Heat and Mass Transfer, 3rd Edition
Yunus A. Cengel
McGraw-Hill, New York, 2007

CHAPTER 7
EXTERNAL FORCED CONVECTION

Prof. Dr. Ali PINARBAŞI
Yildiz Technical University
Mechanical Engineering Department
Yildiz, ISTANBUL

Objectives

- Evaluate the heat transfer associated with flow over a flat plate for both laminar and turbulent flow, and flow over cylinders and spheres
- Distinguish between internal and external flow,
- Develop an intuitive understanding of friction drag and pressure drag, and evaluate the average drag and convection coefficients in external flow,
- Evaluate the drag and heat transfer associated with flow over a flat plate for both laminar and turbulent flow,
- Calculate the drag force exerted on cylinders during cross flow, and the average heat transfer coefficient, and
- Determine the pressure drop and the average heat transfer coefficient associated with flow across a tube bank for both in-line and staggered configurations.

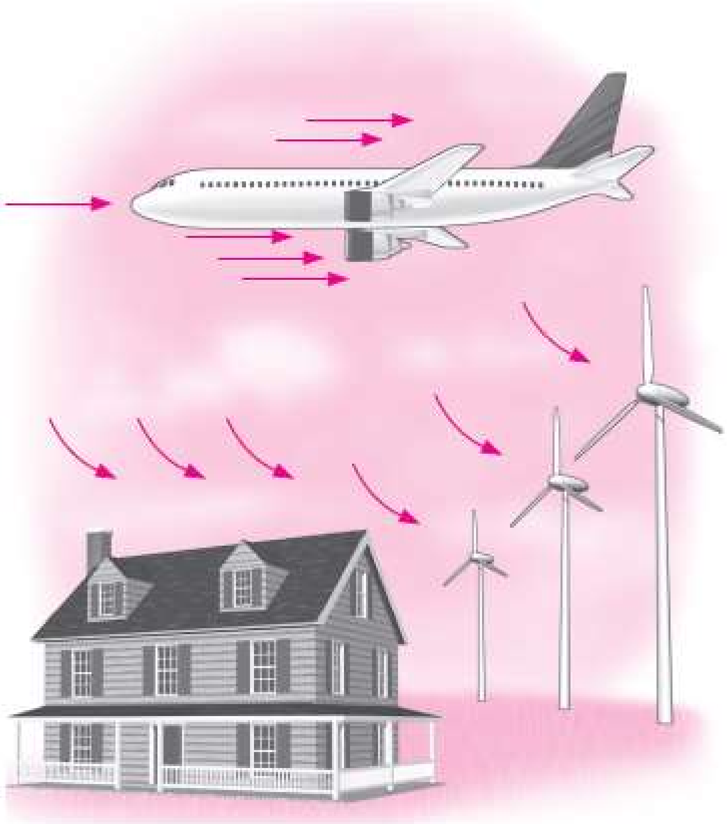
DRAG AND HEAT TRANSFER İN EXTERNAL FLOW

Fluid flow over solid bodies cause physical phenomena such as

- *drag force*
  - automobiles
  - power lines
- *lift force*
  - airplane wings
- *cooling* of metal or plastic sheets.

Free-stream velocity — the velocity of the fluid relative to an immersed solid body sufficiently far from the body.

The fluid velocity ranges from zero at the surface (the no-slip condition) to the free-stream value away from the surface.



FRİCTİON AND PRESSURE DRAG

The force a flowing fluid exerts on a body in the flow direction is called drag.

Drag is composed of:

- pressure drag,
- friction drag (skin friction drag).

The drag force F_D depends on the

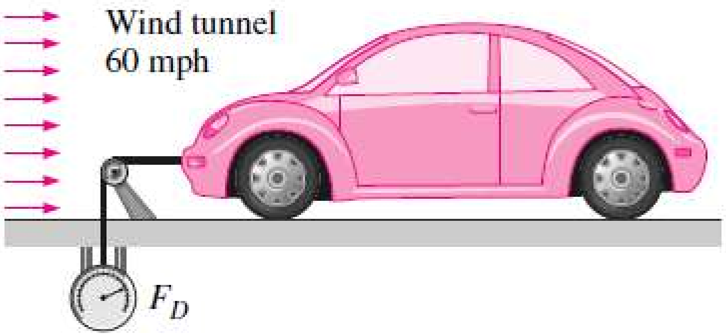
- density ρ of the fluid,
- the upstream velocity V, and
- the size, shape, and orientation of the body.

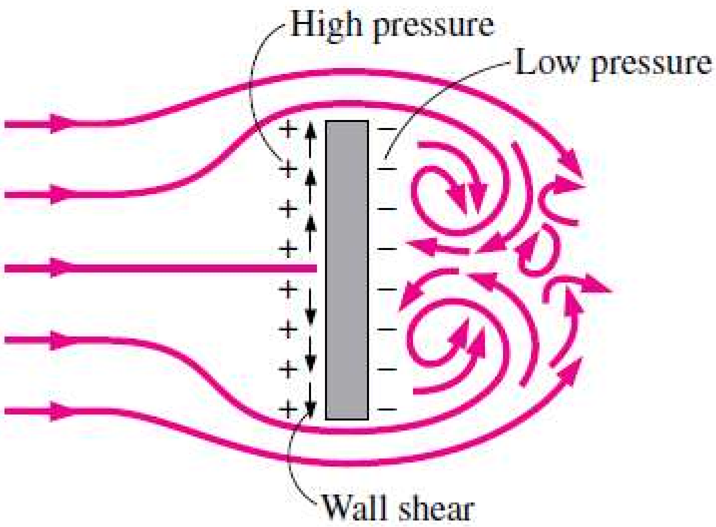
The dimensionless drag coefficient C_D is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

$$C_D = C_{D,\,\text{friction}} + C_{D,\,\text{pressure}}$$

For flat plate: $C_D = C_{D,\,\text{friction}} = C_f$





Drag force acting on a flat plate normal to the flow depends on the pressure only and is independent of the wall shear, which acts normal to the free-stream flow.

At low Reynolds numbers, most drag is due to friction drag.

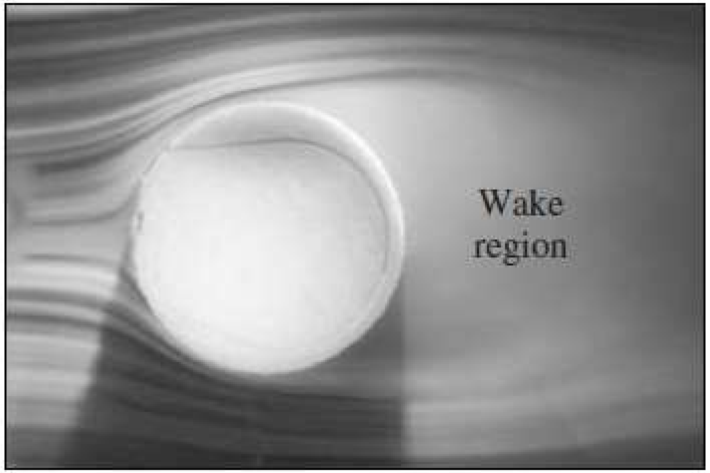
The friction drag is also proportional to the surface area.

