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CHAPTER 4
TRANSIENT HEAT CONDUCTION

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SUMMARY

Lumped System Analysis

- Criteria for Lumped System Analysis

Transient Heat Conduction in Large Plane Walls, Long Cylinders, and Spheres with Spatial Effects

- Nondimensionalized One-Dimensional Transient Conduction Problem
- Exact Solution of One-Dimensional Transient Conduction Problem
- Approximate Analytical and Graphical Solutions

Transient Heat Conduction in Multidimensional Systems

Objectives

- Assess when the spatial variation of temperature is negligible, and temperature varies nearly uniformly with time, making the simplified lumped system analysis applicable

- Obtain analytical solutions for transient one-dimensional conduction problems in rectangular, cylindrical, and spherical geometries using the method of separation of variables, and understand why a one-term solution is usually a reasonable approximation

- Solve the transient conduction problem in large mediums using the similarity variable, and predict the variation of temperature with time and distance from the exposed surface

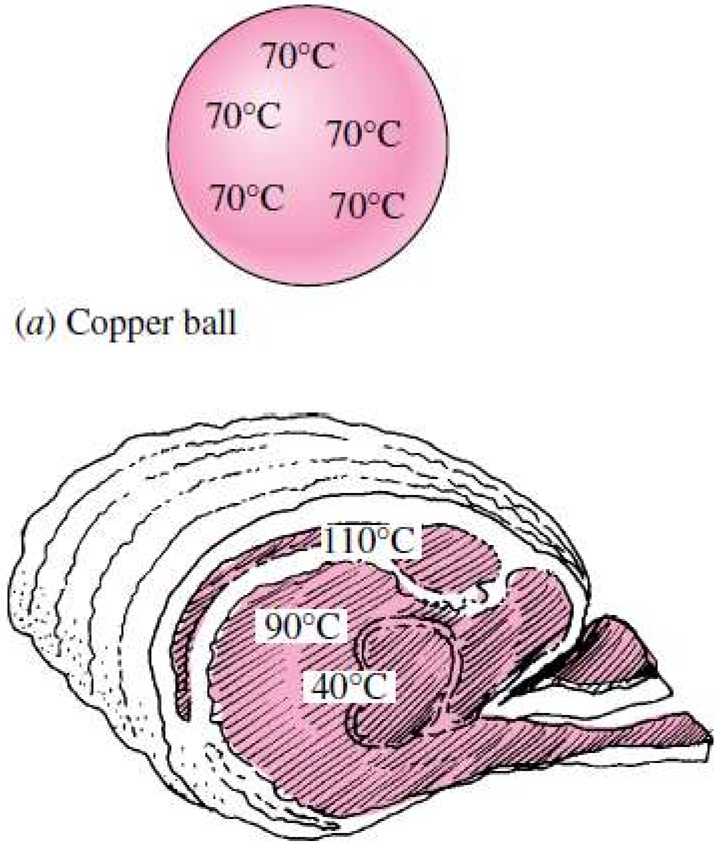
- Construct solutions for multi-dimensional transient conduction problems using the product solution approach.

LUMPED SYSTEM ANALYSIS

Interior temperature of some bodies remains essentially uniform at all times during a heat transfer process.

The temperature of such bodies can be taken to be a function of time only, $T(t)$.

Heat transfer analysis that utilizes this idealization is known as lumped system analysis.



(*a*) Copper ball

(*b*) Roast beef

A small copper ball can be modeled as a lumped system, but a roast beef cannot.

$$\begin{pmatrix}\text{Heat transfer into the body} \\ \text{during } dt\end{pmatrix} = \begin{pmatrix}\text{The increase in the} \\ \text{energy of the body} \\ \text{during } dt\end{pmatrix}$$

$$hA_s(T_\infty - T)\,dt = mc_p\,dT$$

$$m = \rho V \quad dT = d(T - T_\infty)$$

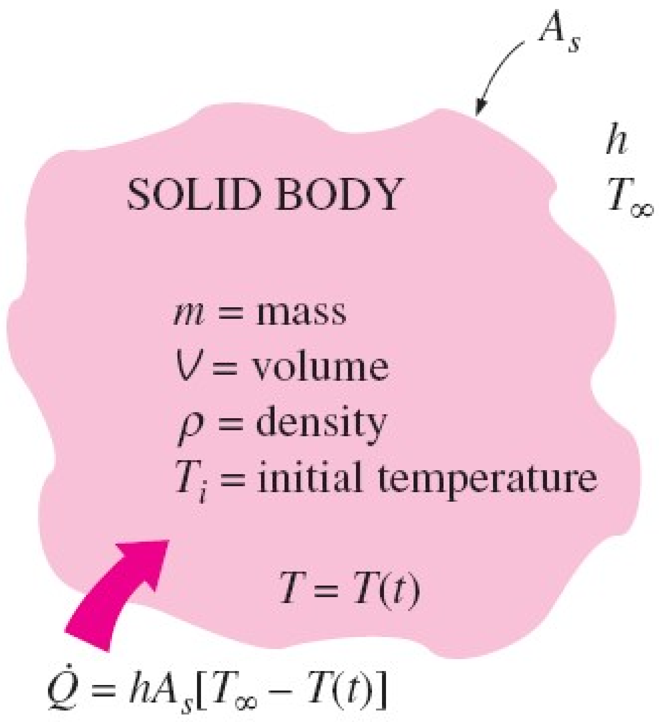
$$\frac{d(T - T_\infty)}{T - T_\infty} = -\frac{hA_s}{\rho V c_p}\,dt$$

Integrating with
$T = T_i$ at $t = 0$
$T = T(t)$ at $t = t$

$$\ln \frac{T(t) - T_\infty}{T_i - T_\infty} = -\frac{hA_s}{\rho V c_p}\,t$$

$$\frac{T(t) - T_\infty}{T_i - T_\infty} = e^{-bt}$$

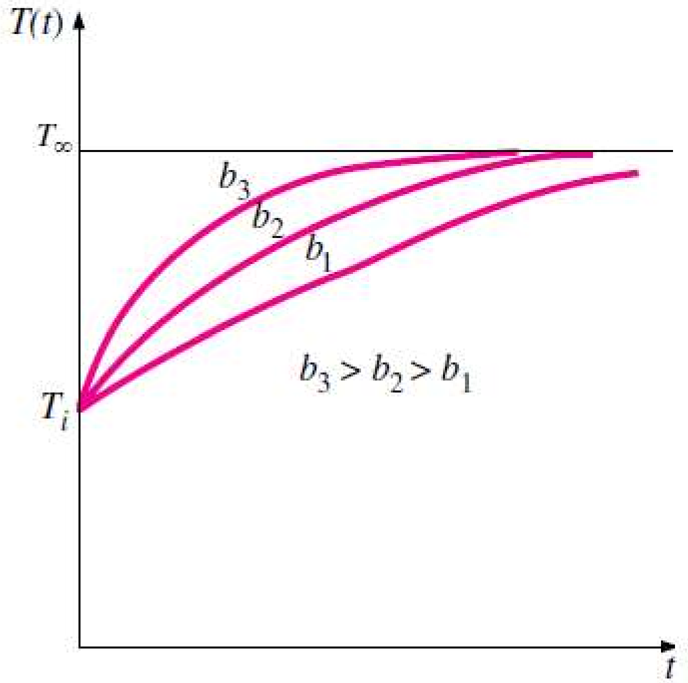
$$b = \frac{hA_s}{\rho V c_p} \qquad (1/\text{s})$$ time constant



The geometry and parameters involved in the lumped system analysis.

$$\frac{T(t)-T_\infty}{T_i-T_\infty}=e^{-bt}$$

$$b=\frac{hA_s}{\rho V c_p}$$



The temperature of a lumped system approaches the environment temperature as time gets larger.

