EXAMPLE 8–68

A water tank filled with solar-heated water at 40°C is to be used for showers in a field using gravity-driven flow. The system includes 20 m of 1.5-cm-diameter galvanized iron piping with four miter bends (90°) without vanes and a wide-open globe valve. If water is to flow at a rate of 0.7 L/s through the shower head, determine how high the water level in the tank must be from the exit level of the shower. Disregard the losses at the entrance and at the shower head, and neglect the effect of the kinetic energy correction factor.

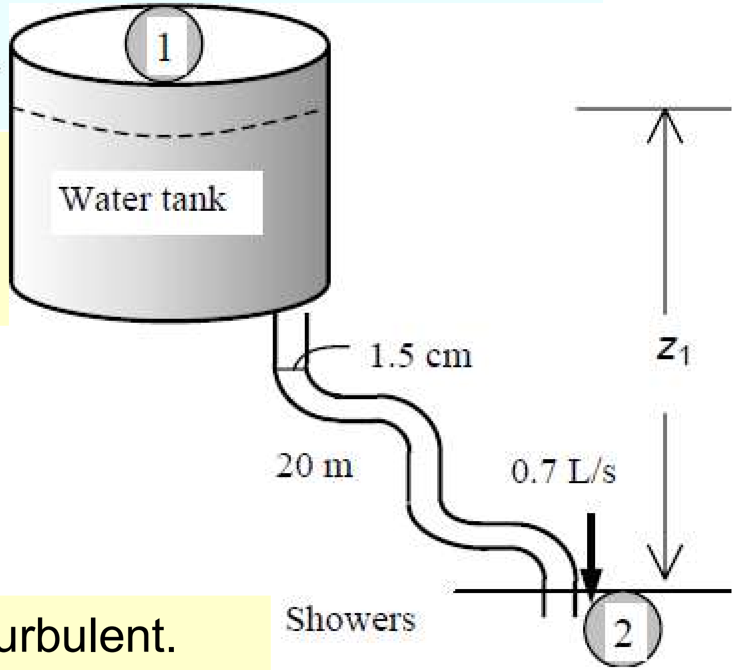
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** The elevation difference between the free surface of water in the tank and the shower head remains constant. **5** There are no pumps or turbines in the piping system. **6** The losses at the entrance and at the showerhead are said to be negligible. **7** The water tank is open to the atmosphere. **8** The effect of the kinetic energy correction factor is negligible, $\alpha = 1$.

Properties The density and dynamic viscosity of water at 40°C are $\rho = 992.1$ kg/m^3 and $\mu = 0.653\times10^{-3}$ kg/m·s, respectively. The loss coefficient is $K_L = 0.5$ for a sharp-edged entrance. The roughness of galvanized iron pipe is $\varepsilon = 0.00015$ m.

Analysis The piping system involves 20 m of 1.5-cm diameter piping, an entrance with negligible loss, 4 miter bends (90°) without vanes (K_L = 1.1 each), and a wide open globe valve (K_L = 10). We choose point 1 at the free surface of water in the tank, and point 2 at the shower exit, which is also taken to be the reference level (z_2 = 0). The fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$), and V_1 = 0. Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad z_1=\alpha_2\frac{V_2^2}{2g}+h_L$$

where $h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$



since the diameter of the piping system is constant. The average velocity in the pipe and the Reynolds number are

$$V_2=\frac{\dot V}{A_c}=\frac{\dot V}{\pi D^2/4}=\frac{0.0007\ \text{m}^3/\text{s}}{\pi(0.015\ \text{m})^2/4}=3.961\ \text{m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(992.1\ \text{kg/m}^3)(3.961\ \text{m/s})(0.015\ \text{m})}{0.653\times10^{-3}\ \text{kg/m}\cdot\text{s}}=90{,}270$$

which is greater than 4000. Therefore, the flow is turbulent.

The relative roughness of the pipe is

$$\varepsilon / D = \frac{0.00015\ \text{m}}{0.015\ \text{m}} = 0.01$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0051}{3.7} + \frac{2.51}{90{,}270\sqrt{f}}\right)$$

It gives f = 0.03857. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}} + 4K_{L,\text{elbow}} + K_{L,\text{valve}} + K_{L,\text{exit}} = 0 + 4\times 1.1 + 10 + 0 = 14.4$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system considered (in this case it is not). Then the total head loss and the elevation of the source become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V_2^2}{2g} = \left((0.03857)\frac{20\ \text{m}}{0.015\ \text{m}} + 14.4\right)\frac{(3.961\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 52.6\ \text{m}$$

$$z_1 = \alpha_2\frac{V_2^2}{2g} + h_L = (1)\frac{(3.961\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} + 52.6\ \text{m} = \mathbf{53.4\ m}$$

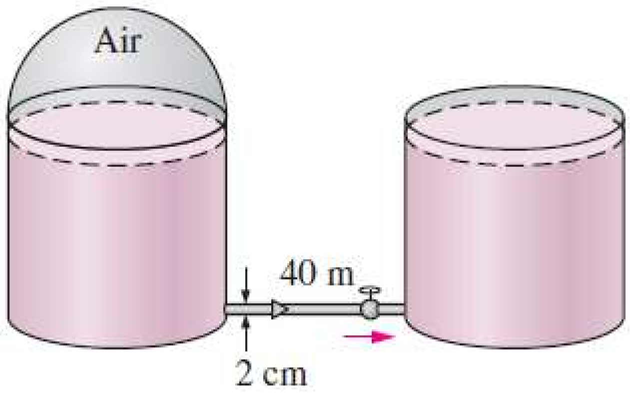
since α_2 = 1. Therefore, the free surface of the tank must be 53.4 m above the shower exit to ensure water flow at the specified rate.