The pressure drag is proportional to the frontal area and to the *difference* between the pressures acting on the front and back of the immersed body.

The pressure drag is usually dominant for blunt bodies and negligible for streamlined bodies.

When a fluid separates from a body, it forms a separated region between the body and the fluid stream.

The larger the separated region, the larger the pressure drag.



Separation during flow over a tennis ball and the wake region.

Heat Transfer

Local and average Nusselt numbers:

$$\text{Nu}_x = f_1(x^*, \text{Re}_x, \text{Pr}) \qquad \text{and} \qquad \text{Nu} = f_2(\text{Re}_L, \text{Pr})$$

Average Nusselt number:

$$\text{Nu} = C\,\text{Re}_L^m \text{Pr}^n$$

Film temperature:

$$T_f = \frac{T_s + T_\infty}{2}$$

Average friction coefficient

$$C_D = \frac{1}{L}\int_0^L C_{D,x}\,dx$$

Average heat transfer coefficient:

$$h = \frac{1}{L}\int_0^L h_x\,dx$$

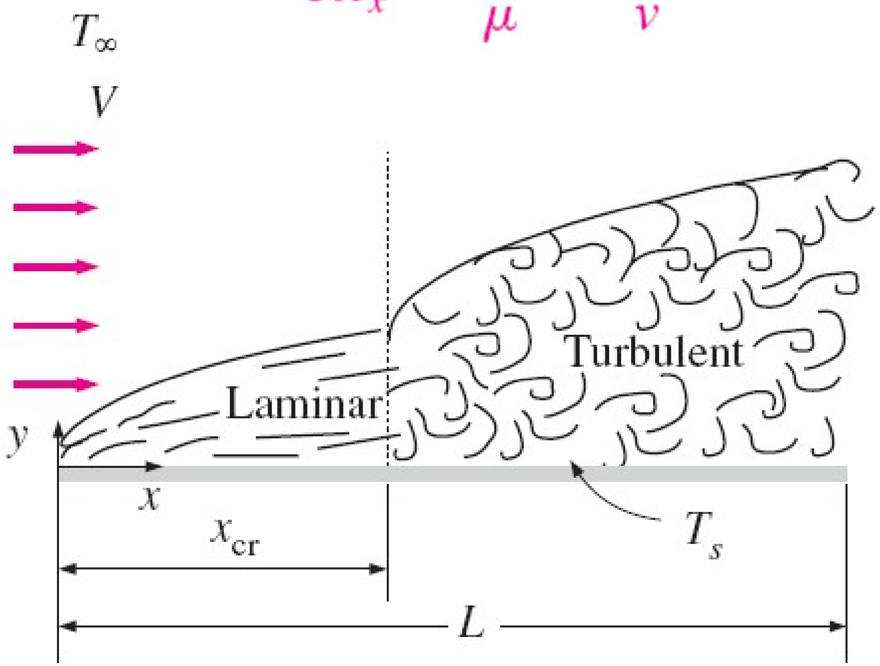
The heat transfer rate:

$$\dot{Q} = hA_s(T_s - T_\infty)$$

PARALLEL FLOW OVER FLAT PLATES

The transition from laminar to turbulent flow depends on the *surface geometry*, *surface roughness*, *upstream velocity*, *surface temperature*, and the *type of fluid*, among other things, and is best characterized by the Reynolds number. The Reynolds number at a distance *x* from the leading edge of a flat plate is expressed as

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{V x}{\nu}$$



Laminar and turbulent regions of the boundary layer during flow over a flat plate.

A generally accepted value for the Critical Reynold number

$$\mathrm{Re}_{cr} = \frac{\rho V x_{cr}}{\mu} = 5 \times 10^5$$

The actual value of the engineering critical Reynolds number for a flat plate may vary somewhat from 10^5 to 3×10^6, depending on the surface roughness, the turbulence level, and the variation of pressure along the surface.

Friction Coefficient

Laminar: $\delta_{v,x} = \dfrac{4.91x}{Re_x^{1/2}}$ and $C_{f,x} = \dfrac{0.664}{Re_x^{1/2}}$, $Re_x < 5 \times 10^5$

Turbulent: $\delta_{v,x} = \dfrac{0.38x}{Re_x^{1/5}}$ and $C_{f,x} = \dfrac{0.059}{Re_x^{1/5}}$, $5 \times 10^5 \le Re_x \le 10^7$

Laminar: $C_f = \dfrac{1.33}{Re_L^{1/2}}$ $Re_L < 5 \times 10^5$

Turbulent: $C_f = \dfrac{0.074}{Re_L^{1/5}}$ $5 \times 10^5 \le Re_L \le 10^7$

$$C_f = \frac{1}{L}\left(\int_0^{x_{cr}} C_{f,x\text{ laminar}}\,dx + \int_{x_{cr}}^{L} C_{f,x,\text{ turbulent}}\,dx\right)$$

$$C_f = \frac{0.074}{Re_L^{1/5}} - \frac{1742}{Re_L} \qquad 5 \times 10^5 \le Re_L \le 10^7$$

Rough surface, turbulent: $C_f = \left(1.89 - 1.62 \log \dfrac{\varepsilon}{L}\right)^{-2.5}$

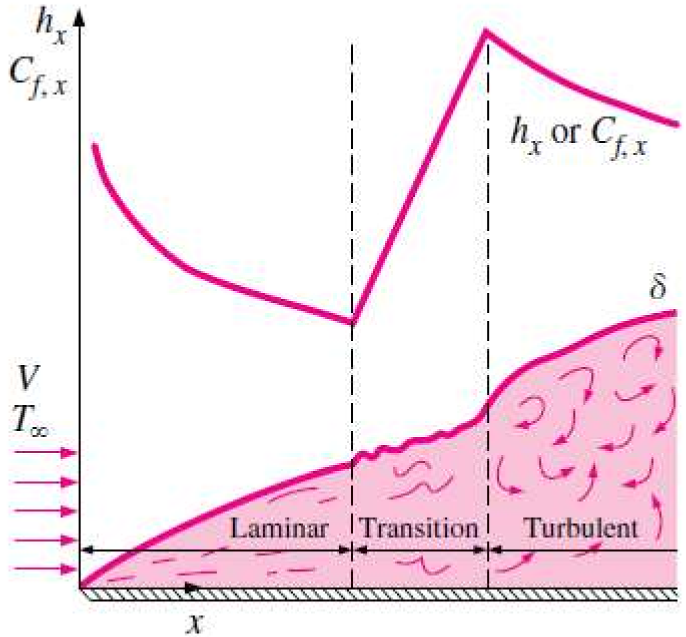
Relative roughness, ε/L	Friction coefficient C_f
0.0*	0.0029
1×10^{-5}	0.0032
1×10^{-4}	0.0049
1×10^{-3}	0.0084

*Smooth surface for Re = 10^7. Others calculated from Eq. 7–18.