- This equation enables us to determine the temperature $T(t)$ of a body at time t, or alternatively, the time t required for the temperature to reach a specified value $T(t)$.
- The temperature of a body approaches the ambient temperature T_∞ exponentially.
- The temperature of the body changes rapidly at the beginning, but rather slowly later on. A large value of b indicates that the body approaches the environment temperature in a short time

$$\dot{Q}(t) = hA_s[T(t) - T_\infty] \qquad \text{(W)}$$

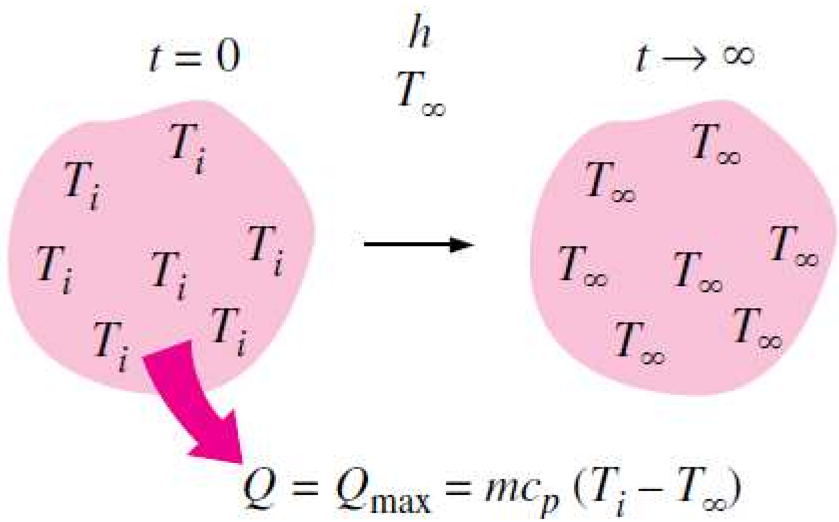
The *rate* of convection heat transfer between the body and its environment at time *t*

$$Q = mc_p[T(t) - T_i] \qquad \text{(kJ)}$$

The *total amount* of heat transfer between the body and the surrounding medium over the time interval *t* = 0 to *t*

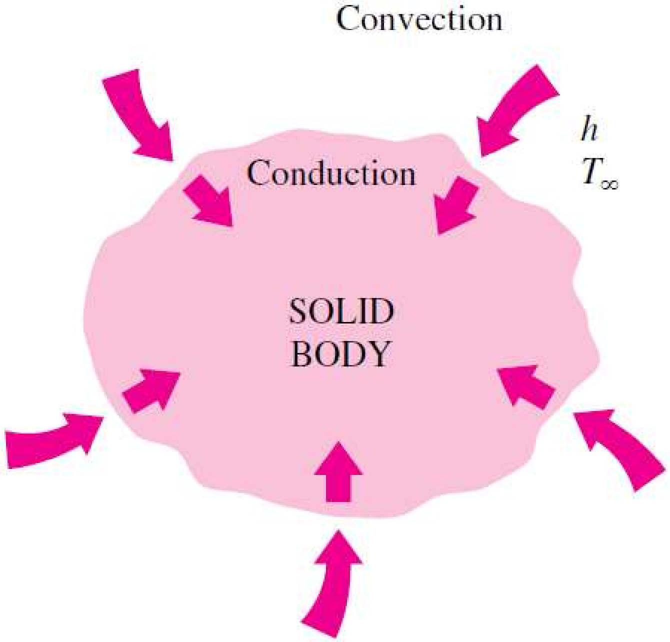
$$Q_{\max} = mc_p(T_\infty - T_i) \qquad \text{(kJ)}$$

The *maximum* heat transfer between the body and its surroundings



Heat transfer to or from a body reaches its maximum value when the body reaches the environment temperature.

Criteria for Lumped System Analysis



$$\mathrm{Bi} = \frac{\text{heat convection}}{\text{heat conduction}}$$

The Biot number can be viewed as the ratio of the convection at the surface to conduction within the body.

$$L_c = \frac{V}{A_s}$$

Characteristic length

$$\mathrm{Bi} = \frac{hL_c}{k}$$

Biot number

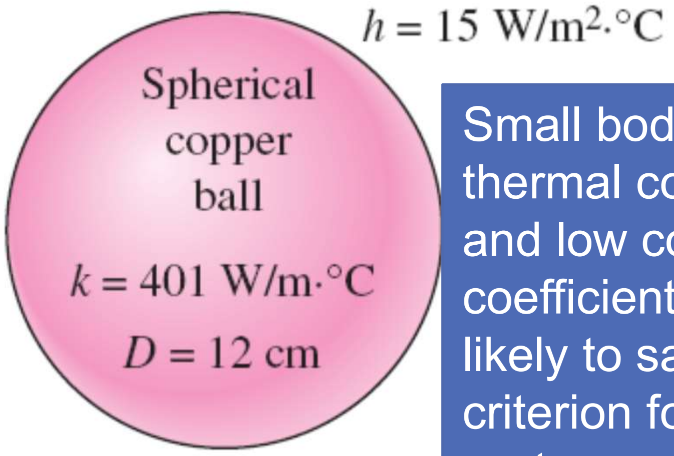
Lumped system analysis is *applicable* if

$$\mathrm{Bi} \le 0.1$$

When Bi ≤ 0.1, the temperatures within the body relative to the surroundings (i.e., $T - T_\infty$) remain within 5 percent of each other.

$$\mathrm{Bi} = \frac{h}{k/L_c}\frac{\Delta T}{\Delta T} = \frac{\text{Convection at the surface of the body}}{\text{Conduction within the body}}$$

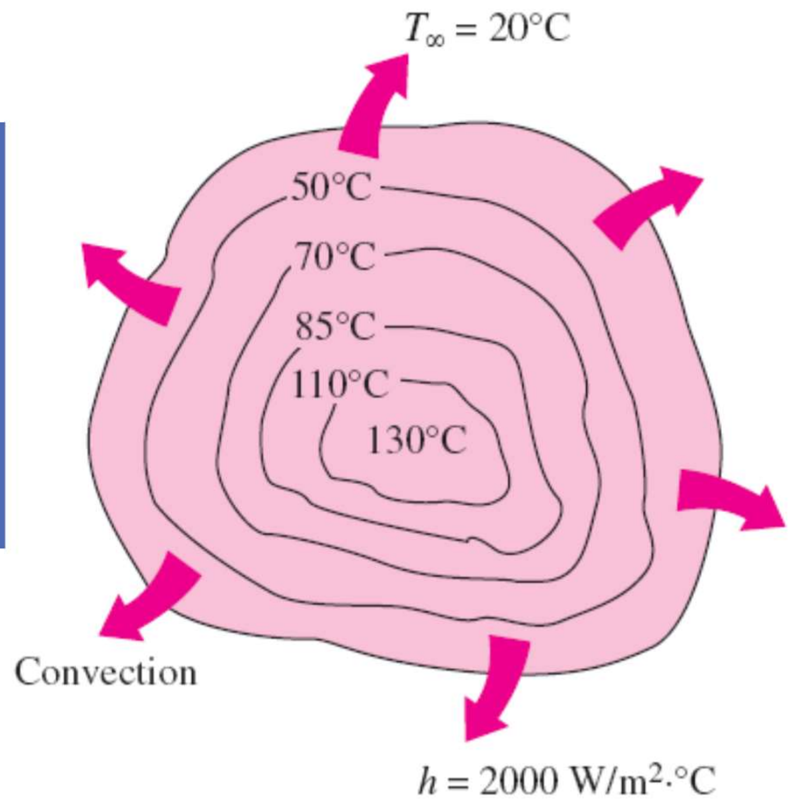
$$\mathrm{Bi} = \frac{L_c/k}{1/h} = \frac{\text{Conduction resistance within the body}}{\text{Convection resistance at the surface of the body}}$$



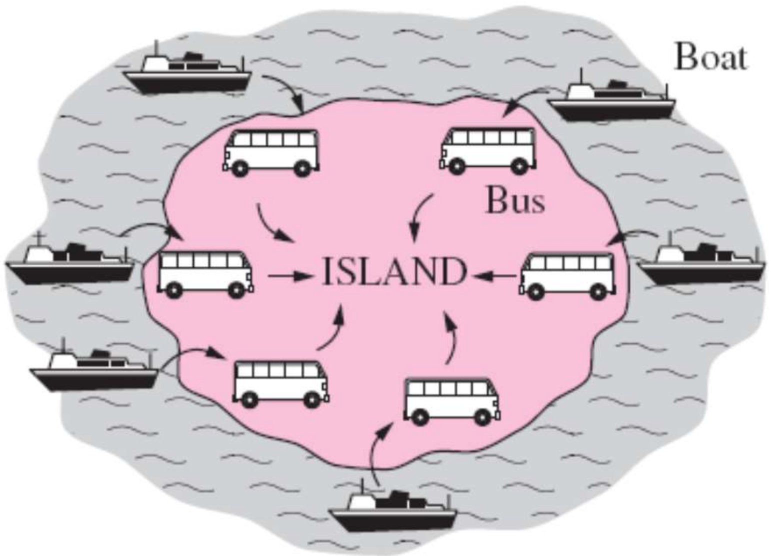
Small bodies with high thermal conductivities and low convection coefficients are most likely to satisfy the criterion for lumped system analysis.

