi(0.1\ \mathrm{m})^2/4]$$

$$= -109\ \mathrm{N}$$

$$F_{Ry} = \beta \dot{m} V = 1.03(25\ \mathrm{kg/s})(3.18\ \mathrm{m/s})\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg \cdot m/s^2}}\right) = 81.9\ \mathrm{N}$$

and $F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{(-109)^2 + 81.9^2} = \mathbf{136\ N},\quad \theta = \tan^{-1}\frac{F_{Ry}}{F_{Rx}} = \tan^{-1}\frac{81.9}{-109} = -37° = \mathbf{143°}$

Discussion Note that the magnitude of the anchoring force is 136 N, and its line of action makes 143° from the positive *x* direction. Also, a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

EXAMPLE 6–23

Repeat Prob. 6–22 for the case of another (identical) elbow being attached to the existing elbow so that the fluid makes a U-turn.

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is negligible. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.
Properties We take the density of water to be $1000\ kg/m^3$.
Analysis (a) We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho A V$, the mean inlet and outlet velocities of water are

$$V_1 = V_2 = V = \frac{\dot{m}}{\rho A} = \frac{\dot{m}}{\rho(\pi D^2/4)} = \frac{25\ \text{kg/s}}{(1000\ \text{kg/m}^3)[\pi(0.1\,\text{m})^2/4]} = 3.18\ \text{m/s}$$

Noting that $V_1 = V_2$ and $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \rightarrow P_1-P_2=\rho g(z_2-z_1) \rightarrow P_{1,\text{gage}}=\rho g(z_2-z_1)$$

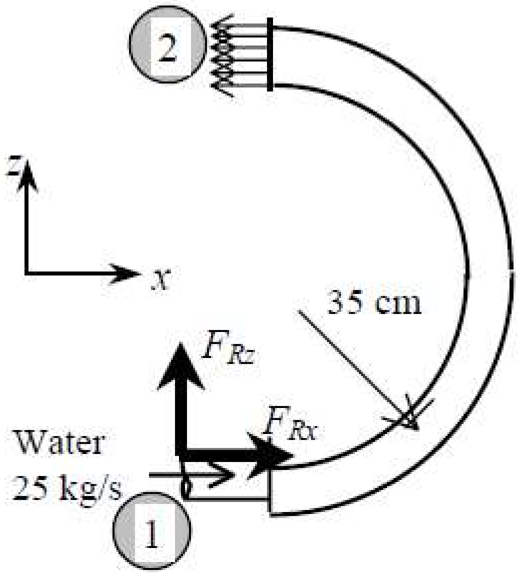
Substituting,

$$P_{1,\text{gage}}=(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.70\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)=6.867\ \text{kN/m}^2=\mathbf{6.867\ kPa}$$

(b) The momentum equation for steady one-dimensional flow is $\sum \vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the x- and z- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx}+P_{1,\text{gage}}A_1=\beta\dot{m}(-V_2)-\beta\dot{m}(+V_1)=-2\beta\dot{m}V$$
$$F_{Rz}=0$$

Solving for F_{Rx} and substituting the given values,



$$F_{Rx} = -2\beta\dot{m}V - P_{1,\,\text{gage}}A_1$$

$$= -2\times 1.03(25\text{ kg/s})(3.18\text{ m/s})\left(\frac{1\text{ N}}{1\text{ kg}\cdot\text{m/s}^2}\right) - (6867\text{ N/m}^2)[\pi(0.1\text{ m})^2/4]$$

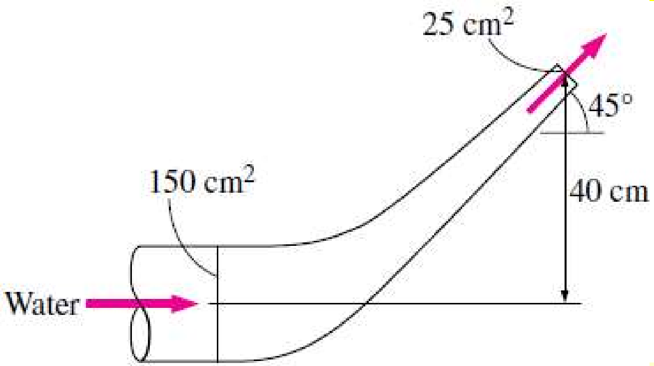
$$= -218\text{ N}$$

and $F_R = F_{Rx}$ = **- 218 N** since the y-component of the anchoring force is zero. Therefore, the anchoring force has a magnitude of 218 N and it acts in the negative *x* direction.

Discussion Note that a negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed.

EXAMPLE 6–24

A reducing elbow is used to deflect water flow at a rate of 30 kg/s in a horizontal pipe upward by an angle θ = 45° from the flow direction while accelerating it. The elbow discharges water into the atmosphere. The cross-sectional area of the elbow is *150 cm²* at the inlet and *25 cm²* at the exit. The elevation difference between the centers of the exit and the inlet is *40 cm*. The mass of the elbow and the water in it is *50 kg*. Determine the anchoring force needed to hold the elbow in place. Take the momentum-flux correction factor to be 1.03.



Assumptions 1 The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\text{ kg})(9.81\text{ m/s}^2) = 490.5\text{ N} = 0.4905\text{ kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0150\text{ m}^2)} = 2.0\text{ m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\text{ kg/s}}{(1000\text{ kg/m}^3)(0.0025\text{ m}^2)} = 12\text{ m/s}$$

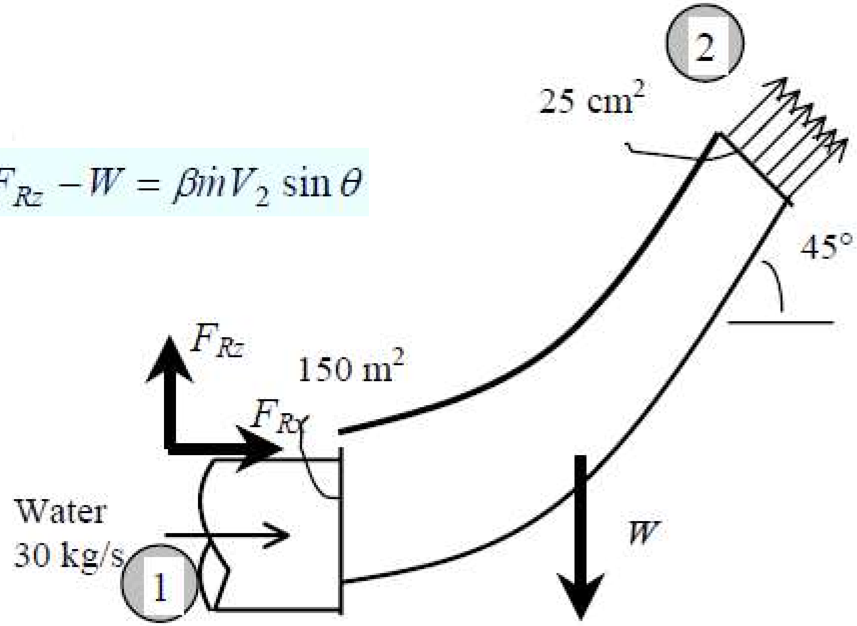
Taking the center of the inlet cross section as the reference level $(z_1 = 0)$ and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2 \rightarrow P_1 - P_2 = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2 - z_1\right) \rightarrow P_{1,\text{gage}} = \rho g\left(\frac{V_2^2 - V_1^2}{2g} + z_2\right)$$

Substituting,

$$P_{1,\text{gage}} = (1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2 - (2\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) = 73.9\text{ kN/m}^2 = 73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become



$$F_{Rx} + P_{1,\text{gage}} A_1 = \beta \dot{m} V_2 \cos\theta - \beta \dot{m} V_1 \quad \text{and} \quad F_{Rz} - W = \beta \dot{m} V_2 \sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given

$$\begin{aligned} F_{Rx} &= \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\text{gage}} A_1 \\ &= 1.03(30\text{ kg/s})[(12\cos 45° - 2)\text{ m/s}]\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) \\ &\quad -(73.9\text{ kN/m}^2)(0.0150\text{ m}^2) \\ &= -0.908\text{ kN} \end{aligned}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta + W = 1.03(30\text{ kg/s})(12\sin 45°\text{ m/s})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right) + 0.4905\text{ kN} = 0.753\text{ kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-0.908)^2 + (0.753)^2} = \mathbf{1.18\ kN}, \qquad \theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.753}{-0.908} = -\mathbf{39.7}°$$

Discussion Note that the magnitude of the anchoring force is *1.18 kN*, and its line of action makes *–39.7°* from +*x* direction. Negative value for F_{Rx} indicates the assumed direction is wrong.

EXAMPLE 6–25

Repeat Prob. 6–25 for the case of u 110°.

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is considered. **3** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. 4 The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis The weight of the elbow and the water in it is

$$W = mg = (50\ \text{kg})(9.81\ \text{m/s}^2) = 490.5\ \text{N} = 0.4905\ \text{kN}$$

We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by *x* (with the direction of flow as being the positive direction) and the vertical coordinate by *z*. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 30$ kg/s. Noting that $\dot{m} = \rho AV$, the inlet and outlet velocities of water are

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0150\ \text{m}^2)} = 2.0\ \text{m/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{30\ \text{kg/s}}{(1000\ \text{kg/m}^3)(0.0025\ \text{m}^2)} = 12\ \text{m/s}$$

Taking the center of the inlet cross section as the reference level *(z_1 = 0)* and noting that $P_2 = P_{atm}$, the Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \;\rightarrow\; P_1-P_2=\rho g\left(\frac{V_2^2-V_1^2}{2g}+z_2-z_1\right) \;\rightarrow\; P_{1,\text{gage}}=\rho g\left(\frac{V_2^2-V_1^2}{2g}+z_2\right)$$

$$\text{or, } P_{1,\text{gage}}=(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(12\text{ m/s})^2-(2\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+0.4\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)=73.9\text{ kN/m}^2=73.9\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum\vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the *x*- and *y*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. We also use gage pressures to avoid dealing with the atmospheric pressure which acts on all surfaces. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx}+P_{1,\text{gage}}A_1=\beta\dot{m}V_2\cos\theta-\beta\dot{m}V_1 \quad\text{and}\quad F_{Ry}-W=\beta\dot{m}V_2\sin\theta$$

Solving for F_{Rx} and F_{Rz}, and substituting the given values,

$$F_{Rx} = \beta \dot{m}(V_2 \cos\theta - V_1) - P_{1,\,\text{gage}} A_1$$

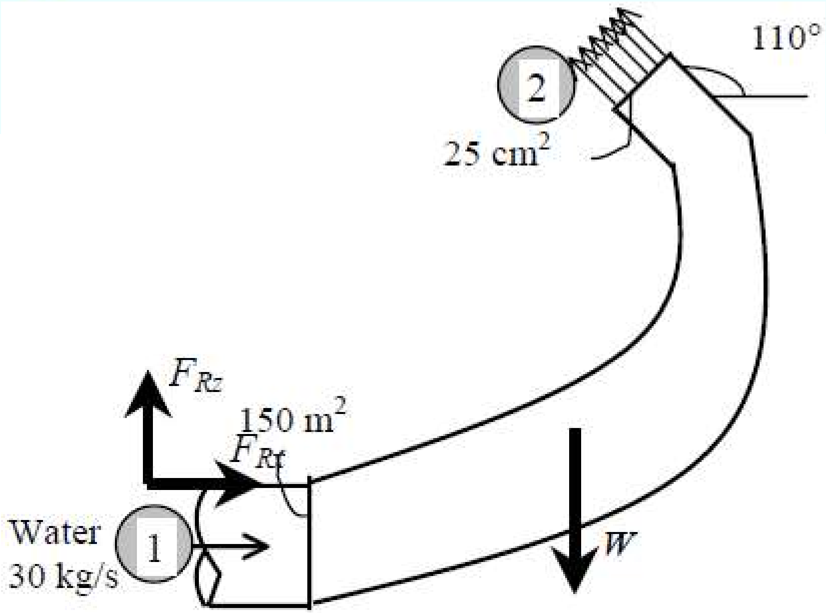
$$= 1.03(30\ \text{kg/s})[(12\cos 110° - 2)\ \text{m/s}]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - (73.9\ \text{kN/m}^2)(0.0150\ \text{m}^2) = -1.297\ \text{kN}$$

$$F_{Rz} = \beta \dot{m} V_2 \sin\theta + W = 1.03(30\ \text{kg/s})(12\sin 110°\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + 0.4905\ \text{kN} = 0.8389\ \text{kN}$$

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-1.297)^2 + 0.8389^2} = \mathbf{1.54\ kN}$$

and

$$\theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{0.8389}{-1.297} = \mathbf{-32.9}°$$

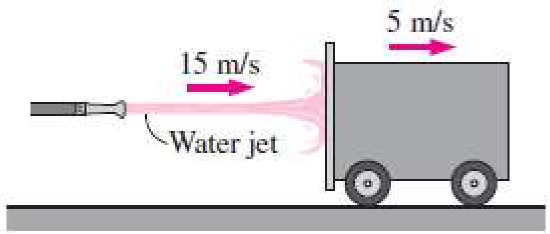


Discussion Note that the magnitude of the anchoring force is 1.54 kN, and its line of action makes –32.9° from +*x* direction. Negative value for F_{Rx} indicates assumed direction is wrong, and should be reversed.

EXAMPLE 6–26

Water accelerated by a nozzle to 15 m/s strikes the vertical back surface of a cart moving horizontally at a constant velocity of 5 m/s in the flow direction. The mass flow rate of water is 25 kg/s. After the strike, the water stream splatters off in all directions in the plane of the back surface. (a) Determine the force that needs to be applied on the brakes of the cart to prevent it from accelerating. (b) If this force were used to generate power instead of wasting it on the brakes, determine the maximum amount of power that can be generated.

Answers: (a) 250 N, (b) 1.25 kW

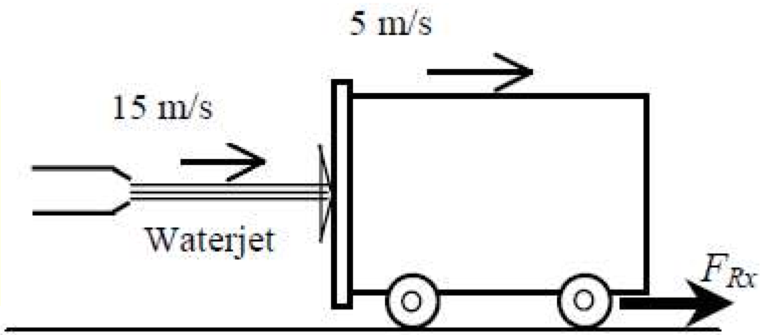


Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in all directions in the plane of the back surface. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** Fiction during motion is negligible. **5** There is no acceleration of the cart. **7** The motions of the water jet and the cart are horizontal. **6** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Analysis We take the cart as the control volume, and the direction of flow as the positive direction of x axis. The relative velocity between the cart and the jet is

$$V_r = V_{\text{jet}} - V_{\text{cart}} = 15 - 10 = 10 \text{ m/s}$$

Therefore, we can assume the cart to be stationary and the jet to move with a velocity of *10 m/s*. The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to



$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{\text{brake}} = -\dot{m} V_r$$

We note that the brake force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

$$F_{\text{brake}} = -\dot{m} V_r = -(25 \text{ kg/s})(+10 \text{ m/s})\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-250 \text{ N}}$$

The negative sign indicates that the braking force acts in the opposite direction to motion, as expected. Noting that work is force times distance and the distance traveled by the cart per unit time is the cart velocity, the power wasted by the brakes is

$$\dot{W} = F_{\text{brake}} V_{\text{cart}} = (250 \text{ N})(5 \text{ m/s})\left(\frac{1 \text{ kW}}{1000 \text{ N}\cdot\text{m/s}}\right) = \mathbf{1.25 \text{ kW}}$$

Discussion Note that the power wasted is equivalent to the maximum power that can be generated as the cart velocity is maintained constant.

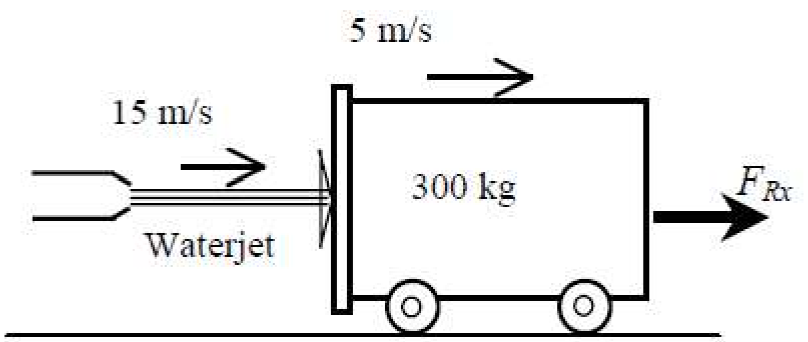
EXAMPLE 6–27

Reconsider Prob. 6–27. If the mass of the cart is 300 kg and the brakes fail, determine the acceleration of the cart when the water first strikes it. Assume the mass of water that wets the back surface is negligible.

Analysis The braking force was determined in previous problem to be 250 N. When the brakes fail, this force will propel the cart forward, and the accelerating will be

$$a = \frac{F}{m_{\text{cart}}} = \frac{250\ \text{N}}{300\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.833\ m/s^2}$$

Discussion This is the acceleration at the moment the brakes fail. The acceleration will decrease as the relative velocity between the water jet and the cart (and thus the force) decreases.



EXAMPLE 6–28

A horizontal 5-cm-diameter water jet with a velocity of 18 m/s impinges normally upon a vertical plate of mass 1000 kg. The plate is held in a nearly frictionless track and is initially stationary. When the jet strikes the plate, the plate begins to move in the direction of the jet. The water always splatters in the plane of the retreating plate. Determine (a) the acceleration of the plate when the jet first strikes it (time = 0), (b) the time it will take for the plate to reach a velocity of 9 m/s, and (c) the plate velocity 20 s after the jet first strikes the plate. Assume the velocity of the jet relative to the plate remains constant.

Assumptions **1** The flow is steady and incompressible. **2** The water always splatters in the plane of the retreating plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on all surfaces. **4** The tract is nearly frictionless, and thus fiction during motion is negligible. **5** The motions of the water jet and the cart are horizontal. **6** The velocity of the jet relative to the plate remains constant, $V_r = V_{jet} = V$. **7** Jet flow is nearly uniform and thus the effect of the momentum-flux correction factor is negligible, $\beta \cong 1$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis (a) We take the vertical plate on the frictionless track as the control volume, and the direction of flow as the positive direction of x axis. The mass flow rate of water in the jet is

$$\dot{m} = \rho VA = (1000\ \mathrm{kg/m^3})(18\ \mathrm{m/s})[\pi(0.05\ \mathrm{m})^2/4] = 35.34\ \mathrm{kg/s}$$

The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{Rx} = -\dot{m} V$$

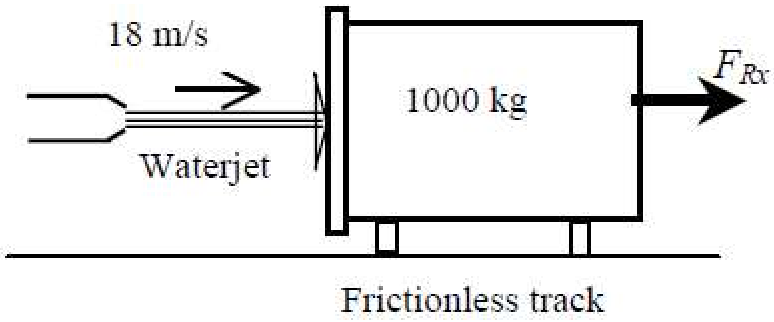
where F_{Rx} is the reaction force required to hold the plate in place. When the plate is released, an equal and opposite impulse force acts on the plate, which is determined to

$$F_{\text{plate}} = -F_{Rx} = \dot{m} V = (35.34 \text{ kg/s})(18 \text{ m/s}) \left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2} \right) = 636 \text{ N}$$

Then the initial acceleration of the plate becomes

$$a = \frac{F_{\text{plate}}}{m_{\text{plate}}} = \frac{636 \text{ N}}{1000 \text{ kg}} \left(\frac{1 \text{ kg} \cdot \text{m/s}^2}{1 \text{ N}} \right) = \mathbf{0.636 \text{ m/s}^2}$$

This acceleration will remain constant during motion since the force acting on the plate remains constant.



(b) Noting that *a = dV/dt = ΔV/Δt* since the acceleration a is constant, the time it takes for the plate to reach a velocity of 9 m/s is

$$\Delta t = \frac{\Delta V_{\text{plate}}}{a} = \frac{(9-0)\ \text{m/s}}{0.636\ \text{m/s}^2} = \mathbf{14.2\ s}$$

(c) Noting that *a = dV/dt* and thus *dV = adt* and that the acceleration a is constant, the plate velocity in 20 s becomes

$$V_{\text{plate}} = V_{0,\ \text{plate}} + a\Delta t = 0 + (0.636\ \text{m/s}^2)(20\ \text{s}) = \mathbf{12.7\ m/s}$$

Discussion The assumption that the relative velocity between the water jet and the plate remains constant is valid only for the initial moments of motion when the plate velocity is low unless the water jet is moving with the plate at the same velocity as the plate.