The local Nusselt number at a location *x* for laminar flow over a flat plate may be obtained by solving the differential energy equation to be

Laminar: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3} \qquad \mathrm{Pr} > 0.6$$

Turbulent: $$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$



The variation of the local friction and heat transfer coefficients for flow over a flat plate.

These relations are for *isothermal* and *smooth* surfaces

The local friction and heat transfer coefficients are higher in turbulent flow than they are in laminar flow.

Also, h_x reaches its highest values when the flow becomes fully turbulent, and then decreases by a factor of $x^{-0.2}$ in the flow direction.

Nusselt numbers for average heat transfer coefficients

Laminar:	$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}$	$\mathrm{Re}_L < 5 \times 10^5$
Turbulent:	$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}$	$0.6 \le \mathrm{Pr} \le 60$ $5 \times 10^5 \le \mathrm{Re}_L \le 10^7$

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\mathrm{Pr}^{1/3} \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

Laminar + turbulent

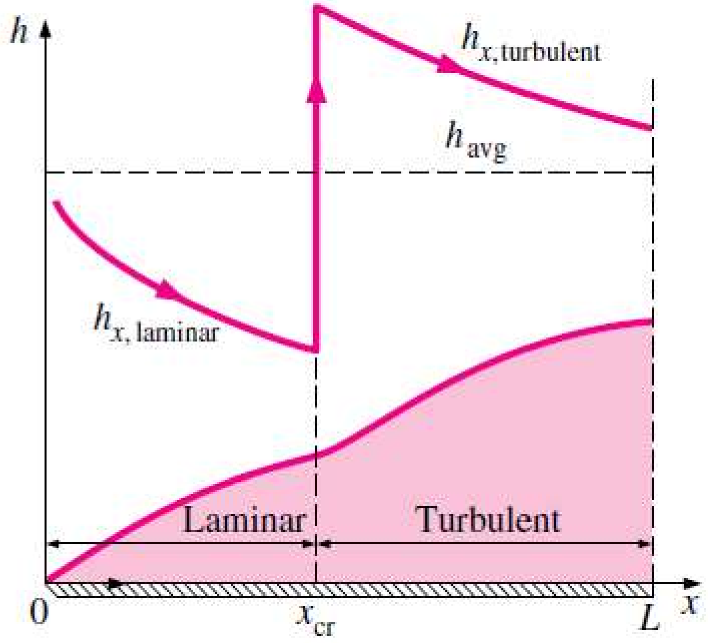
$$h = \frac{1}{L}\left(\int_0^{x_{cr}} h_{x,\,\mathrm{laminar}}\,dx + \int_{x_{cr}}^{L} h_{x,\,\mathrm{turbulent}}\,dx\right)$$

For liquid metals

$$\mathrm{Nu}_x = 0.565(\mathrm{Re}_x\,\mathrm{Pr})^{1/2} \qquad \mathrm{Pr} < 0.05$$

For all liquids, all Prandtl numbers

$$\mathrm{Nu}_x = \frac{h_x x}{k} = \frac{0.3387\,\mathrm{Pr}^{1/3}\,\mathrm{Re}_x^{1/2}}{[1 + (0.0468/\mathrm{Pr})^{2/3}]^{1/4}}$$



the average heat transfer coefficient for a flat plate with combined laminar and turbulent flow.

Flat Plate with Unheated Starting Length

Local Nusselt numbers

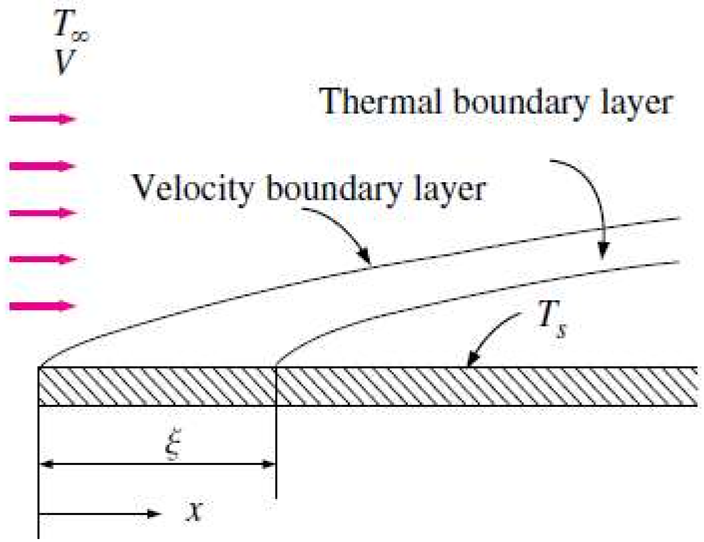
Laminar: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

Turbulent: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

Average heat transfer coefficients

Laminar: $$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent: $$h = \frac{5[1-(\xi/x)^{9/10}]}{4(1-\xi/L)}\,h_{x=L}$$



Flow over a flat plate with an unheated starting length.

Uniform Heat Flux

For a flat plate subjected to *uniform heat flux*

Laminar: $\text{Nu}_x = 0.453\,\text{Re}_x^{0.5}\,\text{Pr}^{1/3}$

Turbulent: $\text{Nu}_x = 0.0308\,\text{Re}_x^{0.8}\,\text{Pr}^{1/3}$

These relations give values that are 36 percent higher for laminar flow and 4 percent higher for turbulent flow relative to the isothermal plate case.