$$L_c = \frac{V}{A_s} = \frac{\frac{1}{6}\pi D^3}{\pi D^2} = \frac{1}{6}D = 0.02 \text{ m}$$

$$\text{Bi} = \frac{hL_c}{k} = \frac{15 \times 0.02}{401} = 0.00075 < 0.1$$



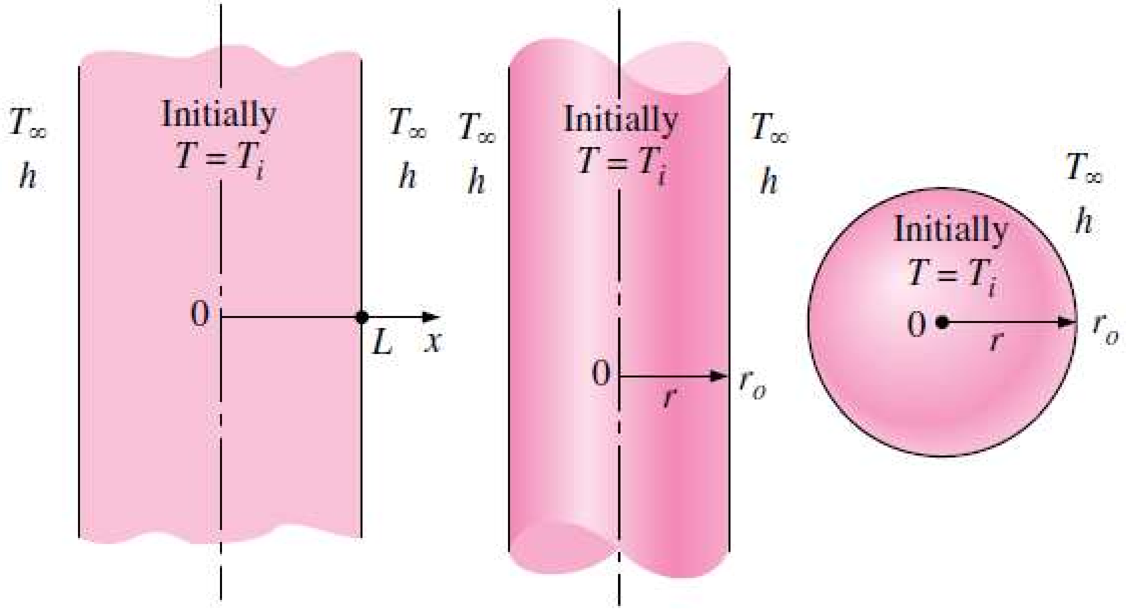
When the convection coefficient h is high and k is low, large temperature differences occur between the inner and outer regions of a large solid.



Analogy between heat transfer to a solid and passenger traffic to an island.

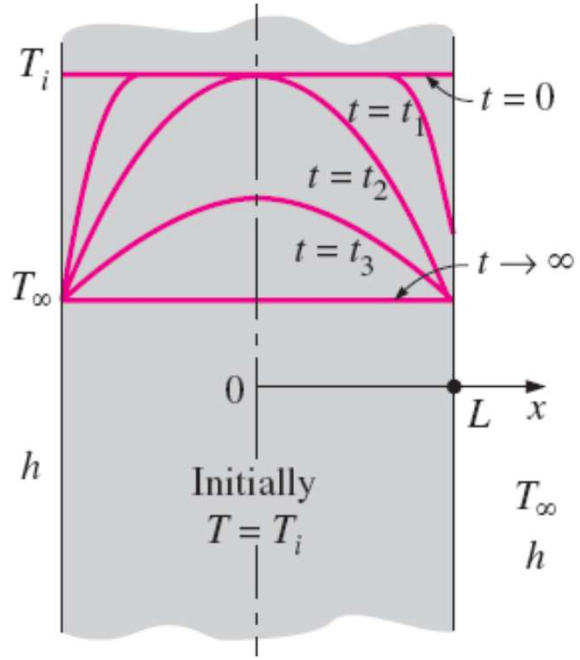
TRANSIENT HEAT CONDUCTION IN LARGE PLANE WALLS, LONG CYLINDERS, AND SPHERES WITH SPATIAL EFFECTS

consider the variation of temperature with *time* and *position* in 1-D problems such as those associated with a large plane wall, a long cylinder, and a sphere.



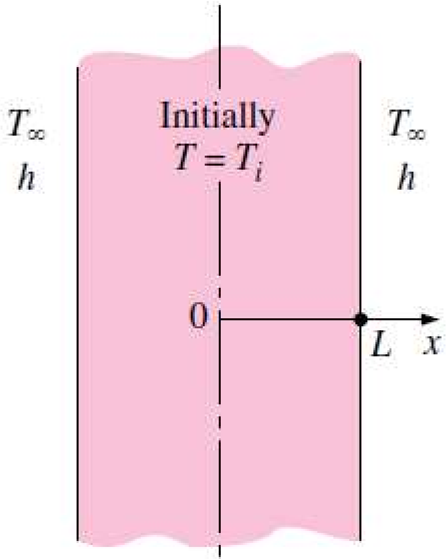
(*a*) A large plane wall (*b*) A long cylinder (*c*) A sphere

Schematic of the simple geometries in which heat transfer is 1-D.



Transient temperature profiles in a plane wall exposed to convection from its surfaces for $T_i > T_\infty$.

Nondimensionalized One-Dimensional Transient Conduction Problem



$\alpha = k/\rho c_p$

$X = x/L$

Differential equation: $\dfrac{\partial^2 T}{\partial x^2} = \dfrac{1}{\alpha}\dfrac{\partial T}{\partial t}$

Boundary conditions: $\dfrac{\partial T(0, t)}{\partial x} = 0$ and $-k\dfrac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$

Initial condition: $T(x, 0) = T_i$

$$\theta(x, t) = [T(x, t) - T_\infty]/[T_i - T_\infty]$$

$$\frac{\partial \theta}{\partial X} = \frac{\partial \theta}{\partial (x/L)} = \frac{L}{T_i - T_\infty}\frac{\partial T}{\partial x}$$

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{T_i - T_\infty}\frac{\partial T}{\partial x} \quad \text{and} \quad \frac{\partial \theta}{\partial t} = \frac{1}{T_i - T_\infty}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{L^2}{\alpha}\frac{\partial \theta}{\partial t} \quad \text{and} \quad \frac{\partial \theta(1, t)}{\partial X} = \frac{hL}{k}\theta(1, t)$$

$$\boxed{\mathrm{Bi} = k/hL} \qquad \boxed{\tau = \alpha t/L^2}$$

Dimensionless differential equation: $$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}$$

Dimensionless BC's:

$$\frac{\partial \theta(0, \tau)}{\partial X} = 0 \quad \text{and} \quad \frac{\partial \theta(1, \tau)}{\partial X} = -\mathrm{Bi}\,\theta(1, \tau)$$

Dimensionless initial condition: $$\theta(X, 0) = 1$$

$$\theta(X, \tau) = \frac{T(x, t) - T_i}{T_\infty - T_i}$$ *Dimensionless temperature*

$$X = \frac{x}{L}$$ *Dimensionless distance from the center*

$$\text{Bi} = \frac{hL}{k}$$ *Dimensionless heat transfer coefficient* (Biot number)

$$\tau = \frac{\alpha t}{L^2} = \text{Fo}$$ *Dimensionless time* (Fourier number)

(*a*) Original heat conduction problem:

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t}, \quad T(x, 0) = T_i$$

$$\frac{\partial T(0, t)}{\partial x} = 0, \quad -k\frac{\partial T(L, t)}{\partial x} = h[T(L, t) - T_\infty]$$

$$T = F(x, L, t, k, \alpha, h, T_i)$$

(*b*) Nondimensionalized problem:

$$\frac{\partial^2 \theta}{\partial X^2} = \frac{\partial \theta}{\partial \tau}; \quad \theta(X, 0) = 1$$

$$\frac{\partial \theta(0, \tau)}{\partial X} = 0, \quad \frac{\partial \theta(1, \tau)}{\partial X} = -\text{Bi}\,\theta(1, \tau)$$

$$\theta = f(X, \text{Bi}, \tau)$$

Nondimensionalization reduces the number of independent variables in 1-D transient conduction problems from 8 to 3, offering great convenience in the presentation of results.