EXAMPLE 8–69

Two water reservoirs *A* and *B* are connected to each other through a 40-m-long, 2-cm-diameter cast iron pipe with a sharp-edged entrance. The pipe also involves a swing check valve and a fully open gate valve. The water level in both reservoirs is the same, but reservoir *A* is pressurized by compressed air while reservoir *B* is open to the atmosphere at 88 kPa. If the initial flow rate through the pipe is 1.2 L/s, determine the absolute air pressure on top of reservoir *A.* Take the water temperature to be 10°C.

Answer: 733 kPa

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** There are no pumps or turbines in the piping system.



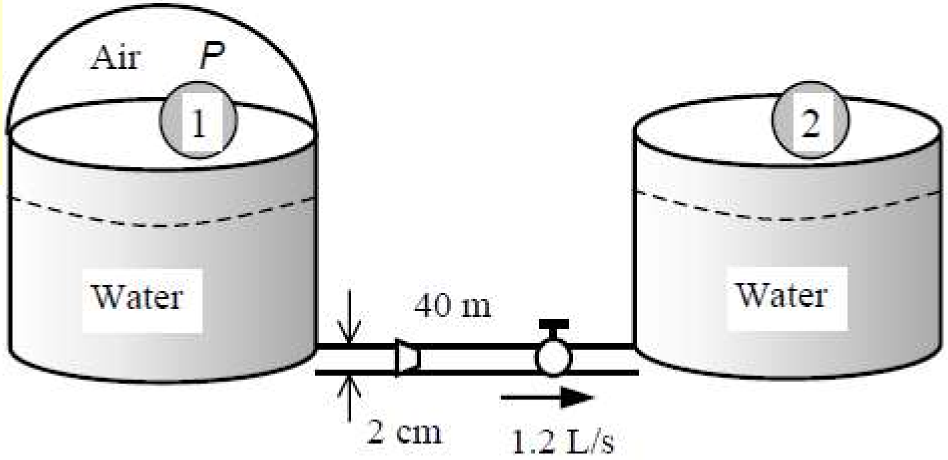
Properties The density and dynamic viscosity of water at 10°C are ρ = 999.7 kg/m^3 and μ =1.307×10^{-3} kg/m·s. The loss coefficient is K_L = 0.5 for a sharp-edged entrance, K_L = 2 for swing check valve, K_L = 0.2 for the fully open gate valve, and K_L = 1 for the exit. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis We choose points 1 and 2 at the free surfaces of the two reservoirs. We note that the fluid velocities at both points are zero ($V_1 = V_2 = 0$), the fluid at point 2 is open to the atmosphere (and thus $P_2 = P_{atm}$), both points are at the same level ($z_1 = z_2$). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \;\rightarrow\; \frac{P_1}{\rho g}=\frac{P_{\text{atm}}}{\rho g}+h_L \;\rightarrow\; P_1=P_{\text{atm}}+\rho g h_L$$

where $h_L = h_{L,\text{total}} = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$

since the diameter of the piping system is constant. The average flow velocity and the Reynolds number are



$$V_2=\frac{\dot{V}}{A_c}=\frac{\dot{V}}{\pi D^2/4}=\frac{0.0012\ \text{m}^3/\text{s}}{\pi(0.02\ \text{m})^2/4}=3.82\ \text{m/s}$$

$$\text{Re}=\frac{\rho V_2 D}{\mu}=\frac{(999.7\ \text{kg/m}^3)(3.82\ \text{m/s})(0.02\ \text{m})}{1.307\times10^{-3}\ \text{kg/m}\cdot\text{s}}$$
$$=58{,}400$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{0.00026\text{ m}}{0.02\text{ m}} = 0.013$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.013}{3.7} + \frac{2.51}{58{,}400\sqrt{f}}\right)$$

It gives f = 0.0424. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}} + K_{L,\text{check valve}} + K_{L,\text{ gate valve}} + K_{L,\text{exit}} = 0.5 + 2 + 0.2 + 1 = 3.7$$

Then the total head loss becomes

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V_2^2}{2g} = \left((0.0424)\frac{40\text{ m}}{0.02\text{ m}} + 3.7\right)\frac{(3.82\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 65.8\text{ m}$$

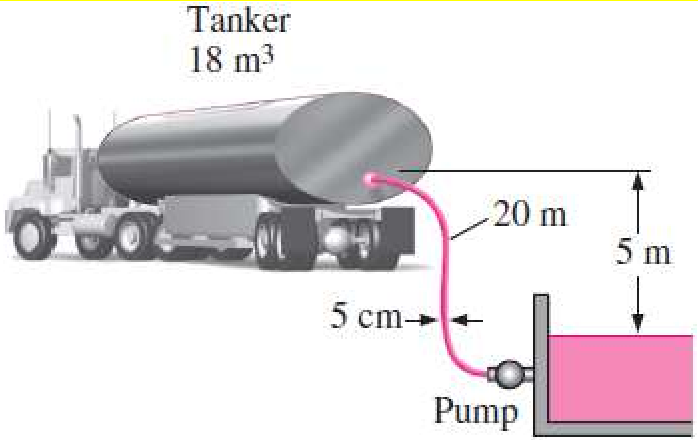
Substituting,

$$P_1 = P_{\text{atm}} + \rho g h_L = (88\text{ kPa}) + (999.7\text{ kg/m}^3)(9.81\text{ m/s}^2)(65.8\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = \mathbf{734\ kPa}$$

Discussion The absolute pressure above the first reservoir must be 734 kPa, which is quite high. Note that the minor losses in this case are negligible (about 4% of total losses). Also, the friction factor could be determined easily from the explicit Haaland relation (it gives the same result, 0.0424). The friction coefficient would drop to 0.0202 if smooth pipes were used.