When heat flux is prescribed, the rate of heat transfer to or from the plate and the surface temperature at a distance *x* are determined from

$$\dot{Q} = \dot{q}_s A_s$$

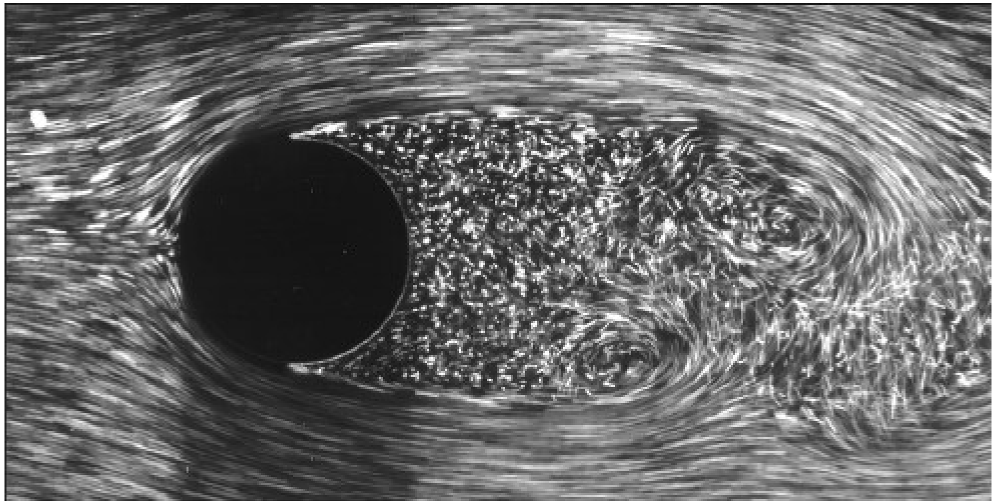
$$\dot{q}_s = h_x[T_s(x) - T_\infty] \quad \rightarrow \quad T_s(x) = T_\infty + \frac{\dot{q}_s}{h_x}$$

FLOW OVER CYLINDERS AND SPHERES

Flow over cylinders and spheres is frequently encountered in practice.
The tubes in a shell-and-tube heat exchanger involve both *internal flow* through the tubes and *external flow* over the tubes.
Many sports such as soccer, tennis, and golf involve flow over spherical balls.

The characteristic length for a circular cylinder or sphere is taken to be the *external diameter D.*

The critical Reynolds number for flow across a circular cylinder or sphere is about $Re_{cr} \cong 2\ \ 105$. That is, the boundary layer remains laminar for about $Re \leq 2x10^5$ and becomes turbulent for $Re \geq 2x10^5$.



Laminar boundary layer separation with a turbulent wake; flow over a circular cylinder at Re=2000.

At very low velocities, the fluid completely wraps around the cylinder.

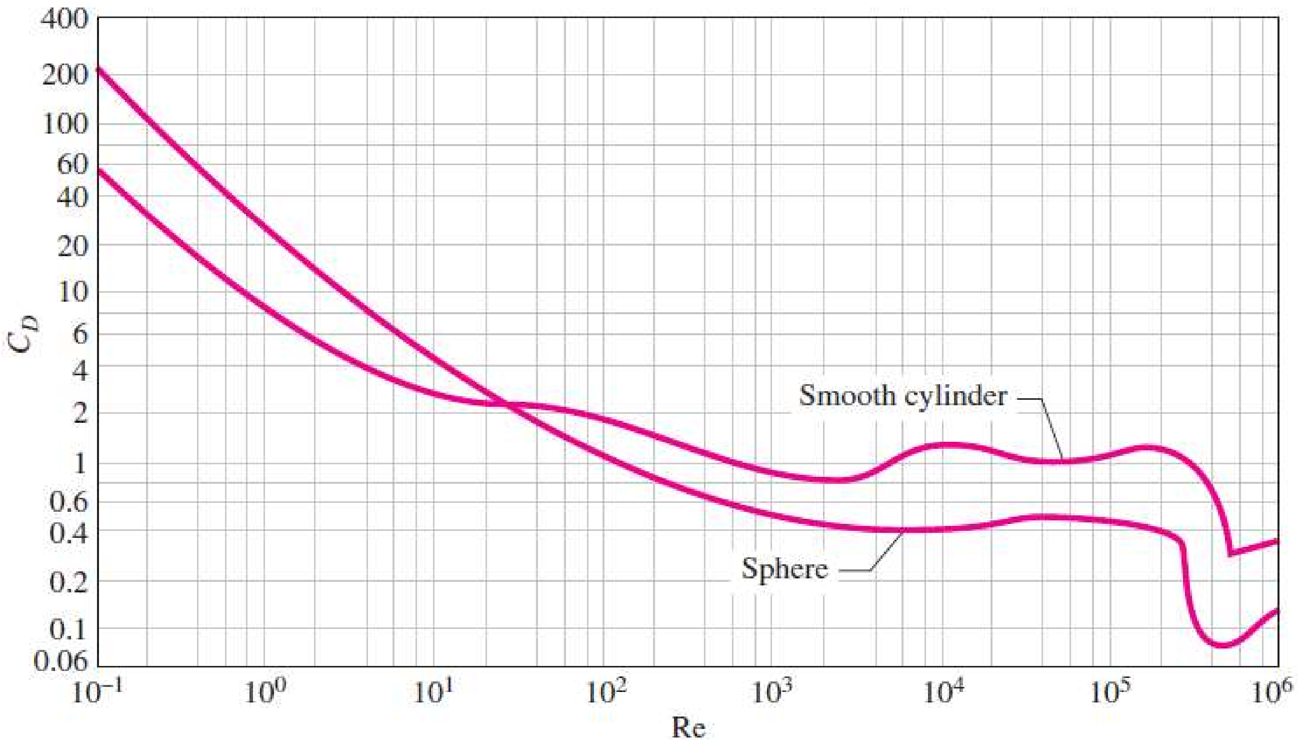
Flow in the wake region is characterized by periodic vortex formation and low pressures.

For flow over cylinder or sphere, both the *friction drag* and the *pressure drag* can be significant.

The high pressure in the vicinity of the stagnation point and the low pressure on the opposite side in the wake produce a net force on the body in the direction of flow.

The drag force is primarily due to friction drag at low Reynolds numbers (Re<10) and to pressure drag at high Reynolds numbers (Re>5000).

