Heat and Mass Transfer, 3rd Edition
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CHAPTER 6
FUNDAMENTALS OF CONVECTION

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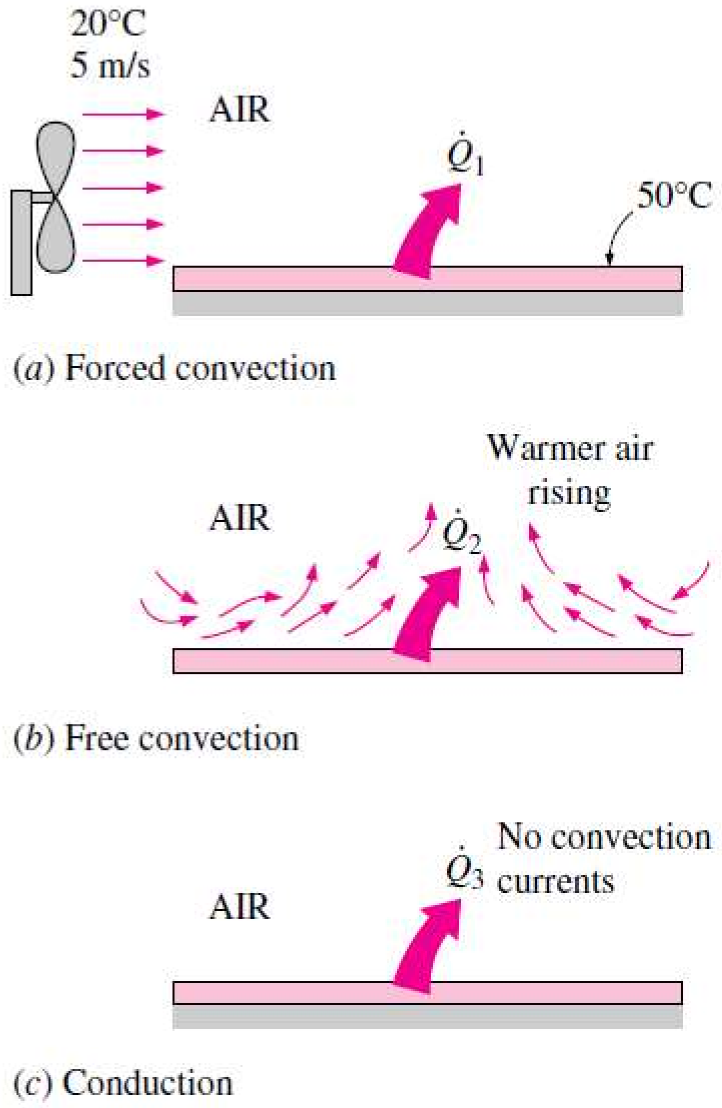
Objectives

- Understand the physical mechanism of convection and its classification
- Visualize the development of thermal boundary layer during flow over surfaces
- Gain a working knowledge of the dimensionless Reynolds, Prandtl, and Nusselt numbers
- Distinguish between laminar and turbulent flows, and gain an understanding of the mechanisms of momentum and heat transfer in turbulent flow,
- Derive the differential equations that govern convection on the basis of mass, momentum, and energy balances, and solve these equations for some simple cases such as laminar flow over a flat plate,
- Nondimensionalize the convection equations and obtain the functional forms of friction and heat transfer coefficients, and
- Use analogies between momentum and heat transfer, and determine heat transfer coefficient from knowledge of friction coefficient.

PHYSICAL MECHANISM OF CONVECTION



Heat transfer from a hot surface to the surrounding fluid by convection and conduction.

Conduction and convection both require the presence of a material medium but convection requires fluid motion.

Convection involves fluid motion as well as heat conduction.

Heat transfer through a solid is always by conduction.

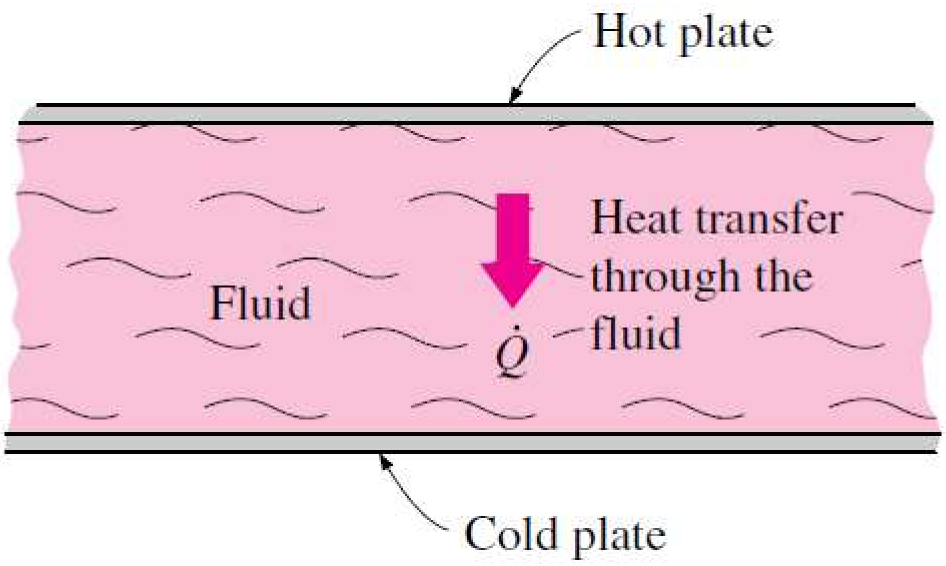
Heat transfer through a fluid is by convection in the presence of bulk fluid motion and by conduction in the absence of it.

Therefore, conduction in a fluid can be viewed as the limiting case of convection, corresponding to the case of quiescent fluid.

The fluid motion enhances heat transfer, since it brings warmer and cooler chunks of fluid into contact, initiating higher rates of conduction at a greater number of sites in a fluid.

The rate of heat transfer through a fluid is much higher by convection than it is by conduction.

In fact, the higher the fluid velocity, the higher the rate of heat transfer.



Heat transfer through a fluid sandwiched between two parallel plates.

Convection heat transfer strongly depends on the fluid properties *dynamic viscosity*, *thermal conductivity, density*, and *specific heat,* as well as the *fluid velocity*.

It also depends on the *geometry* and the *roughness* of the solid surface, in addition to the *type of fluid flow* (such as being streamlined or turbulent).

Newton's law of cooling

$$\dot{q}_{conv} = h(T_s - T_\infty) \qquad (\mathrm{W/m^2})$$

$$\dot{Q}_{conv} = hA_s(T_s - T_\infty) \qquad (\mathrm{W})$$

h = convection heat transfer coefficient, $\mathrm{W/m^2 \cdot {}^\circ C}$

A_s = heat transfer surface area, $\mathrm{m^2}$

T_s = temperature of the surface, °C

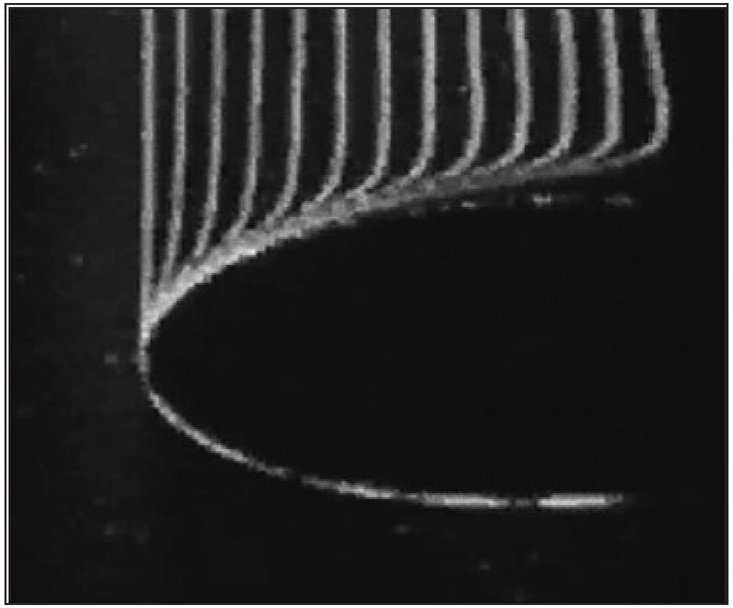
T_∞ = temperature of the fluid sufficiently far from the surface, °C

Convection heat transfer coefficient, *h*: The rate of heat transfer between a solid surface and a fluid per unit surface area per unit temperature difference.

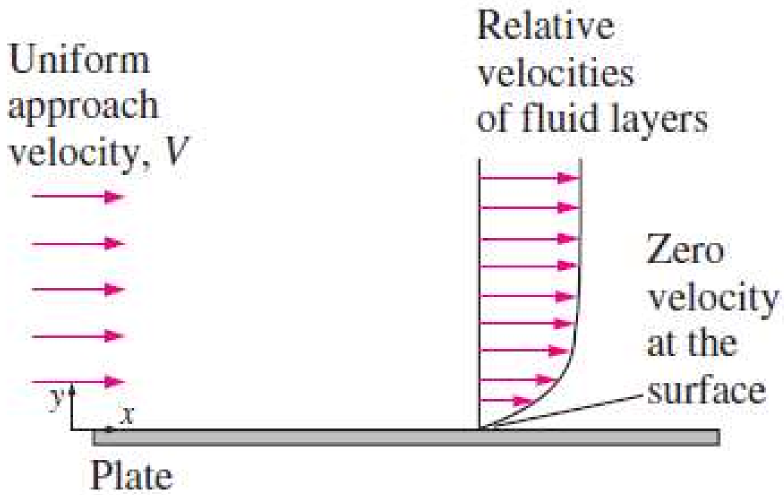
No-slip condition: A fluid in direct contact with a solid “sticks” to the surface due to viscous effects, and there is no slip.

Boundary layer: The flow region adjacent to the wall in which the viscous effects (and thus the velocity gradients) are significant.

The fluid property responsible for the no-slip condition and the development of the boundary layer is *viscosity*.