EXAMPLE 8-70

A vented tanker is to be filled with fuel oil with ρ = 920 kg/m^3 and μ = 0.045 kg/m · s from an underground reservoir using a 20-m-long, 5-cm-diameter plastic hose with a slightly rounded entrance and two 90° smooth bends. The elevation difference between the oil level in the reservoir and the top of the tanker where the hose is discharged is 5 m. The capacity of the tanker is 18 m^3 and the filling time is 30 min. Taking the kinetic energy correction factor at hose discharge to be 1.05 and assuming an overall pump efficiency of 82 percent, determine the required power input to the pump.



Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Fuel oil level remains constant. **5** Reservoir is open to the atmosphere.

Properties The density and dynamic viscosity of fuel oil are given to be ρ = 920 kg/m^3 and μ = 0.045 kg/m·s. The loss coefficient is K_L = 0.12 for a slightly-rounded entrance and K_L = 0.3 for a 90° smooth bend (flanged). The plastic pipe is smooth and thus ε = 0. The kinetic energy correction factor at hose discharge is given to be α = 1.05.

Analysis We choose point 1 at the free surface of oil in the reservoir and point 2 at the exit of the hose in the tanker. We note the fluid at both points is open to the atmosphere (and thus $P_1 = P_2 = P_{atm}$) and the fluid velocity at point 1 is zero (V_1 = 0). We take the free surface of the reservoir as the reference level (z_1 = 0). Then the energy equation for a control volume between these two points simplifies to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=\alpha_2\frac{V_2^2}{2g}+z_2+h_L$$

where

$$h_L=h_{L,\text{total}}=h_{L,\text{major}}+h_{L,\text{minor}}=\left(f\frac{L}{D}+\sum K_L\right)\frac{V_2^2}{2g}$$

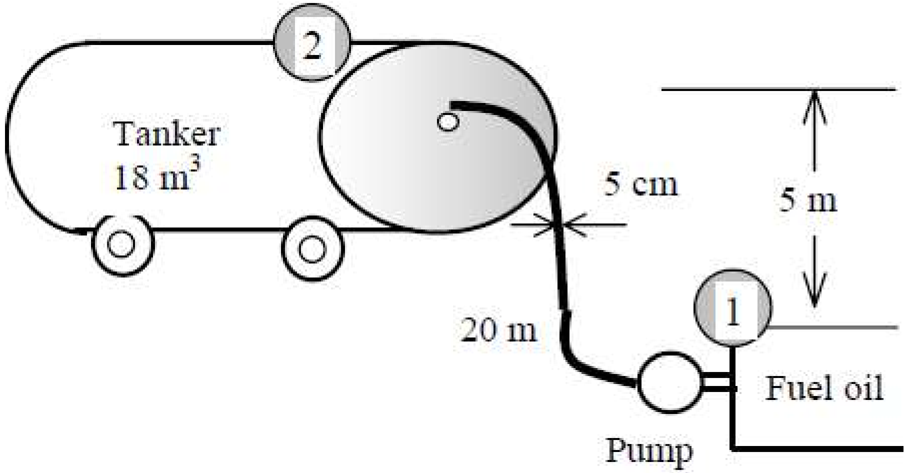
since the diameter of the piping system is constant. The flow rate is determined from the requirement that the tanker must be filled in 30 min,

$$\dot{V}=\frac{V_{\text{tanker}}}{\Delta t}=\frac{18\text{ m}^3}{(30\times 60\text{ s})}=0.01\text{ m}^3/\text{s}$$

Then the average velocity in the pipe and the Reynolds number become

$$V_2 = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.01\,\text{m}^3/\text{s}}{\pi(0.05\,\text{m})^2/4} = 5.093\,\text{m/s}$$

$$\text{Re} = \frac{\rho V_2 D}{\mu} = \frac{(920\,\text{kg/m}^3)(5.093\,\text{m/s})(0.05\,\text{m})}{0.045\,\text{kg/m}\cdot\text{s}} = 5206$$



which is greater than 4000. Therefore, the flow is turbulent. The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(0 + \frac{2.51}{5206\sqrt{f}}\right)$$

It gives f = 0.0370. The sum of the loss coefficients is

$$\sum K_L = K_{L,\text{entrance}} + 2K_{L,\text{bend}} = 0.12 + 2 \times 0.3 = 0.72$$

Note that we do not consider the exit loss unless the exit velocity is dissipated within the system (in this case it is not). Then the total head loss, the useful pump head, and the required pumping power become

$$h_L = \left(f\frac{L}{D} + \sum K_L \right)\frac{V_2^2}{2g} = \left((0.0370)\frac{20\text{ m}}{0.05\text{ m}} + 0.72 \right)\frac{(5.093\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 20.5\text{ m}$$

$$h_{\text{pump, u}} = \frac{V_2^2}{2g} + z_2 + h_L = 1.05\frac{(5.093\text{ m/s})^2}{2(9.81\text{ m/s}^2)} + 5\text{ m} + 20.5\text{ m} = 26.9\text{ m}$$

$$\dot{W}_{\text{pump}} = \frac{\dot{V}\rho g h_{\text{pump, u}}}{\eta_{\text{pump}}} = \frac{(0.01\text{ m}^3/\text{s})(920\text{ kg/m}^3)(9.81\text{ m/s}^2)(26.9\text{ m})}{0.82}\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}^2} \right)\left(\frac{1\text{ kW}}{1\text{ kN}\cdot\text{m/s}} \right) = \mathbf{2.96\text{ kW}}$$

Discussion Note that the minor losses in this case are negligible (0.72/15.52 = 0.046 or about 5% of total losses). Also, the friction factor could be determined easily from the Haaland relation (it gives 0.0372).