Exact Solution of 1-D Transient Conduction Problem

TABLE 18–1

Summary of the solutions for one-dimensional transient conduction in a plane wall of thickness $2L$, a cylinder of radius r_o and a sphere of radius r_o subjected to convention from all surfaces.*

Geometry	Solution	λ_n's are the roots of
Plane wall	$\theta = \sum_{n=1}^{\infty} \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \cos(\lambda_n x / L)$	$\lambda_n \tan \lambda_n = \text{Bi}$
Cylinder	$\theta = \sum_{n=1}^{\infty} \frac{2}{\lambda_n} \frac{J_1(\lambda_n)}{J_0^2(\lambda_n) + J_1^2(\lambda_n)} e^{-\lambda_n^2 \tau} J_0(\lambda_n r / r_o)$	$\lambda_n \frac{J_1(\lambda_n)}{J_0(\lambda_n)} = \text{Bi}$
Sphere	$\theta = \sum_{n=1}^{\infty} \frac{4(\sin \lambda_n - \lambda_n \cos \lambda_n)}{2\lambda_n - \sin(2\lambda_n)} e^{-\lambda_n^2 \tau} \frac{\sin(\lambda_n x / L)}{\lambda_n x / L}$	$1 - \lambda_n \cot \lambda_n = \text{Bi}$

*Here $\theta = (T - T_i)/(T_\infty - T_i)$ is the dimensionless temperature, $\text{Bi} = hL/k$ or hr_o/k is the Biot number, $\text{Fo} = \tau = \alpha t / L^2$ or $\alpha\tau / r_o^2$ is the Fourier number, and J_0 and J_1 are the Bessel functions of the first kind whose values are given in Table 18–3.

$$\theta_n = A_n e^{-\lambda_n^2 \tau} \cos(\lambda_n X)$$

$$A_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$

$$\lambda_n \tan \lambda_n = \text{Bi}$$

For Bi = 5, $X = 1$, and $t = 0.2$:

n	λ_n	A_n	θ_n
1	1.3138	1.2402	0.22321
2	4.0336	−0.3442	0.00835
3	6.9096	0.1588	0.00001
4	9.8928	−0.876	0.00000

The term in the series solution of transient conduction problems decline rapidly as n and thus λ_n increases because of the exponential decay function with the exponent $-\lambda_n\tau$.

The analytical solutions of transient conduction problems typically involve infinite series, and thus the evaluation of an infinite number of terms to determine the temperature at a specified location and time.

Approximate Analytical and Graphical Solutions

The terms in the series solutions converge rapidly with increasing time, and for τ>0.2, keeping the first term and neglecting all the remaining terms in the series results in an error under 2 percent.

Solution with *one-term approximation*

Plane wall: $$\theta_{\text{wall}} = \frac{T(x,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \cos(\lambda_1 x/L), \quad \tau > 0.2$$

Cylinder: $$\theta_{\text{cyl}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} J_0(\lambda_1 r/r_o), \quad \tau > 0.2$$

Sphere: $$\theta_{\text{sph}} = \frac{T(r,t) - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau} \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}, \quad \tau > 0.2$$

Center of plane wall (x = 0): $$\theta_{0,\text{ wall}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

Center of cylinder (r = 0): $$\theta_{0,\text{ cyl}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

Center of sphere (r = 0): $$\theta_{0,\text{ sph}} = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$$

TABLE 18–2

Coefficients used in the one-term approximate solution of transient one-dimensional heat conduction in plane walls, cylinders, and spheres (Bi = hL/k for a plane wall of thickness $2L$, and Bi = hr_o/k for a cylinder or sphere of radius r_o)

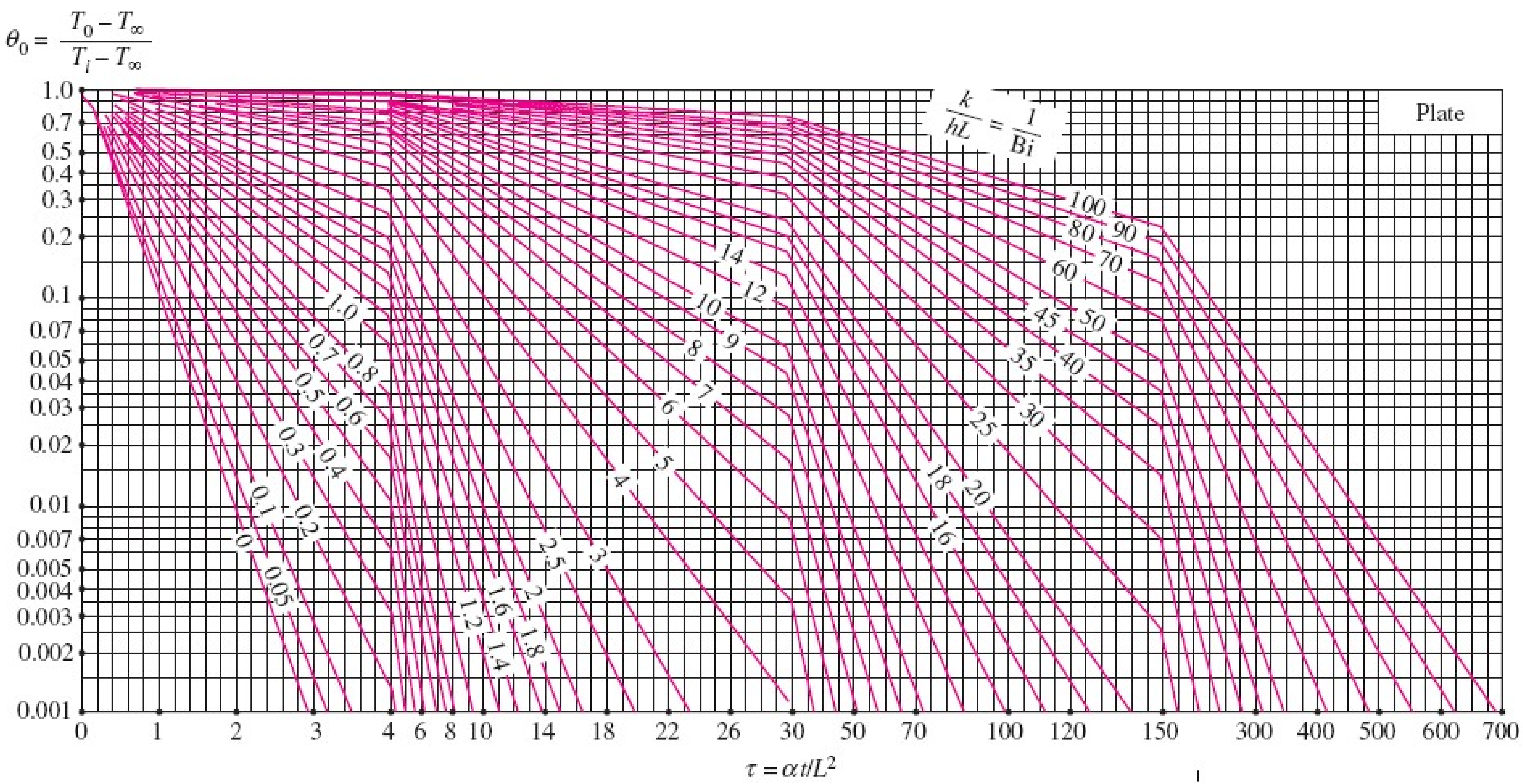
	Plane Wall		Cylinder		Sphere	
Bi	λ_1	A_1	λ_1	A_1	λ_1	A_1
0.01	0.0998	1.0017	0.1412	1.0025	0.1730	1.0030
0.02	0.1410	1.0033	0.1995	1.0050	0.2445	1.0060
0.04	0.1987	1.0066	0.2814	1.0099	0.3450	1.0120
0.06	0.2425	1.0098	0.3438	1.0148	0.4217	1.0179
0.08	0.2791	1.0130	0.3960	1.0197	0.4860	1.0239
0.1	0.3111	1.0161	0.4417	1.0246	0.5423	1.0298
0.2	0.4328	1.0311	0.6170	1.0483	0.7593	1.0592
0.3	0.5218	1.0450	0.7465	1.0712	0.9208	1.0880
0.4	0.5932	1.0580	0.8516	1.0931	1.0528	1.1164
0.5	0.6533	1.0701	0.9408	1.1143	1.1656	1.1441
0.6	0.7051	1.0814	1.0184	1.1345	1.2644	1.1713
0.7	0.7506	1.0918	1.0873	1.1539	1.3525	1.1978
0.8	0.7910	1.1016	1.1490	1.1724	1.4320	1.2236
0.9	0.8274	1.1107	1.2048	1.1902	1.5044	1.2488
1.0	0.8603	1.1191	1.2558	1.2071	1.5708	1.2732
2.0	1.0769	1.1785	1.5995	1.3384	2.0288	1.4793
3.0	1.1925	1.2102	1.7887	1.4191	2.2889	1.6227
4.0	1.2646	1.2287	1.9081	1.4698	2.4556	1.7202
5.0	1.3138	1.2403	1.9898	1.5029	2.5704	1.7870
6.0	1.3496	1.2479	2.0490	1.5253	2.6537	1.8338
7.0	1.3766	1.2532	2.0937	1.5411	2.7165	1.8673
8.0	1.3978	1.2570	2.1286	1.5526	2.7654	1.8920
9.0	1.4149	1.2598	2.1566	1.5611	2.8044	1.9106
10.0	1.4289	1.2620	2.1795	1.5677	2.8363	1.9249
20.0	1.4961	1.2699	2.2880	1.5919	2.9857	1.9781
30.0	1.5202	1.2717	2.3261	1.5973	3.0372	1.9898
40.0	1.5325	1.2723	2.3455	1.5993	3.0632	1.9942
50.0	1.5400	1.2727	2.3572	1.6002	3.0788	1.9962
100.0	1.5552	1.2731	2.3809	1.6015	3.1102	1.9990
∞	1.5708	1.2732	2.4048	1.6021	3.1416	2.0000