EXAMPLE 6–29

Water flowing in a horizontal 30-cm-diameter pipe at 5 m/s and 300 kPa gage enters a 90° bend reducing section, which connects to a 15-cm-diameter vertical pipe. The inlet of the bend is 50 cm above the exit. Neglecting any frictional and gravitational effects, determine the net resultant force exerted on the reducer by the water. Take the momentum-flux correction factor to be 1.04.

Assumptions **1** The flow is steady, frictionless, one-dimensional, incompressible, and irrotational (so that the Bernoulli equation is applicable). **2** The weight of the elbow and the water in it is disregarded since the gravitational effects are negligible. **3** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.04$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis We take the elbow as the control volume, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction) and the vertical coordinate by z. The continuity equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m} = 353.4$ kg/s. Noting that $\dot{m} = \rho A V$, the mass flow rate of water and its outlet velocity are

$$\dot{m} = \rho V_1 A_1 = \rho V_1 (\pi D_1^2 / 4) = (1000\ \mathrm{kg/m^3})(5\ \mathrm{m/s})[\pi (0.3\ \mathrm{m})^2 / 4] = 353.4\ \mathrm{kg/s}$$

$$V_2 = \frac{\dot{m}}{\rho A_2} = \frac{\dot{m}}{\rho \pi D_2^2 / 4} = \frac{353.4\ \mathrm{kg/s}}{(1000\ \mathrm{kg/m^3})[\pi (0.15\ \mathrm{m})^2 / 4]} = 20\ \mathrm{m/s}$$

The Bernoulli equation for a streamline going through the center of the reducing elbow is expressed as

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad \rightarrow \quad P_2=P_1+\rho g\left(\frac{V_1^2-V_2^2}{2g}+z_1-z_2\right)$$

Substituting, the gage pressure at the outlet becomes

$$P_2=(300\text{ kPa})+(1000\text{ kg/m}^3)(9.81\text{ m/s}^2)\left(\frac{(5\text{ m/s})^2-(20\text{ m/s})^2}{2(9.81\text{ m/s}^2)}+0.5\right)\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right)=117.4\text{ kPa}$$

The momentum equation for steady one-dimensional flow is $\sum\vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. We let the *x*- and *z*- components of the anchoring force of the elbow be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Then the momentum equations along the *x* and *z* axes become

$$F_{Rx}+P_{1,\text{gage}}A_1=0-\beta\dot{m}V_1$$
$$F_{Rz}-P_{2,\text{gage}}A_2=\beta\dot{m}(-V_2)-0$$

Note that we should not forget the negative sign for forces and velocities in the negative x or z direction. Solving for FRx and FRz, and substituting the given values,

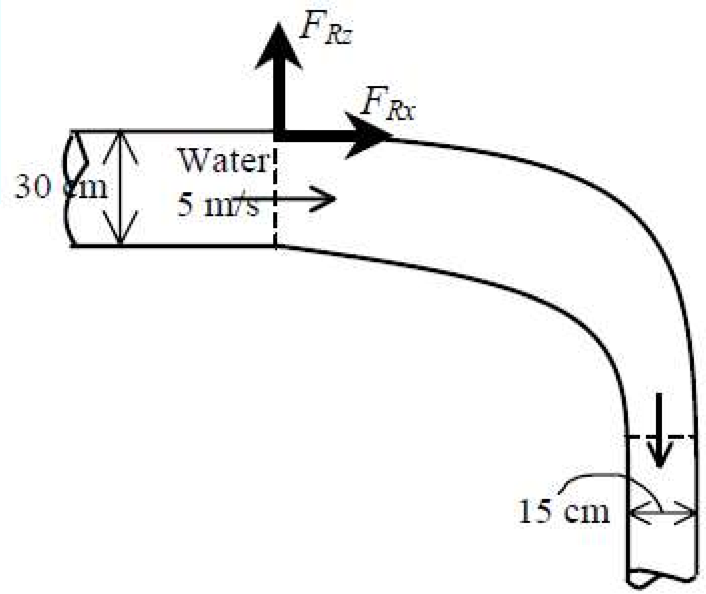
$$F_{Rx} = -\beta \dot{m} V_1 - P_{1,\text{gage}} A_1 = -1.04(353.4\ \text{kg/s})(5\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) - (300\ \text{kN/m}^2)\frac{\pi(0.3\ \text{m})^2}{4} = -23.0\ \text{kN}$$

$$F_{Rz} = -\beta \dot{m} V_2 + P_{2,\text{gage}} A_1 = -1.04(353.4\ \text{kg/s})(20\ \text{m/s})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) + (117.4\ \text{kN/m}^2)\frac{\pi(0.15\ \text{m})^2}{4} = -5.28\ \text{kN}$$

and

$$F_R = \sqrt{F_{Rx}^2 + F_{Rz}^2} = \sqrt{(-23.0)^2 + (-5.28)^2} = \mathbf{23.6\ kN}$$

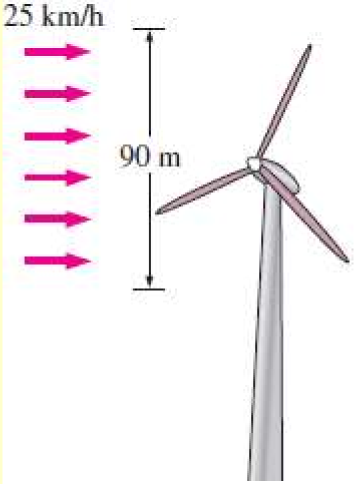
$$\theta = \tan^{-1}\frac{F_{Rz}}{F_{Rx}} = \tan^{-1}\frac{-5.28}{-23.0} = \mathbf{12.9°}$$



Discussion The magnitude of the anchoring force is 23.6 kN, and its line of action makes 12.9° from +x direction. Negative values for F_{Rx} and F_{Ry} indicate that the assumed directions are wrong, and should be reversed.

EXAMPLE 6–30

Commercially available large wind turbines have blade span diameters as large as 100 m and generate over 3 MW of electric power at peak design conditions. Consider a wind turbine with a 90-m blade span subjected to 25-km/h steady winds. If the combined turbine– generator efficiency of the wind turbine is 32 percent, determine (a) the power generated by the turbine and (b) the horizontal force exerted by the wind on the supporting mast of the turbine. Take the density of air to be *1.25 kg/m³*, and disregard frictional effects.



Assumptions **1** The wind flow is steady and incompressible. **2** The efficiency of the turbine-generator is independent of wind speed. **3** The frictional effects are negligible, and thus none of the incoming kinetic energy is converted to thermal energy. **4** Wind flow is uniform and thus the momentum-flux correction factor is nearly unity, $\beta \cong 1$.

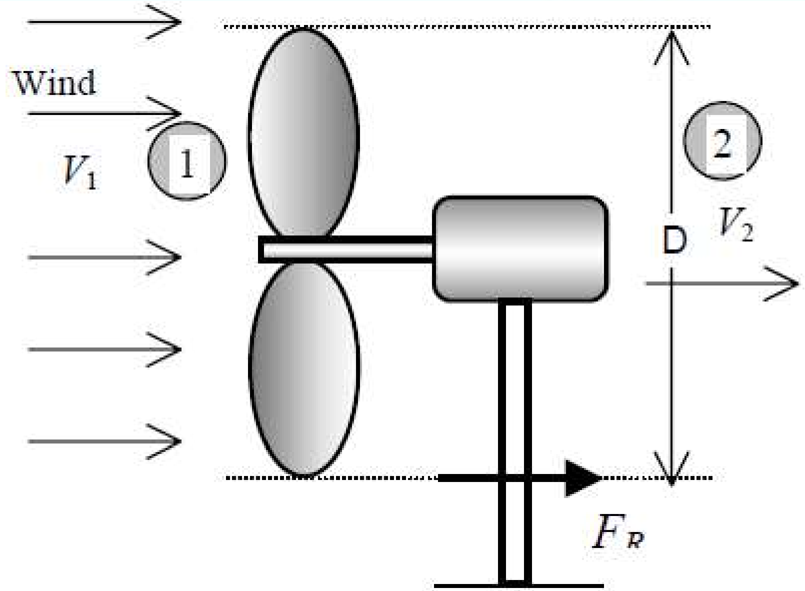
Properties The density of air is given to be $1.25\ kg/m^3$.

Analysis (a) The power potential of the wind is its kinetic energy, which is $\dot{m}V^2/2$ per unit mass, and for a given mass flow rate:

$$V_1 = (25\ \text{km/h})\left(\frac{1\ \text{m/s}}{3.6\ \text{km/h}}\right) = 6.94\ \text{m/s}$$

$$\dot{m} = \rho_1 V_1 A_1 = \rho_1 V_1 \frac{\pi D^2}{4} = (1.25\ \text{kg/m}^3)(6.94\ \text{m/s})\frac{\pi (90\ \text{m})^2}{4} = 55{,}200\ \text{kg/s}$$

$$\dot{W}_{\max} = \dot{m}ke_1 = \dot{m}\frac{V_1^2}{2} = (55{,}200\ \text{kg/s})\frac{(6.94\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = 1330\ \text{kW}$$



Then the actual power produced becomes

$$\dot{W}_{\text{act}} = \eta_{\text{wind turbine}}\dot{W}_{\max} = (0.32)(1330\ \text{kW}) = \mathbf{426\ kW}$$

(b) The frictional effects are assumed to be negligible, and thus the portion of incoming kinetic energy not converted to electric power leaves the wind turbine as outgoing kinetic energy. Therefore,

$$\dot{m}ke_2 = \dot{m}ke_1(1-\eta_{\text{wind turbine}}) \quad \rightarrow \quad \dot{m}\frac{V_2^2}{2} = \dot{m}\frac{V_1^2}{2}(1-\eta_{\text{wind turbine}})$$

or

$$V_2 = V_1\sqrt{1-\eta_{\text{wind turbine}}} = (6.94 \text{ m/s})\sqrt{1-0.32} = 5.72 \text{ m/s}$$

We choose the control volume around the wind turbine such that the wind is normal to the control surface at the inlet and the outlet, and the entire control surface is at the atmospheric pressure. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$. Writing it along the x-direction (without forgetting the negative sign for forces and velocities in the negative x-direction) and assuming the flow velocity through the turbine to be equal to the wind velocity give

$$F_R = \dot{m}V_2 - \dot{m}V_1 = \dot{m}(V_2 - V_1) = (55{,}200 \text{ kg/s})(5.72 - 6.94 \text{ m/s})\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right) = \mathbf{-67.3 \text{ kN}}$$

The negative sign indicates that the reaction force acts in the negative x direction, as expected.

Discussion This force acts on top of the tower where the wind turbine is installed, and the bending moment it generates at the bottom of the tower is obtained by multiplying this force by the tower height.

EXAMPLE 6–31

Firefighters are holding a nozzle at the end of a hose while trying to extinguish a fire. If the nozzle exit diameter is *6 cm* and the water flow rate is $5\ m^3/min$, determine (*a*) the average water exit velocity and (*b*) the horizontal resistance force required of the firefighters to hold the nozzle.

Answers: (*a*) 29.5 m/s, (*b*) 2457 N

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined. **5** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be $1000\ kg/m^3$.

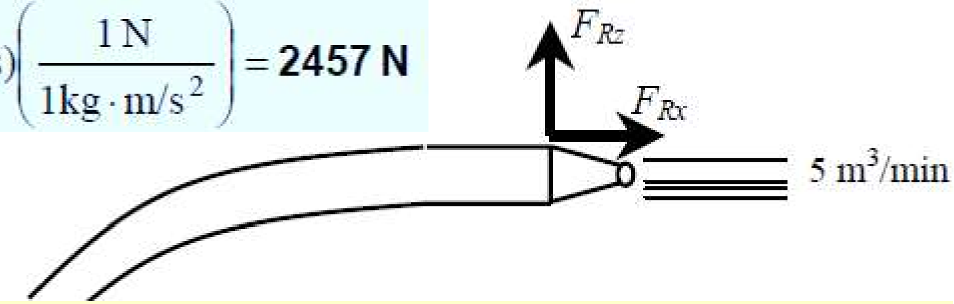
Analysis (a) We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and outlets horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction), and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction). The average outlet velocity and the mass flow rate of water are determined from

$$V = \frac{\dot{V}}{A} = \frac{\dot{V}}{\pi D^2/4} = \frac{5\ \mathrm{m^3/min}}{\pi(0.06\ \mathrm{m})^2/4} = 1768\ \mathrm{m/min} = \mathbf{29.5\ m/s}$$

$$\dot{m} = \rho\dot{V} = (1000\ \mathrm{kg/m^3})(5\ \mathrm{m^3/min}) = 5000\ \mathrm{kg/min} = 83.3\ \mathrm{kg/s}$$

(b) The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum_{out}\beta\dot{m}\vec{V} - \sum_{in}\beta\dot{m}\vec{V}$. We let horizontal force applied by the firemen to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction gives

$$F_{Rx} = \dot{m}V_e - 0 = \dot{m}V = (83.3\ \mathrm{kg/s})(29.5\ \mathrm{m/s})\left(\frac{1\ \mathrm{N}}{1\mathrm{kg\cdot m/s^2}}\right) = \mathbf{2457\ N}$$



Therefore, the firemen must be able to resist a force of 2457 N to hold the nozzle in place.

Discussion The force of 2457 N is equivalent to the weight of about 250 kg. That is, holding the nozzle requires the strength of holding a weight of 250 kg, which cannot be done by a single person. This demonstrates why several firemen are used to hold a hose with a high flow rate.

EXAMPLE 6–32

A 5-cm-diameter horizontal jet of water with a velocity of 30 m/s strikes a flat plate that is moving in the same direction as the jet at a velocity of 10 m/s. The water splatters in all directions in the plane of the plate. How much force does the water stream exert on the plate?

Assumptions **1** The flow is steady and incompressible. **2** The water splatters in all directions in the plane of the plate. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure, which is disregarded since it acts on all surfaces. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal force exerted on the plate. **5** The velocity of the plate, and the velocity of the water jet relative to the plate, are constant. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be $1000\ kg/m^3$.

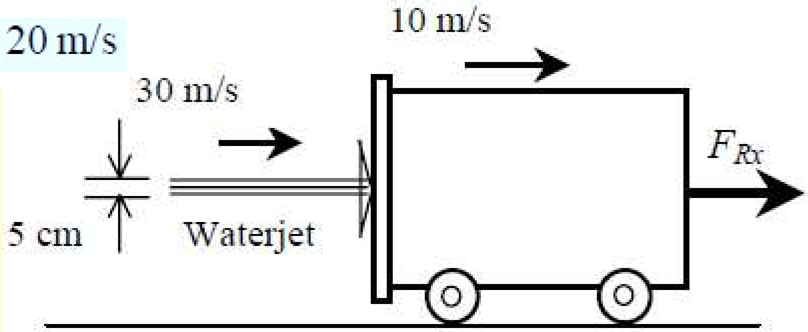
Analysis We take the plate as the control volume, and the flow direction as the positive direction of x axis. The mass flow rate of water in the jet is 6-20

$$\dot{m} = \rho V_{jet} A = \rho V_{jet} \frac{\pi D^2}{4} = (1000\ \mathrm{kg/m^3})(30\ \mathrm{m/s}) \frac{\pi (0.05\ \mathrm{m})^2}{4} = 58.9\ \mathrm{kg/s}$$

The relative velocity between the plate and the jet is

$$V_r = V_{\text{jet}} - V_{\text{plate}} = 30 - 10 = 20 \text{ m/s}$$

Therefore, we can assume the plate to be stationary and the jet to move with a velocity of 20 m/s. The momentum equation for steady one-dimensional flow is



$\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. We let the horizontal reaction force applied to the plate in the negative x direction to counteract the impulse of the water jet be F_{Rx}. Then the momentum equation along the *x* direction gives

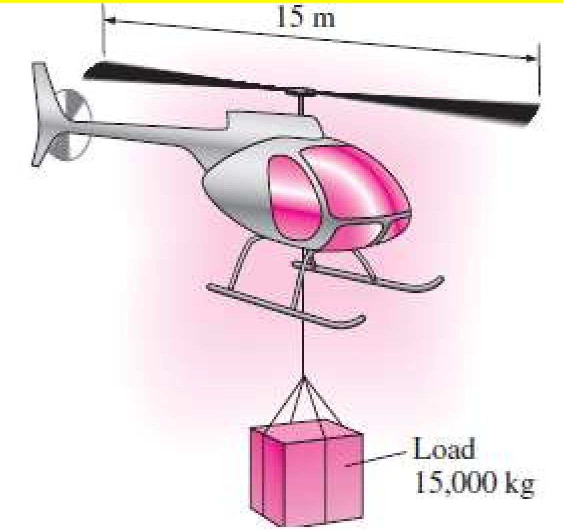
$$-F_{Rx} = 0 - \dot{m}V_i \quad \rightarrow \quad F_{Rx} = \dot{m}V_r = (58.9 \text{ kg/s})(20 \text{ m/s})\left(\frac{1\text{ N}}{1\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{1178\ N}$$

Therefore, the water jet applies a force of 1178 N on the plate in the direction of motion, and an equal and opposite force must be applied on the plate if its velocity is to remain constant.

Discussion Note that we used the relative velocity in the determination of the mass flow rate of water in the momentum analysis since water will enter the control volume at this rate. (In the limiting case of the plate and the water jet moving at the same velocity, the mass flow rate of water relative to the plate will be zero since no water will be able to strike the plate).

EXAMPLE 6–33

An unloaded helicopter of mass *10,000 kg* hovers at sea level while it is being loaded. In the unloaded hover mode, the blades rotate at *400 rpm*. The horizontal blades above the helicopter cause a 15-m-diameter air mass to move downward at an average velocity proportional to the overhead blade rotational velocity (rpm). A load of *15,000 kg* is loaded onto the helicopter, and the helicopter slowly rises. Determine (*a*) the volumetric airflow rate downdraft that the helicopter generates during unloaded hover and the required power input and (*b*) the rpm of the helicopter blades to hover with the *15,000-kg* load and the required power input. Take the density of atmospheric air to be *1.18 kg/m³*. Assume air approaches the blades from the top through a large area with negligible velocity and air is forced by the blades to move down with a uniform velocity through an imaginary cylinder whose base is the blade span area.

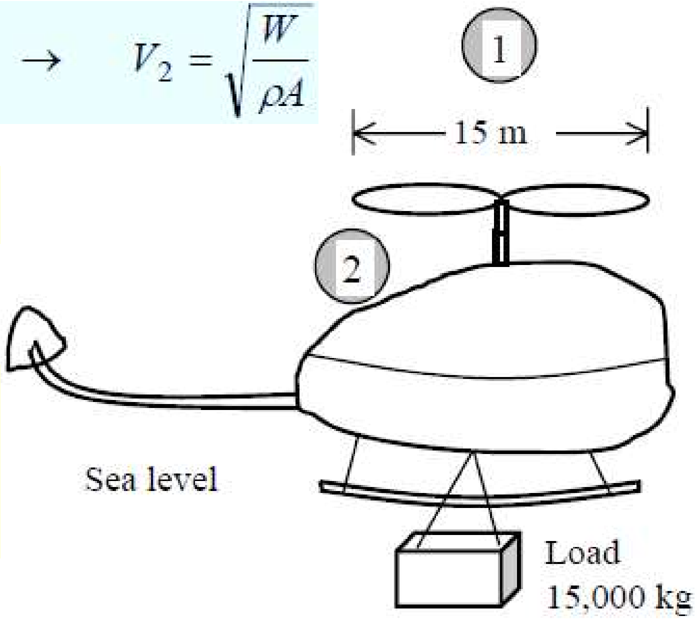


Properties The density of air is given to be *1.18 kg/m³*.