EXAMPLE 8–71

Two pipes of identical length and material are connected in parallel. The diameter of pipe A is twice the diameter of pipe B. Assuming the friction factor to be the same in both cases and disregarding minor losses, determine the ratio of the flow rates in the two pipes.

Assumptions **1** The flow is steady and incompressible. **2** The friction factor is given to be the same for both pipes. **3** The minor losses are negligible.

Analysis When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D can be expressed as

$$h_L = f\frac{L}{D}\frac{V^2}{2g} = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{A_c}\right)^2 = f\frac{L}{D}\frac{1}{2g}\left(\frac{\dot{V}}{\pi D^2/4}\right)^2 = 8f\frac{L}{D}\frac{1}{g}\frac{\dot{V}^2}{\pi^2 D^4} = 8f\frac{L}{g\pi^2}\frac{\dot{V}^2}{D^5}$$

Solving for the flow rate gives

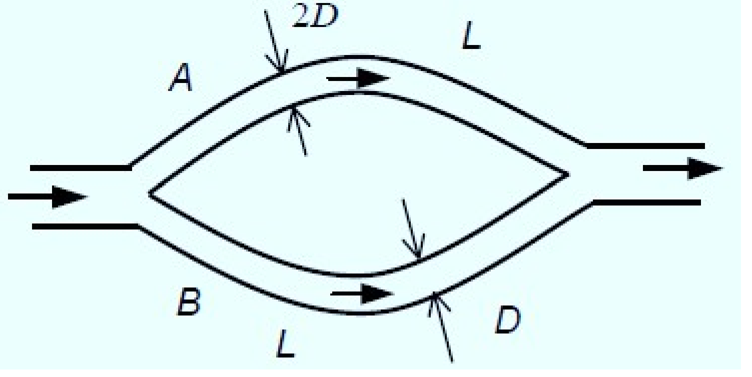
$$\dot{V} = \sqrt{\frac{\pi^2 h_L g}{8fL}}D^{2.5} = kD^{2.5} \qquad (k = \text{constant of proportionality})$$

When the pipe length, friction factor, and the head loss is constant, which is the case here for parallel connection, the flow rate becomes proportional to the 2.5th power of diameter. Therefore, when the diameter is doubled, the flow rate will increase by a factor of $2^{2.5} = 5.66$ since

If $\dot{V}_A = kD_A^{2.5}$

Then $\dot{V}_B = kD_B^{2.5} = k(2D_A)^{2.5} = 2^{2.5}kD_A^{2.5} = 2^{2.5}\dot{V}_A = 5.66\dot{V}_A$

Therefore, the ratio of the flow rates in the two pipes is **5.66**.

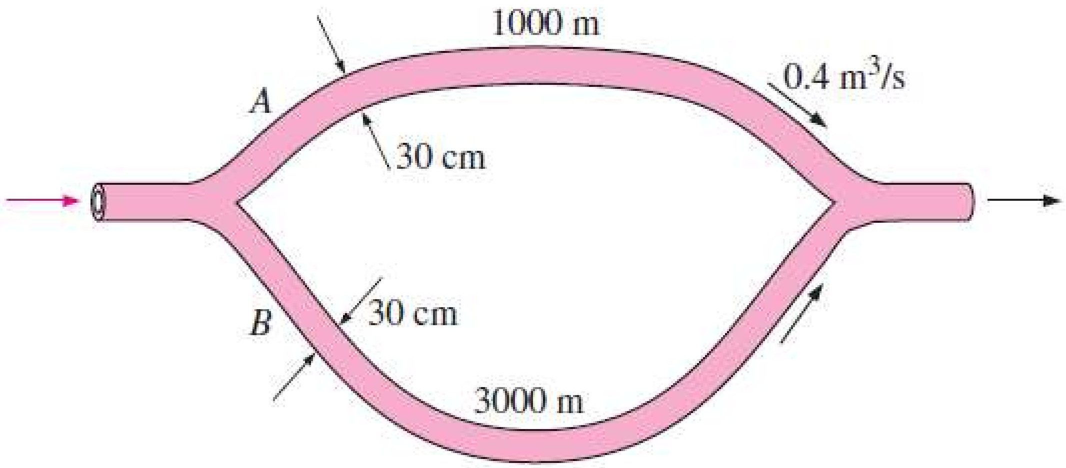


EXAMPLE 8–72

A certain part of cast iron piping of a water distribution system involves a parallel section. Both parallel pipes have a diameter of 30 cm, and the flow is fully turbulent. One of the branches (pipe A) is 1000 m long while the other branch (pipe B) is 3000 m long. If the flow rate through pipe A is 0.4 m^3/s, determine the flow rate through pipe B. Disregard minor losses and assume the water temperature to be 15°C. Show that the flow is fully turbulent, and thus the friction factor is independent of Reynolds number.