TABLE 18–3

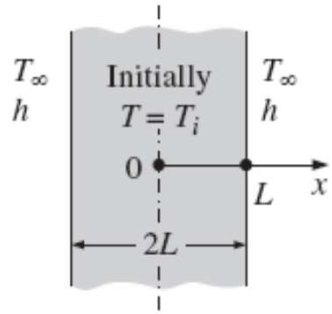
The zeroth- and first-order Bessel functions of the first kind

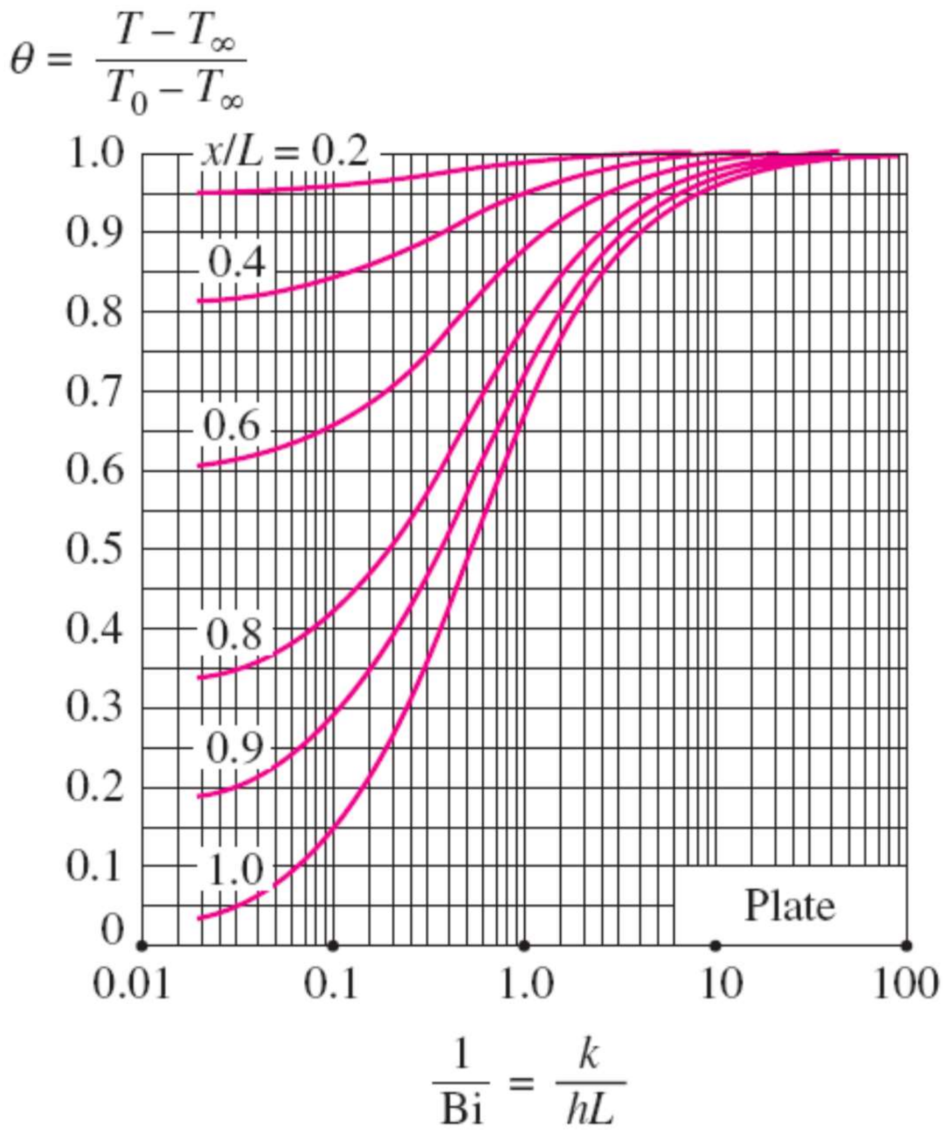
η	$J_0(\eta)$	$J_1(\eta)$
0.0	1.0000	0.0000
0.1	0.9975	0.0499
0.2	0.9900	0.0995
0.3	0.9776	0.1483
0.4	0.9604	0.1960
0.5	0.9385	0.2423
0.6	0.9120	0.2867
0.7	0.8812	0.3290
0.8	0.8463	0.3688
0.9	0.8075	0.4059
1.0	0.7652	0.4400
1.1	0.7196	0.4709
1.2	0.6711	0.4983
1.3	0.6201	0.5220
1.4	0.5669	0.5419
1.5	0.5118	0.5579
1.6	0.4554	0.5699
1.7	0.3980	0.5778
1.8	0.3400	0.5815
1.9	0.2818	0.5812
2.0	0.2239	0.5767
2.1	0.1666	0.5683
2.2	0.1104	0.5560
2.3	0.0555	0.5399
2.4	0.0025	0.5202
2.6	−0.0968	−0.4708
2.8	−0.1850	−0.4097
3.0	−0.2601	−0.3391
3.2	−0.3202	−0.2613