Analysis (a) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the z axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative z direction, the momentum equation along the z axis gives

$$-W = \dot{m}(-V_2) - 0 \quad \rightarrow \quad W = \dot{m}V_2 = (\rho A V_2)V_2 = \rho A V_2^2 \quad \rightarrow \quad V_2 = \sqrt{\frac{W}{\rho A}}$$



where A is the blade span area,

$$A = \pi D^2 / 4 = \pi (15\ \text{m})^2 / 4 = 176.7\ \text{m}^2$$

Then the discharge velocity, volume flow rate, and the mass flow rate of air in the unloaded mode become

$$V_{2,\text{unloaded}} = \sqrt{\frac{m_{\text{unloaded}}\, g}{\rho A}} = \sqrt{\frac{(10{,}000 \text{ kg})(9.81 \text{ m/s}^2)}{(1.18 \text{ kg/m}^3)(176.7 \text{ m}^2)}} = 21.7 \text{ m/s}$$

$$\dot{V}_{\text{unloaded}} = A V_{2,\text{unloaded}} = (176.7 \text{ m}^2)(21.7 \text{ m/s}) = \mathbf{3834\ m^3/s}$$

$$\dot{m}_{\text{unloaded}} = \rho \dot{V}_{\text{unloaded}} = (1.18 \text{ kg/m}^3)(3834 \text{ m}^3\text{/s}) = 4524 \text{ kg/s}$$

Noting that $P_1 = P_2 = P_{atm}$, $V_1 \cong 0$, the elevation effects are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho} + \frac{V_1^2}{2} + gz_1\right) + \dot{W}_{\text{pump, u}} = \dot{m}\left(\frac{P_2}{\rho} + \frac{V_2^2}{2} + gz_2\right) + \dot{W}_{\text{turbine}} + \dot{E}_{\text{mech,loss}} \quad \rightarrow \quad \dot{W}_{\text{fan, u}} = \dot{m}\frac{V_2^2}{2}$$

Substituting,

$$\dot{W}_{\text{unloaded fan,u}} = \left(\dot{m}\frac{V_2^2}{2}\right)_{\text{unloaded}} = (4524 \text{ kg/s})\frac{(21.7 \text{ m/s})^2}{2}\left(\frac{1 \text{ kN}}{1000 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ kW}}{1 \text{ kN}\cdot\text{m/s}}\right) = \mathbf{1065\ kW}$$

(b) We now repeat the calculations for the *loaded* helicopter, whose mass is 10,000+15,000 = 25,000 kg:

$$V_{2,\text{loaded}} = \sqrt{\frac{m_{\text{loaded}} g}{\rho A}} = \sqrt{\frac{(25{,}000\text{ kg})(9.81\text{ m/s}^2)}{(1.18\text{ kg/m}^3)(176.7\text{ m}^2)}} = 34.3\text{ m/s}$$

$$\dot{m}_{\text{loaded}} = \rho \dot{V}_{\text{loaded}} = \rho A V_{2,\text{ loaded}} = (1.18\text{ kg/m}^3)(176.7\text{ m}^2)(34.3\text{ m/s}) = 7152\text{ kg/s}$$

$$\dot{W}_{\text{loaded fan,u}} = \left(\dot{m}\frac{V_2^2}{2}\right)_{\text{loaded}} = (7152\text{ kg/s})\frac{(34.3\text{ m/s})^2}{2}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}}\right) = \mathbf{4207\ kW}$$

Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the loaded helicopter blades becomes

$$V_2 = k\dot{n} \quad \rightarrow \quad \frac{V_{2,\text{loaded}}}{V_{2,\text{unloaded}}} = \frac{\dot{n}_{\text{loaded}}}{\dot{n}_{\text{unloaded}}} \quad \rightarrow \quad \dot{n}_{\text{loaded}} = \frac{V_{2,\text{loaded}}}{V_{2,\text{unloaded}}}\dot{n}_{\text{unloaded}} = \frac{34.3}{21.7}(400\text{ rpm}) = \mathbf{632\ rpm}$$

Discussion The actual power input to the helicopter blades will be considerably larger than the calculated power input because of the fan inefficiency in converting mechanical power to kinetic energy.

EXAMPLE 6–34

Reconsider the helicopter in Prob. 6–40, except that it is hovering on top of a 3000-m-high mountain where the air density is *0.79 kg/m³*. Noting that the unloaded helicopter blades must rotate at 400 rpm to hover at sea level, determine the blade rotational velocity to hover at the higher altitude. Also determine the percent increase in the required power input to hover at 3000-m altitude relative to that at sea level.

Answers: 489 rpm, 22 percent

Assumptions **1** The flow of air is steady and incompressible. **2** The air leaves the blades at a uniform velocity at atmospheric pressure. **3** Air approaches the blades from the top through a large area at atmospheric pressure with negligible velocity. **4** The frictional effects are negligible, and thus the entire mechanical power input is converted to kinetic energy of air. **5** The change in air pressure with elevation while hovering at a given location is negligible because of the low density of air. **6** There is no acceleration of the helicopter, and thus the lift generated is equal to the total weight. **7** Air flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties The density of air is given to be $1.18\ kg/m^3$ at sea level, and $0.79\ kg/m^3$ on top of the mountain.

Analysis (a) We take the control volume to be a vertical hyperbolic cylinder bounded by streamlines on the sides with air entering through the large cross-section (section 1) at the top and the fan located at the narrow cross-section at the bottom (section 2), and let its centerline be the *z* axis with upwards being the positive direction.

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$. Noting that the only force acting on the control volume is the total weight W and it acts in the negative *z* direction, the momentum equation along the *z* axis gives

$$-W = \dot{m}(-V_2) - 0 \quad \rightarrow \quad W = \dot{m}V_2 = (\rho A V_2)V_2 = \rho A V_2^2 \quad \rightarrow \quad V_2 = \sqrt{\frac{W}{\rho A}}$$

where A is the blade span area. Then for a given weight W, the ratio of discharge velocities becomes

$$\frac{V_{2,\text{mountain}}}{V_{2,\text{sea}}} = \frac{\sqrt{W/\rho_{\text{mountain}}A}}{\sqrt{W/\rho_{\text{sea}}A}} = \sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}} = \sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}} = 1.222$$

Noting that the average flow velocity is proportional to the overhead blade rotational velocity, the rpm of the helicopter blades on top of the mountain becomes

$$\dot{n} = kV_2 \quad \rightarrow \quad \frac{\dot{n}_{\text{mountain}}}{\dot{n}_{\text{sea}}} = \frac{V_{2,\text{mountain}}}{V_{2,\text{sea}}} \quad \rightarrow \quad \dot{n}_{\text{mountain}} = \frac{V_{2,\text{mountain}}}{V_{2,\text{sea}}}\dot{n}_{\text{sea}} = 1.222(400\ \text{rpm}) = \mathbf{489\ rpm}$$

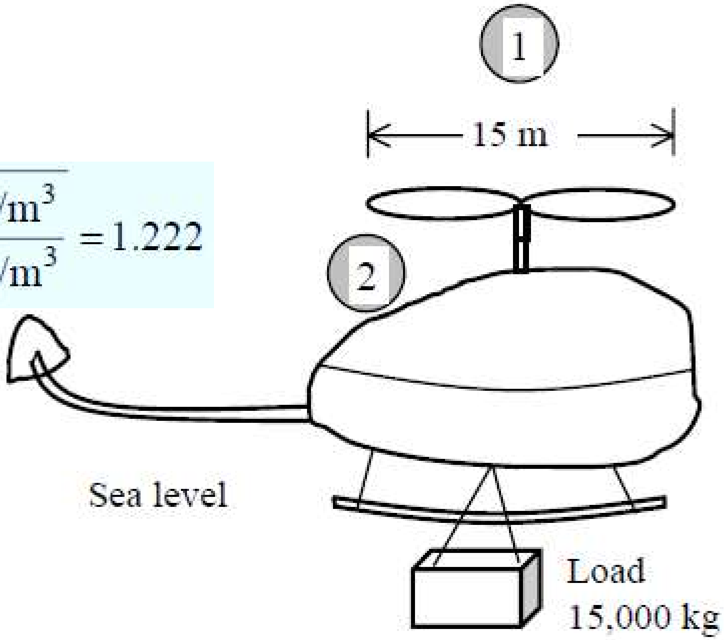
Noting that $P_1 = P_2 = P_{atm}$, $V_1 \cong 0$, the elevation effect are negligible, and the frictional effects are disregarded, the energy equation for the selected control volume reduces to

$$\dot{m}\left(\frac{P_1}{\rho}+\frac{V_1^2}{2}+gz_1\right)+\dot{W}_{\text{pump, u}}=\dot{m}\left(\frac{P_2}{\rho}+\frac{V_2^2}{2}+gz_2\right)+\dot{W}_{\text{turbine}}+\dot{E}_{\text{mech,loss}} \quad\rightarrow\quad \dot{W}_{\text{fan, u}}=\dot{m}\frac{V_2^2}{2}$$

$$\text{or}\quad \dot{W}_{\text{fan,u}}=\dot{m}\frac{V_2^2}{2}=\rho A V_2\frac{V_2^2}{2}=\rho A\frac{V_2^3}{2}=\tfrac{1}{2}\rho A\left(\sqrt{\frac{W}{\rho A}}\right)^3=\tfrac{1}{2}\rho A\left(\frac{W}{\rho A}\right)^{1.5}=\frac{W^{1.5}}{2\sqrt{\rho A}}$$

Then the ratio of the required power input on top of the mountain to that at sea level becomes

$$\frac{\dot{W}_{\text{mountain fan,u}}}{\dot{W}_{\text{sea fan,u}}}=\frac{0.5W^{1.5}/\sqrt{\rho_{\text{mountain}}A}}{0.5W^{1.5}/\sqrt{\rho_{\text{sea}}A}}=\sqrt{\frac{\rho_{\text{sea}}}{\rho_{\text{mountain}}}}=\sqrt{\frac{1.18\ \text{kg/m}^3}{0.79\ \text{kg/m}^3}}=1.222$$



Therefore, the required power input will increase by 22.2% on top of the mountain relative to the sea level.

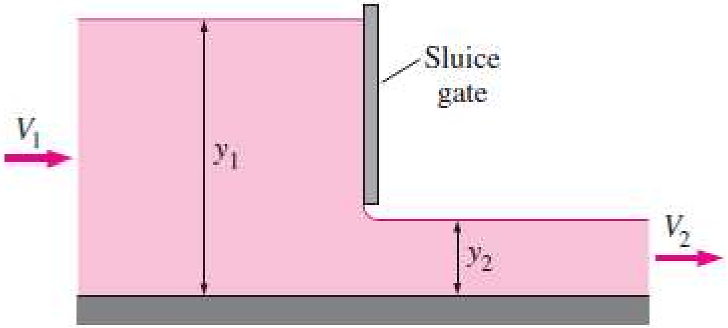
Discussion Note that both the rpm and the required power input to the helicopter are inversely proportional to the square root of air density. Therefore, more power is required at higher elevations for the helicopter to operate because air is less dense, and more air must be forced by the blades into the downdraft.

EXAMPLE 6–35

A sluice gate, which controls flow rate in a channel by simply raising or lowering a vertical plate, is commonly used in irrigation systems. A force is exerted on the gate due to the difference between the water heights y_1 and y_2 and the flow velocities V_1 and V_2 upstream and downstream from the gate, respectively. Disregarding the wall shear forces at the channel surfaces, develop relations for V_1, V_2, and the force acting on a sluice gate of width w during steady and uniform flow.

Answer: $F_R = \dot{m}(V_1 - V_2) + \frac{W}{2}\rho g(y_1^2 - y_2^2)$

Assumptions **1** The flow is steady, incompressible, frictionless, and uniform (and thus the Bernoulli equation is applicable.) **2** Wall shear forces at surfaces are negligible. **3** The channel is exposed to the atmosphere, and thus the pressure at free surfaces is the atmospheric pressure. **4** The flow is horizontal. **5** Water flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.



Analysis We take point 1 at the free surface of the upstream flow before the gate and point 2 at the free surface of the downstream flow after the gate. We also take the bottom surface of the channel as the reference level so that the elevations of points 1 and 2 are y_1 and y_2, respectively. The application of the Bernoulli equation between points 1 and 2 gives

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+y_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+y_2 \quad \rightarrow \quad V_2^2-V_1^2=2\,g(y_1-y_2) \qquad (1)$$

The flow is assumed to be incompressible and thus the density is constant. Then the conservation of mass relation for this single stream steady flow device can be expressed as

$$\dot{V}_1=\dot{V}_2=\dot{V} \quad \rightarrow \quad A_1V_1=A_2V_2=\dot{V} \quad \rightarrow \quad V_1=\frac{\dot{V}}{A_1}=\frac{\dot{V}}{wy_1} \quad \text{and} \quad V_2=\frac{\dot{V}}{A_2}=\frac{\dot{V}}{wy_2} \qquad (2)$$

Substituting into Eq. (1),

$$\left(\frac{\dot{V}}{wy_2}\right)^2-\left(\frac{\dot{V}}{wy_1}\right)^2=2g(y_1-y_2) \rightarrow \dot{V}=w\sqrt{\frac{2g(y_1-y_2)}{1/y_2^2-1/y_1^2}} \quad \rightarrow \dot{V}=wy_2\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (3)$$

Substituting Eq. (3) into Eqs. (2) gives the following relations for velocities,

$$V_1=\frac{y_2}{y_1}\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \quad \text{and} \quad V_2=\sqrt{\frac{2g(y_1-y_2)}{1-y_2^2/y_1^2}} \qquad (4)$$

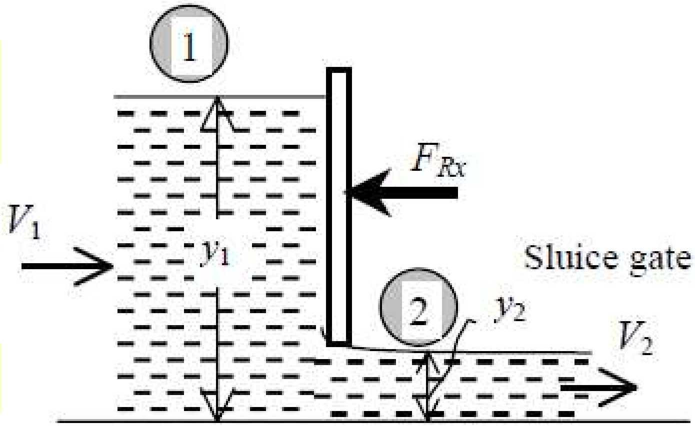
We choose the control volume as the water body surrounded by the vertical cross-sections of the upstream and downstream flows, free surfaces of water, the inner surface of the sluice gate, and the bottom surface of the channel. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m}\vec{V} - \sum_{in} \beta \dot{m}\vec{V}$. The force acting on the sluice gate F_{Rx} is horizontal since the wall shear at the surfaces is negligible, and it is equal and opposite to the force applied on water by the sluice gate. Noting that the pressure force acting on a vertical surface is equal to the product of the pressure at the centroid of the surface and the surface area, the momentum equation along the x direction gives

$$-F_{Rx} + P_1 A_1 - P_2 A_2 = \dot{m}V_2 - \dot{m}V_1 \quad \rightarrow \quad -F_{Rx} + \left(\rho g \frac{y_1}{2}\right)(wy_1) - \left(\rho g \frac{y_2}{2}\right)(wy_2) = \dot{m}(V_2 - V_1)$$

Rearranging, the force acting on the sluice gate is determined to be

$$F_{Rx} = \dot{m}(V_1 - V_2) + \frac{w}{2}\rho g(y_1^2 - y_2^2) \qquad (5)$$

where V_1 and V_2 are given in Eq. (4).



Discussion Note that for $y_1 >> y_2$, Eq. (3) simplifies to $\dot{V} = y_2 w\sqrt{2gy_1}$ or $V_2 = \sqrt{2gy_1}$ which is the Toricelli equation for frictionless flow from a tank through a hole a distance y_1 below the free surface.

EXAMPLE 6–36

Water enters a centrifugal pump axially at atmospheric pressure at a rate of *0.12 m^3/s* and at a velocity of *7 m/s*, and leaves in the normal direction along the pump casing, as shown in Fig. P6–43. Determine the force acting on the shaft (which is also the force acting on the bearing of the shaft) in the axial direction.

Properties We take the density of water to be *1000 kg/m^3*.

Assumptions **1** The flow is steady and incompressible. **2** The forces acting on the piping system in the horizontal direction are negligible. **3** The atmospheric pressure is disregarded since it acts on all surfaces.

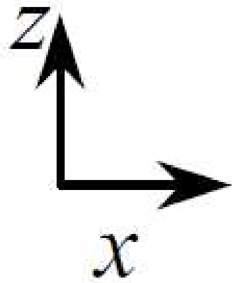
Analysis We take the pump as the control volume, and the inlet direction of flow as the positive direction of *x* axis. The momentum equation for steady one-dimensional flow in the *x* (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad -F_{Rx} = -\dot{m} V_i \quad \rightarrow \quad F_{Rx} = \dot{m} V_i = \rho \dot{V} V_i$$

Note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. Substituting the given values,

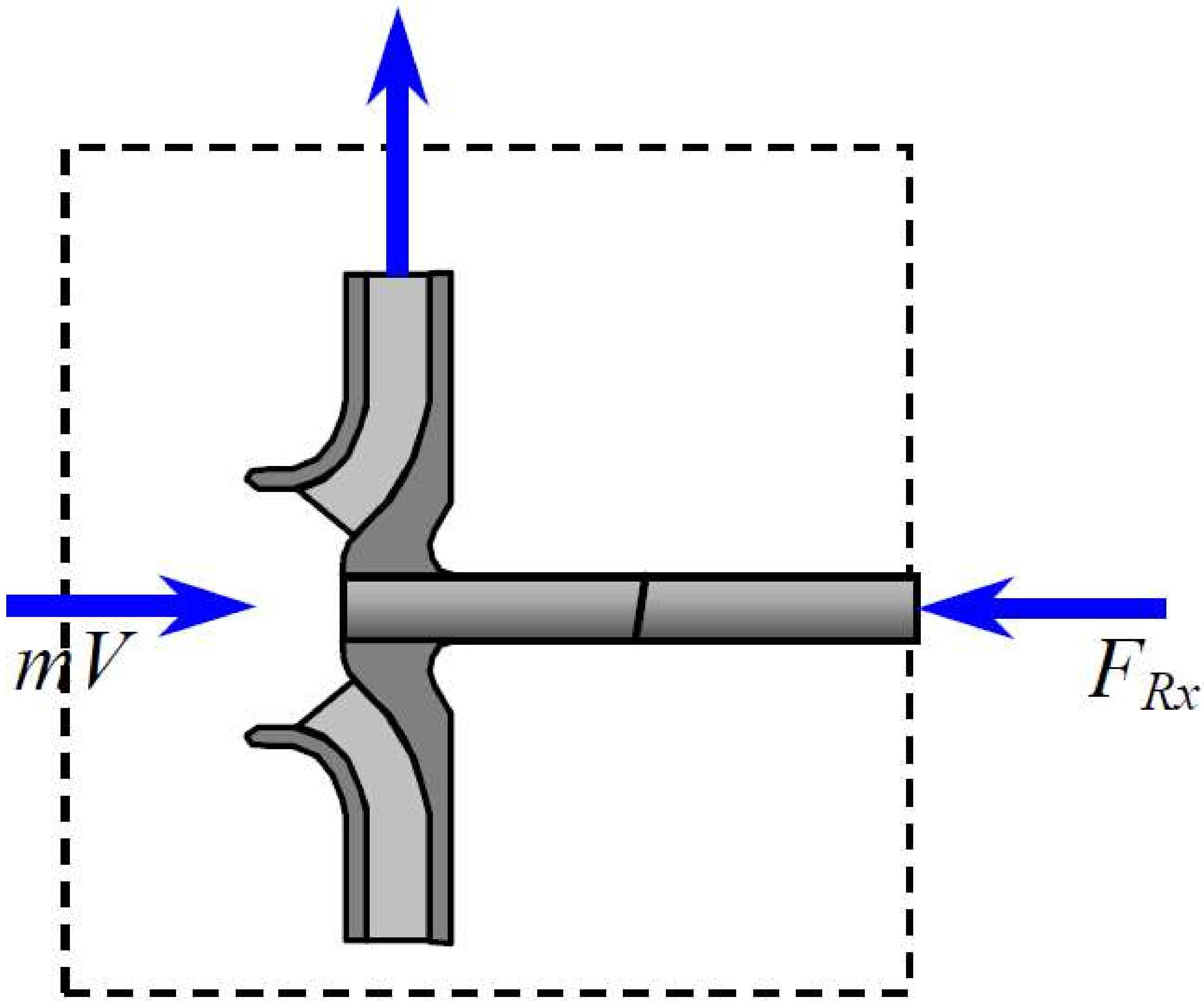
$$F_{\text{brake}} = (1000\ \text{kg/m}^3)(0.12\ \text{m}^3/\text{s})(7\ \ \text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{840\ N}$$

Discussion To find the total force acting on the shaft, we also need to do a force balance for the vertical direction, and find the vertical component of the reaction force.



$0.12\ \text{m}^3/\text{s}$

$7\ \text{m/s}$



Angular Momentum Equation
Sample Questions and Answers

EXAMPLE 6–37

How is the angular momentum equation obtained from Reynolds transport equations?

The angular momentum equation is obtained by replacing *B* in the Reynolds transport theorem by the total angular momentum $\vec{H}_{\text{sys}}$, and b by the angular momentum per unit mass $\vec{r} \times \vec{V}$.

EXAMPLE 6–38

Express the unsteady angular momentum equation in vector form for a control volume that has a constant moment of inertia *I*, no external moments applied, one outgoing uniform flow stream of velocity $\vec{V}$, and mass flow rate $\dot{m}$.

The angular momentum equation in this case is expressed as $I\vec{a} = -\vec{r} \times \dot{m}\vec{V}$ where $\vec{a}$ is the angular acceleration of the control volume, and $\vec{r}$ is the position vector from the axis of rotation to any point on the line of action of $\vec{F}$.

EXAMPLE 6–39

Express the angular momentum equation in scalar form about a specified axis of rotation for a fixed control volume for steady and uniform flow.