The development of a velocity profile due to the no-slip condition as a fluid flows over a blunt nose.



A fluid flowing over a stationary surface comes to a complete stop at the surface because of the no-slip condition.

An implication of the no-slip condition is that heat transfer from the solid surface to the fluid layer adjacent to the surface is by *pure conduction,* since the fluid layer is motionless, and can be expressed as

$$\dot{q}_{\text{conv}} = \dot{q}_{\text{cond}} = -k_{\text{fluid}} \left.\frac{\partial T}{\partial y}\right|_{y=0} \qquad (\text{W/m}^2)$$

The determination of the *convection heat transfer coefficient* when the temperature distribution within the fluid is known

$$h = \frac{-k_{\text{fluid}}(\partial T/\partial y)_{y=0}}{T_s - T_\infty} \qquad (\text{W/m}^2 \cdot {}^\circ\text{C})$$

The convection heat transfer coefficient, in general, varies along the flow (or *x*-) direction. The ***average or mean*** convection heat transfer coefficient for a surface in such cases is determined by properly averaging the *local* convection heat transfer coefficients over the entire surface area A_s or length L as

$$h = \frac{1}{A_s}\int_{A_s} h_{\text{local}}\, dA_s \quad \text{and} \quad h = \frac{1}{L}\int_0^L h_x\, dx$$

Nusselt Number

In convection studies, it is common practice to nondimensionalize the governing equations and combine the variables, which group together into *dimensionless numbers* in order to reduce the number of total variables.

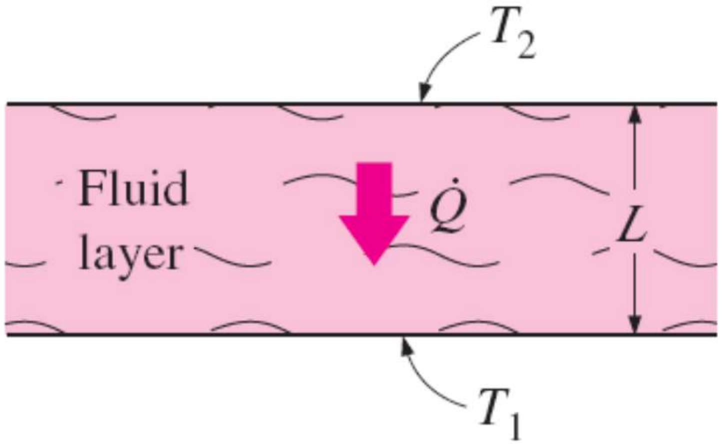
Nusselt number: Dimensionless convection heat transfer coefficient

$$\mathrm{Nu} = \frac{hL_c}{k}$$

L_c characteristic length

$$\dot{q}_{\mathrm{conv}} = h\Delta T \qquad \dot{q}_{\mathrm{cond}} = k\frac{\Delta T}{L}$$

$$\boxed{\frac{\dot{q}_{\mathrm{conv}}}{\dot{q}_{\mathrm{cond}}} = \frac{h\Delta T}{k\Delta T/L} = \frac{hL}{k} = \mathrm{Nu}}$$



$$\Delta T = T_2 - T_1$$

Heat transfer through a fluid layer of thickness L and temperature difference ΔT.

The Nusselt number represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer.

The larger the Nusselt number, the more effective the convection.

A Nusselt number of Nu = 1 for a fluid layer represents heat transfer across the layer by pure conduction.

Convection in daily life

We resort to forced convection whenever we need to increase the rate of heat transfer.

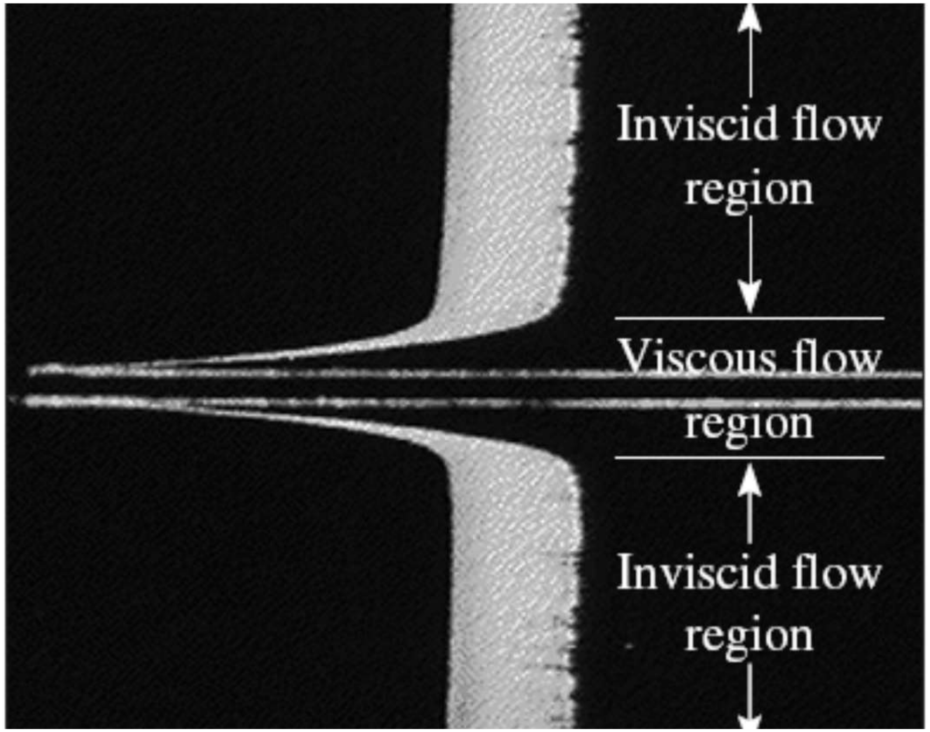
- We turn on the fan on hot summer days to help our body cool more effectively. The higher the fan speed, the better we feel.
- We *stir* our soup and *blow* on a hot slice of pizza to make them cool faster.
- The air on windy winter days feels much colder than it actually is.
- The simplest solution to heating problems in electronics packaging is to use a large enough fan.

CLASSIFICATION OF FLUID FLOWS

Viscous versus Inviscid Regions of Flow

Viscous flows: Flows in which the frictional effects are significant.

Inviscid flow regions: In many flows of practical interest, there are *regions* (typically regions not close to solid surfaces) where viscous forces are negligibly small compared to inertial or pressure forces.

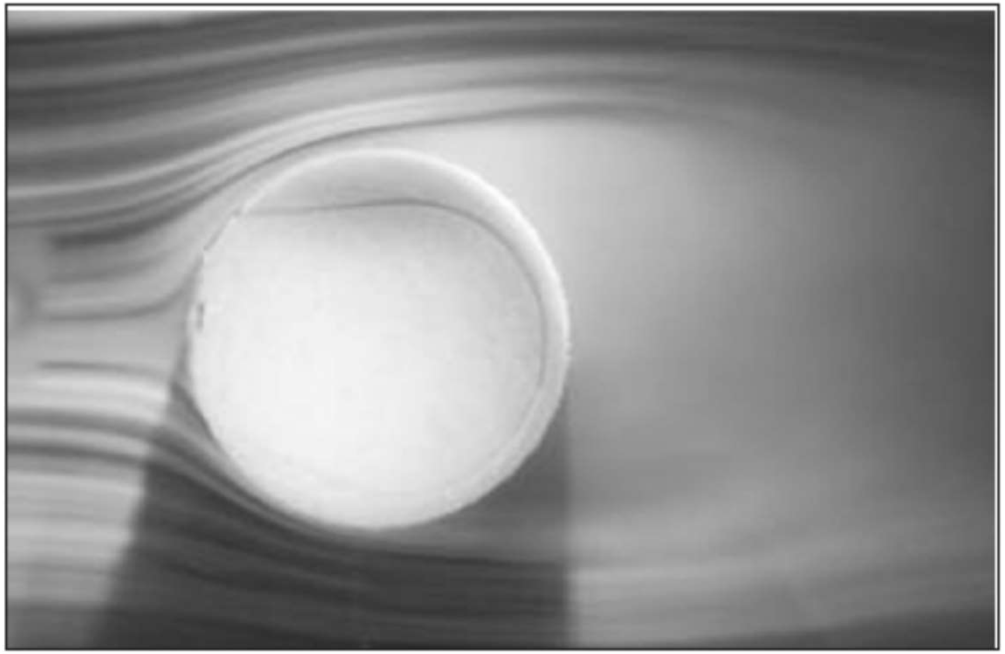


The flow of an originally uniform fluid stream over a flat plate, and the regions of viscous flow (next to the plate on both sides) and inviscid flow (away from the plate).

Internal versus External Flow

External flow: The flow of an unbounded fluid over a surface such as a plate, a wire, or a pipe.

Internal flow: The flow in a pipe or duct if the fluid is completely bounded by solid surfaces.



External flow over a tennis ball, and the turbulent wake region behind.

- Water flow in a pipe is internal flow, and airflow over a ball is external flow .
- The flow of liquids in a duct is called *open-channel flow* if the duct is only partially filled with the liquid and there is a free surface.

Compressible versus Incompressible Flow

Incompressible flow: If the density of flowing fluid remains nearly constant throughout (e.g., liquid flow).