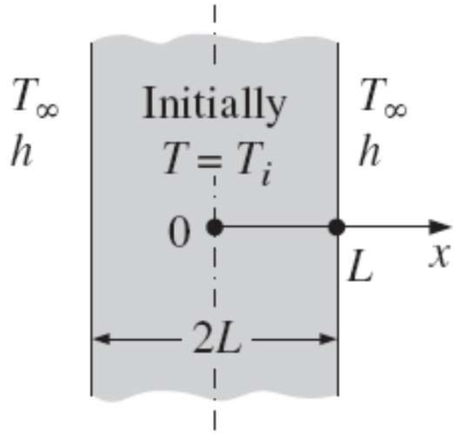
(*a*) Midplane temperature



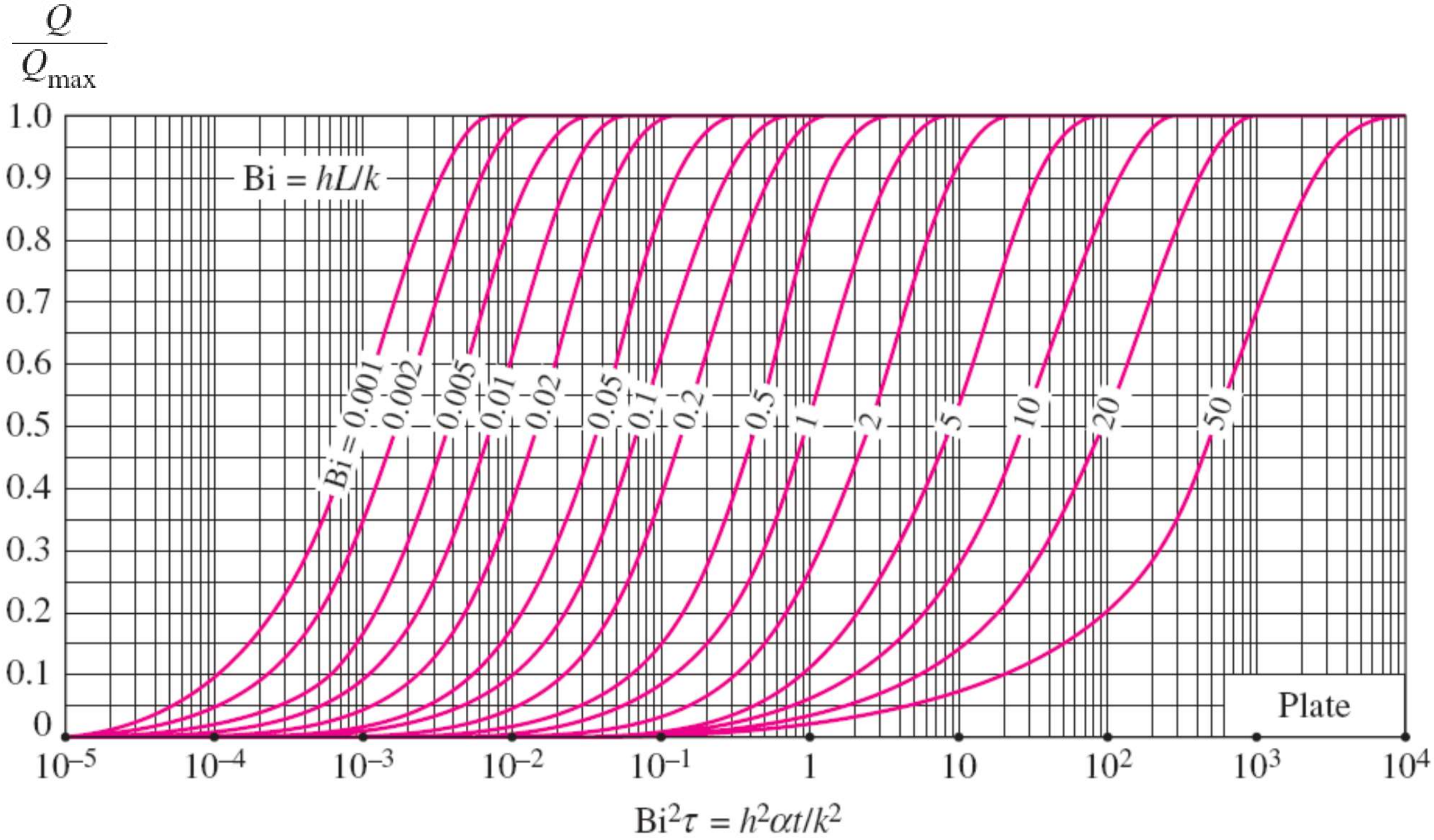
Transient temperature and heat transfer charts (Heisler and Grober charts) for a plane wall of thickness 2*L* initially at a uniform temperature T_i subjected to convection from both sides to an environment at temperature T_∞ with a convection coefficient of *h.*

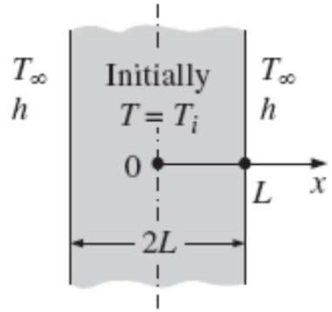






(*b*) Temperature distribution

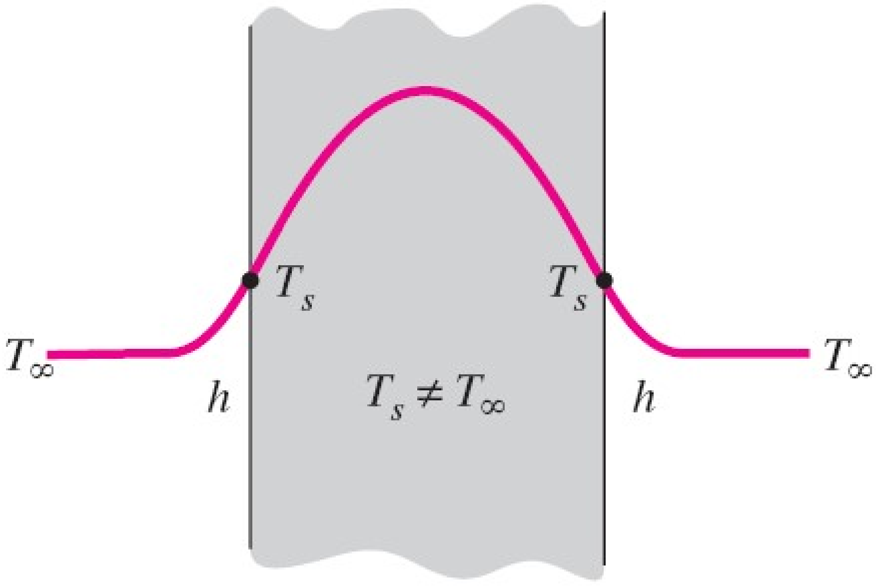




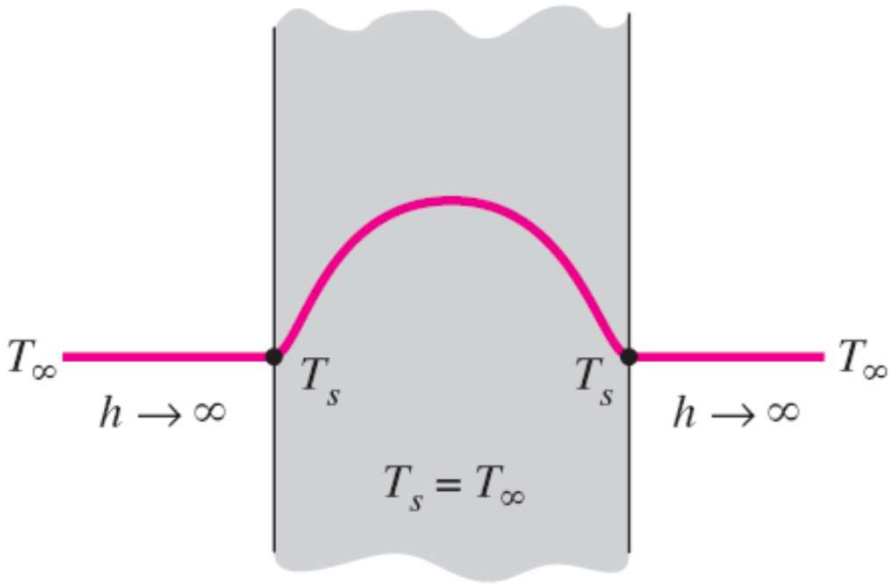
(*c*) Heat transfer

The dimensionless temperatures anywhere in a plane wall, cylinder, and sphere are related to the center temperature by

$$\frac{\theta_{\text{wall}}}{\theta_{0,\,\text{wall}}} = \cos\left(\frac{\lambda_1 x}{L}\right), \quad \frac{\theta_{\text{cyl}}}{\theta_{0,\,\text{cyl}}} = J_0\left(\frac{\lambda_1 r}{r_o}\right), \quad \text{and} \quad \frac{\theta_{\text{sph}}}{\theta_{0,\,\text{sph}}} = \frac{\sin(\lambda_1 r/r_o)}{\lambda_1 r/r_o}$$



(a) Finite convection coefficient



(b) Infinite convection coefficient

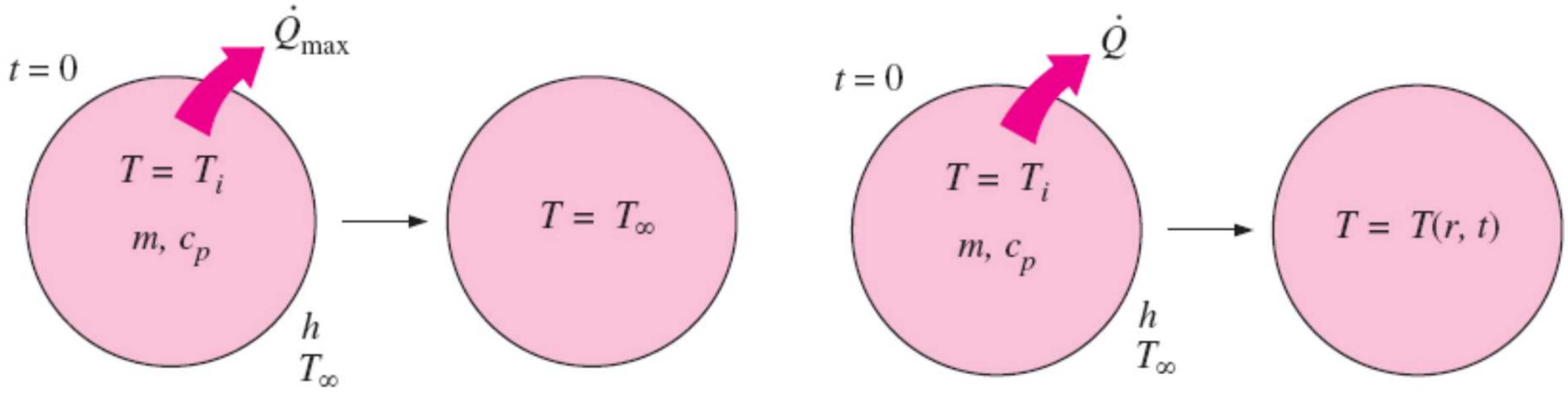
The specified surface temperature corresponds to the case of convection to an environment at T_∞ with with a convection coefficient h that is *infinite.*

$$\boxed{Q_{max} = mc_p(T_\infty - T_i) = \rho V c_p(T_\infty - T_i) \qquad \text{(kJ)}}$$