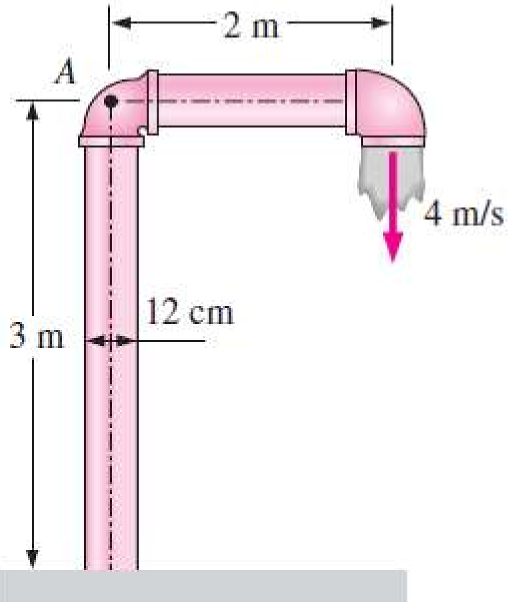
The angular momentum equation in this case is expressed as $I\vec{a} = -\vec{r} \times \dot{m}\vec{V}$ where $\vec{a}$ is the angular acceleration of the control volume, and $\vec{r}$ is the position vector from the axis of rotation to any point on the line of action of $\vec{F}$.

EXAMPLE 6–40

Water is flowing through a 12-cm-diameter pipe that consists of a 3-m-long vertical and 2-m-long horizontal section with a 90° elbow at the exit to force the water to be discharged downward, as shown in Fig. P6–47, in the vertical direction. Water discharges to atmospheric air at a velocity of 4 m/s, and the mass of the pipe section when filled with water is 15 kg per meter length. Determine the moment acting at the intersection of the vertical and horizontal sections of the pipe (point A). What would your answer be if the flow were discharged upward instead of downward?

Assumptions **1** The flow is steady and incompressible. **2** The water is discharged to the atmosphere, and thus the gage pressure at the outlet is zero. **3** Effects of water falling down during upward discharge is disregarded. **4** Pipe outlet diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be *1000 kg/m³*.

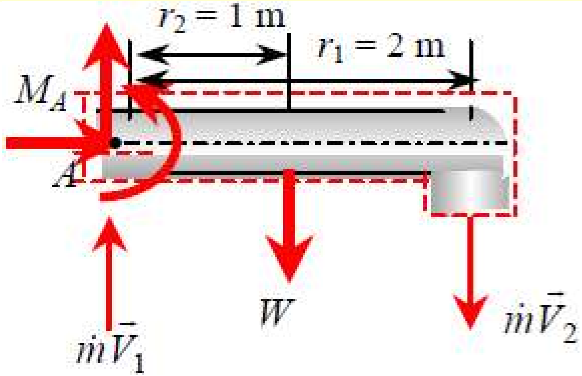


Analysis We take the entire pipe as the control volume, and designate the inlet by 1 and the outlet by 2. We also take the x and y coordinates as shown. The control volume and the reference frame are fixed.

The conservation of mass equation for this one-inlet one-outlet steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and $V_1 = V_2 = V$ since A_c = constant. The mass flow rate and the weight of the horizontal section of the pipe are

$$\dot{m} = \rho A_c V = (1000\ \text{kg/m}^3)[\pi(0.12\ \text{m})^2/4](4\ \text{m/s}) = 45.24\ \text{kg/s}$$

$$W = mg = (15\ \text{kg/m})(2\ \text{m})(9.81\ \text{m/s}^2)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 294.3\ \text{N/m}$$



(a) **Downward discharge**: To determine the moment acting on the pipe at point A, we need to take the moment of all forces and momentum flows about that point. This is a steady and uniform flow problem, and all forces and momentum flows are in the same plane. Therefore, the angular momentum equation in this case can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where *r* is the moment arm, all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative.

The free body diagram of the pipe section is given in the figure. Noting that the moments of all forces and momentum flows passing through point A are zero, the only force that will yield a moment about point A is the weight W of the horizontal pipe section, and the only momentum flow that will yield a moment is the outlet stream (both are negative since both moments are in the clockwise direction). Then the angular momentum equation about point A becomes

$$M_A - r_1 W = -r_2 \dot{m} V_2$$

Solving for M_A and substituting,

$$M_A = r_1 W - r_2 \dot{m} V_2 = (1\,\text{m})(294.3\,\text{N}) - (2\,\text{m})(45.54\,\text{kg/s})(4\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{-70.0\,N\cdot m}$$

The negative sign indicates that the assumed direction for M_A is wrong, and should be reversed. Therefore, a moment of 70 N·m acts at the stem of the pipe in the clockwise direction.

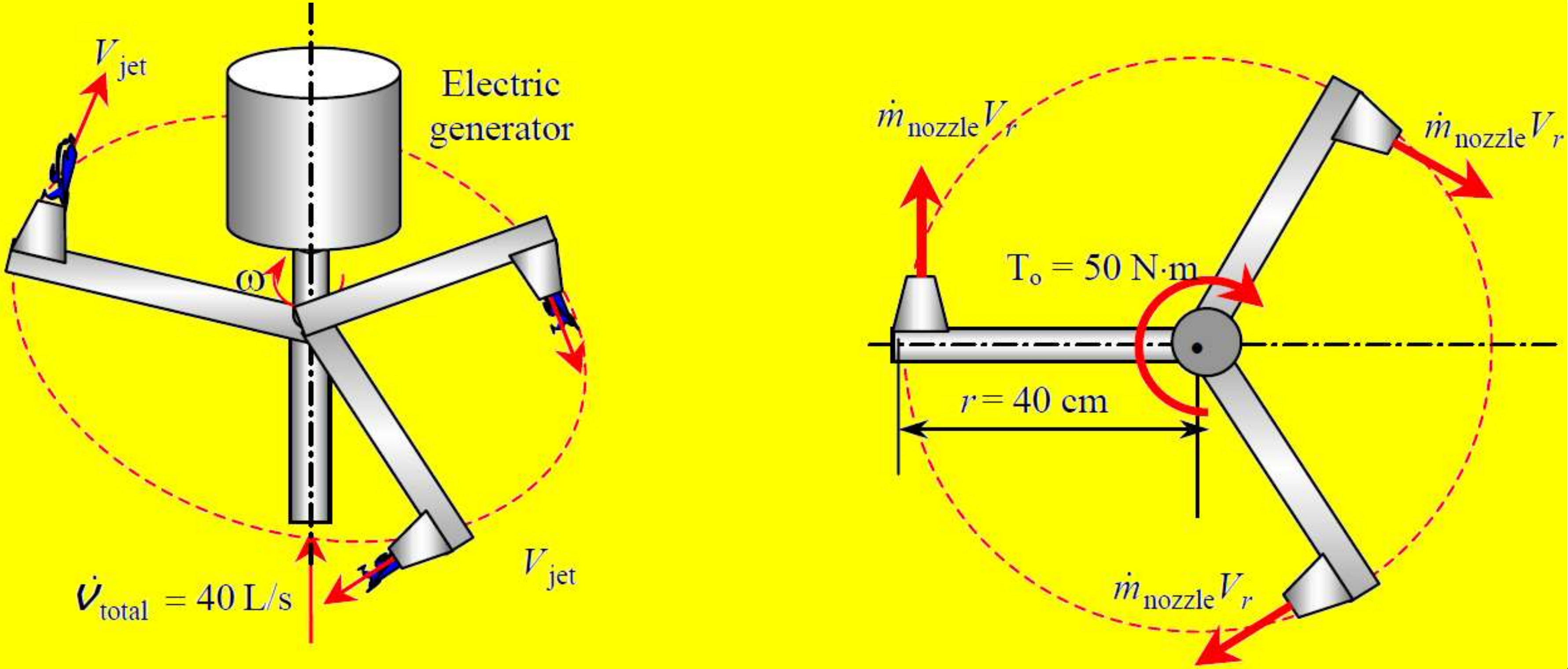
(b) **Upward discharge**: The moment due to discharge stream is positive in this case, and the moment acting on the pipe at point A is

$$M_A = r_1 W + r_2 \dot{m} V_2 = (1\,\text{m})(294.3\,\text{N}) + (2\,\text{m})(45.54\,\text{kg/s})(4\,\text{m/s})\left(\frac{1\,\text{N}}{1\,\text{kg}\cdot\text{m/s}^2}\right) = \mathbf{659\,N\cdot m}$$

Discussion Note direction of discharge can make a big difference in the moments applied on a piping system. This problem also shows the importance of accounting for the moments of momentums of flow streams when performing evaluating the stresses in pipe materials at critical cross-sections.

EXAMPLE 6–41

A lawn sprinkler with three identical arms is used to water a garden by rotating in a horizontal plane by the impulse caused by water flow. Water enters the sprinkler along the axis of rotation at a rate of 40 L/s and leaves the 1.2-cm-diameter nozzles in the tangential direction. The bearing applies a retarding torque of T_0 = 50 N · m due to friction at the anticipated operating speeds. For a normal distance of 40 cm between the axis of rotation and the center of the nozzles, determine the angular velocity of the sprinkler shaft.



Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be *1000 kg/m³ = 1 kg/L.*

Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the three nozzles are identical, we have $\dot{m}_{nozzle} = \dot{m}/3$ or $\dot{V}_{nozzle} = \dot{V}_{total} / 3$ since the density of water is constant. The average jet outlet velocity relative to the nozzle and the mass flow rate are

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{40 \text{ L/s}}{3[\pi(0.012 \text{ m})^2/4]}\left(\frac{1 \text{ m}^3}{1000 \text{ L}}\right) = 117.9 \text{ m/s}$$

$$\dot{m}_{total} = \rho\dot{V}_{total} = (1 \text{ kg/L})(40 \text{ L/s}) = 40 \text{ kg/s}$$

The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-T_0 = -3r\dot{m}_{\text{nozzle}}V_r \qquad \text{or} \qquad T_0 = r\dot{m}_{\text{total}}V_r$$

Solving for the relative velocity V_r and substituting,

$$V_r = \frac{T_0}{r\dot{m}_{\text{total}}} = \frac{50\ \text{N}\cdot\text{m}}{(0.40\ \text{m})(40\ \text{kg/s})}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 3.1\ \text{m/s}$$

Then the tangential and angular velocity of the nozzles become

$$V_{\text{nozzle}} = V_{\text{jet}} - V_r = 117.9 - 3.1 = 114.8\ \text{m/s}$$

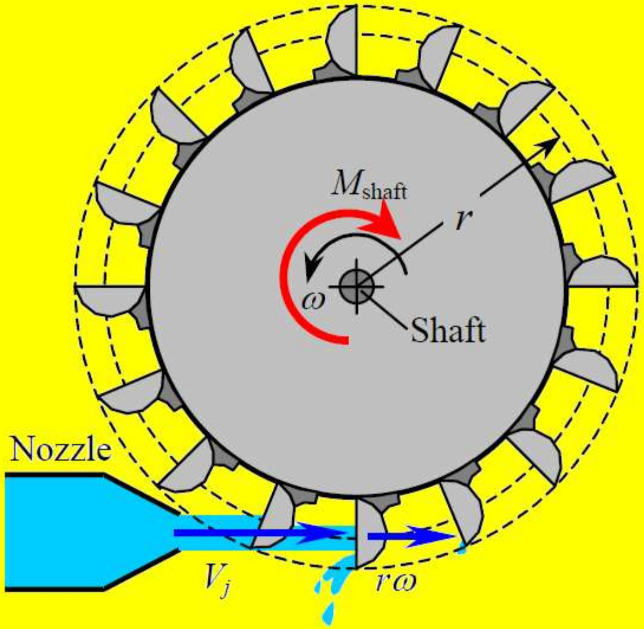
$$\omega = \frac{V_{\text{nozzle}}}{r} = \frac{114.8\ \text{m/s}}{0.4\ \text{m}} = \mathbf{287\ rad/s}$$

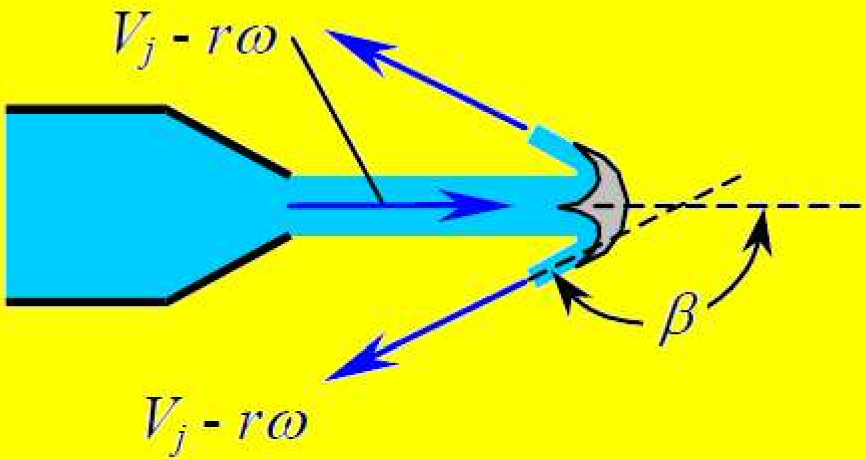
$$\dot{n} = \frac{\omega}{2\pi} = \frac{287\ \text{rad/s}}{2\pi}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{2741\ rpm}$$

Therefore, this sprinkler will rotate at 2741 revolutions per minute.

EXAMPLE 6–42

Pelton wheel turbines are commonly used in hydroelectric power plants to generate electric power. In these turbines, a high-speed jet at a velocity of V_j impinges on buckets, forcing the wheel to rotate. The buckets reverse the direction of the jet, and the jet leaves the bucket making an angle β with the direction of the jet, as shown in Fig. P6–51. Show that the power produced by a Pelton wheel of radius r rotating steadily at an angular velocity of ω is $\dot{W}_{\text{shaft}} = \rho\omega r\dot{V}(V_j - \omega r)(1 - \cos\beta)$, where ρ is the density and $\dot{V}$ İs the volume flow rate of the fluid. Obtain the numerical value for $\rho = 1000\ kg/m^3$, $r = 2$ m, $\dot{V} = 10\ m^3/s$, $\dot{V} = 150$ rpm, $\beta = 160°$, and $V_j = 50\ m/s$.





Assumptions **1** The flow is uniform and cyclically steady. **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Generator losses and air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be $1000\ kg/m^3 = 1\ kg/L$.

Analysis The tangential velocity of buckets corresponding to an angular velocity of $\omega = 2\pi\dot{n}$ is $V_{bucket} = r\omega$. Then the relative velocity of the jet (relative to the bucket) becomes

$$V_r = V_j - V_{\text{bucket}} = V_j - r\omega$$

We take the imaginary disk that contains the Pelton wheel as the control volume. The inlet velocity of the fluid into this control volume is V_r, and the component of outlet velocity normal to the moment arm is $V_r \cos\beta$. The angular momentum equation can be expressed as $\sum M = \sum_{out} r\dot{m}V - \sum_{in} r\dot{m}V$ where all moments in the counterclockwise direction are positive, and all in the clockwise direction are negative. Then the angular momentum equation about the axis of rotation becomes

$$-M_{\text{shaft}} = r\dot{m}V_r \cos\beta - r\dot{m}V_r \quad \text{or} \quad M_{\text{shaft}} = r\dot{m}V_r(1-\cos\beta) = r\dot{m}(V_j - r\omega)(1-\cos\beta)$$

Noting that $\dot{W}_{shaft} = 2\pi\dot{n}M_{shaft} = \omega M_{shaft}$ and $\dot{m} = \rho\dot{V}$, the power output of a Pelton turbine becomes

$$\dot{W}_{\text{shaft}} = \rho\dot{V}r\omega(V_j - r\omega)(1-\cos\beta)$$

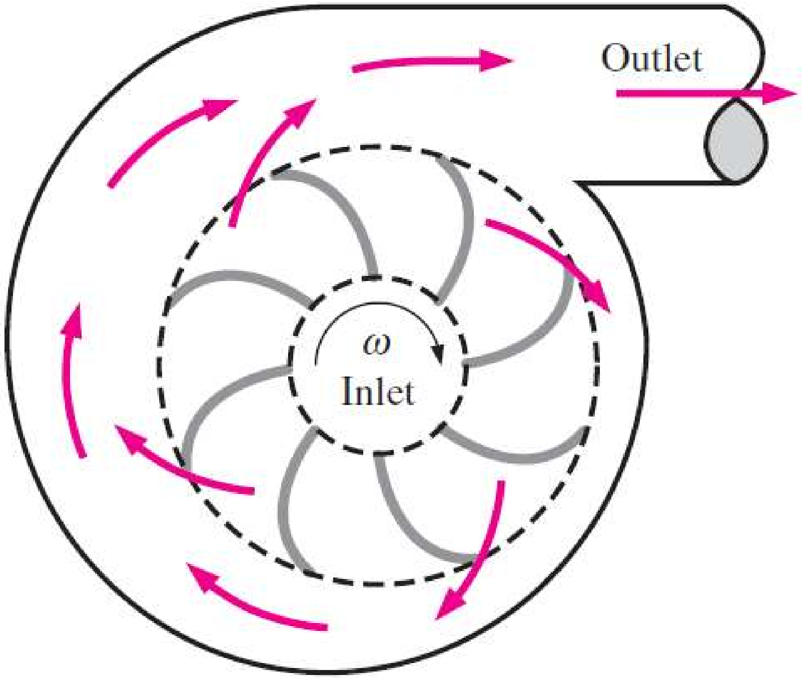
which is the desired relation. For given values, the power output is determined to be

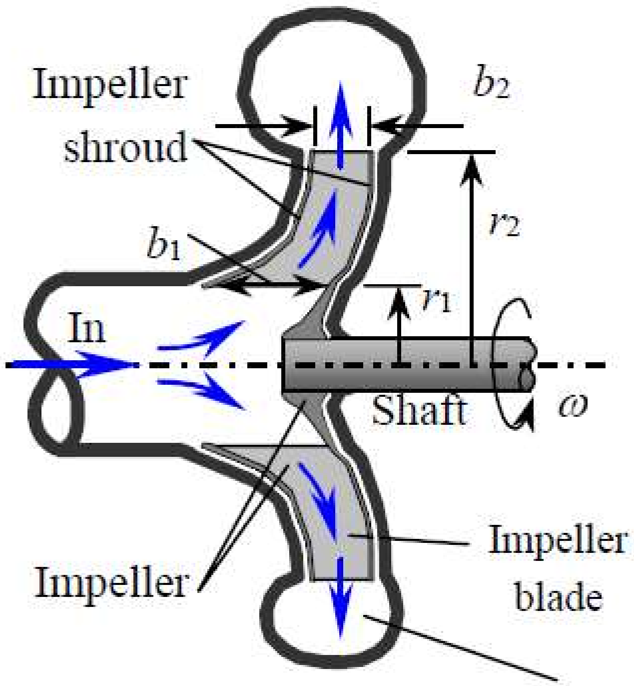
$$\dot{W}_{\text{shaft}} = (1000\ \text{kg/m}^3)(10\ \text{m}^3/\text{s})(2\ \text{m})(15.71\ \text{rad/s})(50 - 2\times 15.71\ \text{m/s})(1-\cos 160°)\left(\frac{1\ \text{MW}}{10^6\ \text{N}\cdot\text{m/s}}\right) = \mathbf{11.3\ MW}$$

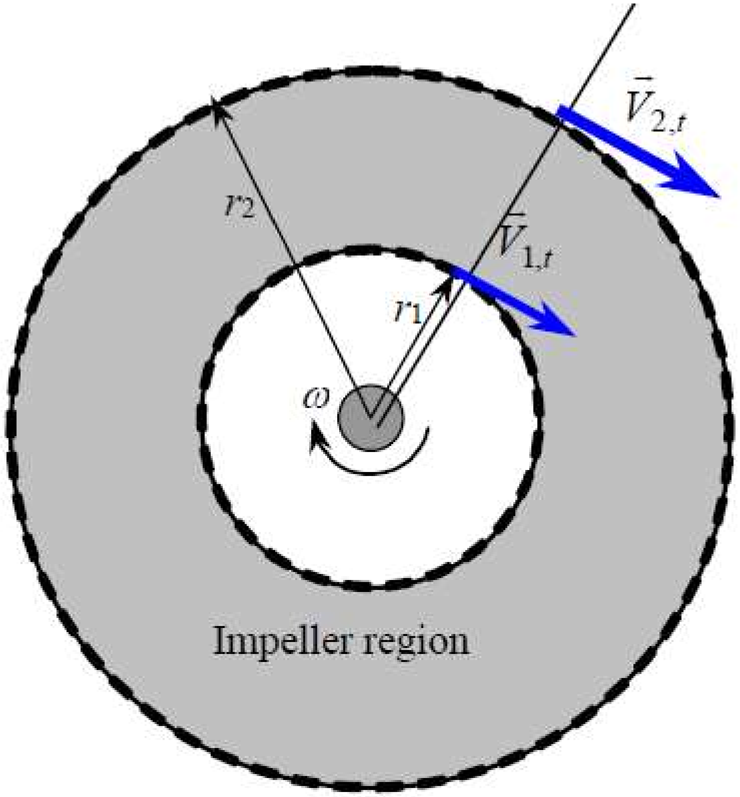
where $\omega = 2\pi\dot{n} = 2\pi(150\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 15.71\ \text{rad/s}$

EXAMPLE 6–43

The impeller of a centrifugal blower has a radius of 15 cm and a blade width of 6.1 cm at the inlet, and a radius of 30 cm and a blade width of 3.4 cm at the outlet. The blower delivers atmospheric air at 20°C and 95 kPa. Disregarding any losses and assuming the tangential components of air velocity at the inlet and the outlet to be equal to the impeller velocity at respective locations, determine the volumetric flow rate of air when the rotational speed of the shaft is 800 rpm and the power consumption of the blower is 120 W. Also determine the normal components of velocity at the inlet and outlet of the impeller.







Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible. **3** The tangential components of air velocity at the inlet and the outlet are said to be equal to the impeller velocity at respective locations.

Properties The gas constant of air is *0.287 kPa·m³/kg·K*. The density of air at *20°C* and *95 kPa* is

$$\rho = \frac{P}{RT} = \frac{95 \text{ kPa}}{(0.287 \text{ kPa} \cdot \text{m}^3/\text{kg} \cdot \text{K})(293 \text{ K})} = 1.130 \text{ kg/m}^3$$

Analysis In the idealized case of the tangential fluid velocity being equal to the blade angular velocity both at the inlet and the outlet, we have $V_{1,t} = \omega r_1$ and $V_{2,t} = \omega r_2$, and the torque is expressed as

$$T_{shaft} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = \dot{m}\omega(r_2^2 - r_1^2) = \rho \dot{V}\omega(r_2^2 - r_1^2)$$

where the angular velocity is

$$\omega = 2\pi\dot{n} = 2\pi(800\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 83.78\ \text{rad/s}$$

Then the shaft power becomes

$$\dot{W}_{shaft} = \omega T_{shaft} = \rho \dot{V}\omega^2(r_2^2 - r_1^2)$$

Solving for $\dot{V}$ and substituting, the volume flow rate of air is determined to

$$\dot{V} = \frac{\dot{W}_{shaft}}{\rho\omega^2(r_2^2 - r_1^2)} = \frac{120\ \text{N}\cdot\text{m/s}}{(1.130\ \text{kg/m}^3)(83.78\ \text{rad/s})^2[(0.30\ \text{m})^2 - (0.15\ \text{m})^2]}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = \mathbf{0.224\ m^3/s}$$

The normal velocity components at the inlet and the outlet are

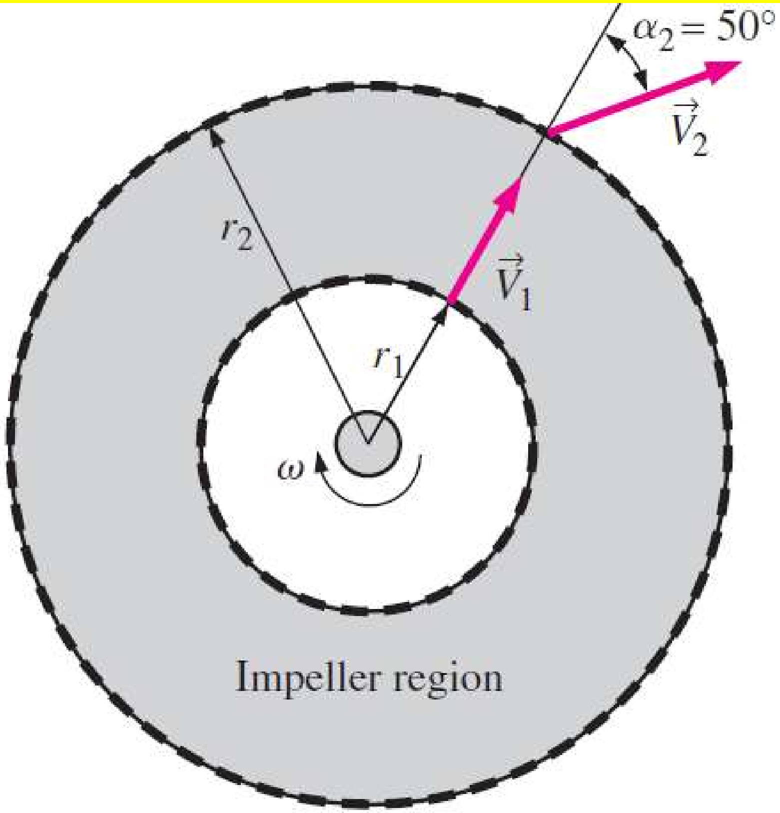
$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.224\ \text{m}^3/\text{s}}{2\pi(0.15\ \text{m})(0.061\ \text{m})} = \mathbf{3.90\ m/s}$$

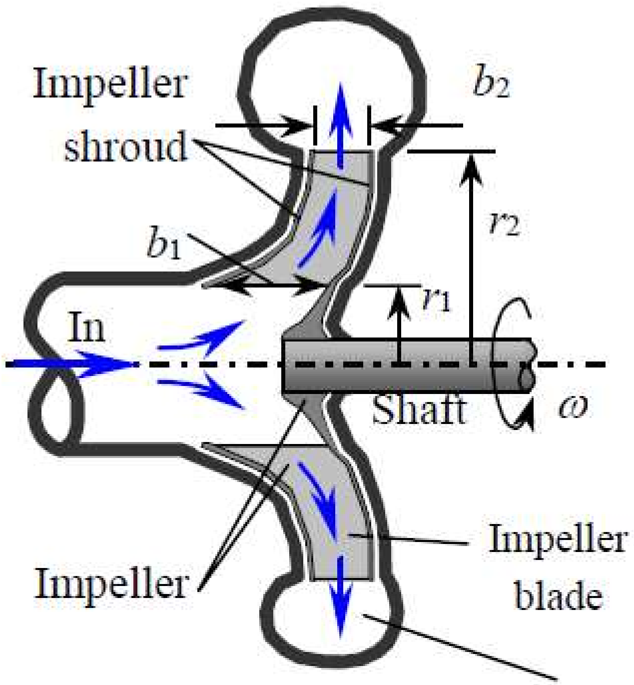
$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.224\ \text{m}^3/\text{s}}{2\pi(0.30\ \text{m})(0.034\ \text{m})} = \mathbf{3.50\ m/s}$$

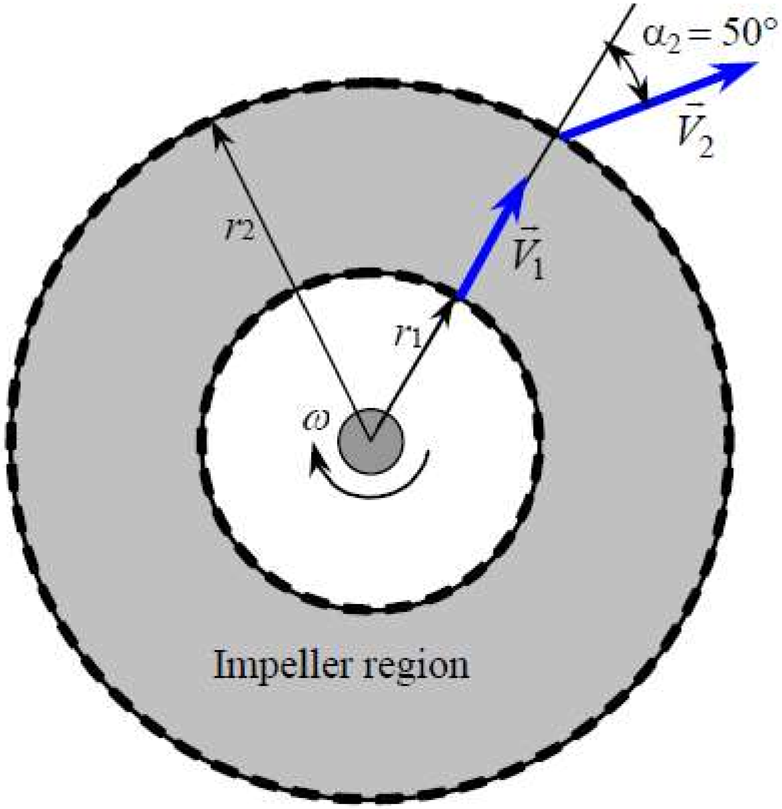
Discussion Note that the irreversible losses are not considered in analysis. In reality, the flow rate and the normal components of velocities will be smaller.

EXAMPLE 6–44

Consider a centrifugal blower that has a radius of 20 cm and a blade width of 8.2 cm at the impeller inlet, and a radius of 45 cm and a blade width of 5.6 cm at the outlet. The blower delivers air at a rate of *0.70 m^3/s* at a rotational speed of 700 rpm. Assuming the air to enter the impeller in radial direction and to exit at an angle of 50° from the radial direction, determine the minimum power consumption of the blower. Take the density of air to be *1.25 kg/m^3*.







Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.
Properties The density of air is given to be *1.25 kg/m³*.
Analysis We take the impeller region as the control volume. The normal velocity components at the inlet and the outlet are

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.70\ \text{m}^3/\text{s}}{2\pi(0.20\ \text{m})(0.082\ \text{m})} = 6.793\ \text{m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.70\ \text{m}^3/\text{s}}{2\pi(0.45\ \text{m})(0.056\ \text{m})} = 4.421\ \text{m/s}$$

The tangential components of absolute velocity are:

$$\alpha_1 = 0°: \qquad V_{1,t} = V_{1,n}\tan\alpha_1 = 0$$

$$\alpha_2 = 60°: \qquad V_{2,t} = V_{2,n}\tan\alpha_1 = (4.421\ \text{m/s})\tan 50° = 5.269\ \text{m/s}$$

The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(700\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 73.30\ \text{rad/s}$$

$$\dot{m} = \rho\dot{V} = (1.25\ \text{kg/m}^3)(0.7\ \text{m}^3/\text{s}) = 0.875\ \text{kg/s}$$

Normal velocity components $V_{1,n}$ and $V_{2,n}$ as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

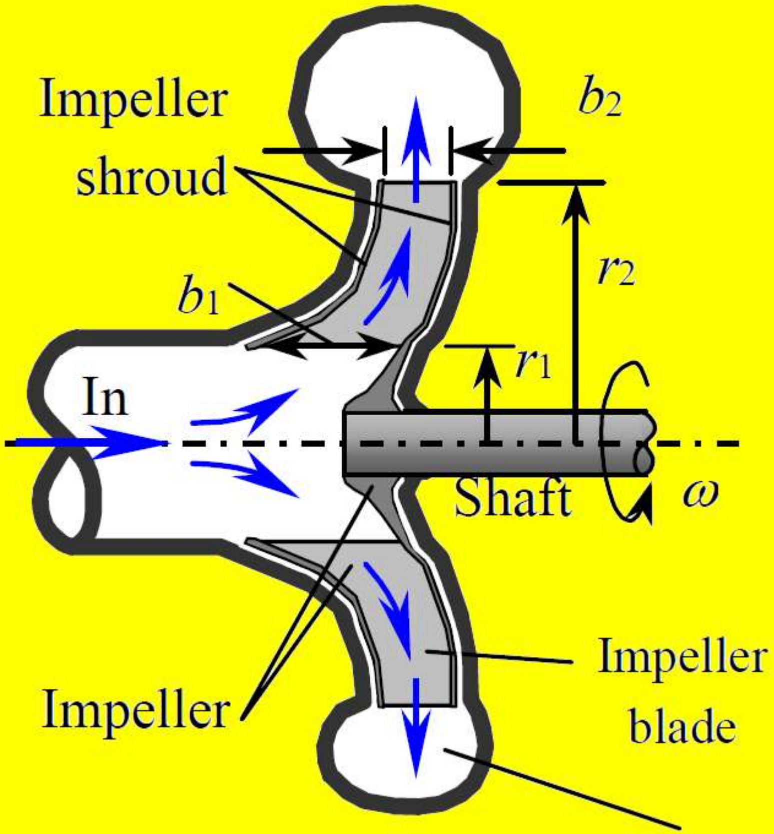
$$\text{T}_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = (0.875\ \text{kg/s})[(0.45\ \text{m})(5.269\ \text{m/s}) - 0]\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 2.075\ \text{N}\cdot\text{m}$$

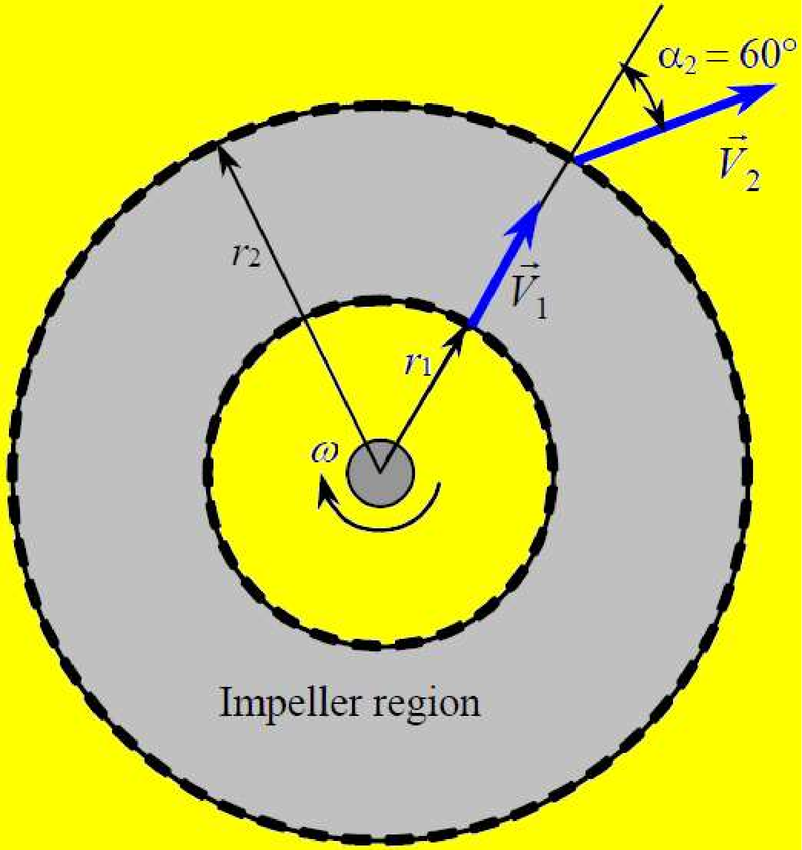
Then the shaft power becomes

$$\dot{W} = \omega\text{T}_{\text{shaft}} = (73.30\ \text{rad/s})(2.075\ \text{N}\cdot\text{m})\left(\frac{1\ \text{W}}{1\ \text{N}\cdot\text{m/s}}\right) = \mathbf{152\ W}$$

EXAMPLE 6–45

The impeller of a centrifugal pump has inner and outer diameters of 13 and 30 cm, respectively, and a flow rate of $0.15\ m^3/s$ at a rotational speed of 1200 rpm. The blade width of the impeller is 8 cm at the inlet and 3.5 cm at the outlet. If water enters the impeller in the radial direction and exits at an angle of 60° from the radial direction, determine the minimum power requirement for the pump.





Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.
Properties We take the density of water to be *1000 kg/m³*.
Analysis We take the impeller region as the control volume. The normal velocity components at the inlet and the outlet are

$$V_{1,n} = \frac{\dot{V}}{2\pi r_1 b_1} = \frac{0.15\ \text{m}^3/\text{s}}{2\pi(0.13\ \text{m})(0.080\ \text{m})} = 2.296\ \text{m/s}$$

$$V_{2,n} = \frac{\dot{V}}{2\pi r_2 b_2} = \frac{0.15\ \text{m}^3/\text{s}}{2\pi(0.30\ \text{m})(0.035\ \text{m})} = 2.274\ \text{m/s}$$

The tangential components of absolute velocity are:

$$\alpha_1 = 0°: \qquad V_{1,t} = V_{1,n}\tan\alpha_1 = 0$$

$$\alpha_2 = 60°: \qquad V_{2,t} = V_{2,n}\tan\alpha_1 = (2.274\ \text{m/s})\tan 60° = 3.938\ \text{m/s}$$

The angular velocity of the propeller is

$$\omega = 2\pi\dot{n} = 2\pi(1200\ \text{rev/min})\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 125.7\ \text{rad/s}$$

$$\dot{m} = \rho\dot{V} = (1000\ \text{kg/m}^3)(0.15\ \text{m}^3/\text{s}) = 150\ \text{kg/s}$$

Normal velocity components $V_{1,n}$ and $V_{2,n}$ as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$T_{shaft} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = (150\ \text{kg/s})[(0.30\ \text{m})(3.938\ \text{m/s}) - 0]\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right) = 177.2\ \text{kN}\cdot\text{m}$$

Then the shaft power becomes

$$\dot{W} = \omega T_{shaft} = (125.7\ \text{rad/s})(177.2\ \text{kN}\cdot\text{m})\left(\frac{1\ \text{kW}}{1\ \text{kN}\cdot\text{m/s}}\right) = \mathbf{22.3\ kW}$$

Discussion Note that the irreversible losses are not considered in analysis. In reality, the required power input will be larger.

Review Problems
Sample Questions and Answers

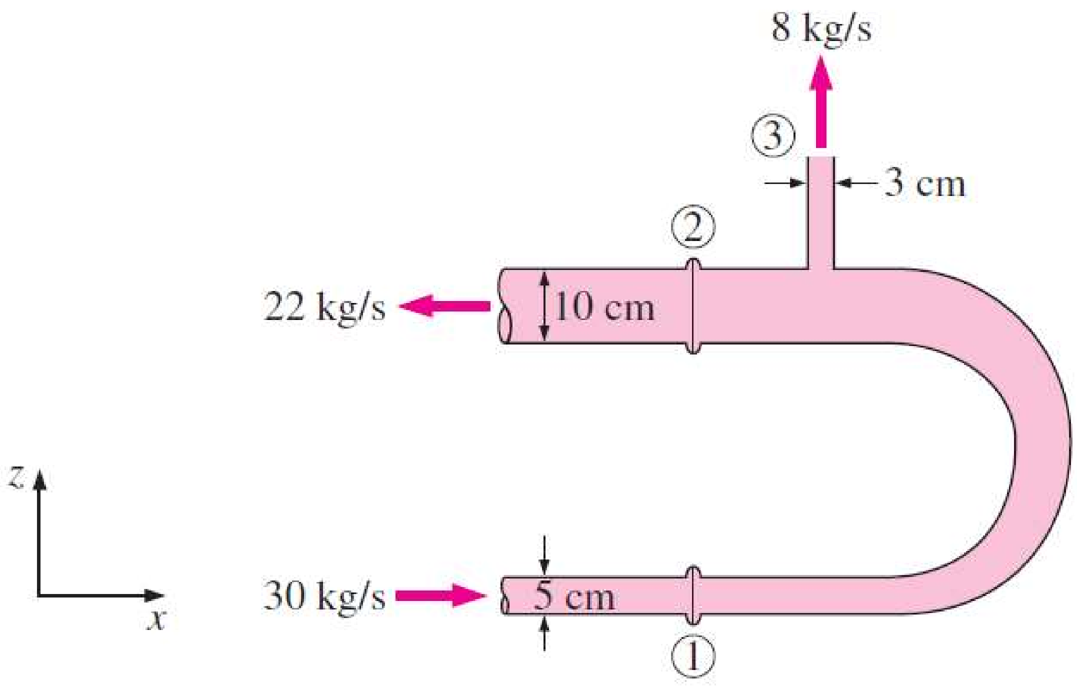
EXAMPLE 6–46

Water is flowing into and discharging from a pipe Usection as shown in Fig. P6–58. At flange (1), the total absolute pressure is 200 kPa, and 30 kg/s flows into the pipe. At flange (2), the total pressure is 150 kPa. At location (3), 8 kg/s of water discharges to the atmosphere, which is at 100 kPa. Determine the total *x*- and *z*-forces at the two flanges connecting the pipe. Discuss the significance of gravity force for this problem. Take the momentum-flux correction factor to be 1.03.

Assumptions **1** The flow is steady and incompressible. **2** The weight of the U-turn and the water in it is negligible. **4** The momentum-flux correction factor for each inlet and outlet is given to be $\beta = 1.03$.

Properties We take the density of water to be *1000 kg/m³*.