Answer: 0.231 m^3/s

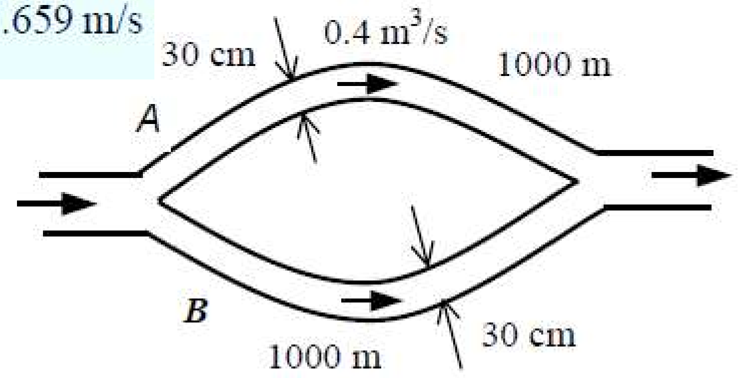
Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number (to be verified).



Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ =1.138×10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m.

Analysis The average velocity in pipe A is

$$V_A = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.4\ \text{m}^3/\text{s}}{\pi (0.30\ \text{m})^2/4} = 5.659\ \text{m/s}$$



When two pipes are parallel in a piping system, the head loss for each pipe must be same. When the minor losses are disregarded, the head loss for fully developed flow in a pipe of length L and diameter D is

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

Writing this for both pipes and setting them equal to each other, and noting that $D_A = D_B$ (given) and $f_A = f_B$ (to be verified) gives

$$f_A\frac{L_A}{D_A}\frac{V_A^2}{2g} = f_B\frac{L_B}{D_B}\frac{V_B^2}{2g} \quad \rightarrow \quad V_B = V_A\sqrt{\frac{L_A}{L_B}} = (5.659\ \text{m/s})\sqrt{\frac{1000\ \text{m}}{3000\ \text{m}}} = 3.267\ \text{m/s}$$

Then the flow rate in pipe B becomes

$$\dot{V}_B = A_c V_B = [\pi D^2/4]V_B = [\pi (0.30\ \text{m})^2/4](3.267\ \text{m/s}) = \mathbf{0.231\ m^3/s}$$

Proof *that flow is fully turbulent and thus friction factor is independent of Reynolds number:*

The velocity in pipe *B* is lower. Therefore, if the flow is fully turbulent in pipe *B*, then it is also fully turbulent in pipe *A*. The Reynolds number in pipe *B* is

$$\mathrm{Re}_B = \frac{\rho V_B D}{\mu} = \frac{(999.1\,\mathrm{kg/m^3})(3.267\,\mathrm{m/s})(0.30\,\mathrm{m})}{1.138\times10^{-3}\,\mathrm{kg/m\cdot s}} = 0.860\times10^6$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{0.00026\,\mathrm{m}}{0.30\,\mathrm{m}} = 0.00087$$

From Moody's chart, we observe that for a relative roughness of 0.00087, the flow is fully turbulent for Reynolds number greater than about 10^6. Therefore, the flow in both pipes is fully turbulent, and thus the assumption that the friction factor is the same for both pipes is valid.

Discussion Note that the flow rate in pipe B is less than the flow rate in pipe A because of the larger losses due to the larger length.

EXAMPLE 8–73

Repeat Prob. 8–82 assuming pipe *A* has a halfway-closed gate valve (K_L = 2.1) while pipe *B* has a fully open globe valve (K_L = 10), and the other minor losses are negligible.

Assume the flow to be fully turbulent.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses other than those for the valves are negligible. **4** The flow is fully turbulent and thus the friction factor is independent of the Reynolds number.

Properties The density and dynamic viscosity of water at 15°C are ρ = 999.1 kg/m^3 and μ =1.138×10^{-3} kg/m·s. The roughness of cast iron pipe is ε = 0.00026 m.

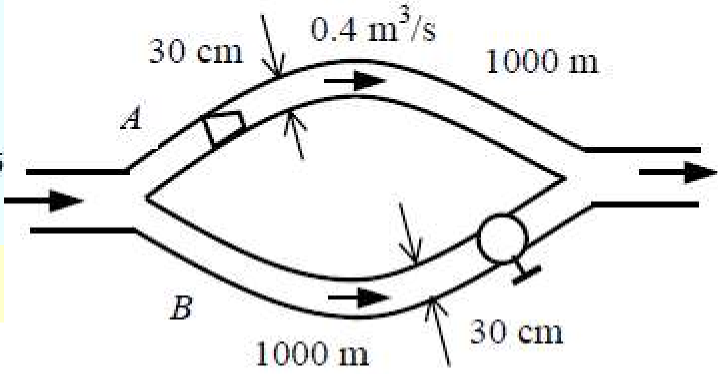
Analysis For pipe A, the average velocity and the Reynolds number are

$$V_A = \frac{\dot{V}_A}{A_c} = \frac{\dot{V}_A}{\pi D^2/4} = \frac{0.4\ \mathrm{m^3/s}}{\pi(0.30\ \mathrm{m})^2/4} = 5.659\ \mathrm{m/s}$$

$$\mathrm{Re}_A = \frac{\rho V_A D}{\mu} = \frac{(999.1\ \mathrm{kg/m^3})(5.659\ \mathrm{m/s})(0.30\ \mathrm{m})}{1.138\times10^{-3}\ \mathrm{kg/m\cdot s}} = 1.49\times10^6$$

The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026\ \mathrm{m}}{0.30\ \mathrm{m}} = 8.667\times10^{-4}$$



The friction factor corresponding to this relative roughness and the Reynolds number can simply be determined from the Moody chart. To avoid the reading error, we determine it from the Colebrook equation

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{8.667\times10^{-4}}{3.7} + \frac{2.51}{1.49\times10^{6}\sqrt{f}}\right)$$