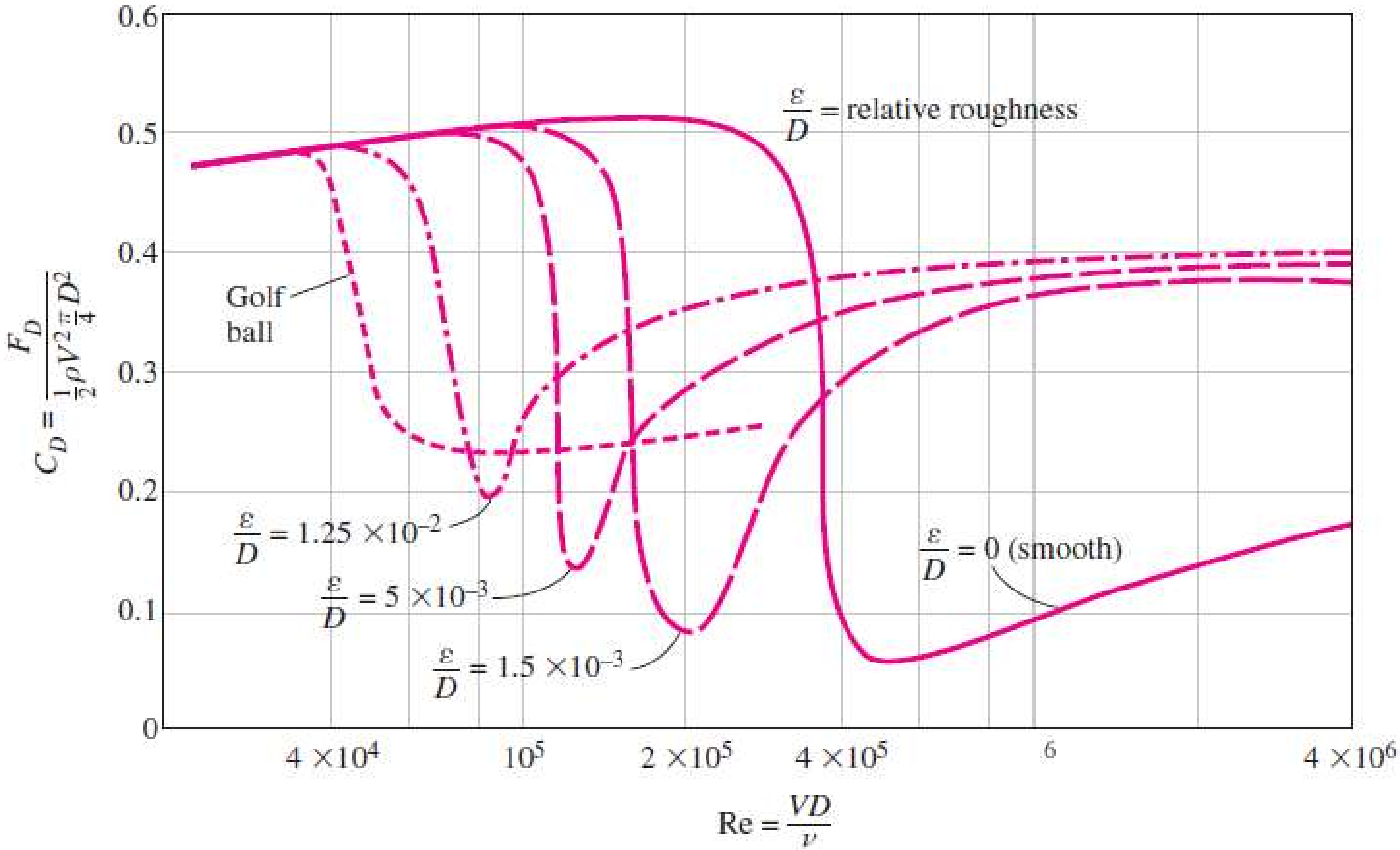
Both effects are significant at intermediate Reynolds numbers.



Average drag coefficient for cross-flow over a smooth circular cylinder and a smooth sphere.

Effect of Surface Roughness

Surface roughness, in general, increases the drag coefficient in turbulent flow.
This is especially the case for streamlined bodies.
For blunt bodies such as a circular cylinder or sphere, however, an increase in the surface roughness may *increase* or *decrease* the drag coefficient depending on Reynolds number.

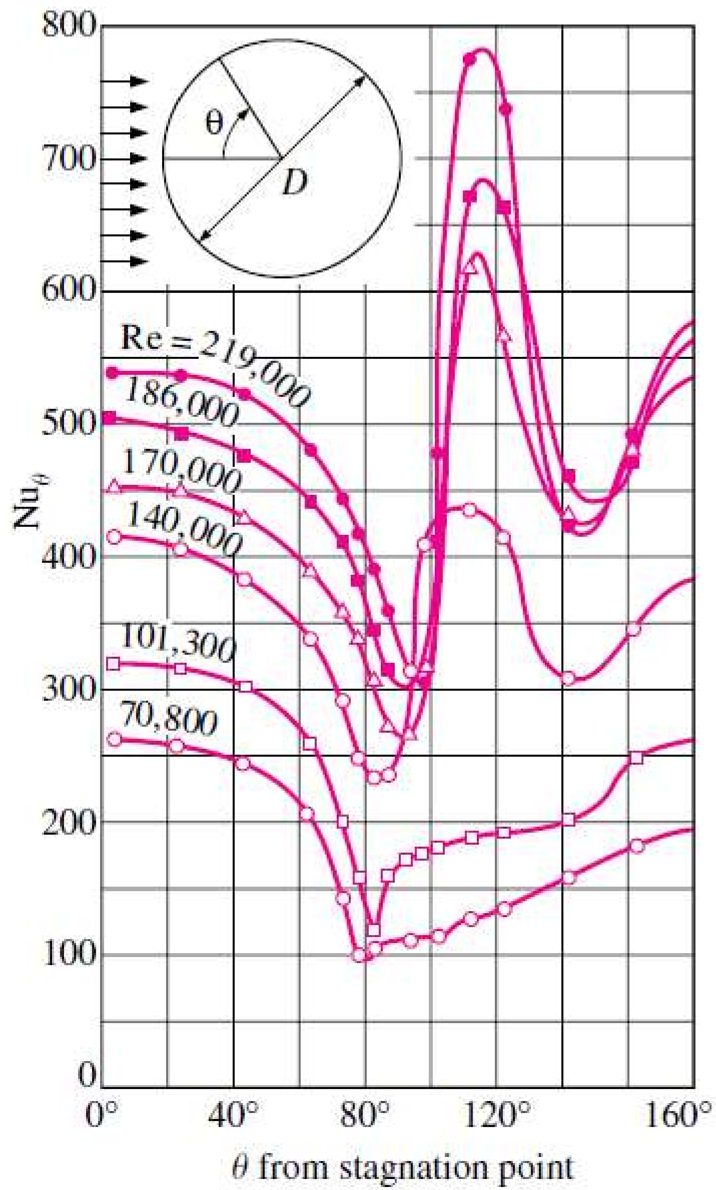


The effect of surface roughness on the drag coefficient of a sphere.

Heat Transfer Coefficient

- Flows across cylinders and spheres, in general, involve *flow separation,* which is difficult to handle analytically.

- Flow across cylinders and spheres has been studied experimentally by numerous investigators, and several empirical correlations have been developed for the heat transfer coefficient.