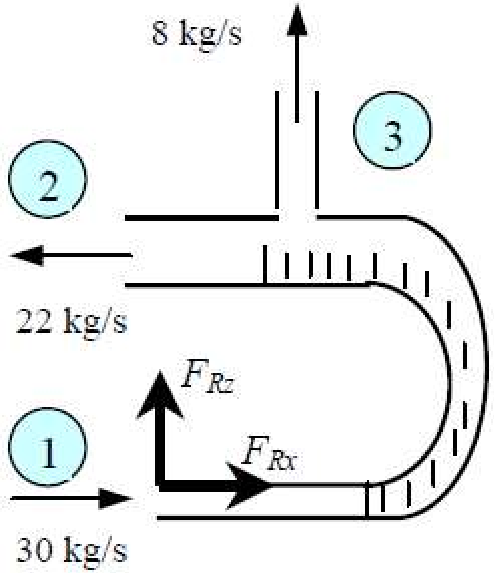
Analysis The flow velocities of the 3 streams are



$$V_1 = \frac{\dot{m}_1}{\rho A_1} = \frac{\dot{m}_1}{\rho(\pi D_1^2/4)} = \frac{30 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi(0.05 \text{ m})^2/4]} = 15.3 \text{ m/s}$$

$$V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_2}{\rho(\pi D_2^2/4)} = \frac{22 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi(0.10 \text{ m})^2/4]} = 2.80 \text{ m/s}$$

$$V_3 = \frac{\dot{m}_3}{\rho A_3} = \frac{\dot{m}_3}{\rho(\pi D_3^2/4)} = \frac{8 \text{ kg/s}}{(1000 \text{ kg/m}^3)[\pi(0.03 \text{ m})^2/4]} = 11.3 \text{ m/s}$$



We take the entire U-section as the control volume. We designate the horizontal coordinate by x with the direction of incoming flow as being the positive direction and the vertical coordinate by *z*. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$. We let the *x*- and *z*- components of the anchoring force of the cone be F_{Rx} and F_{Rz}, and assume them to be in the positive directions. Then the momentum equations along the x and z axes become

$$F_{Rx} + P_1 A_1 + P_2 A_2 = \beta \dot{m}_2(-V_2) - \beta \dot{m}_1 V_1 \quad \rightarrow \quad F_{Rx} = -P_1 A_1 - P_2 A_2 - \beta(\dot{m}_2 V_2 + \dot{m}_1 V_1)$$

$$F_{Rz} + 0 = \dot{m}_3 V_3 - 0 \quad \rightarrow \quad F_{Rz} = \beta \dot{m}_3 V_3$$

Substituting the given values,

$$F_{Rx} = -[(200-100)\ \mathrm{kN/m^2}]\frac{\pi(0.05\ \mathrm{m})^2}{4} - [(150-100)\ \mathrm{kN/m^2}]\frac{\pi(0.10\ \mathrm{m})^2}{4}$$

$$-1.03\left[(22\ \mathrm{kg/s})(2.80\ \mathrm{m/s})\left(\frac{1\ \mathrm{kN}}{1000\ \mathrm{kg\cdot m/s^2}}\right) + (30\ \mathrm{kg/s})(15.3\ \mathrm{m/s})\left(\frac{1\ \mathrm{kN}}{1000\ \mathrm{kg\cdot m/s^2}}\right)\right]$$

$$= -0.733\ \mathrm{kN} = \mathbf{-733\ N}$$

$$F_{Rz} = 1.03(8\ \mathrm{kg/s})(11.3\ \mathrm{m/s})\left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg\cdot m/s^2}}\right) = \mathbf{93.1\ N}$$

The negative value for F_{Rx} indicates the assumed direction is wrong, and should be reversed. Therefore, a force of 733 N acts on the flanges in the opposite direction. A vertical force of 93.1 N acts on the flange in the vertical direction.

Discussion To assess the significance of gravity forces, we estimate the weight of the weight of water in the U-turn and compare it to the vertical force. Assuming the length of the U-turn to be 0.5 m and the average diameter to be 7.5 cm, the mass of the water becomes

$$m = \rho V = \rho A L = \rho\frac{\pi D^2}{4}L = (1000\ \mathrm{kg/m^3})\frac{\pi(0.075\ \mathrm{m})^2}{4}(0.5\ \mathrm{m}) = 2.2\ \mathrm{kg}$$

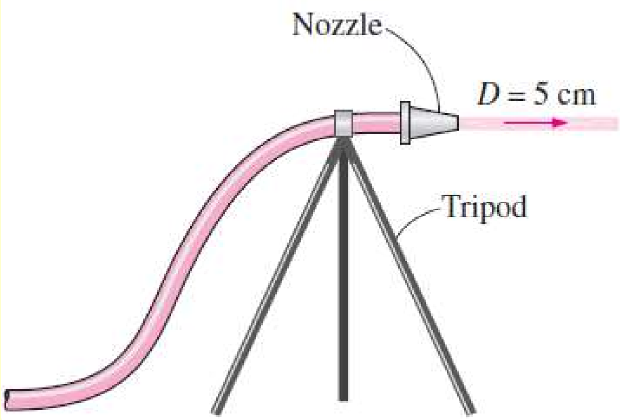
whose weight is 2.2×9.81 = 22 N, which is much less than 93.1, but still significant. Therefore, disregarding the gravitational effects is a reasonable assumption if great accuracy is not required.

EXAMPLE 6–47

A tripod holding a nozzle, which directs a 5-cm-diameter stream of water from a hose, is shown in Fig. P6–59. The nozzle mass is 10 kg when filled with water. The tripod is rated to provide 1800 N of holding force. A firefighter was standing 60 cm behind the nozzle and was hit by the nozzle when the tripod suddenly failed and released the nozzle. You have been hired as an accident reconstructionist and, after testing the tripod, have determined that as water flow rate increased, it did collapse at 1800 N. In your final report you must state the water velocity and the flow rate consistent with the failure and the nozzle velocity when it hit the firefighter.

Answers: 30.2 m/s, 0.0593 m^3/s, 14.7 m/s

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet is the atmospheric pressure, which is disregarded since it acts on all surfaces. **3** Gravitational effects and vertical forces are disregarded since the horizontal resistance force is to be determined. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.



Properties We take the density of water to be *1000 kg/m³*.

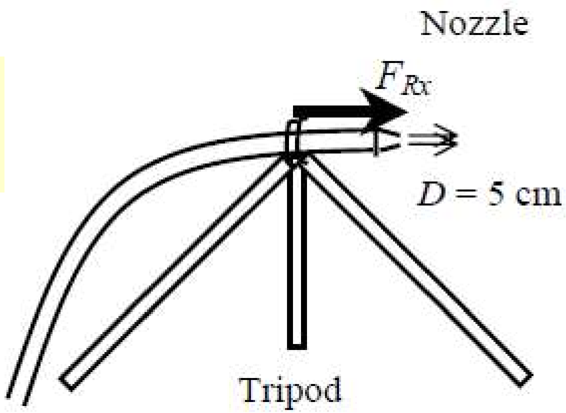
Analysis We take the nozzle and the horizontal portion of the hose as the system such that water enters the control volume vertically and outlets horizontally (this way the pressure force and the momentum flux at the inlet are in the vertical direction, with no contribution to the force balance in the horizontal direction, and designate the entrance by 1 and the outlet by 2. We also designate the horizontal coordinate by x (with the direction of flow as being the positive direction).

The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$. We let the horizontal force applied by the tripod to the nozzle to hold it be F_{Rx}, and assume it to be in the positive x direction. Then the momentum equation along the x direction becomes

$$F_{Rx} = \dot{m}V_e - 0 = \dot{m}V = \rho AVV = \rho \frac{\pi D^2}{4} V^2 \quad \rightarrow \quad (1800\text{ N})\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = (1000\text{ kg/m}^3)\frac{\pi (0.05\text{ m})^2}{4} V^2$$

Solving for the water outlet velocity gives V = **30.3 m/s**. Then the water flow rate becomes

$$\dot{V} = AV = \frac{\pi D^2}{4} V = \frac{\pi (0.05\text{ m})^2}{4}(30.3\text{ m/s}) = \mathbf{0.0595\ m^3/s}$$



When the nozzle was released, its acceleration must have been

$$a_{\text{nozzle}} = \frac{F}{m_{\text{nozzle}}} = \frac{1800\ \text{N}}{10\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 180\ \text{m/s}^2$$

Assuming the reaction force acting on the nozzle and thus its acceleration to remain constant, the time it takes for the nozzle to travel 60 cm and the nozzle velocity at that moment were (note that both the distance *x* and the velocity *V* are zero at time *t = 0*)

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(0.6\ \text{m})}{180\ \text{m/s}^2}} = 0.0816\ s$$

$$V = at = (180\ \text{m/s}^2)(0.0816\ \text{s}) = \mathbf{14.7\ m/s}$$

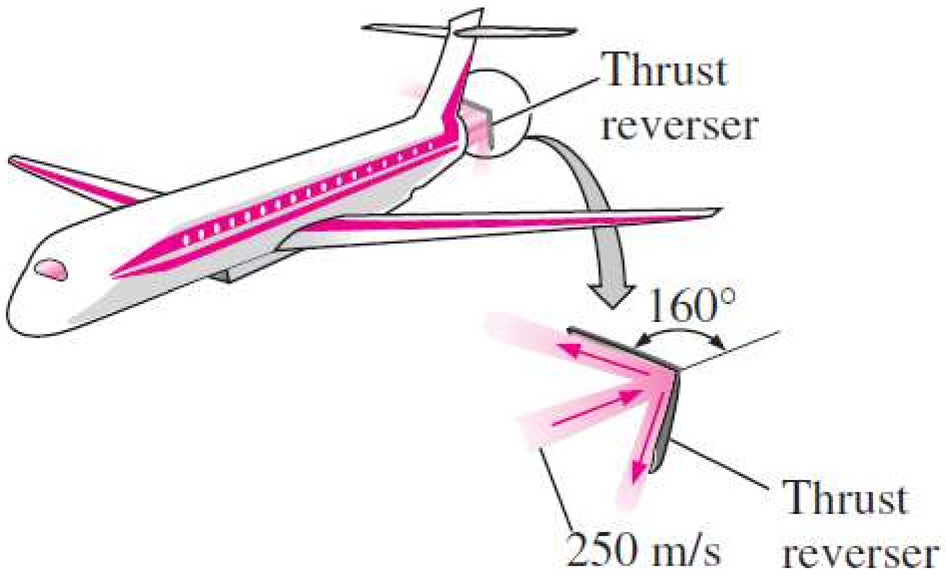
Thus we conclude that the nozzle hit the fireman with a velocity of 14.7 m/s.

Discussion Engineering analyses such as this one are frequently used in accident reconstruction cases, and they often form the basis for judgment in courts.

EXAMPLE 6–48

Consider an airplane with a jet engine attached to the tail section that expels combustion gases at a rate of 18 kg/s with a velocity of V = 250 m/s relative to the plane. During landing, a thrust reverser (which serves as a brake for the aircraft and facilitates landing on a short runway) is lowered in the path of the exhaust jet, which deflects the exhaust from rearward to 160°. Determine (a) the thrust (forward force) that the engine produces prior to the insertion of the thrust reverser and (b) the braking force produced after the thrust reverser is deployed.

Assumptions 1 The flow of exhaust gases is steady and one-dimensional. **2** The exhaust gas stream is exposed to the atmosphere, and thus its pressure is the atmospheric pressure. **3** The velocity of exhaust gases remains constant during reversing. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.



Analysis (a) The thrust exerted on an airplane is simply the momentum flux of the combustion gases in the reverse direction,

$$\text{Thrust} = \dot{m}_{ex} V_{ex} = (18\ \text{kg/s})(250\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = \mathbf{4500\ N}$$

(b) We take the thrust reverser as the control volume such that it cuts through both exhaust streams normally and the connecting bars to the airplane, and the direction of airplane as the positive direction of x axis. The momentum equation for steady one-dimensional flow in the x direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad F_{Rx} = \dot{m}(V)\cos 20° - \dot{m}(-V) \quad \rightarrow \quad F_{Rx} = (1+\cos 20°)\dot{m} V_i$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (1+\cos 20°)(18\ \text{kg/s})(250\ \text{m/s}) = 8729\ \text{N}$$

The breaking force acting on the plane is equal and opposite to this force,

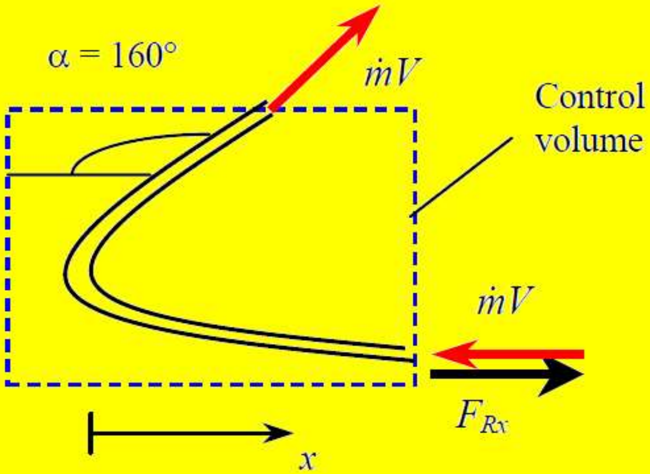
$$F_{\text{breaking}} = \mathbf{8729\ \ N}$$

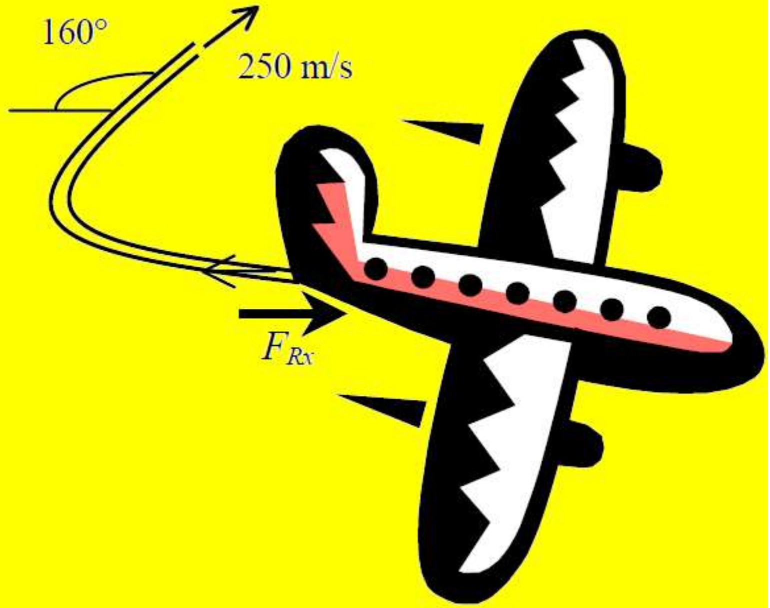
Therefore, a braking force of 8729 N develops in the opposite direction tot flight.

Discussion This problem can be solved more generally by measuring the reversing angle from the direction of exhaust gases (*α = 0* when there is no reversing). When *α < 90°*, the reversed gases are discharged in the negative x direction, and the momentum equation reduces to

$$F_{Rx} = \dot{m}(-V)\cos\alpha - \dot{m}(-V) \quad \rightarrow \quad F_{Rx} = (1-\cos\alpha)\dot{m}V_i$$

This equation is also valid for α >90° since *cos(180°-α) = - cosα*. Using *α = 160°*, for example, gives $F_{Rx} = (1 - cos160)\dot{m}V_i = (1 + cos20)\dot{m}V_i$, which is identical to the solution above.





EXAMPLE 6–49

A 5-cm-diameter horizontal water jet having a velocity of 30 m/s strikes a vertical stationary flat plate. The water splatters in all directions in the plane of the plate. How much force is required to hold the plate against the water stream?

Assumptions **1** The flow is steady and incompressible. **2** The water splatters off the sides of the plate in a plane normal to the jet. **3** The water jet is exposed to the atmosphere, and thus the pressure of the water jet and the splattered water is the atmospheric pressure which is disregarded since it acts on the entire control surface. **4** The vertical forces and momentum fluxes are not considered since they have no effect on the horizontal reaction force. **5** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis We take the plate as the control volume such that it contains the entire plate and cuts through the water jet and the support bar normally, and the direction of flow as the positive direction of x axis. We take the reaction force to be in the negative x direction. The momentum equation for steady one-dimensional flow in the x (flow) direction reduces in this case to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad -F_{Rx} = -\dot{m}_i V_i \quad \rightarrow \quad F_{Rx} = \dot{m} V$$

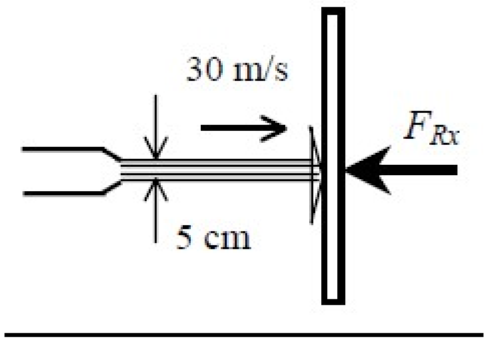
We note that the reaction force acts in the opposite direction to flow, and we should not forget the negative sign for forces and velocities in the negative x-direction. The mass flow rate of water is

$$\dot{m} = \rho\dot{V} = \rho AV = \rho\frac{\pi D^2}{4}V = (1000\ \mathrm{kg/m^3})\frac{\pi(0.05\ \mathrm{m})^2}{4}(30\ \mathrm{m/s}) = 58.90\ \mathrm{kg/s}$$

Substituting, the reaction force is determined to be

$$F_{Rx} = (58.90\ \mathrm{kg/s})(30\ \mathrm{m/s}) = \mathbf{1767\ N}$$

Therefore, a force of 1767 N must be applied to the plate in the opposite direction to flow to hold it in place.



Discussion In reality, some water will be scattered back, and this will add to the reaction force of water.

EXAMPLE 6–50

A 5-cm-diameter horizontal jet of water, with velocity 30 m/s, strikes the tip of a horizontal cone, which deflects the water by 45° from its original direction. How much force is required to hold the cone against the water stream?

Assumptions **1** The flow is steady and incompressible. **2** The water jet is exposed to the atmosphere, and thus the pressure of the water jet before and after the split is the atmospheric pressure which is disregarded since it acts on all surfaces. **3** The gravitational effects are disregarded. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be $1000\ kg/m^3$.

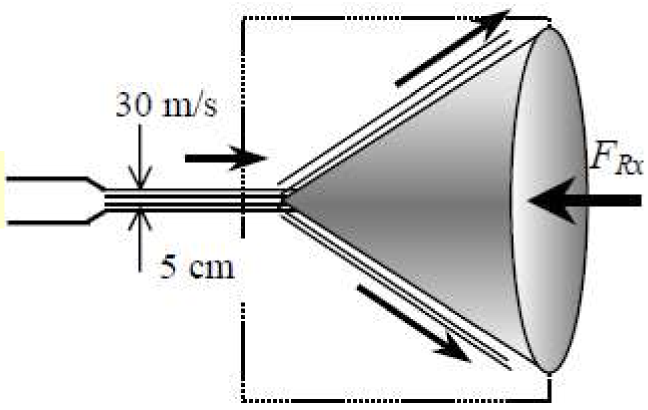
Analysis The mass flow rate of water jet is

$$\dot{m} = \rho\dot{V} = \rho AV = \rho\frac{\pi D^2}{4}V = (1000\ \text{kg/m}^3)\frac{\pi(0.05\ \text{m})^2}{4}(30\ \text{m/s}) = 58.90\ \text{kg/s}$$

We take the diverting section of water jet, including the cone as the control volume, and designate the entrance by 1 and the outlet after divergence by 2. We also designate the horizontal coordinate by x with the direction of flow as being the positive direction and the vertical coordinate by y.

The momentum equation for steady one-dimensional flow is $\sum\vec{F} = \sum_{out}\beta\dot{m}V - \sum_{in}\beta\dot{m}V$. We let the x- and y- components of the anchoring force of the cone be F_{Rx} and F_{Ry}, and assume them to be in the positive directions. Noting that $V_2 = V_1 = V$ and $\dot{m}_2 = \dot{m}_1 = \dot{m}$, the momentum equations along the x and y axes become

$$F_{Rx} = \dot{m}V_2 \cos\theta - \dot{m}V_1 = \dot{m}V(\cos\theta - 1)$$
$$F_{Ry} = 0 \quad \text{(because of symmetry about x axis)}$$



Substituting the given values,

$$F_{Rx} = (58.90\ \text{kg/s})(30\ \text{m/s})(\cos 45° - 1)\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)$$
$$= -\mathbf{518\ N}$$
$$F_{Ry} = \mathbf{0}$$

The negative value for F_{Rx} indicates that the assumed direction is wrong, and should be reversed. Therefore, a force of 518 N must be applied to the cone in the opposite direction to flow to hold it in place. No holding force is necessary in the vertical direction due to symmetry and neglecting gravitational effects.

Discussion In reality, the gravitational effects will cause the upper part of flow to slow down and the lower part to speed up after the split. But for short distances, these effects are negligible.

EXAMPLE 6–51

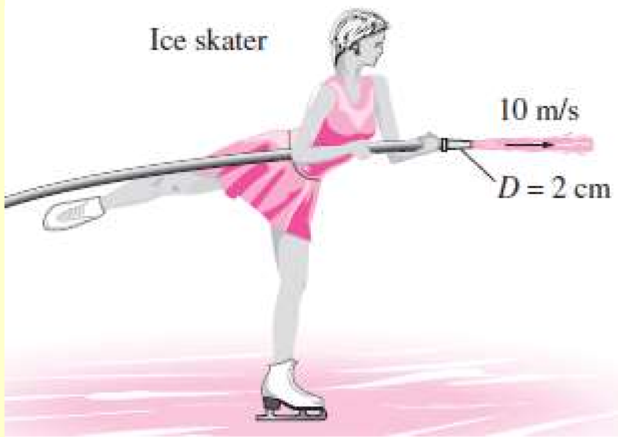
A 60-kg ice skater is standing on ice with ice skates (negligible friction). She is holding a flexible hose (essentially weightless) that directs a 2-cm-diameter stream of water horizontally parallel to her skates. The water velocity at the hose outlet is 10 m/s. If she is initially standing still, determine (a) the velocity of the skater and the distance she travels in 5 s and (b) how long it will take to move 5 m and the velocity at that moment.