Compressible flow: If the density of fluid changes during flow (e.g., high-speed gas flow)

When analyzing rockets, spacecraft, and other systems that involve high-speed gas flows, the flow speed is often expressed by Mach number

$$\mathrm{Ma} = \frac{V}{c} = \frac{\text{Speed of flow}}{\text{Speed of sound}}$$

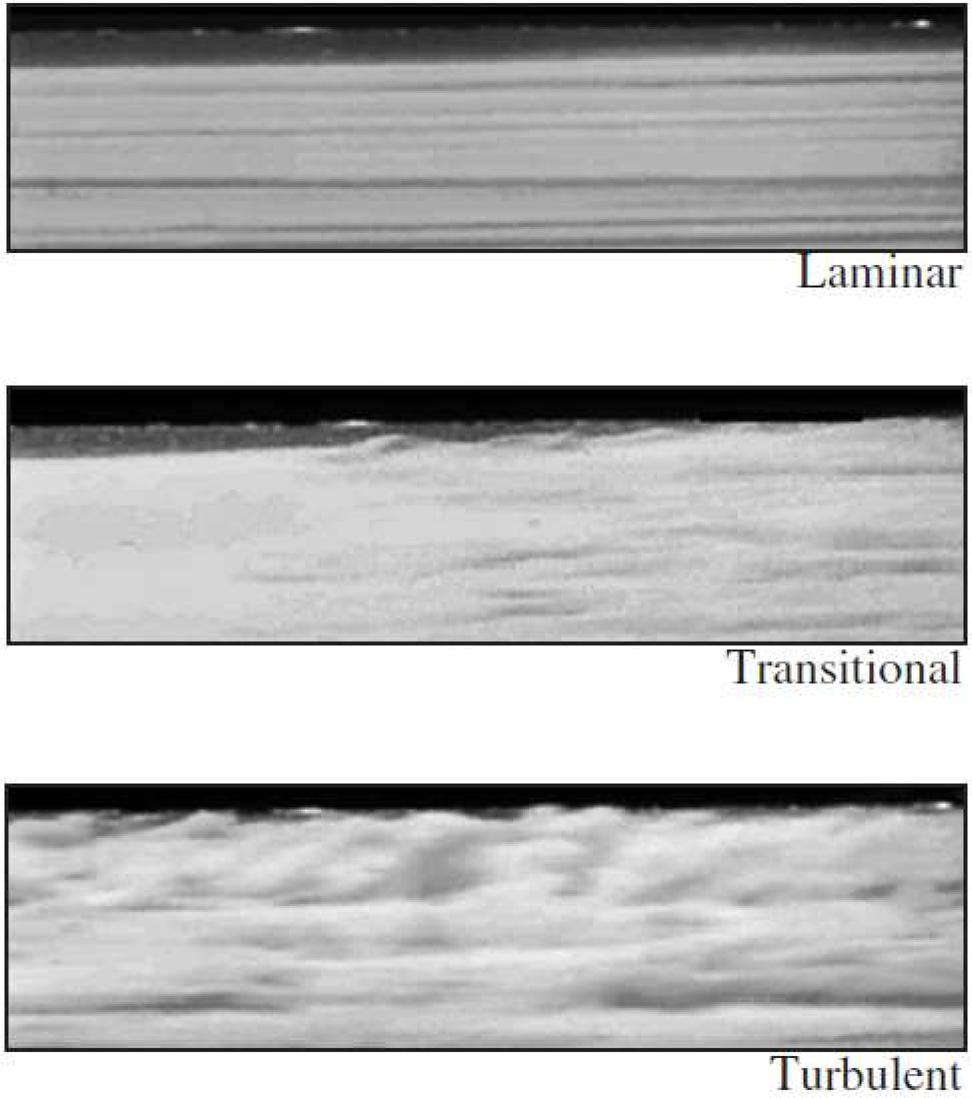
Ma = 1	Sonic flow
Ma < 1	Subsonic flow
Ma > 1	Supersonic flow
Ma >> 1	Hypersonic flow

Laminar versus Turbulent Flow

Laminar flow: The highly ordered fluid motion characterized by smooth layers of fluid. The flow of high-viscosity fluids such as oils at low velocities is typically laminar.

Turbulent flow: The highly disordered fluid motion that typically occurs at high velocities and is characterized by velocity fluctuations. The flow of low-viscosity fluids such as air at high velocities is typically turbulent.

Transitional flow: A flow that alternates between being laminar and turbulent.



Laminar, transitional, and turbulent flows.

Natural (or Unforced) versus Forced Flow

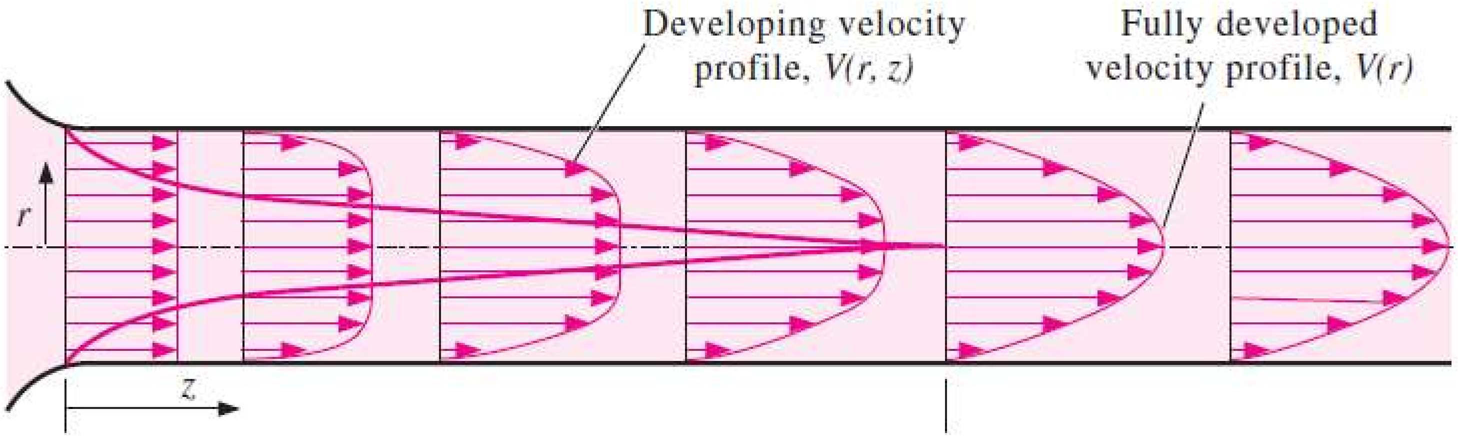
Forced flow: A fluid is forced to flow over a surface or in a pipe by external means such as a pump or a fan.

Natural flow: Fluid motion is due to natural means such as the buoyancy effect, which manifests itself as the rise of warmer (and thus lighter) fluid and the fall of cooler (and thus denser) fluid.

In this schlieren image, the rise of lighter, warmer air adjacent to her body indicates that humans and warm-blooded animals are surrounded by thermal plumes of rising warm air.

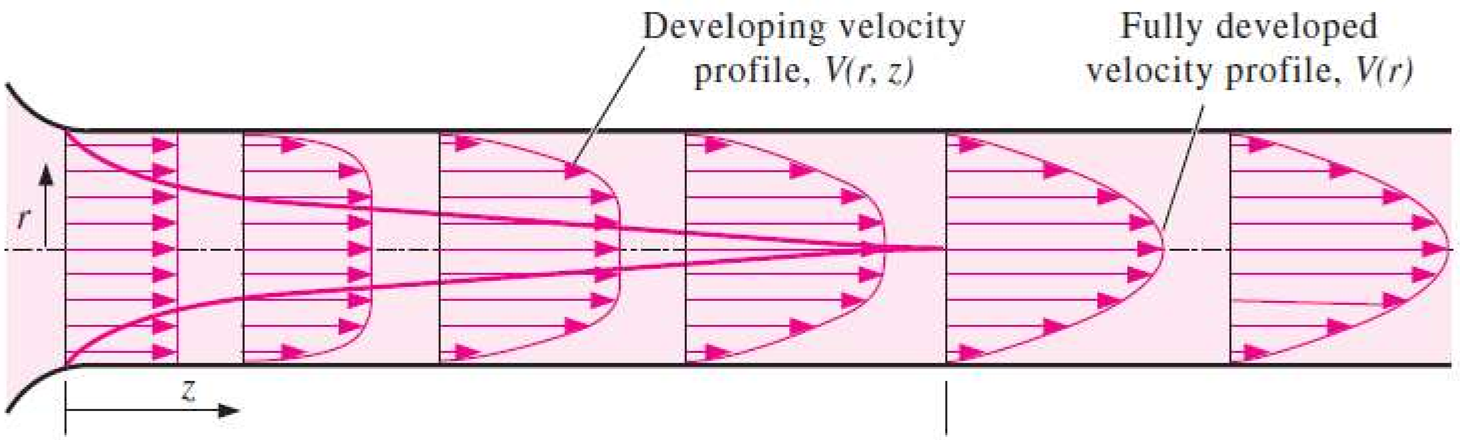
One-, Two-, and Three-Dimensional Flows

- A flow field is best characterized by its velocity distribution.
- A flow is said to be one-, two-, or three-dimensional if the flow velocity varies in one, two, or three dimensions, respectively.
- However, the variation of velocity in certain directions can be small relative to the variation in other directions and can be ignored.

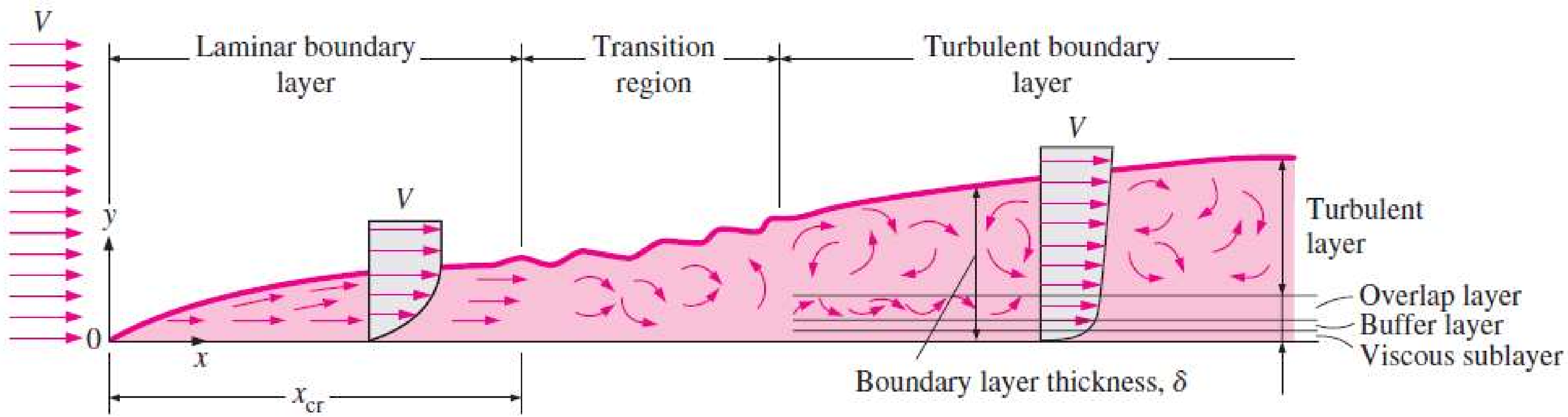


The development of the velocity profile in a circular pipe. $V = V(r, z)$ and thus the flow is two-dimensional in the entrance region, and becomes one-dimensional downstream when the velocity profile fully develops and remains unchanged in the flow direction, $V = V(r)$.