It gives f = 0.0192. Then the total head loss in pipe A becomes

$$h_{L,A} = \left(f\frac{L_A}{D} + K_L\right)\frac{V_A^2}{2g} = \left((0.0192)\frac{1000\text{ m}}{0.30\text{ m}} + 2.1\right)\frac{(5.659\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 107.9\text{ m}$$

When two pipes are parallel in a piping system, the head loss for each pipe must be same. Therefore, the head loss for pipe B must also be 107.9 m. Then the average velocity in pipe B and the flow rate become

$$h_{L,B} = \left(f\frac{L_B}{D} + K_L\right)\frac{V_B^2}{2g} \rightarrow 107.9\text{ m} = \left((0.0192)\frac{3000\text{ m}}{0.30\text{ m}} + 10\right)\frac{V_B^2}{2(9.81\text{ m/s}^2)} \rightarrow V_B = 3.24\text{ m/s}$$

$$\dot{V}_B = A_c V_B = [\pi D^2 / 4]V_B = [\pi(0.30\text{ m})^2 / 4](3.24\text{ m/s}) = \mathbf{0.229\ m^3/s}$$

Discussion Note that the flow rate in pipe B decreases slightly (from 0.231 to 0.229 m^3/s) due to the larger minor loss in that pipe. Also, minor losses constitute just a few percent of the total loss, and they can be neglected if great accuracy is not required.

EXAMPLE 8–74

A geothermal district heating system involves the transport of geothermal water at 110°C from a geothermal well to a city at about the same elevation for a distance of 12 km at a rate of 1.5 m^3/s in 60-cm-diameter stainless-steel pipes. The fluid pressures at the wellhead and the arrival point in the city are to be the same. The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. (a) Assuming the pump–motor efficiency to be 74 percent, determine the electric power consumption of the system for pumping. Would you recommend the use of a single large pump or several smaller pumps of the same total pumping power scattered along the pipeline? Explain. (b) Determine the daily cost of power consumption of the system if the unit cost of electricity is \$0.06/kWh. (c) The temperature of geothermal water is estimated to drop 0.5°C during this long flow. Determine if the frictional heating during flow can make up for this drop in temperature.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m·s, and C_p = 4.229 kJ/kg·°C. The roughness of stainless steel pipes is 2×10^{-6} m.

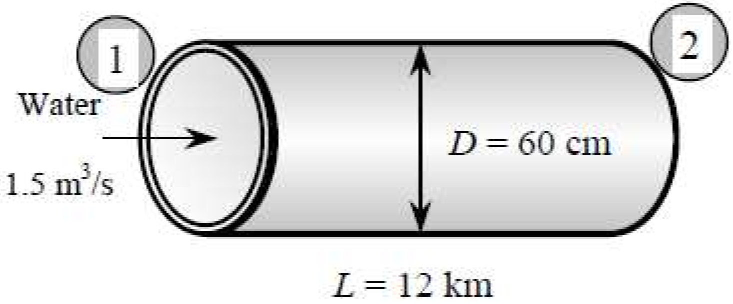
Analysis (a) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The average velocity and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{1.5\ \text{m}^3/\text{s}}{\pi(0.60\ \text{m})^2/4} = 5.305\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(950.6\ \text{kg/m}^3)(5.305\ \text{m/s})(0.60\ \text{m})}{0.255\times10^{-3}\ \text{kg/m}\cdot\text{s}} = 1.187\times10^7$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{2\times10^{-6}\ \text{m}}{0.60\ \text{m}} = 3.33\times10^{-6}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{3.33\times10^{-6}}{3.7} + \frac{2.51}{1.187\times10^7\sqrt{f}}\right)$$

It gives f = 0.00829. Then the pressure drop, the head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.00829\frac{12{,}000\ \text{m}}{0.60\ \text{m}}\frac{(950.6\ \text{kg/m}^3)(5.305\ \text{m/s})^2}{2}\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}}\right)\left(\frac{1\ \text{kPa}}{1\ \text{kN/m}^2}\right) = 2218\ \text{kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = (0.00829)\frac{12{,}000\text{ m}}{0.60\text{ m}}\frac{(5.305\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 238\text{ m}$$

$$\dot{W}_{\text{electric, in}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(1.5\text{ m}^3\text{/s})(2218\text{ kPa})}{0.74}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3\text{/s}}\right) = \mathbf{4496\ kW}$$

Therefore, the pumps will consume 4496 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 2218 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(b) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\Delta t = (4496\text{ kW})(24\text{ h/day}) = 107{,}900\text{ kWh/day}$$

$$\text{Cost} = \text{Amount}\times\text{Unit cost} = (107{,}900\text{ kWh/day})(\$0.06\text{/kWh}) = \mathbf{\$6474/day}$$

(c) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 4496 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{mech}} = \rho \dot{V} c_p \Delta T \rightarrow \Delta T = \frac{\eta_{\text{pump-motor}} \dot{W}_{\text{elect,in}}}{\rho \dot{V} c_p} = \frac{0.74 \times (4496 \text{ kJ/s})}{(950.6 \text{ kg/m}^3)(1.5 \text{ m}^3\text{/s})(4.229 \text{ kJ/kg} \cdot {}^\circ\text{C})} = \mathbf{0.55^\circ C}$$

Therefore, the temperature of water will rise at least 0.55°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

EXAMPLE 8-75

Repeat Prob. 8–84 for cast iron pipes of the same diameter.