Answers: (a) 2.62 m/s, 6.54 m, (b) 4.4 s, 2.3 m/s

Assumptions **1** Friction between the skates and ice is negligible. **2** The flow of water is steady and one-dimensional (but the motion of skater is unsteady). **3** The ice skating arena is level, and the water jet is discharged horizontally. **4** The mass of the hose and the water in it is negligible. **5** The skater is standing still initially at t = 0. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be *1000 kg/m³*.

Analysis (a) The mass flow rate of water through the hose is



$$\dot{m} = \rho A V = \rho \frac{\pi D^2}{4} V = (1000\ \mathrm{kg/m^3}) \frac{\pi (0.02\ \mathrm{m})^2}{4} (10\ \mathrm{m/s}) = 3.14\ \mathrm{kg/s}$$

The thrust exerted on the skater by the water stream is simply the momentum flux of the water stream, and it acts in the reverse direction,

$$F = \text{Thrust} = \dot{m}V = (3.14\ \text{kg/s})(10\ \text{m/s})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right) = 31.4\ \ \text{N}\ \ \text{(constant)}$$

The acceleration of the skater is determined from Newton's 2nd law of motion $F = ma$ where m is the mass of the skater,

$$a = \frac{F}{m} = \frac{31.4\ \text{N}}{60\ \text{kg}}\left(\frac{1\ \text{kg}\cdot\text{m/s}^2}{1\ \text{N}}\right) = 0.523\ \text{m/s}^2$$

Note that thrust and thus the acceleration of the skater is constant. The velocity of the skater and the distance traveled in 5 s are

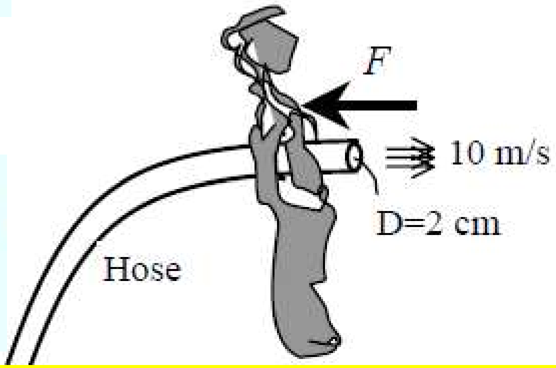
$$V_{\text{skater}} = at = (0.523\ \text{m/s}^2)(5\ \text{s}) = \mathbf{2.62\ m/s}$$

$$x = \tfrac{1}{2}at^2 = \tfrac{1}{2}(0.523\ \text{m/s}^2)(5\ \text{s})^2 = \mathbf{6.54\ m}$$

(b) The time it will take to move 5 m and the velocity at that moment are

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(5\ \text{m})}{0.523\ \text{m/s}^2}} = \mathbf{4.4\ s}$$

$$V_{\text{skater}} = at = (0.523\ \text{m/s}^2)(4.4\ \text{s}) = \mathbf{2.3\ m/s}$$

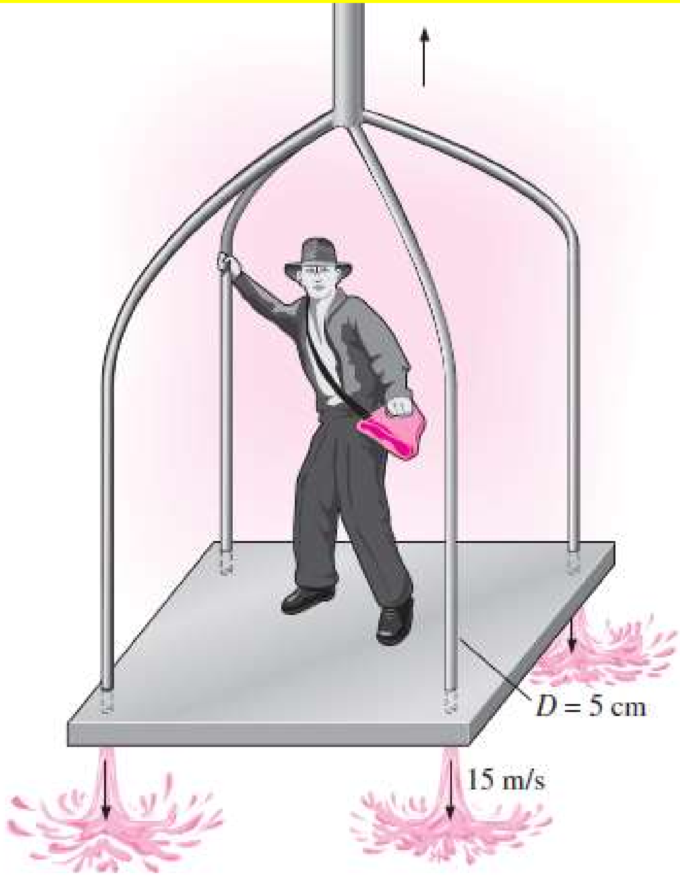


Discussion In reality, the velocity of the skater will be lower because of friction on ice and the resistance of the hose to follow the skater.

EXAMPLE 6–52

Indiana Jones needs to ascend a 10-m-high building. There is a large hose filled with pressurized water hanging down from the building top. He builds a square platform and mounts four 5-cm-diameter nozzles pointing down at each corner. By connecting hose branches, a water jet with 15-m/s velocity can be produced from each nozzle. Jones, the platform, and the nozzles have a combined mass of 150 kg. Determine (a) the minimum water jet velocity needed to raise the system, (b) how long it takes for the system to rise 10 m when the water jet velocity is 15 m/s and the velocity of the platform at that moment, and (c) how much higher will the momentum raise Jones if he shuts off the water at the moment the platform reaches 10 m above the ground. How much time does he have to jump from the platform to the roof?

Answers: (a) 13.7 m/s, (b) 3.2 s, (c) 2.1 m, 1.3 s



Assumptions **1** The air resistance is negligible. **2** The flow of water is steady and one-dimensional (but the motion of platform is unsteady). **3** The platform is still initially at t = 0. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Properties We take the density of water to be $1000\ kg/m^3$.

Analysis (a) The total mass flow rate of water through the 4 hoses and the total weight of the platform are

$$\dot{m} = \rho A V = 4\rho \frac{\pi D^2}{4} V = 4(1000\ \mathrm{kg/m^3}) \frac{\pi (0.05\ \mathrm{m})^2}{4} (15\ \mathrm{m/s}) = 118\ \mathrm{kg/s}$$

$$W = mg = (150\ \mathrm{kg})(9.81\ \mathrm{m/s^2}) \left(\frac{1\ \mathrm{N}}{1\ \mathrm{kg \cdot m/s^2}} \right) = 1472\ \ \mathrm{N}$$

We take the platform as the system. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} V - \sum_{in} \beta \dot{m} V$. The minimum water jet velocity needed to raise the platform is determined by setting the net force acting on the platform equal to zero,

$$-W = \dot{m}(-V_{\min}) - 0 \quad \rightarrow \quad W = \dot{m} V_{\min} = \rho A V_{\min} V_{\min} = 4\rho \frac{\pi D^2}{4} V_{\min}^2$$

Solving for $V_{\min}$ and substituting,

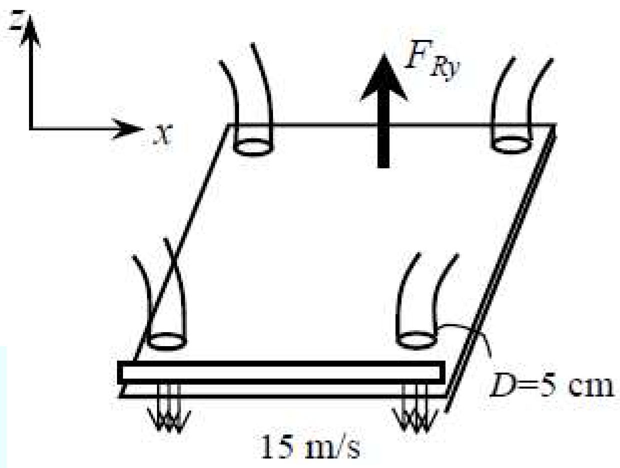
$$F_{Rz} - W = \dot{m}(-V) - 0 = \dot{m}V \quad \rightarrow \quad F_{Rz} = W - \dot{m}V = (1472\text{ N}) - (118\text{ kg/s})(15\text{ m/s})\left(\frac{1\text{kg}\cdot\text{m/s}^2}{1\text{N}}\right) = -298\text{ N}$$

(b) We let the vertical reaction force (assumed upwards) acting on the platform be F_{Rz}. Then the momentum equation in the vertical direction becomes

$$a = \frac{F}{m} = \frac{298\text{ N}}{150\text{ kg}}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = 2.0\text{ m/s}^2$$

$$x = \tfrac{1}{2}at^2 \quad \rightarrow \quad t = \sqrt{\frac{2x}{a}} = \sqrt{\frac{2(10\text{ m})}{2\text{ m/s}^2}} = \mathbf{3.2\text{ s}}$$

$$V = at = (2\text{ m/s}^2)(3.2\text{ s}) = \mathbf{6.4\text{ m/s}}$$



The upward thrust acting on the platform is equal and opposite to this reaction force, and thus *F = 298 N*. Then the acceleration and the ascending time to rise 10 m and the velocity at that moment become

$$V = V_0 - gt = 0 \quad \rightarrow \quad t = \frac{V_0}{g} = \frac{6.4\text{ m/s}}{9.81\text{ m/s}^2} = \mathbf{0.65\text{ s}}$$

$$z = V_0 t - \tfrac{1}{2}gt^2 = (6.4\text{ m/s})(0.65\text{ s}) - \tfrac{1}{2}(9.81\text{m/s}^2)(0.65\text{ s})^2 = \mathbf{2.1\text{m}}$$

Therefore, Jones has 2×0.65 = 1.3 s to jump off from the platform to the roof since it takes another 0.65 s for the platform to descend to the 10 m level.

EXAMPLE 6-53

A soldier jumps from a plane and opens his parachute when his velocity reaches the terminal velocity V_T. The parachute slows him down to his landing velocity of V_F. After the parachute is deployed, the air resistance is proportional to the velocity squared (i.e., $F = kV^2$). The soldier, his parachute, and his gear have a total mass of m. Show that $k = mg/V^2_F$ and develop a relation for the soldier's velocity after he opens the parachute at time $t = 0$.

Answer: $V = V_F \frac{V_T+V_F+(V_T-V_F)e^{-2gt/V_F}}{V_T+V_F-(V_T-V_F)e^{-2gt/V_F}}$

Assumptions **1** The air resistance is proportional to the velocity squared (*i.e.* $F = -kV^2$). **2** The variation of the air properties with altitude is negligible. **3** The buoyancy force applied by air to the person (and the parachute) is negligible because of the small volume occupied and the low density of air. **4** The final velocity of the soldier is equal to its terminal velocity with his parachute open.

Analysis The terminal velocity of a free falling object is reached when the air resistance (or air drag) equals the weight of the object, less the buoyancy force applied by the fluid, which is negligible in this case,

$$F_{\text{air resistance}} = W \quad \rightarrow \quad kV_F^2 = mg \quad \rightarrow \quad k = \frac{mg}{V_F^2}$$

This is the desired relation for the constant of proportionality k. When the parachute is deployed and the soldier starts to decelerate, the net downward force acting on him is his weight less the air resistance,

$$F_{\text{net}} = W - F_{\text{air resistance}} = mg - kV^2 = mg - \frac{mg}{V_F^2}V^2 = mg\left(1 - \frac{V^2}{V_F^2}\right)$$

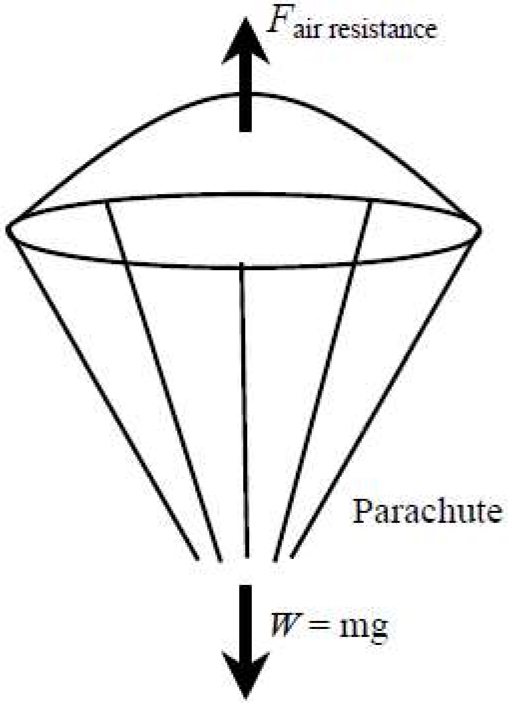
Substituting it into Newton's 2nd law relation $F_{net} = ma = m\frac{dV}{dt}$ gives

$$mg\left(1 - \frac{V^2}{V_F^2}\right) = m\frac{dV}{dt}$$

Canceling m and separating variables, and integrating from *t = 0* when *V* = V_T to *t* = *t* when *V* = *V* gives

$$\frac{dV}{1 - V^2/V_F^2} = g\,dt \quad \rightarrow \quad \int_{V_T}^{V} \frac{dV}{V_F^2 - V^2} = \frac{g}{V_F^2}\int_0^t dt$$

Using $\int \frac{dx}{a^2-x^2} = \frac{1}{2a} \ln \frac{a+x}{a-x}$ from integral tables and applying the integration limits,



$$\frac{1}{2V_F}\left(\ln\frac{V_F+V}{V_F-V} - \ln\frac{V_F+V_T}{V_F-V_T}\right) = \frac{gt}{V_F^2}$$

Rearranging, the velocity can be expressed explicitly as a function of time as

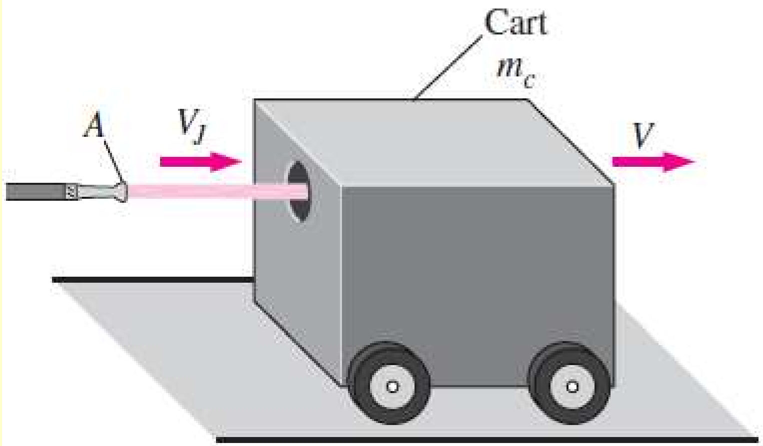
$$V = V_F \frac{V_T + V_F + (V_T - V_F)e^{-2gt/V_F}}{V_T + V_F - (V_T - V_F)e^{-2gt/V_F}}$$

Discussion Note that as t→∞, the velocity approaches the landing velocity of V_F, as expected.

EXAMPLE 6–54

A horizontal water jet with a flow rate of $\dot{V}$ and cross-sectional area of A drives a covered cart of mass m_c along a level and nearly frictionless path. The jet enters a hole at the rear of the cart and all water that enters the cart is retained, increasing the system mass. The relative velocity between the jet of constant velocity V_J and the cart of variable velocity V is $V_J - V$. If the cart is initially empty and stationary when the jet action is initiated, develop a relation (integral form is acceptable) for cart velocity versus time.

Assumptions **1** The flow of water is steady, one-dimensional, incompressible, and horizontal. **2** All the water which enters the cart is retained. **3** The path of the cart is level and frictionless. **4** The cart is initially empty and stationary, and thus $V = 0$ at time $t = 0$. **5** Friction between water jet and air is negligible, and the entire momentum of water jet is used to drive the cart with no losses. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

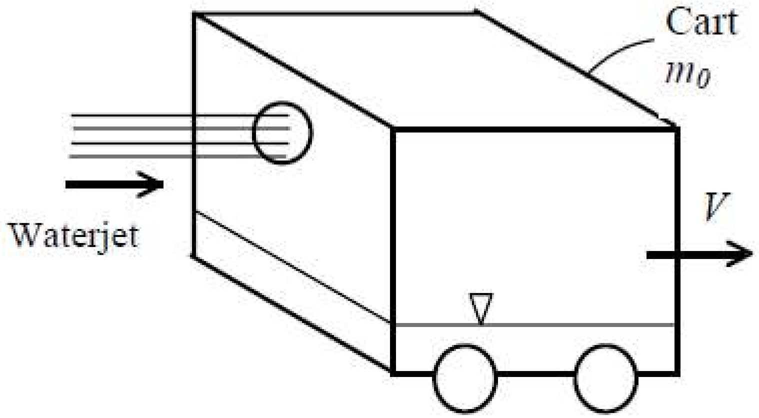


Analysis We note that the water jet velocity V_J is constant, but the car velocity V is variable. Noting that $\dot{m} = \rho A(V_j - V)$ where A is the cross-sectional area of the water jet and V_J - V is the velocity of the water jet relative to the cart, the mass of water in the cart at any time t is

$$m_w = \int_0^t \dot{m}dt = \int_0^t \rho A(V_J - V)dt = \rho A V_J t - \rho A \int_0^t V dt \quad (1)$$

Also,

$$\frac{dm_w}{dt} = \dot{m} = \rho A(V_J - V)$$



We take the cart as the system. The net force acting on the cart in this case is equal to the momentum flux of the water jet. Newton's *2nd* law $F = ma = d(mV)/dt$ in this case can be expressed as

$$F = \frac{d(m_{\text{total}}V)}{dt} \quad \text{where} \quad F = \dot{m}(V_J - V) = \rho A(V_J - V)^2$$

and

$$\frac{d(m_{\text{total}}V)}{dt}=\frac{d[(m_c+m_w)V]}{dt}=m_c\frac{dV}{dt}+\frac{d(m_wV)}{dt}=m_c\frac{dV}{dt}+m_w\frac{dV}{dt}+V\frac{dm_w}{dt}$$
$$=(m_c+m_w)\frac{dV}{dt}+\rho A(V_J-V)V$$

Substituting,

$$\rho A(V_J-V)^2=(m_c+m_w)\frac{dV}{dt}+\rho A(V_J-V)V \quad\rightarrow\quad \rho A(V_J-V)(V_J-2V)=(m_c+m_w)\frac{dV}{dt}$$

Noting that m^w is a function of t (as given by Eq. 1) and separating variables,

$$\frac{dV}{\rho A(V_J-V)(V_J-2V)}=\frac{dt}{m_c+m_w} \quad\rightarrow\quad \frac{dV}{\rho A(V_J-V)(V_J-2V)}=\frac{dt}{m_c+\rho AV_Jt-\rho A\int_0^t Vdt}$$

Integrating from *t = 0* when *V = 0* to *t = t* when *V = V* gives the desired integral,

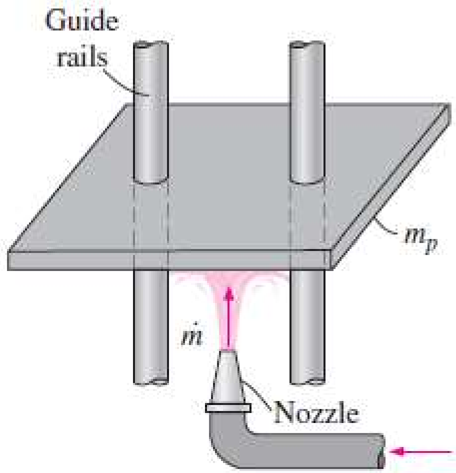
$$\int_0^V\frac{dV}{\rho A(V_J-V)(V_J-2V)}=\int_o^t\frac{dt}{m_c+\rho AV_Jt-\rho A\int_0^t Vdt}$$

Discussion Note that the time integral involves the integral of velocity, which complicates the solution.

EXAMPLE 6–55

Nearly frictionless vertical guide rails maintain a plate of mass m_p in a horizontal position, such that it can slide freely in the vertical direction. A nozzle directs a water stream of area A against the plate underside. The water jet splatters in the plate plane, applying an upward force against the plate. The water flow rate $\dot{m}$ (kg/s) can be controlled. Assume that distances are short, so the velocity of the rising jet can be considered constant with height. (a) Determine the minimum mass flow rate $\dot{m}_{min}$ necessary to just levitate the plate and obtain a relation for the steady-state velocity of the upward moving plate for $\dot{m} > \dot{m}_{min}$. (b) At time $t = 0$, the plate is at rest, and the water jet with $\dot{m} > \dot{m}_{min}$ is suddenly turned on. Apply a force balance to the plate and obtain the integral that relates velocity to time (do not solve).

Assumptions **1** The flow of water is steady and one-dimensional. **2** The water jet splatters in the plane of he plate. **3** The vertical guide rails are frictionless. **4** Times are short, so the velocity of the rising jet can be considered to remain constant with height. **5** At time $t = 0$, the plate is at rest. **6** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.