Variation of the local heat transfer coefficient along the circumference of a circular cylinder in cross flow of air

For flow over a cylinder

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

$$\mathrm{RePr} > 0.2$$

The fluid properties are evaluated at the film temperature $T_f = \frac{1}{2}(T_\infty + T_s)$

For flow over a *sphere*

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\,\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

$$3.5 \le \mathrm{Re} \le 80{,}000 \text{ and } 0.7 \le \mathrm{Pr} \le 380$$

The fluid properties are evaluated at the free-stream temperature T_∞, except for μ_s, which is evaluated at the surface temperature T_s.

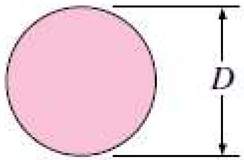
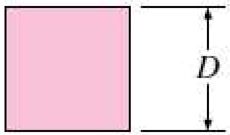
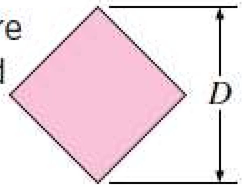
$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = C\,\mathrm{Re}^m\,\mathrm{Pr}^n$$

$$\boxed{n = \frac{1}{3}}$$

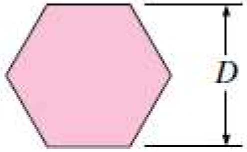
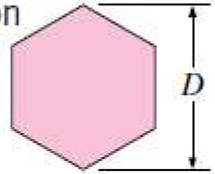
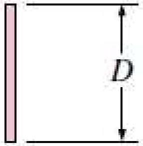
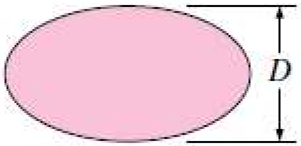
Constants *C* and *m* are given in the table.

The relations for cylinders above are for *single* cylinders or cylinders oriented such that the flow over them is not affected by the presence of others. They are applicable to *smooth* surfaces.

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square 	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°) 	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, 1972 and Jakob, 1949)

Hexagon 	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\,Pr^{1/3}$
Hexagon (tilted 45°) 	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\,Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\,Pr^{1/3}$
Vertical plate 	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\,Pr^{1/3}$
Ellipse 	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\,Pr^{1/3}$

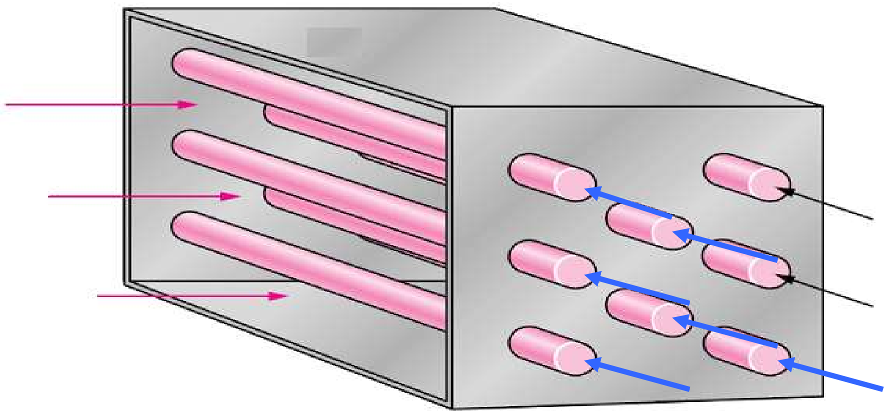
FLOW ACROSS TUBE BANKS

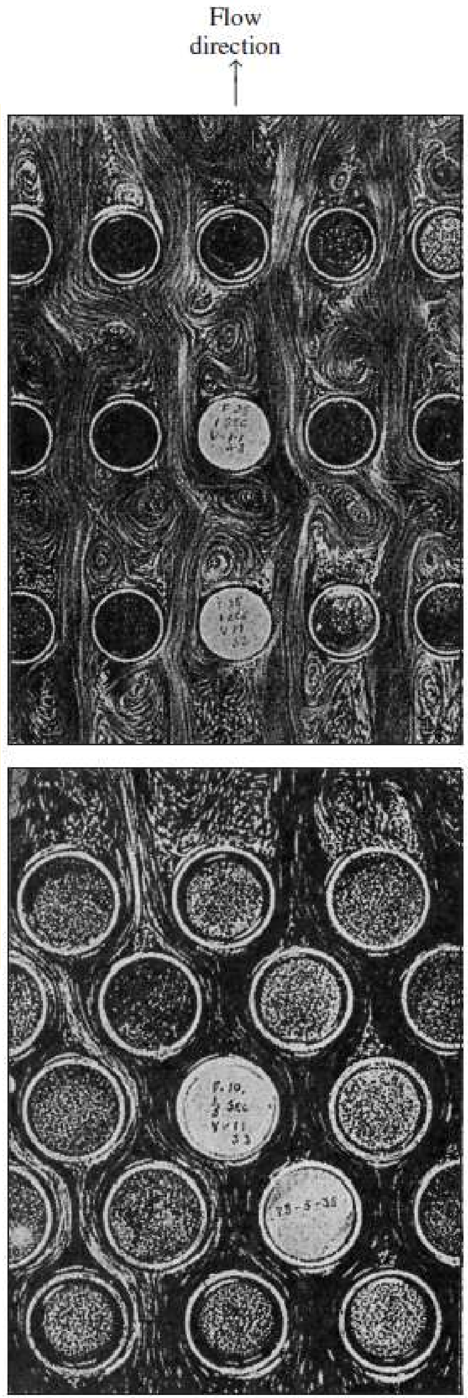
Cross-flow over tube banks is commonly encountered in practice in heat transfer equipment, e.g., heat exchangers.

In such equipment, one fluid moves through the tubes while the other moves over the tubes in a perpendicular direction.

Flow *through* the tubes can be analyzed by considering flow through a single tube, and multiplying the results by the number of tubes.

For flow *over* the tubes the tubes affect the flow pattern and turbulence level downstream, and thus heat transfer to or from them are altered.