VELOCİTY BOUNDARY LAYER



The region of the flow above the plate bounded by δ is called the velocity boundary layer.

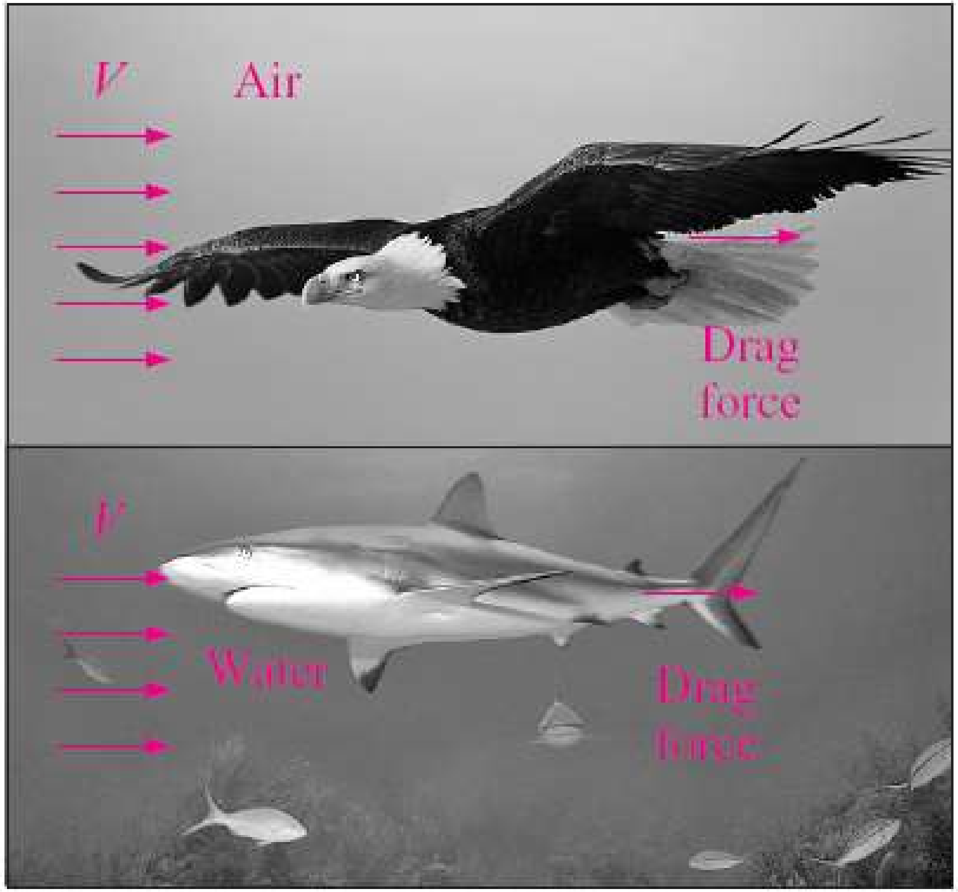


The development of the boundary layer for flow over a flat plate, and the different flow regimes.

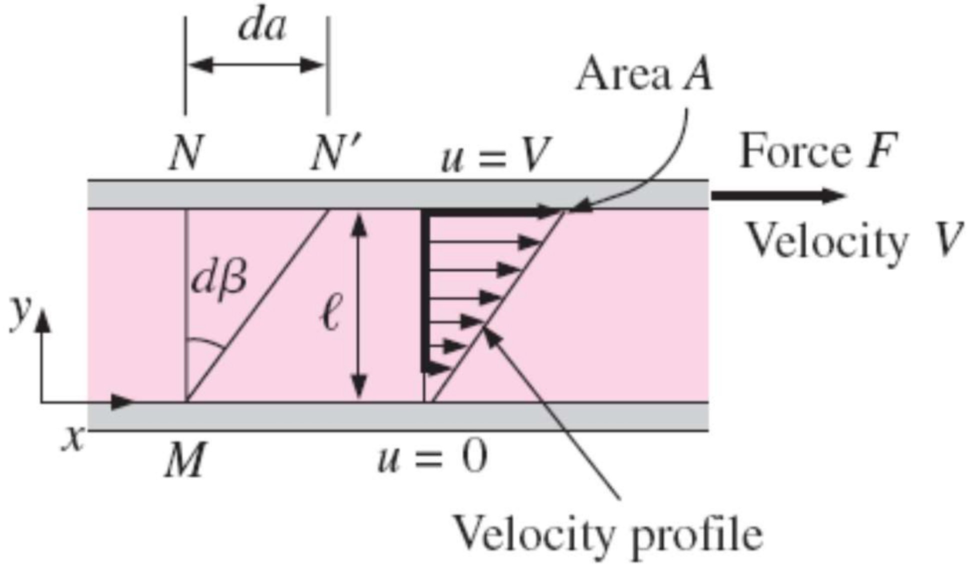
VISCOSITY

Viscosity: A property that represents the internal resistance of a fluid to motion or the "fluidity".

Drag force: The force a flowing fluid exerts on a body in the flow direction. The magnitude of this force depends, in part, on viscosity



A fluid moving relative to a body exerts a drag force on the body, partly because of friction caused by viscosity.



$$u(y) = \frac{y}{\ell} V$$

The behavior of a fluid in laminar flow between two parallel plates when the upper plate moves with a constant velocity.

$$\tau \propto \frac{d(d\beta)}{dt} \quad \text{or} \quad \tau \propto \frac{du}{dy}$$

Fluids for which the rate of deformation is proportional to the shear stress are called Newtonian fluids.

Shear stress

$$\tau = \mu \frac{du}{dy} \qquad (\mathrm{N/m^2})$$

Shear force

$$F = \tau A = \mu A \frac{du}{dy} \qquad (\mathrm{N})$$

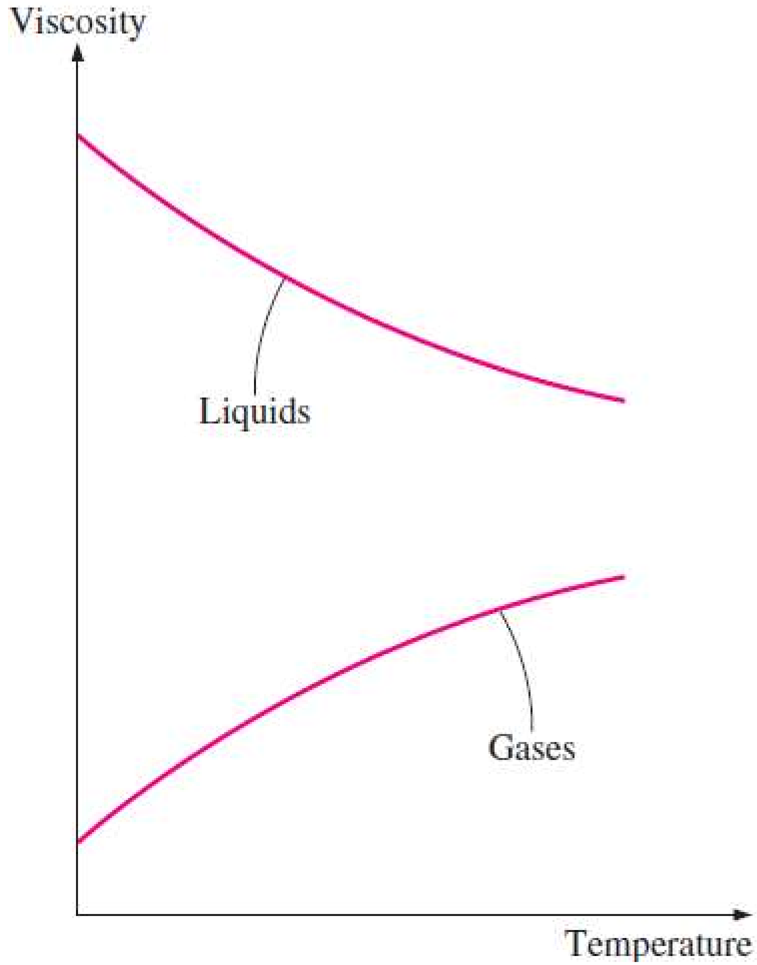
μ **dynamic viscosity**

kg/m · s or N · s/m^2 or Pa · s

1 poise = 0.1 Pa · s

Kinematic viscosity, $\nu = \mu/\rho$

m^2/s or stoke (1 stoke = 1 cm^2/s)



The viscosity of liquids decreases and the viscosity of gases increases with temperature.