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The minor losses are negligible because of the large length-to-diameter ratio and the relatively small number of components that cause minor losses. **4** The geothermal well and the city are at about the same elevation. **5** The properties of geothermal water are the same as fresh water. **6** The fluid pressures at the wellhead and the arrival point in the city are the same.

Properties The properties of water at 110°C are ρ = 950.6 kg/m^3, μ = 0.255×10^{-3} kg/m·s, and c_p = 4.229 kJ/kg·°C. The roughness of cast iron pipes is 0.00026 m.

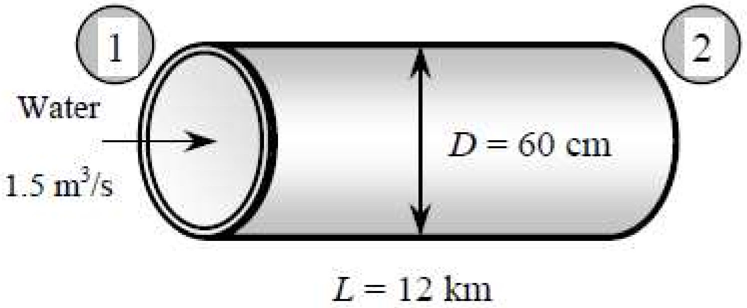
Analysis (a) We take point 1 at the well-head of geothermal resource and point 2 at the final point of delivery at the city, and the entire piping system as the control volume. Both points are at the same elevation ($z_2 = z_2$) and the same velocity ($V_1 = V_2$) since the pipe diameter is constant, and the same pressure ($P_1 = P_2$). Then the energy equation for this control volume simplifies to

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, e}} + h_L \quad \rightarrow \quad h_{\text{pump, u}} = h_L$$

That is, the pumping power is to be used to overcome the head losses due to friction in flow. The average velocity and the Reynolds number are

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{1.5\ \mathrm{m^3/s}}{\pi(0.60\ \mathrm{m})^2/4} = 5.305\ \mathrm{m/s}$$

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(950.6\ \mathrm{kg/m^3})(5.305\ \mathrm{m/s})(0.60\ \mathrm{m})}{0.255\times10^{-3}\ \mathrm{kg/m\cdot s}} = 1.187\times10^{7}$$



which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon / D = \frac{0.00026\ \mathrm{m}}{0.60\ \mathrm{m}} = 4.33\times10^{-4}$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{4.33\times10^{-4}}{3.7} + \frac{2.51}{1.187\times10^{7}\sqrt{f}}\right)$$

It gives f = 0.0162. Then the pressure drop, the head loss, and the required power input become

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V^2}{2} = 0.0162\,\frac{12{,}000\ \mathrm{m}}{0.60\ \mathrm{m}}\,\frac{(950.6\ \mathrm{kg/m^3})(5.305\ \mathrm{m/s})^2}{2}\left(\frac{1\ \mathrm{kN}}{1000\ \mathrm{kg\cdot m/s}}\right)\left(\frac{1\ \mathrm{kPa}}{1\ \mathrm{kN/m^2}}\right) = 4334\ \mathrm{kPa}$$

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V^2}{2g} = (0.0162)\frac{12{,}000\ \text{m}}{0.60\ \text{m}}\frac{(5.305\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 465\ \text{m}$$

$$\dot{W}_{\text{elect, in}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(1.5\ \text{m}^3/\text{s})(4334\ \text{kPa})}{0.74}\left(\frac{1\ \text{kW}}{1\ \text{kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{8785\ kW}$$

Therefore, the pumps will consume 8785 kW of electric power to overcome friction and maintain flow. The pumps must raise the pressure of the geothermal water by 4334 kPa. Providing a pressure rise of this magnitude at one location may create excessive stress in piping at that location. Therefore, it is more desirable to raise the pressure by smaller amounts at a several locations along the flow. This will keep the maximum pressure in the system and the stress in piping at a safer level.

(b) The daily cost of electric power consumption is determined by multiplying the amount of power used per day by the unit cost of electricity,

$$\text{Amount} = \dot{W}_{\text{elect,in}}\Delta t = (8785\ \text{kW})(24\ \text{h/day}) = 210{,}800\ \text{kWh/day}$$

$$\text{Cost} = \text{Amount} \times \text{Unit cost} = (210{,}800\ \text{kWh/day})(\$0.06/\text{kWh}) = \mathbf{\$12{,}650/day}$$

(c) The energy consumed by the pump (except the heat dissipated by the motor to the air) is eventually dissipated as heat due to the frictional effects. Therefore, this problem is equivalent to heating the water by a 8785 kW of resistance heater (again except the heat dissipated by the motor). To be conservative, we consider only the useful mechanical energy supplied to the water by the pump. The temperature rise of water due to this addition of energy is

$$\dot{W}_{\text{mech}} = \rho \dot{V} c_p \Delta T \;\rightarrow\; \Delta T = \frac{\eta_{\text{pump-motor}} \dot{W}_{\text{elect,in}}}{\rho \dot{V} c_p} = \frac{0.74 \times (8785\ \text{kJ/s})}{(950.6\ \text{kg/m}^3)(1.5\ \text{m}^3/\text{s})(4.229\ \text{kJ/kg}\cdot{}^\circ\text{C})} = \mathbf{1.1^\circ C}$$

Therefore, the temperature of water will rise at least 1.1°C, which is more than the 0.5°C drop in temperature (in reality, the temperature rise will be more since the energy dissipation due to pump inefficiency will also appear as temperature rise of water). Thus we conclude that the frictional heating during flow can more than make up for the temperature drop caused by heat loss.

Discussion The pumping power requirement and the associated cost can be reduced by using a larger diameter pipe. But the cost savings should be compared to the increased cost of larger diameter pipe.