Plane wall: $$\left(\frac{Q}{Q_{max}}\right)_{wall} = 1 - \theta_{0,\,wall}\frac{\sin\lambda_1}{\lambda_1}$$

Cylinder: $$\left(\frac{Q}{Q_{max}}\right)_{cyl} = 1 - 2\theta_{0,\,cyl}\frac{J_1(\lambda_1)}{\lambda_1}$$

Sphere: $$\left(\frac{Q}{Q_{max}}\right)_{sph} = 1 - 3\theta_{0,\,sph}\frac{\sin\lambda_1 - \lambda_1\cos\lambda_1}{\lambda_1^3}$$



(*a*) Maximum heat transfer ($t \to \infty$)

$$\left.\begin{array}{l} \text{Bi} = \ldots \\ \dfrac{h^2\alpha t}{k^2} = \text{Bi}^2\tau = \cdots \end{array}\right\} \quad \frac{Q}{Q_{max}} = \cdots$$

(Gröber chart)

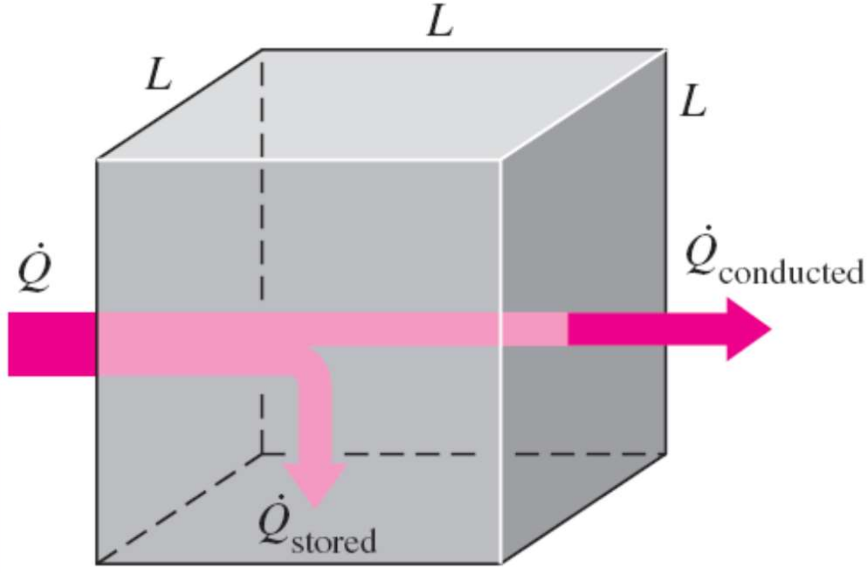
(*b*) Actual heat transfer for time t

The fraction of total heat transfer Q/Q_{max} up to a specified time t is determined using the Gröber charts.

The physical significance of the *Fourier number*

$$\tau = \frac{\alpha t}{L^2} = \frac{kL^2\,(1/L)}{\rho c_p L^3/t}\,\frac{\Delta T}{\Delta T} = \frac{\text{The rate at which heat is \textit{conducted} across } L \text{ of a body of volume } L^3}{\text{The rate at which heat is \textit{stored} in a body of volume } L^3}$$

- The Fourier number is a measure of *heat conducted* through a body relative to *heat stored.*
- A large value of the Fourier number indicates faster propagation of heat through a body.



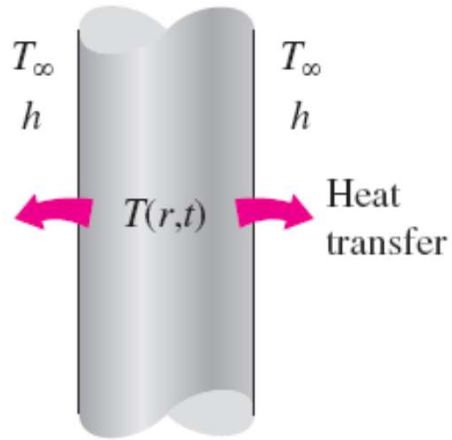
$$\text{Fourier number: } \tau = \frac{\alpha t}{L^2} = \frac{\dot{Q}_{\text{conducted}}}{\dot{Q}_{\text{stored}}}$$

Fourier number at time *t* can be viewed as the ratio of the rate of heat conducted to the rate of heat stored at that time.

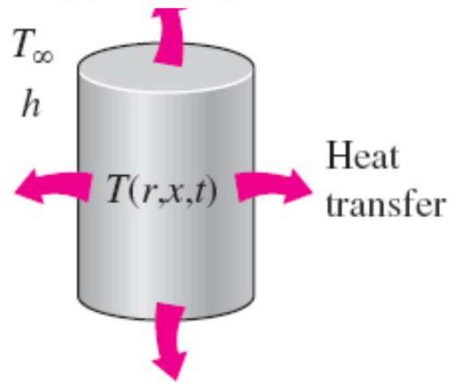
TRANSIENT HEAT CONDUCTION IN MULTIDIMENSIONAL SYSTEMS

- Using a superposition approach called the product solution, the transient temperature charts and solutions can be used to construct solutions for the *2-D* and *3-D* transient heat conduction problems encountered in geometries such as a short cylinder, a long rectangular bar, a rectangular prism or a semi-infinite rectangular bar, provided that *all* surfaces of the solid are subjected to convection to the *same* fluid at temperature T_∞, with the *same* heat transfer coefficient h, and the body involves no heat generation.

- The solution in such multidimensional geometries can be expressed as the *product* of the solutions for the one-dimensional geometries whose intersection is the multidimensional geometry.



(*a*) Long cylinder



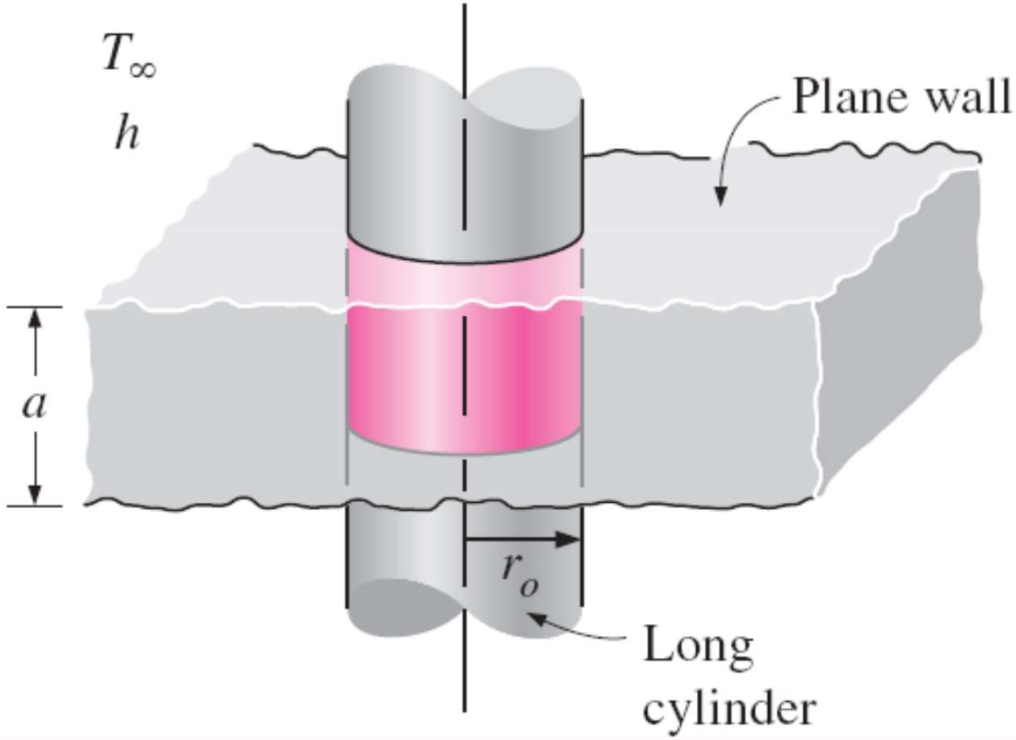
(*b*) Short cylinder (two-dimensional)

The temperature in a short cylinder exposed to convection from all surfaces varies in both the radial and axial directions, and thus heat is transferred in both directions.