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

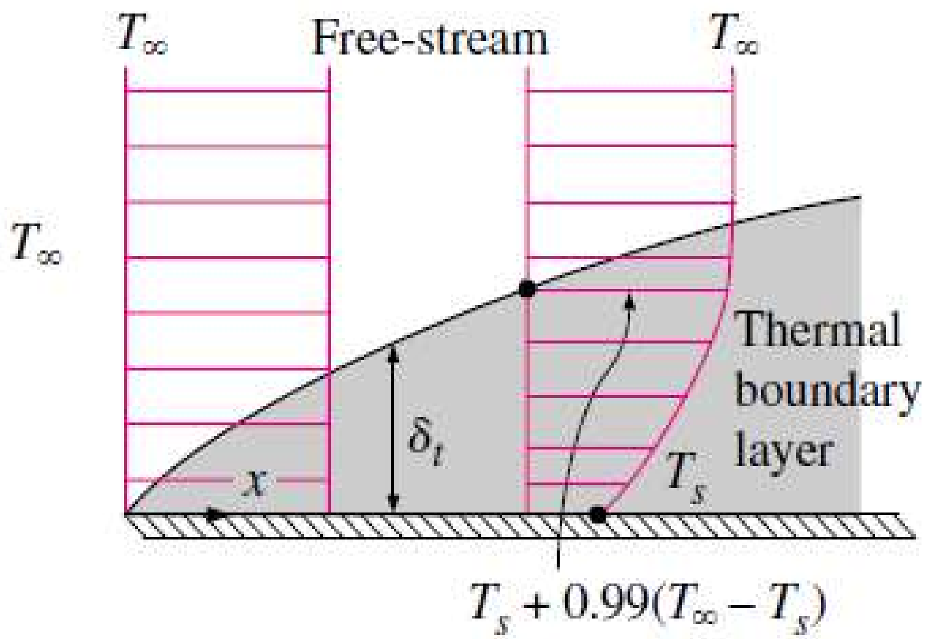
Fluid	Dynamic Viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

THERMAL BOUNDARY LAYER

A *thermal boundary layer* develops when a fluid at a specified temperature flows over a surface that is at a different temperature.

Thermal boundary layer: The flow region over the surface in which the temperature variation in the direction normal to the surface is significant.

The *thickness* of the thermal boundary layer δ_t at any location along the surface is defined as *the distance from the surface at which the temperature difference $T - T_s$ equals* $0.99(T_\infty - T_s)$.



Thermal boundary layer on a flat plate (the fluid is hotter than the plate surface).

The thickness of the thermal boundary layer increases in the flow direction, since the effects of heat transfer are felt at greater distances from the surface further down stream.

The shape of the temperature profile in the thermal boundary layer dictates the convection heat transfer between a solid surface and the fluid flowing over it.

Prandtl Number

The relative thickness of the velocity and the thermal boundary layers is best described by the ***dimensionless*** parameter Prandtl number

$$\mathrm{Pr} = \frac{\text{Molecular diffusivity of momentum}}{\text{Molecular diffusivity of heat}} = \frac{\nu}{\alpha} = \frac{\mu c_p}{k}$$

TABLE 6–2

Typical ranges of Prandtl numbers for common fluids

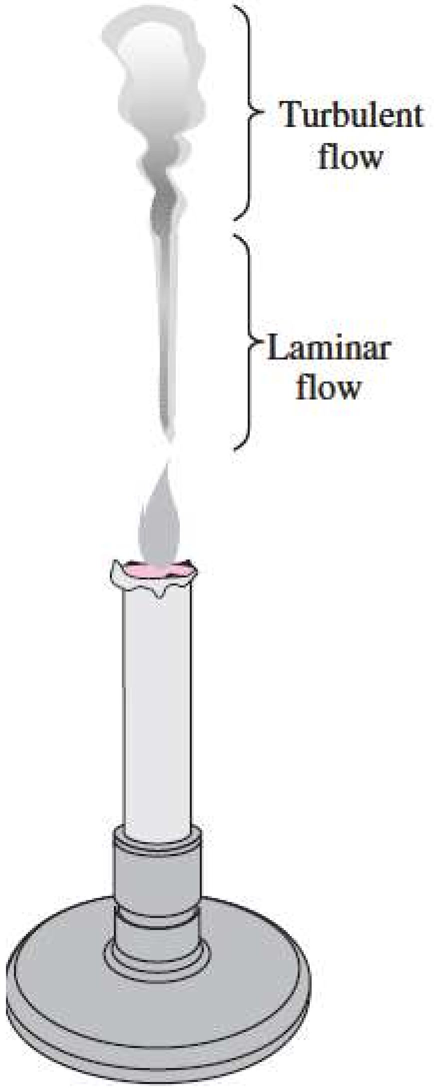
Fluid	Pr
Liquid metals	0.004–0.030
Gases	0.7–1.0
Water	1.7–13.7
Light organic fluids	5–50
Oils	50–100,000
Glycerin	2000–100,000

The Prandtl numbers of gases are about 1, which indicates that both momentum and heat dissipate through the fluid at about the same rate.

Heat diffuses very quickly in liquid metals (Pr << 1) and very slowly in oils (Pr >> 1) relative to momentum.

Consequently the thermal boundary layer is much thicker for liquid metals and much thinner for oils relative to the velocity boundary layer.

LAMINAR AND TURBULENT FLOWS



Laminar and turbulent flow regimes of candle smoke.

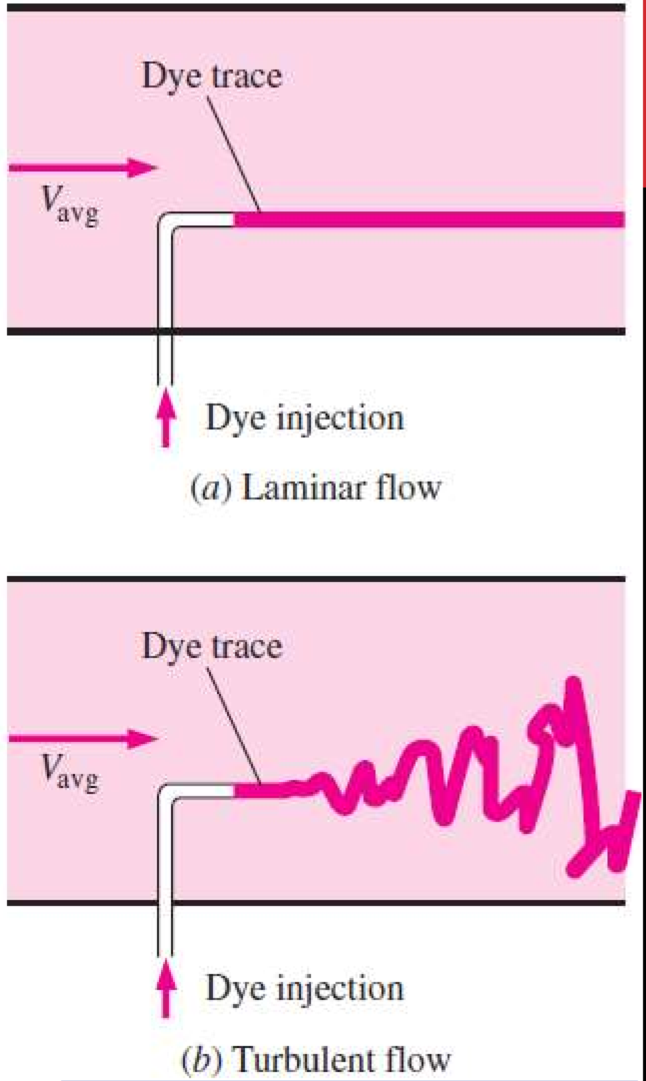
Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.

Laminar: Smooth streamlines and highly ordered motion.

Turbulent: Velocity fluctuations and highly disordered motion.

Transition: The flow fluctuates between laminar and turbulent flows.

Most flows encountered in practice are turbulent.



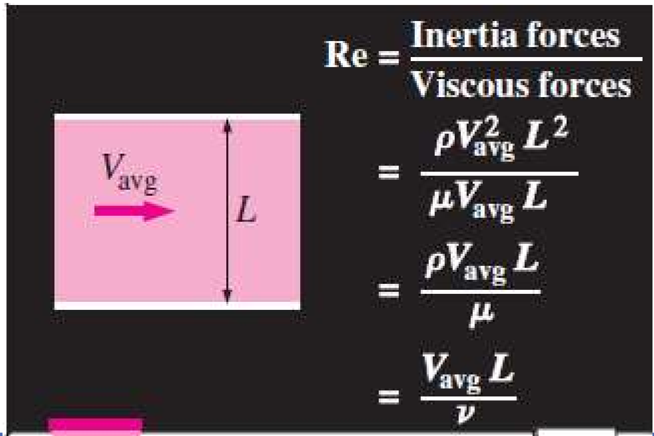
The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry, surface roughness, flow velocity, surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous forces* (Reynolds number).

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$



The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (laminar).

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

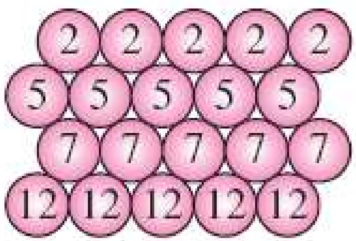
The value of the critical Reynolds number is different for different geometries and flow conditions.

TURBULENT FLOW IN PIPES

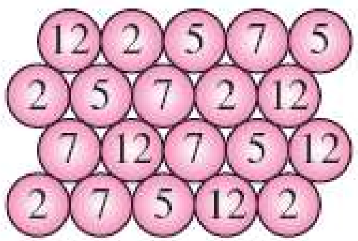
Most flows encountered in engineering practice are turbulent, and thus it is important to understand how turbulence affects wall shear stress.

Turbulent flow is a complex mechanism dominated by fluctuations, and it is still not fully understood.

We must rely on experiments and the empirical or semi-empirical correlations developed for various situations.