EXAMPLE 8–76

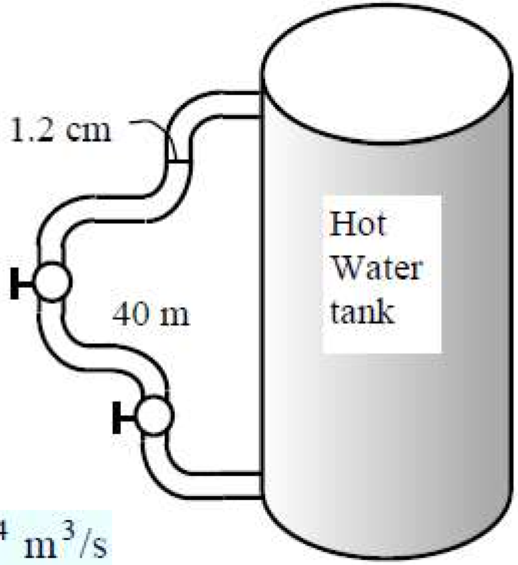
In large buildings, hot water in a water tank is circulated through a loop so that the user doesn't have to wait for all the water in long piping to drain before hot water starts coming out. A certain recirculating loop involves 40-m-long, 1.2-cm-diameter cast iron pipes with six 90° threaded smooth bends and two fully open gate valves. If the average flow velocity through the loop is 2.5 m/s, determine the required power input for the recirculating pump. Take the average water temperature to be 60°C and the efficiency of the pump to be 70 percent.
Answer: 0.217 kW

Assumptions **1** The flow is steady and incompressible. **2** The flow is fully developed. **3** The flow is turbulent so that the tabulated value of the loss coefficients can be used (to be verified). **4** Minor losses other than those for elbows and valves are negligible.
Properties The density and dynamic viscosity of water at 60°C are ρ = 983.3 kg/m^3, μ = 0.467×10^{-3} kg/m·s. The roughness of cast iron pipes is 0.00026 m. The loss coefficient is K^L = 0.9 for a threaded 90° smooth bend and K_L = 0.2 for a fully open gate valve.
Analysis Since the water circulates continually and undergoes a cycle, we can take the entire recirculating system as the control volume, and choose points 1 and 2 at any location at the same point. Then the properties (pressure, elevation, and velocity) at 1 and 2 will be identical, and the energy equation will simplify to

$$\frac{P_1}{\rho g}+\alpha_1\frac{V_1^2}{2g}+z_1+h_{\text{pump, u}}=\frac{P_2}{\rho g}+\alpha_2\frac{V_2^2}{2g}+z_2+h_{\text{turbine, e}}+h_L \quad\rightarrow\quad h_{\text{pump, u}}=h_L$$

where

$$h_L = h_{L,\text{major}} + h_{L,\text{minor}} = \left(f\frac{L}{D} + \sum K_L \right) \frac{V_2^2}{2g}$$



since the diameter of the piping system is constant. Therefore, the pumping power is to be used to overcome the head losses in the flow. The flow rate and the Reynolds number are

$$\dot{V} = VA_c = V(\pi D^2/4) = (2.5\text{ m/s})[\pi(0.012\text{ m})^2/4] = 2.83\times10^{-4}\text{ m}^3/\text{s}$$

$$\text{Re} = \frac{\rho VD}{\mu} = \frac{(983.3\text{ kg/m}^3)(2.5\text{ m/s})(0.012\text{ m})}{0.467\times10^{-3}\text{ kg/m}\cdot\text{s}} = 63{,}200$$

which is greater than 4000. Therefore, the flow is turbulent. The relative roughness of the pipe is

$$\varepsilon/D = \frac{0.00026\text{ m}}{0.012\text{ m}} = 0.0217$$

The friction factor can be determined from the Moody chart, but to avoid the reading error, we determine it from the Colebrook equation using an equation solver (or an iterative scheme),

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon / D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \rightarrow \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0217}{3.7} + \frac{2.51}{63200\sqrt{f}}\right)$$

It gives f = 0.05075. Then the total head loss, pressure drop, and the required pumping power input become

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V_2^2}{2g} = \left((0.05075)\frac{40\text{ m}}{0.012\text{ m}} + 6\times 0.9 + 2\times 0.2\right)\frac{(2.5\text{ m/s})^2}{2(9.81\text{ m/s}^2)} = 55.8\text{ m}$$

$$\Delta P = \Delta P_L = \rho g h_L = (983.3\text{ kg/m}^3)(9.81\text{ m/s}^2)(55.8\text{ m})\left(\frac{1\text{ kN}}{1000\text{ kg}\cdot\text{m/s}}\right)\left(\frac{1\text{ kPa}}{1\text{ kN/m}^2}\right) = 538\text{ kPa}$$

$$\dot{W}_{\text{elect}} = \frac{\dot{W}_{\text{pump, u}}}{\eta_{\text{pump-motor}}} = \frac{\dot{V}\Delta P}{\eta_{\text{pump-motor}}} = \frac{(2.83\times 10^{-4}\text{ m}^3/\text{s})(538\text{ kPa})}{0.70}\left(\frac{1\text{ kW}}{1\text{ kPa}\cdot\text{m}^3/\text{s}}\right) = \mathbf{0.217\text{ kW}}$$

Therefore, the required power input of the recirculating pump is 0.217 kW

Discussion It can be shown that the required pumping power input for the recirculating pump is 0.210 kW when the minor losses are not considered. Therefore, the minor losses can be neglected in this case without a major loss in accuracy.