Typical arrangement

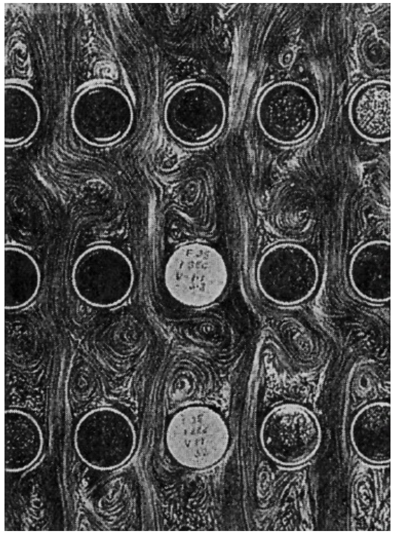
- in-line
- staggered

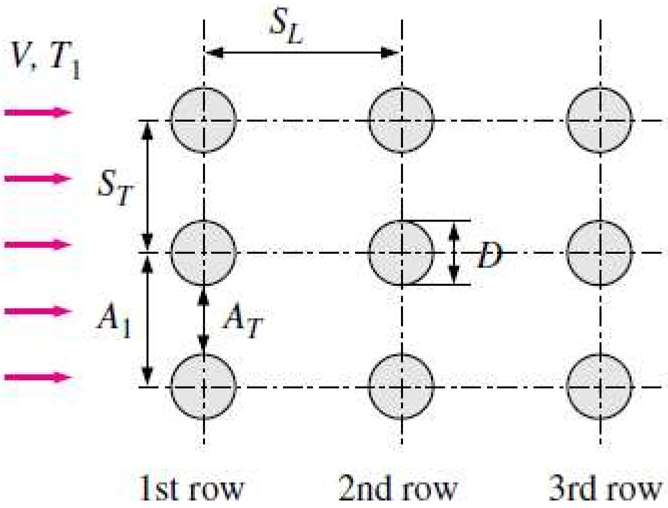
The outer tube diameter *D* is the characteristic length.

The arrangement of the tubes are characterized by the

- *transverse pitch* S_T,
- *longitudinal pitch* S_L , and the
- *diagonal pitch* S_D between tube centers.

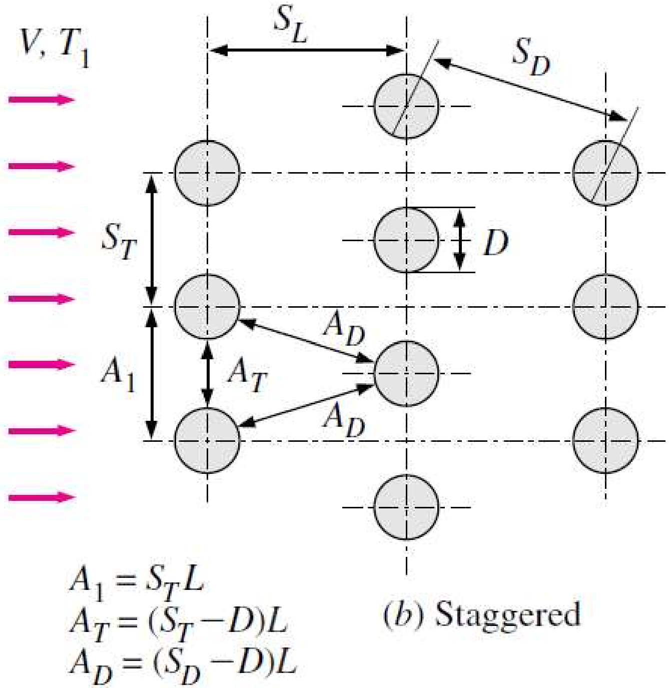
In-line





(*a*) In-line

Staggered



(b) Staggered

The diagonal pitch:

$$S_D = \sqrt{S_L^2 + (S_T/2)^2}$$

Re number based on max. velocity:

$$\mathrm{Re}_D = \frac{\rho V_{\max} D}{\mu} = \frac{V_{\max} D}{\nu}$$

Max. velocity (in-line):

$$V_{\max} = \frac{S_T}{S_T - D} V$$

Max. velocity (staggered):

$$V_{\max} = \frac{S_T}{2(S_D - D)} V$$

$$S_D < (S_T + D)/2$$

Nusselt number

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\,\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

Average temperature of inlet and exit (for property evaluation):

$$T_m = \frac{T_i + T_e}{2}$$

Nusselt number (< 16 rows):

$$\mathrm{Nu}_{D,N_L} = F\,\mathrm{Nu}_D$$

Log mean temp. dif.

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

Exit temperature:

$$T_e = T_s - (T_s - T_i)\exp\left(\pm\frac{A_s h}{\dot{m} c_p}\right)$$

Heat transfer rate:

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}c_p(T_e - T_i)$$

Nusselt number correlations for cross flow over tube banks for $N > 16$ and $0.7 < \Pr < 500$ (from Zukauskas, 1987)*

Arrangement	Range of Re_D	Correlation
In-line	0–100	$\mathrm{Nu}_D = 0.9\,\mathrm{Re}_D^{0.4}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	100–1000	$\mathrm{Nu}_D = 0.52\,\mathrm{Re}_D^{0.5}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\mathrm{Nu}_D = 0.27\,\mathrm{Re}_D^{0.63}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\mathrm{Nu}_D = 0.033\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.4}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
Staggered	0–500	$\mathrm{Nu}_D = 1.04\,\mathrm{Re}_D^{0.4}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	500–1000	$\mathrm{Nu}_D = 0.71\,\mathrm{Re}_D^{0.5}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$1000–2 \times 10^5$	$\mathrm{Nu}_D = 0.35(S_T/S_L)^{0.2}\,\mathrm{Re}_D^{0.6}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$
	$2 \times 10^5–2 \times 10^6$	$\mathrm{Nu}_D = 0.031(S_T/S_L)^{0.2}\,\mathrm{Re}_D^{0.8}\mathrm{Pr}^{0.36}(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$

*All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and outlet temperatures of the fluid (Pr_s is to be evaluated at T_s).

Correction factor F to be used in $\mathrm{Nu}_{D,\,N_L} = F\,\mathrm{Nu}_D$ for $N_L < 16$ and $\mathrm{Re}_D > 1000$ (from Zukauskas, 1987)

N_L	1	2	3	4	5	7	10	13
In-line	0.70	0.80	0.86	0.90	0.93	0.96	0.98	0.99
Staggered	0.64	0.76	0.84	0.89	0.93	0.96	0.98	0.99

The force a flowing fluid exerts on a body in the flow direction is called *drag*. The part of drag that is due directly to wall shear stress τ_w is called the *skin friction drag* since it is caused by frictional effects, and the part that is due directly to pressure is called the *pressure drag* or *form drag* because of its strong dependence on the form or shape of the body.