(a) Before turbulence



(b) After turbulence

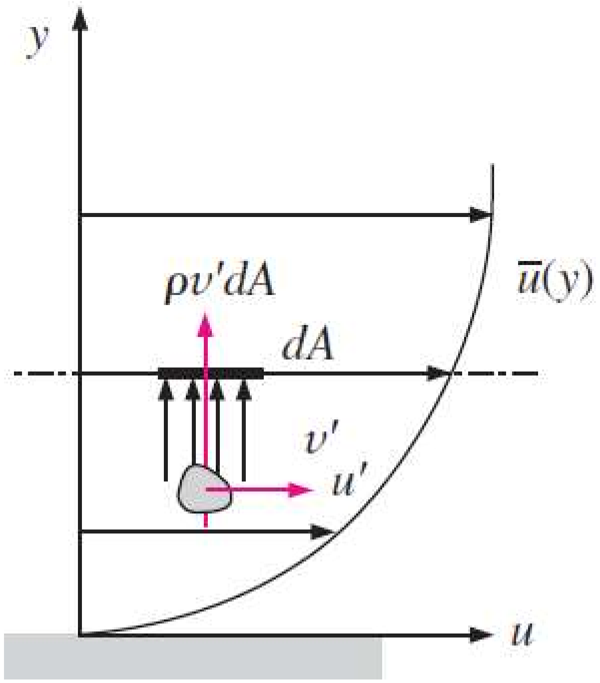
The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called eddies, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

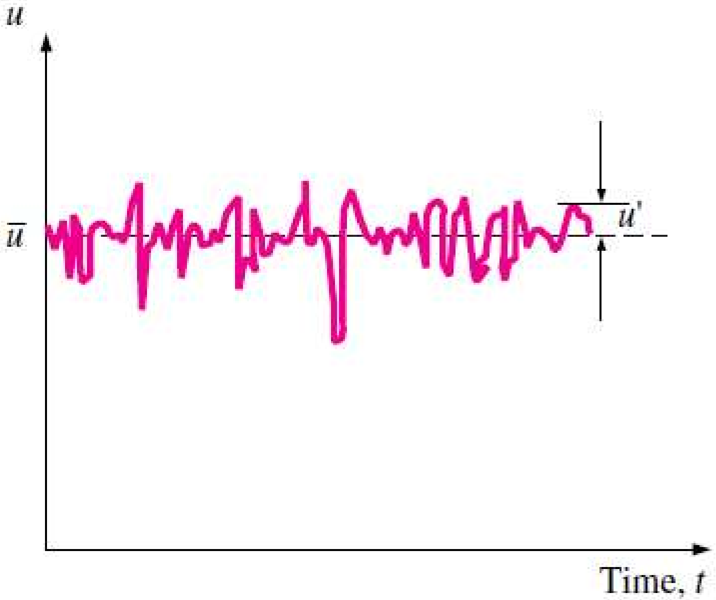
In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients



Fluid particle moving upward through a differential area *dA* as a result of the velocity fluctuation v'.

Turbulent flow is characterized by random and rapid fluctuations of swirling regions of fluid, called **eddies**, throughout the flow.



Fluctuations of the velocity component u with time at a specified location in turbulent flow.

$$u = \overline{u} + u' \qquad v = \overline{v} + v'$$

$$P = \overline{P} + P' \qquad T = \overline{T} + T'$$

$$\boxed{\tau_{\text{turb}} = -\rho\,\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y}}$$

$$\boxed{\dot{q}_{\text{turb}} = \rho c_p\,\overline{v'T'} = -k_t \frac{\partial \overline{T}}{\partial y}}$$

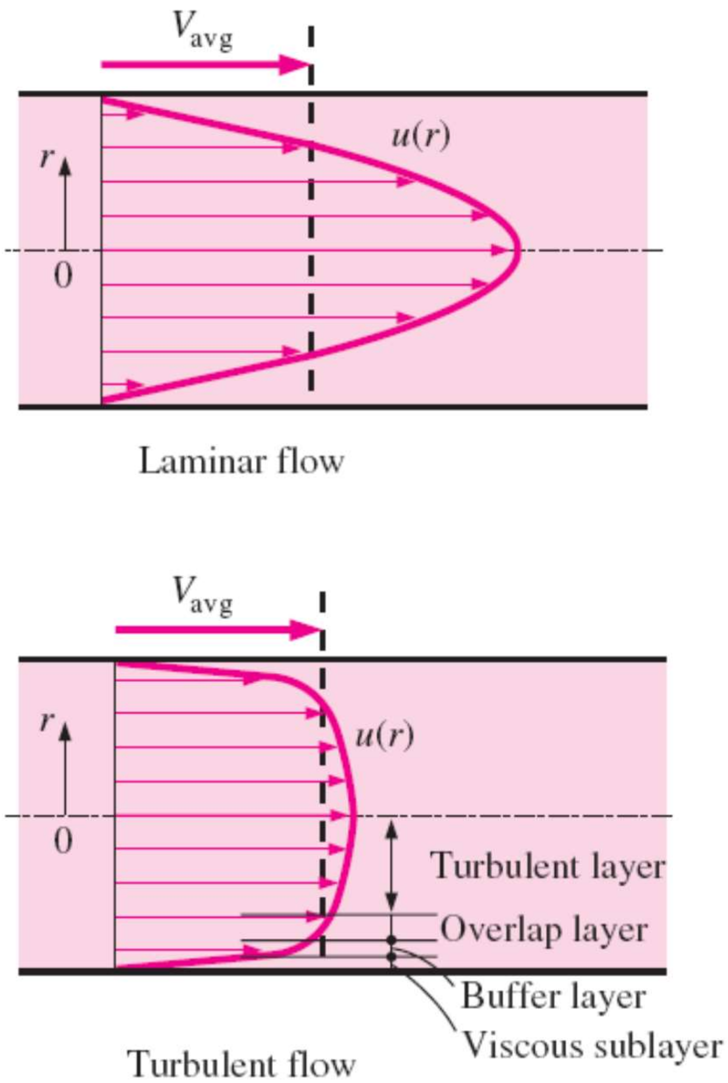
$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \overline{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \overline{u}}{\partial y}$$

$$\dot{q}_{\text{total}} = -(k + k_t)\frac{\partial \overline{T}}{\partial y} = -\rho c_p(\alpha + \alpha_t)\frac{\partial \overline{T}}{\partial y}$$

where μ_t is called the **turbulent** (or **eddy**) **viscosity**, which accounts for momentum transport by turbulent eddies, and k_t is called the **turbulent thermal conductivity**, which accounts for thermal energy transport by turbulent eddies.

$\nu_t = \mu_t/\rho$ is the **kinematic eddy viscosity** (or **eddy diffusivity of momentum**)
$\alpha_t = k_t/\rho c_p$ is the **eddy thermal diffusivity** (or **eddy diffusivity of heat**).

Turbulent Velocity Profile



The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow.

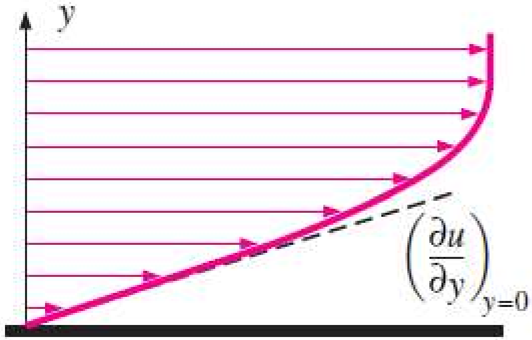
The very thin layer next to the wall where viscous effects are dominant is the viscous (or laminar or linear or wall) sublayer.

The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

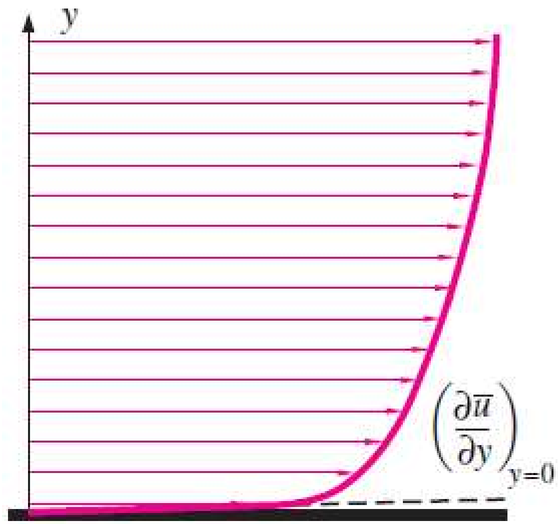
Next to the viscous sublayer is the buffer layer, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

Above the buffer layer is the overlap (or transition) layer, also called the inertial sublayer, in which the turbulent effects are much more significant, but still not dominant.

Above that is the outer (or turbulent) layer in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.



Laminar flow

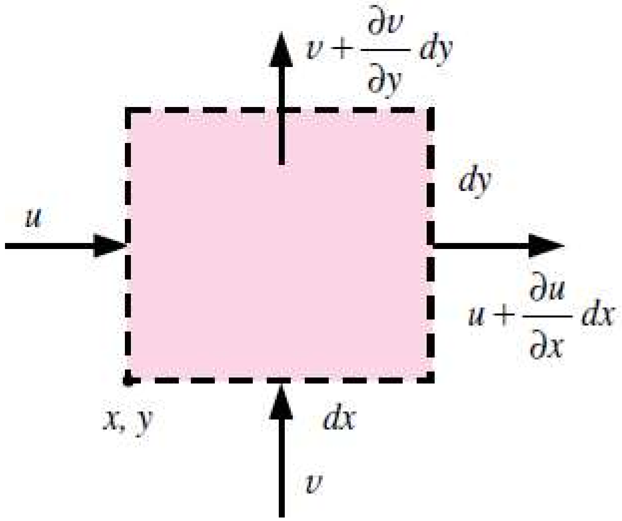


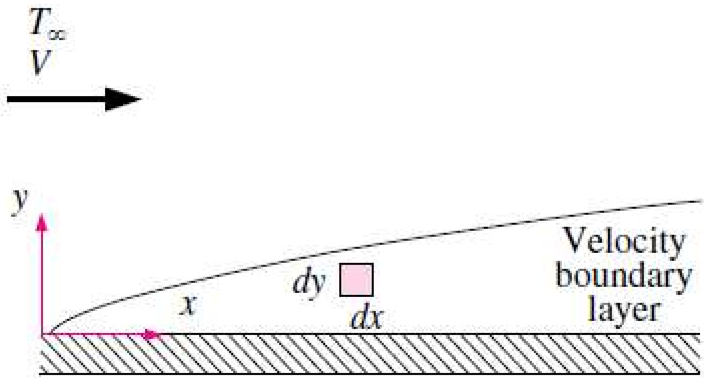
Turbulent flow

The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free-stream velocity.