Analysis (a) We take the plate as the system. The momentum equation for steady one-dimensional flow is $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. Noting that $\dot{m} = \rho A V_j$ where A is the cross-sectional area of the water jet and $W = m_p g$, the minimum mass flow rate of water needed to raise the plate is determined by setting the net force acting on the plate equal to zero,

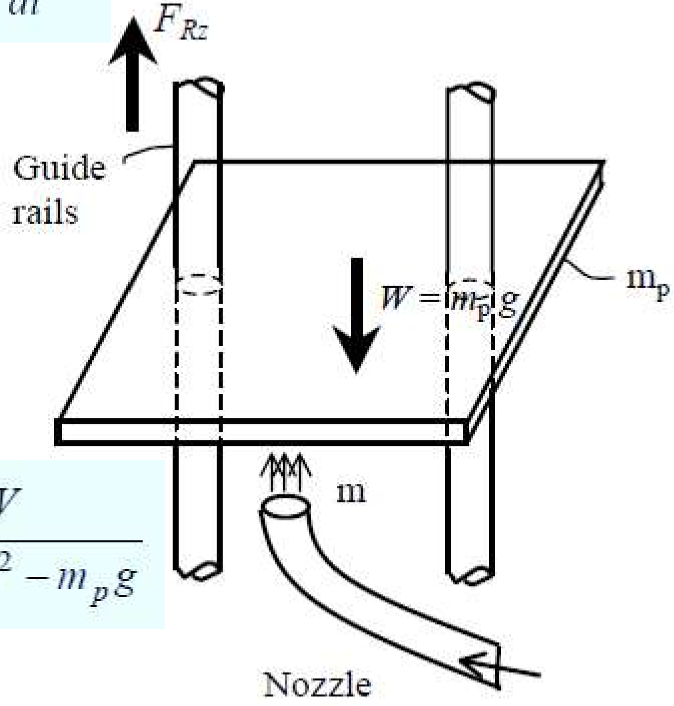
$$-W = 0 - \dot{m}_{\min} V_J \quad \rightarrow \quad W = \dot{m}_{\min} V_J \rightarrow \quad m_p g = \dot{m}_{\min} (\dot{m}_{\min} / A V_J) \rightarrow \quad \dot{m}_{\min} = \sqrt{\rho A m_p g}$$

For $\dot{m} > \dot{m}_{\min}$, a relation for the steady state upward velocity V is obtained setting the upward impulse applied by water jet to the weight of the plate (during steady motion, the plate velocity V is constant, and the velocity of water jet relative to plate is $V_J - V$),

$$W = \dot{m}(V_J - V) \quad \rightarrow \quad m_p g = \rho A (V_J - V)^2 \quad \rightarrow \quad V_J - V = \sqrt{\frac{m_p g}{\rho A}} \quad \rightarrow \quad V = \frac{\dot{m}}{\rho A} - \sqrt{\frac{m_p g}{\rho A}}$$

(b) At time $t = 0$ the plate is at rest $(V = 0)$, and it is subjected to water jet with $\dot{m} > \dot{m}_{\min}$ thus the net force acting on it is greater than the weight of the plate, and the difference between the jet impulse and the weight will accelerate the plate upwards. Therefore, Newton's 2nd law $F = ma = mdV/dt$ in this case can be expressed as

$$\dot{m}(V_J - V) - W = m_p a \quad \rightarrow \quad \rho A(V_J - V)^2 - m_p g = m_p \frac{dV}{dt}$$



Separating the variables and integrating from *t = 0* when *V = 0* to *t = t* when *V = V* gives the desired integral,

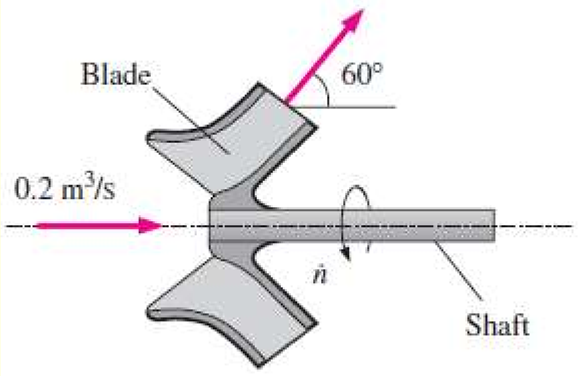
$$\int_0^V \frac{m_p dV}{\rho A(V_J - V)^2 - m_p g} = \int_{t=0}^{t} dt \quad \rightarrow \quad t = \int_0^V \frac{m_p dV}{\rho A(V_J - V)^2 - m_p g}$$

Discussion This integral can be performed with the help of integral tables. But the relation obtained will be implicit in *V*.

EXAMPLE 6–56

Water enters a mixed flow pump axially at a rate of *0.2 m^3/s* and at a velocity of 5 m/s, and is discharged to the atmosphere at an angle of 60° from the horizontal, as shown in Fig. P6–71. If the discharge flow area is half the inlet area, determine the force acting on the shaft in the axial direction.

Assumptions **1** The flow is steady and incompressible. **2** The forces acting on the piping system in the horizontal direction are negligible. **3** The atmospheric pressure is disregarded since it acts on all surfaces. **4** Water flow is nearly uniform at the outlet and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.



Properties We take the density of water to be *1000 kg/m^3*.

Analysis From conservation of mass we have $\dot{m}_1 = \dot{m}_2 = \dot{m}$, and $A_{c1}V_1$ = Ac2V2 thus and . Noting that the discharge area is half the inlet area, the discharge velocity is twice the inlet velocity. That is,

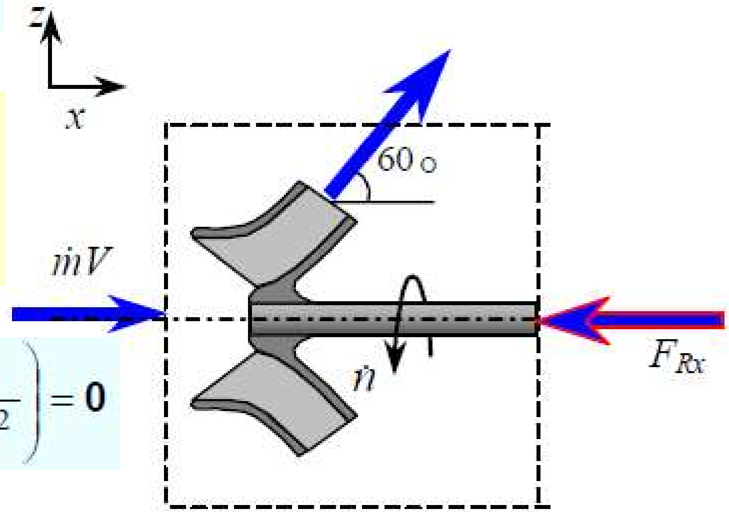
$$A_{c1}V_2 = \frac{A_{c1}}{A_{c2}}V_1 = 2V_1 = 2(5\text{ m/s}) = 10\text{ m/s}$$

We take the pump as the control volume, and the inlet direction of flow as the positive direction of x axis. The linear momentum equation in this case in the x direction reduces to

$$\sum \vec{F} = \sum_{\text{out}} \beta \dot{m} \vec{V} - \sum_{\text{in}} \beta \dot{m} \vec{V} \quad \rightarrow \quad -F_{Rx} = \dot{m} V_2 \cos\theta - \dot{m} V_1 \quad \rightarrow \quad F_{Rx} = \dot{m}(V_1 - V_2 \cos\theta)$$

where the mass flow rate it

$$\dot{m} = \rho \dot{V} = (1000 \text{ kg/m}^3)(0.20 \text{ m}^3/\text{s}) = 200 \text{ kg/s}$$



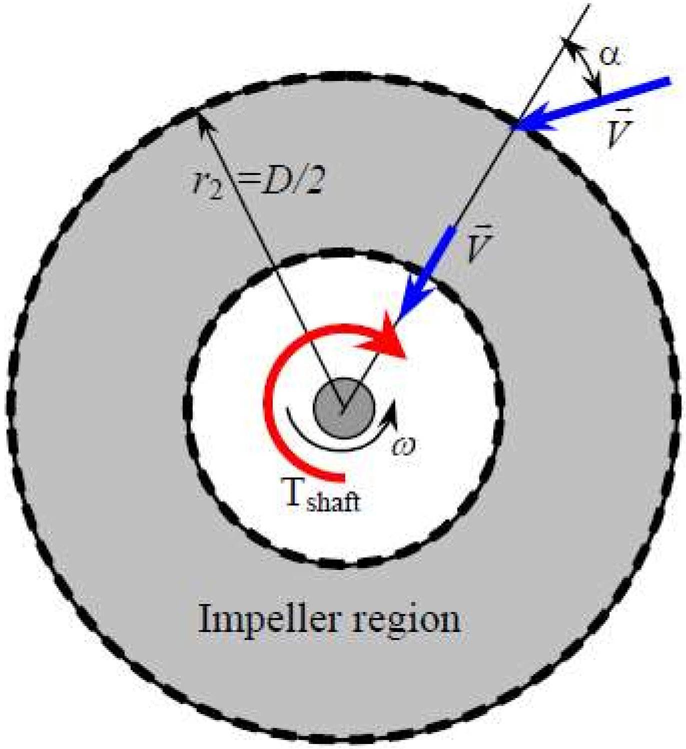
Substituting the known quantities, the reaction force is determined to be (note that cos60° = 0.5)

$$F_{\text{Rx}} = (200 \text{ kg/s})[(5 \text{ m/s}) - (10 \text{ m/s})\cos 60]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right) = \mathbf{0}$$

Discussion Note that at this angle of discharge, the bearing is not subjected to any horizontal loading. Therefore, the loading in the system can be controlled by adjusting the discharge angle.

EXAMPLE 6–57

Water accelerated by a nozzle enters the impeller of a turbine through its outer edge of diameter D with a velocity of V making an angle α with the radial direction at a mass flow rate of $\dot{m}$. Water leaves the impeller in the radial direction. If the angular speed of the turbine shaft is $\dot{n}$, show that the maximum power that can be generated by this radial turbine is $\dot{W}_{shaft} = \pi \dot{n} \dot{m} D V \sin \alpha$.



Assumptions **1** The flow is steady in the mean. **2** Irreversible losses are negligible.
Analysis We take the impeller region as the control volume. The tangential velocity components at the inlet and the outlet are $V_{1,t} = 0$ and $V_{2,t} = V \sin \alpha$.
Normal velocity components as well pressure acting on the inner and outer circumferential areas pass through the shaft center, and thus they do not contribute to torque. Only the tangential velocity components contribute to torque, and the application of the angular momentum equation gives

$$\mathrm{T}_{\text{shaft}} = \dot{m}(r_2 V_{2,t} - r_1 V_{1,t}) = \dot{m} r_2 V_{2,t} - 0 = \dot{m} D (V \sin \alpha)/2$$

The angular velocity of the propeller is $\omega = 2\pi\dot{n}$. Then the shaft power becomes

$$\dot{W}_{\text{shaft}} = \omega \mathrm{T}_{\text{shaft}} = 2\pi \dot{n} \dot{m} D (V \sin \alpha)/2$$

Simplifying, the maximum power generated becomes

$$\dot{W}_{\text{shaft}} = \pi \dot{n} \dot{m} D V \sin \alpha$$

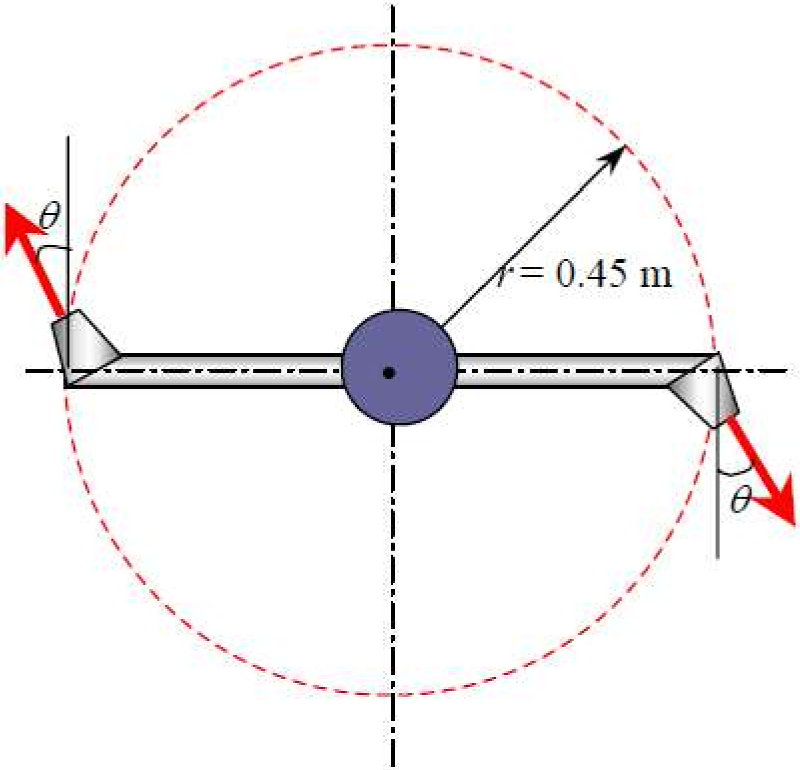
which is the desired relation.

EXAMPLE 6–58

Water enters a two-armed lawn sprinkler along the vertical axis at a rate of 60 L/s, and leaves the sprinkler nozzles as 2-cm diameter jets at an angle of θ from the tangential direction, as shown in Fig. P6–73. The length of each sprinkler arm is 0.45 m. Disregarding any frictional effects, determine the rate of rotation $\dot{n}$ of the sprinkler in rev/min for (*a*) θ = 0°, (*b*) θ = 30°, and (*c*) θ = 60°.

Assumptions **1** The flow is uniform and cyclically steady (i.e., steady from a frame of reference rotating with the sprinkler head). **2** The water is discharged to the atmosphere, and thus the gage pressure at the nozzle outlet is zero. **3** Frictional effects and air drag of rotating components are neglected. **4** The nozzle diameter is small compared to the moment arm, and thus we use average values of radius and velocity at the outlet.

Properties We take the density of water to be *1000 kg/m³ = 1 kg/L*.



Analysis We take the disk that encloses the sprinkler arms as the control volume, which is a stationary control volume. The conservation of mass equation for this steady flow system is $\dot{m}_1 = \dot{m}_2 = \dot{m}$. Noting that the two nozzles are identical, we have $\dot{m}_{nozzle} = \dot{m}/2$ or $\dot{\boldsymbol{V}}_{nozzle} = \dot{\boldsymbol{V}}_{total}/2$ since the density of water is constant. The average jet outlet velocity relative to the nozzle is

$$V_{jet} = \frac{\dot{V}_{nozzle}}{A_{jet}} = \frac{60\ \mathrm{L/s}}{2[\pi(0.02\ \mathrm{m})^2/4]}\left(\frac{1\ \mathrm{m}^3}{1000\ \mathrm{L}}\right) = 95.49\ \mathrm{m/s}$$

The angular momentum equation can be expressed as $\sum \vec{F} = \sum_{out} \beta \dot{m} \vec{V} - \sum_{in} \beta \dot{m} \vec{V}$. Noting that there are no external moments acting, the angular momentum equation about the axis of rotation becomes

$$0 = -2r\dot{m}_{nozzle} V_r \cos\theta \quad \rightarrow \quad V_r = 0 \quad \rightarrow \quad V_{jet,t} - V_{nozzle} = 0$$

Noting that the tangential component of jet velocity is $V_{jet,t} = V_{jet} \cos\theta$, we have

$$V_{nozzle} = V_{jet} \cos\theta = (95.49\ \mathrm{m/s})\cos\theta$$

Also noting that $V_{nozzle} = \omega r = 2\pi\dot{n}r$, and angular speed and the rate of rotation of sprinkler head become

$$1)\ \theta = 0°:\ \omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\ \text{m/s})\cos 0}{0.45\ \text{m}} = \mathbf{212\ rad/s} \quad \text{and} \quad \dot{n} = \frac{\omega}{2\pi} = \frac{212\ \text{rad/s}}{2\pi}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{2026\ rpm}$$

$$2)\ \theta = 30°:\ \omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\ \text{m/s})\cos 30°}{0.45\ \text{m}} = \mathbf{184\ rad/s} \quad \text{and} \quad \dot{n} = \frac{\omega}{2\pi} = \frac{184\ \text{rad/s}}{2\pi}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{1755\ rpm}$$

$$3)\ \theta = 60°:\ \omega = \frac{V_{\text{nozzle}}}{r} = \frac{(95.49\ \text{m/s})\cos 60°}{0.45\ \text{m}} = \mathbf{106\ rad/s} \quad \text{and} \quad \dot{n} = \frac{\omega}{2\pi} = \frac{106\ \text{rad/s}}{2\pi}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{1013\ rpm}$$

Discussion The rate of rotation in reality will be lower because of frictional effects and air drag.

EXAMPLE 6–59

A stationary water tank of diameter *D* is mounted on wheels and is placed on a nearly frictionless level surface. A smooth hole of diameter D_o near the bottom of the tank allows water to jet horizontally and rearward and the water jet force propels the system forward. The water in the tank is much heavier than the tank-and-wheel assembly, so only the mass of water remaining in the tank needs to be considered in this problem. Considering the decrease in the mass of water with time, develop relations for (*a*) the acceleration, (*b*) the velocity, and (*c*) the distance traveled by the system as a function of time.

Assumptions **1** The orifice has a smooth entrance, and thus the frictional losses are negligible. **2** The flow is steady, incompressible, and irrotational (so that the Bernoulli equation is applicable). **3** The surface under the wheeled tank is level and frictionless. **4** The water jet is discharged horizontally and rearward. **5** The mass of the tank and wheel assembly is negligible compared to the mass of water in the tank. **4** Jet flow is nearly uniform and thus the momentum-flux correction factor can be taken to be unity, $\beta \cong 1$.

Analysis *(a)* We take point 1 at the free surface of the tank, and point 2 at the outlet of the hole, which is also taken to be the reference level ($z_2 = 0$) so that the water height above the hole at any time is z. Noting that the fluid velocity at the free surface is very low ($V_1 \cong 0$), it is open to the atmosphere ($P_1 = P_{atm}$), and water discharges into the atmosphere (and thus $P_2 = P_{atm}$), the Bernoulli equation simplifies to

$$\frac{P_1}{\rho g}+\frac{V_1^2}{2g}+z_1=\frac{P_2}{\rho g}+\frac{V_2^2}{2g}+z_2 \quad\rightarrow\quad z=\frac{V_J^2}{2g}+0 \quad\rightarrow\quad V_J=\sqrt{2gz}$$

The discharge rate of water from the tank through the hole is

$$\dot{m}=\rho A V_J=\rho\frac{\pi D_0^2}{4}V_J=\rho\frac{\pi D_0^2}{4}\sqrt{2gz}$$

The momentum equation for steady one-dimensional flow is $\sum\vec{F}=\sum_{out}\beta\dot{m}\vec{V}-\sum_{in}\beta\dot{m}\vec{V}$. Applying it to the water tank, the horizontal force that acts on the tank is determined to be

$$F=\dot{m}V_e-0=\dot{m}V_J=\rho\frac{\pi D_0^2}{4}2gz=\rho gz\frac{\pi D_0^2}{2}$$

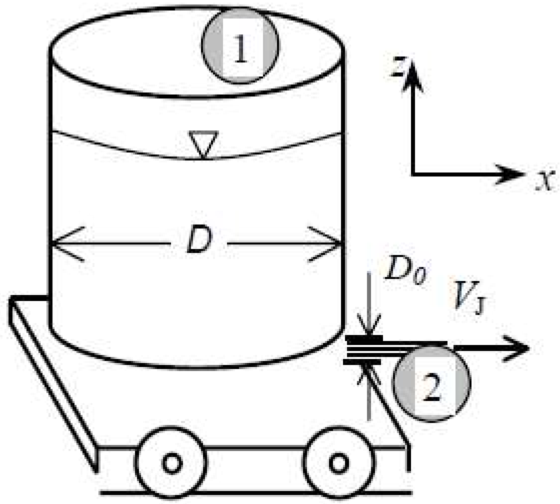
The acceleration of the water tank is determined from Newton's *2nd* law of motion F = ma where m is the mass of water in the tank, $m = \rho \boldsymbol{V}_{\text{tank}} = \rho(\pi D^2/4)z$,

$$a=\frac{F}{m}=\frac{\rho gz(\pi D_0^2/2)}{\rho z(\pi D^2/4)} \quad\rightarrow\quad a=2g\frac{D_0^2}{D^2}$$

Note that the acceleration of the tank is constant.

(b) Noting that *a*= *dV/dt* and thus *dV* = *adt* and acceleration *a* is constant, the velocity is expressed as

$$V = at \qquad \rightarrow \qquad V = 2g\frac{D_0^2}{D^2}t$$



(c) Noting that *V* = *dx/dt* and thus dx = Vdt, the distance traveled by the water tank is determined by integration to be

$$dx = Vdt \qquad \rightarrow \qquad dx = 2g\frac{D_0^2}{D^2}tdt \qquad \rightarrow \qquad x = g\frac{D_0^2}{D^2}t^2$$

since $x = 0$ at $t = 0$.

Discussion In reality, the flow rate discharge velocity and thus the force acting on the water tank will be less because of the frictional losses at the hole. But these losses can be accounted for by incorporating a discharge coefficient.