The *drag coefficient* C_D is a dimensionless number that represents the drag characteristics of a body, and is defined as

$$C_D = \frac{F_D}{\frac{1}{2}\rho V^2 A}$$

where A is the *frontal area* for blunt bodies, and surface area for parallel flow over flat plates or thin airfoils. For flow over a flat plate, the Reynolds number is

$$\mathrm{Re}_x = \frac{\rho V x}{\mu} = \frac{Vx}{\nu}$$

Transition from laminar to turbulent occurs at the *critical Reynolds number* of

$$\mathrm{Re}_{x,\mathrm{cr}} = \frac{\rho V x_{\mathrm{cr}}}{\mu} = 5 \times 10^5$$

For parallel flow over a flat plate, the local friction and convection coefficients are

Laminar: $$C_{f,x} = \frac{0.664}{\mathrm{Re}_x^{1/2}}, \qquad \mathrm{Re}_x < 5 \times 10^5$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}, \qquad \mathrm{Pr} > 0.6$$

Turbulent: $$C_{f,x} = \frac{0.059}{\mathrm{Re}_x^{1/5}}, \qquad 5 \times 10^5 \le \mathrm{Re}_x \le 10^7$$

$$\mathrm{Nu}_x = \frac{h_x x}{k} = 0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_x \le 10^7 \end{array}$$

The *average* friction coefficient relations for flow over a flat plate are:

Laminar: $$C_f = \frac{1.33}{\mathrm{Re}_L^{1/2}}, \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}}, \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Combined: $$C_f = \frac{0.074}{\mathrm{Re}_L^{1/5}} - \frac{1742}{\mathrm{Re}_L}, \qquad 5 \times 10^5 \le \mathrm{Re}_L \le 10^7$$

Rough surface, turbulent: $$C_f = \left(1.89 - 1.62 \log \frac{\varepsilon}{L}\right)^{-2.5}$$

The average Nusselt number relations for flow over a flat plate are:

Laminar: $$\mathrm{Nu} = \frac{hL}{k} = 0.664\,\mathrm{Re}_L^{0.5}\,\mathrm{Pr}^{1/3}, \qquad \mathrm{Re}_L < 5 \times 10^5$$

Turbulent:

$$\mathrm{Nu} = \frac{hL}{k} = 0.037\,\mathrm{Re}_L^{0.8}\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

Combined:

$$\mathrm{Nu} = \frac{hL}{k} = (0.037\,\mathrm{Re}_L^{0.8} - 871)\,\mathrm{Pr}^{1/3}, \qquad \begin{array}{l} 0.6 \le \mathrm{Pr} \le 60 \\ 5 \times 10^5 \le \mathrm{Re}_L \le 10^7 \end{array}$$

For isothermal surfaces with an unheated starting section of length ξ, the local Nusselt number and the average convection coefficient relations are

Laminar: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{3/4}]^{1/3}} = \frac{0.332\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{3/4}]^{1/3}}$$

Turbulent: $$\mathrm{Nu}_x = \frac{\mathrm{Nu}_{x\,(\text{for } \xi=0)}}{[1-(\xi/x)^{9/10}]^{1/9}} = \frac{0.0296\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}}{[1-(\xi/x)^{9/10}]^{1/9}}$$

Laminar: $$h = \frac{2[1-(\xi/x)^{3/4}]}{1-\xi/L}\,h_{x=L}$$

Turbulent: $$h = \frac{5[1-(\xi/x)^{9/10}]}{(1-\xi/L)}\,h_{x=L}$$

These relations are for the case of *isothermal* surfaces. When a flat plate is subjected to *uniform heat flux,* the local Nusselt number is given by

Laminar: $\mathrm{Nu}_x = 0.453\,\mathrm{Re}_x^{0.5}\,\mathrm{Pr}^{1/3}$

Turbulent: $\mathrm{Nu}_x = 0.0308\,\mathrm{Re}_x^{0.8}\,\mathrm{Pr}^{1/3}$

The average Nusselt numbers for cross flow over a *cylinder* and *sphere* are

$$\mathrm{Nu}_{\mathrm{cyl}} = \frac{hD}{k} = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{[1 + (0.4/\mathrm{Pr})^{2/3}]^{1/4}}\left[1 + \left(\frac{\mathrm{Re}}{282{,}000}\right)^{5/8}\right]^{4/5}$$

which is valid for Re Pr > 0.2, and

$$\mathrm{Nu}_{\mathrm{sph}} = \frac{hD}{k} = 2 + [0.4\,\mathrm{Re}^{1/2} + 0.06\,\mathrm{Re}^{2/3}]\mathrm{Pr}^{0.4}\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$

which is valid for $3.5 \le \mathrm{Re} \le 80{,}000$ and $0.7 \le \mathrm{Pr} \le 380$. The fluid properties are evaluated at the film temperature $T_f = (T_\infty + T_s)/2$ in the case of a cylinder, and at the free-stream temperature T_∞ (except for μ_s, which is evaluated at the surface temperature T_s) in the case of a sphere.

In tube banks, the Reynolds number is based on the maximum velocity $V_{\max}$ that is related to the approach velocity V as

In-line and *Staggered* with $S_D < (S_T + D)/2$:

$$V_{\max} = \frac{S_T}{S_T - D}V$$

Staggered with $S_D < (S_T + D)/2$:

$$V_{\max} = \frac{S_T}{2(S_D - D)}V$$

where S_T the transverse pitch and S_D is the diagonal pitch. The average Nusselt number for cross flow over tube banks is expressed as

$$\mathrm{Nu}_D = \frac{hD}{k} = C\,\mathrm{Re}_D^m\mathrm{Pr}^n(\mathrm{Pr}/\mathrm{Pr}_s)^{0.25}$$

where the values of the constants C, m, and n depend on Reynolds number. Such correlations are given in Table 7–2. All properties except Pr_s are to be evaluated at the arithmetic mean of the inlet and exit temperatures of the fluid defined as $T_m = (T_i + T_e)/2$.

The average Nusselt number for tube banks with less than 16 rows is expressed as

$$\mathrm{Nu}_{D,N_L} = F\mathrm{Nu}_D$$

where F is the *correction factor* whose values are given in Table 7–3. The heat transfer rate to or from a tube bank is determined from

$$\dot{Q} = hA_s\Delta T_{\ln} = \dot{m}c_p(T_e - T_i)$$

where $\Delta T_{\ln}$ is the logarithmic mean temperature difference defined as

$$\Delta T_{\ln} = \frac{(T_s - T_e) - (T_s - T_i)}{\ln[(T_s - T_e)/(T_s - T_i)]} = \frac{\Delta T_e - \Delta T_i}{\ln(\Delta T_e/\Delta T_i)}$$

and the exit temperature of the fluid T_e is

$$T_e = T_s - (T_s - T_i)\exp\left(-\frac{A_s h}{\dot{m}c_p}\right)$$

where $A_s = N\pi DL$ is the heat transfer surface area and $\dot{m} = \rho V(N_T S_T L)$ is the mass flow rate of the fluid. The pressure drop ΔP for a tube bank is expressed as

$$\Delta P = N_L f\chi\frac{\rho V_{\max}^2}{2}$$

where f is the friction factor and χ is the correction factor, both given in Fig. 7–27.

SUMMARY

Parallel Flow Over Flat Plates

- Flat Plate with Unheated Starting Length, Uniform Heat Flux

Flow Across Cylinders and Spheres

Flow across Tube Banks