DERIVATION OF DIFFERENTIAL CONVECTION EQUATIONS

The Continuity Equation





$$\begin{pmatrix}\text{Rate of mass flow} \\ \text{into the control volume}\end{pmatrix} = \begin{pmatrix}\text{Rate of mass flow} \\ \text{out of the control volume}\end{pmatrix}$$

$$\rho u(dy \cdot 1) + \rho v(dx \cdot 1) = \rho\left(u + \frac{\partial u}{\partial x} dx\right)(dy \cdot 1) + \rho\left(v + \frac{\partial v}{\partial y} dy\right)(dx \cdot 1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$\rho\left(u + \frac{\partial u}{\partial x} dx\right)(dy \cdot 1)$$

The Momentum Equations

Newton's second law is an expression for momentum balance and can be stated as ***the net force acting on the control volume is equal to the mass times the acceleration of the fluid element within the control volume, which is also equal to the net rate of momentum outflow from the control volume.***

$$(\text{Mass})\begin{pmatrix}\text{Acceleration}\\ \text{in a specified direction}\end{pmatrix} = \begin{pmatrix}\text{Net force (body and surface)}\\ \text{acting in that direction}\end{pmatrix}$$

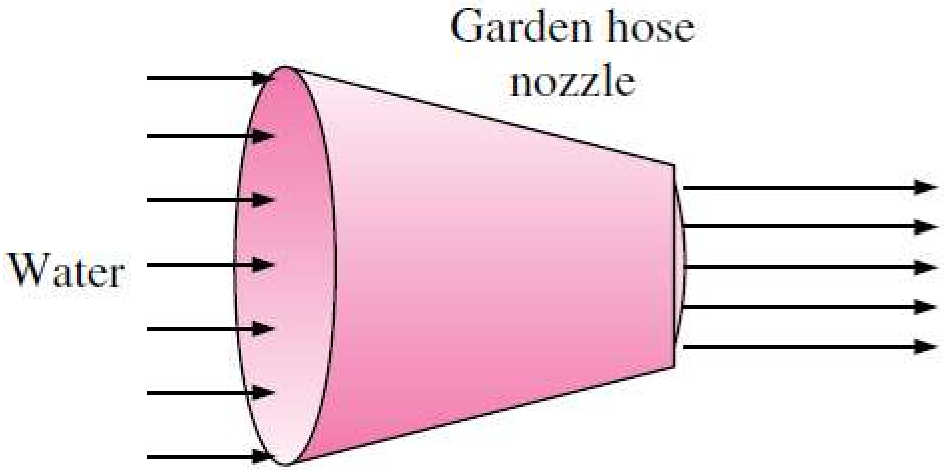
$$\delta m \cdot a_x = F_{\text{surface}, x} + F_{\text{body}, x} \qquad \delta m = \rho(dx \cdot dy \cdot 1)$$

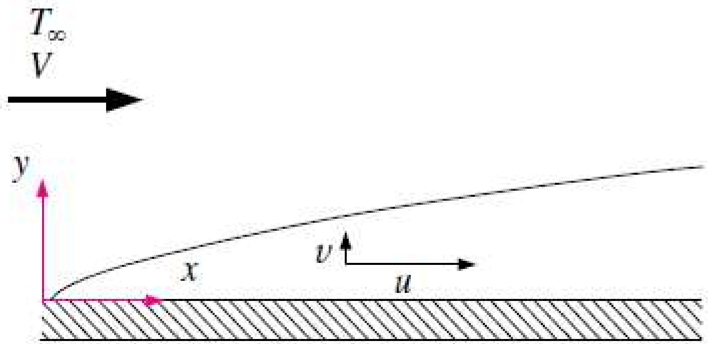
$$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy \qquad a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + \text{v}\frac{\partial u}{\partial y}$$

$$F_{\text{surface}, x} = \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) = \left(\frac{\partial \tau}{\partial y} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

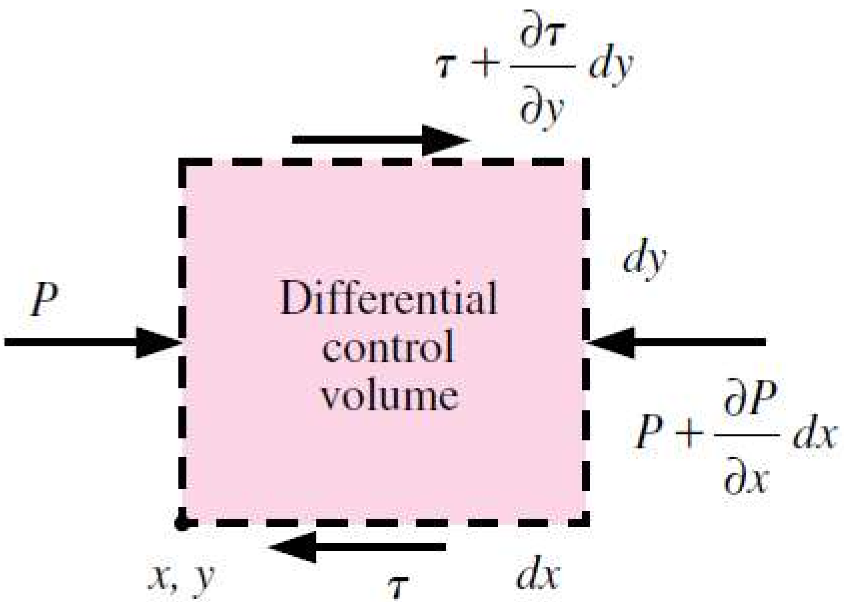
$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}\right)(dx \cdot dy \cdot 1)$$

$$\boxed{\rho\left(u\frac{\partial u}{\partial x} + \text{v}\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x}}$$





During steady flow, a fluid may not accelerate in time at a fixed point, but it may accelerate in space



1) Velocity components:

$v \ll u$

2) Velocity grandients:

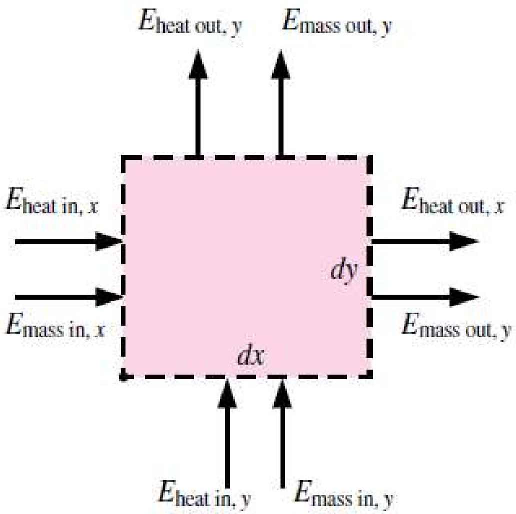
$\frac{\partial v}{\partial x} \ll 0, \frac{\partial v}{\partial y} \ll 0$

$\frac{\partial u}{\partial x} \ll \frac{\partial u}{\partial y}$

3) Temperature gradients:

$\frac{\partial T}{\partial x} \ll \frac{\partial T}{\partial y}$

Conservation of Energy Equation



$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat}} + (\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by work}} + (\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by mass}} = 0$$

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by mass, }x} = (\dot{m}e_{\text{stream}})_x - \left[(\dot{m}e_{\text{stream}})_x + \frac{\partial(\dot{m}e_{\text{stream}})_x}{\partial x}dx\right]$$
$$= -\frac{\partial[\rho u(dy\cdot 1)c_pT]}{\partial x}dx = -\rho c_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy$$

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by mass}} = -\rho c_p\left(u\frac{\partial T}{\partial x} + T\frac{\partial u}{\partial x}\right)dx\,dy - \rho c_p\left(\text{v}\frac{\partial T}{\partial y} + T\frac{\partial \text{v}}{\partial y}\right)dx\,dy$$
$$= -\rho c_p\left(u\frac{\partial T}{\partial x} + \text{v}\frac{\partial T}{\partial y}\right)dx\,dy$$

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat, }x} = \dot{Q}_x - \left(\dot{Q}_x + \frac{\partial \dot{Q}_x}{\partial x}dx\right) = -\frac{\partial}{\partial x}\left(-k(dy\cdot 1)\frac{\partial T}{\partial x}\right)dx = k\frac{\partial^2 T}{\partial x^2}dx\,dy$$

$$(\dot{E}_{\text{in}} - \dot{E}_{\text{out}})_{\text{by heat}} = k\frac{\partial^2 T}{\partial x^2}dx\,dy + k\frac{\partial^2 T}{\partial y^2}dx\,dy = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)dx\,dy$$

$$\boxed{\rho c_p\left(u\frac{\partial T}{\partial x} + \text{v}\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right)}$$

the net energy convected by the fluid out of the control volume is equal to the net energy transferred into the control volume by heat conduction.

$$\rho c_p\left(u\frac{\partial T}{\partial x} + \mathrm{v}\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) + \mu\Phi$$

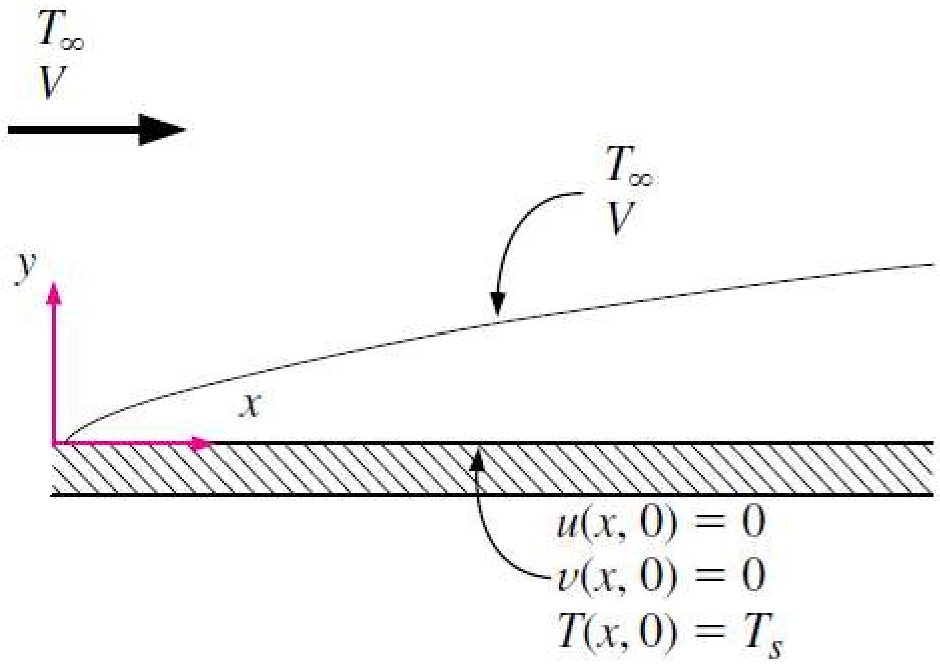
viscous dissipation function Φ

$$\Phi = 2\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial \mathrm{v}}{\partial y}\right)^2\right] + \left(\frac{\partial u}{\partial y} + \frac{\partial \mathrm{v}}{\partial x}\right)^2$$

For the special case of a stationary fluid, *u*=*v*= 0

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

SOLUTIONS OF CONVECTION EQUATIONS FOR A FLAT PLATE



Continuity: $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Momentum: $$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

Energy: $$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}$$

At $x = 0$: $u(0, y) = V,\quad T(0, y) = T_\infty$

At $y = 0$: $u(x, 0) = 0,\quad v(x, 0) = 0,\ T(x, 0) = T_s$

As $y \to \infty$: $u(x, \infty) = V,\quad T(x, \infty) = T_\infty$

dimensionless similarity variable

$$\eta = y\sqrt{\frac{V}{\nu x}}$$

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad \mathrm{v} = -\frac{\partial \psi}{\partial x} \qquad f(\eta) = \frac{\psi}{V\sqrt{\nu x/V}}$$

$$u = \frac{\partial \psi}{\partial y} = \frac{\partial \psi}{\partial \eta}\frac{\partial \eta}{\partial y} = V\sqrt{\frac{\nu x}{V}}\frac{df}{d\eta}\sqrt{\frac{V}{\nu x}} = V\frac{df}{d\eta}$$

$$\mathrm{v} = -\frac{\partial \psi}{\partial x} = -V\sqrt{\frac{\nu x}{V}}\frac{df}{d\eta} - \frac{V}{2}\sqrt{\frac{\nu}{Vx}}f = \frac{1}{2}\sqrt{\frac{V\nu}{x}}\left(\eta\frac{df}{d\eta} - f\right)$$

$$\frac{\partial u}{\partial x} = -\frac{V}{2x}\eta\frac{d^2 f}{d\eta^2}, \qquad \frac{\partial u}{\partial y} = V\sqrt{\frac{V}{\nu x}}\frac{d^2 f}{d\eta^2}, \qquad \frac{\partial^2 u}{\partial y^2} = \frac{V^2}{\nu x}\frac{d^3 f}{d\eta^3}$$

$$2\frac{d^3f}{d\eta^3} + f\frac{d^2f}{d\eta^2} = 0$$

$$f(0) = 0, \qquad \left.\frac{df}{d\eta}\right|_{\eta=0} = 0, \qquad \text{and} \qquad \left.\frac{df}{d\eta}\right|_{\eta=\infty} = 1$$

$$\boxed{\delta = \frac{4.91}{\sqrt{V/\nu x}} = \frac{4.91x}{\sqrt{\mathrm{Re}_x}}}$$

$$\tau_w = \mu\left.\frac{\partial u}{\partial y}\right|_{y=0} = \mu V\sqrt{\frac{V}{\nu x}}\left.\frac{d^2f}{d\eta^2}\right|_{\eta=0}$$

$$\tau_w = 0.332V\sqrt{\frac{\rho\mu V}{x}} = \frac{0.332\rho V^2}{\sqrt{\mathrm{Re}_x}}$$

$$\boxed{C_{f,x} = \frac{\tau_w}{\rho V^2/2} = 0.664\ \mathrm{Re}_x^{-1/2}}$$

Similarity function f and its derivatives for laminar boundary layer along a flat plate.

η	f	$\frac{df}{d\eta} = \frac{u}{V}$	$\frac{d^2f}{d\eta^2}$
0	0	0	0.332
0.5	0.042	0.166	0.331
1.0	0.166	0.330	0.323
1.5	0.370	0.487	0.303
2.0	0.650	0.630	0.267
2.5	0.996	0.751	0.217
3.0	1.397	0.846	0.161
3.5	1.838	0.913	0.108
4.0	2.306	0.956	0.064
4.5	2.790	0.980	0.034
5.0	3.283	0.992	0.016
5.5	3.781	0.997	0.007
6.0	4.280	0.999	0.002
∞	∞	1	0

The Energy Equation

$$\theta(x, y) = \frac{T(x, y) - T_s}{T_\infty - T_s} \qquad u\frac{\partial \theta}{\partial x} + \mathrm{v}\frac{\partial \theta}{\partial y} = \alpha \frac{\partial^2 \theta}{\partial y^2}$$

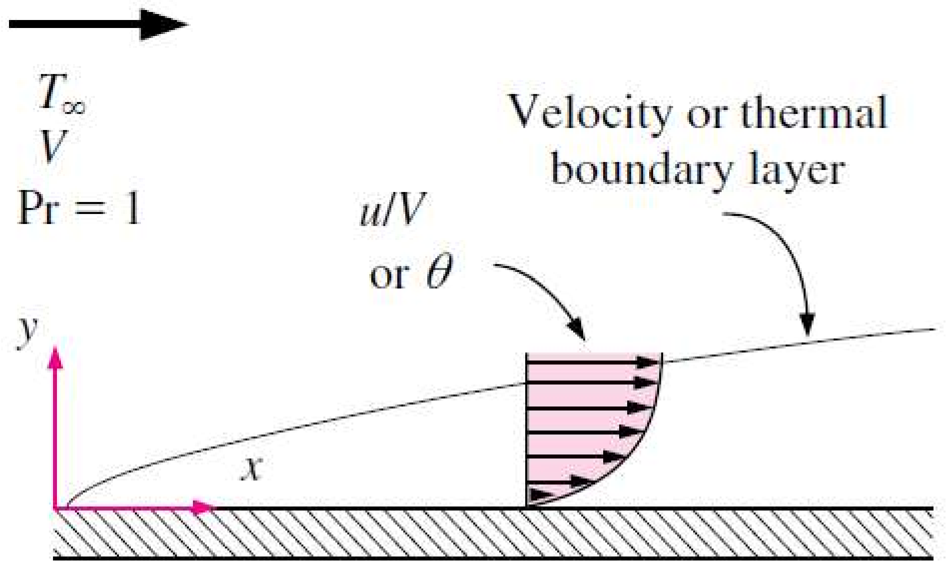
$$V\frac{df}{d\eta}\frac{d\theta}{d\eta}\frac{\partial \eta}{\partial x} + \frac{1}{2}\sqrt{\frac{Vy}{x}}\left(\eta\frac{df}{d\eta}f\right)\frac{d\theta}{d\eta}\frac{\partial \eta}{\partial y} = \alpha\frac{d^2\theta}{d\eta^2}\left(\frac{\partial \eta}{\partial y}\right)^2$$

$$2\frac{d^2\theta}{d\eta^2} + \mathrm{Pr}\, f\frac{d\theta}{d\eta} = 0 \qquad \left.\frac{d\theta}{d\eta}\right|_{\eta=0} = 0.332\ \mathrm{Pr}^{1/3}$$

$$\left.\frac{\partial T}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{\partial \theta}{\partial y}\right|_{y=0} = (T_\infty - T_s)\left.\frac{d\theta}{d\eta}\right|_{\eta=0}\left.\frac{\partial \eta}{\partial y}\right|_{y=0}$$

$$= 0.332\ \mathrm{Pr}^{1/3}(T_\infty - T_s)\sqrt{\frac{V}{\nu x}}$$

$$h_x = \frac{\dot{q}_s}{T_s - T_\infty} = \frac{-k(\partial T/\partial y)|_{y=0}}{T_s - T_\infty} = 0.332\ Pr^{1/3}k\sqrt{\frac{V}{\nu x}}$$



When Pr 1, the velocity and thermal boundary layers coincide, and the nondimensional velocity and temperature profiles are identical for steady, incompressible, laminar flow over a flat plate.

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6$$

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{4.91x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}}$$

SOME İMPORTANT RESULTS FROM CONVECTİON EQUATİONS

The velocity boundary layer thickness

$$\delta = \frac{4.91}{\sqrt{V/\nu x}} = \frac{4.91x}{\sqrt{\mathrm{Re}_x}}$$

The average local skin friction coefficient

$$C_{f,x} = \frac{\tau_w}{\rho V^2/2} = 0.664\,\mathrm{Re}_x^{-1/2}$$

Local Nusselt number

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Pr}^{1/3}\mathrm{Re}_x^{1/2} \qquad \mathrm{Pr} > 0.6$$

The thermal boundary layer thickness

$$\delta_t = \frac{\delta}{\mathrm{Pr}^{1/3}} = \frac{4.91x}{\mathrm{Pr}^{1/3}\sqrt{\mathrm{Re}_x}}$$

Reynold analogy

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x \qquad (\mathrm{Pr} = 1)$$

Modified Reynold analogy

$$C_{f,x}\frac{\mathrm{Re}_L}{2} = \mathrm{Nu}_x\,\mathrm{Pr}^{\pm 1/3} \qquad \text{or} \qquad \frac{C_{f,x}}{2} = \frac{h_x}{\rho c_p V}\mathrm{Pr}^{2/3} \equiv j_H$$

SUMMARY

Physical Mechanism of Convection

- Nusselt Number

Classification of flows

Thermal Boundary Layer

- Prandtl Number

Laminar and Turbulent Flow

- Reynolds Number