The solution for a multidimensional geometry is the product of the solutions of the one-dimensional geometries whose intersection is the multidimensional body.

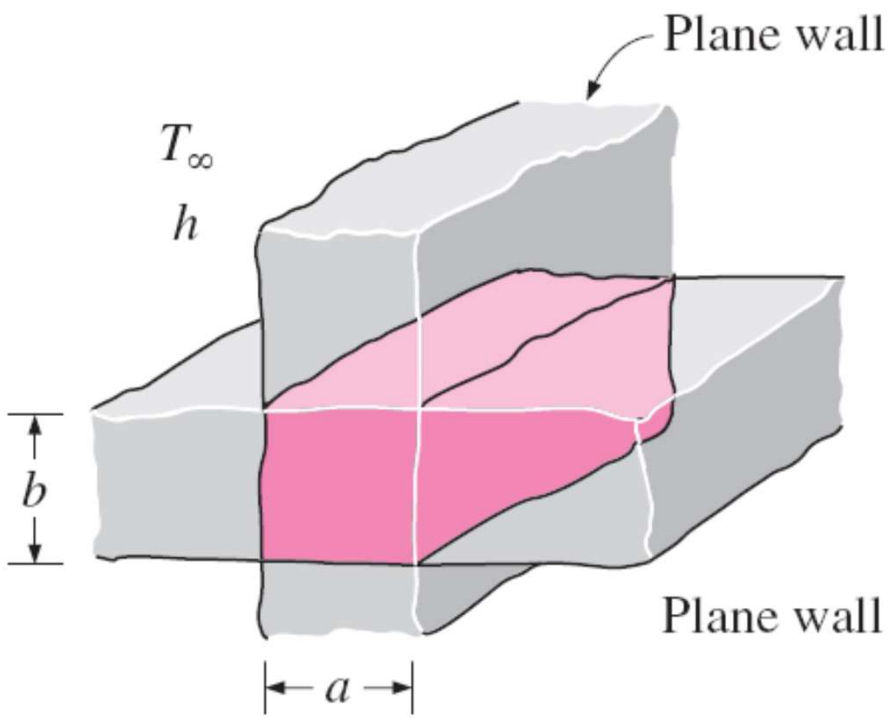
The solution for the two-dimensional short cylinder of height a and radius r_o is equal to the *product* of the nondimensionalized solutions for the one-dimensional plane wall of thickness a and the long cylinder of radius r_o.

$$\left(\frac{T(r,x,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{short}\\ \text{cylinder}}} = \left(\frac{T(x,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{plane}\\ \text{wall}}} \left(\frac{T(r,t)-T_\infty}{T_i-T_\infty}\right)_{\substack{\text{infinite}\\ \text{cylinder}}}$$



A short cylinder of radius r_o and height a is the *intersection* of a long cylinder of radius r_o and a plane wall of thickness a.

$$\left(\frac{T(x, y, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{rectangular}\\ \text{bar}}} = \theta_{\text{wall}}(x, t)\,\theta_{\text{wall}}(y, t)$$



$$\theta_{\text{wall}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{plane}\\ \text{wall}}}$$

$$\theta_{\text{cyl}}(r, t) = \left(\frac{T(r, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{infinite}\\ \text{cylinder}}}$$

$$\theta_{\text{semi-inf}}(x, t) = \left(\frac{T(x, t) - T_\infty}{T_i - T_\infty}\right)_{\substack{\text{semi-infinite}\\ \text{solid}}}$$

A long solid bar of rectangular profile $a \times b$ is the *intersection* of two plane walls of thicknesses a and b.

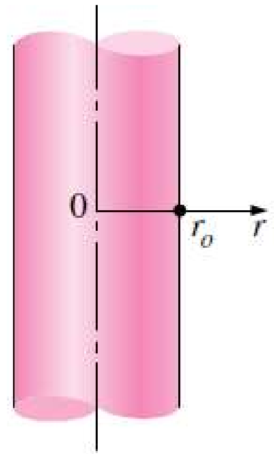
The transient heat transfer for a two-dimensional geometry formed by the intersection of two one-dimensional geometries 1 and 2 is

$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 2D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]$$

Transient heat transfer for a three-dimensional body formed by the intersection of three one-dimensional bodies 1, 2, and 3 is

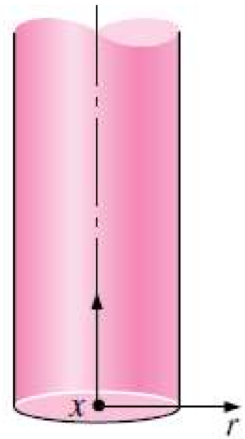
$$\left(\frac{Q}{Q_{\max}}\right)_{\text{total, 3D}} = \left(\frac{Q}{Q_{\max}}\right)_1 + \left(\frac{Q}{Q_{\max}}\right)_2 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right] + \left(\frac{Q}{Q_{\max}}\right)_3 \left[1 - \left(\frac{Q}{Q_{\max}}\right)_1\right]\left[1 - \left(\frac{Q}{Q_{\max}}\right)_2\right]$$

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature T_i and exposed to convection from all surfaces to a medium at T_∞



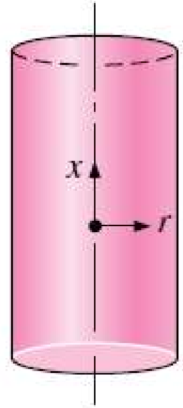
$\theta(r,t) = \theta_{cyl}(r,t)$

Infinite cylinder



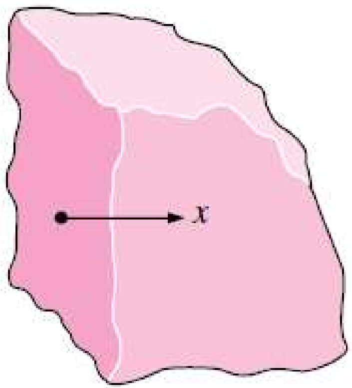
$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{semi\text{-}inf}(x,t)$

Semi-infinite cylinder



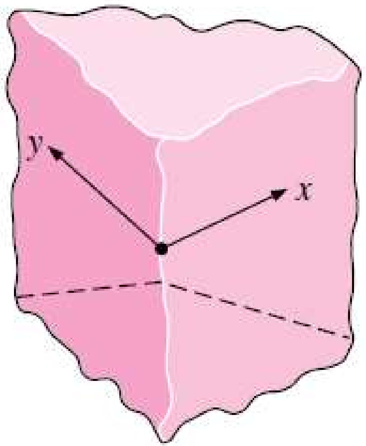
$\theta(x,r,t) = \theta_{cyl}(r,t)\,\theta_{wall}(x,t)$

Short cylinder



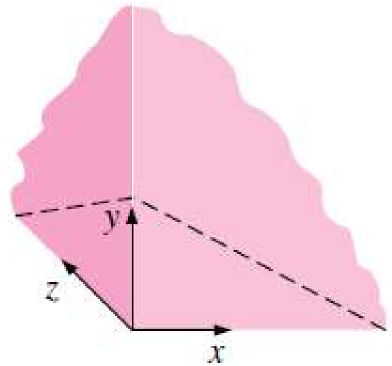
$\theta(x,t) = \theta_{semi\text{-}inf}(x,t)$

Semi-infinite medium



$\theta(x,y,t) = \theta_{semi\text{-}inf}(x,t)\,\theta_{semi\text{-}inf}(y,t)$

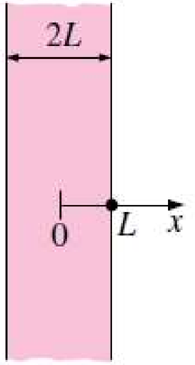
Quarter-infinite medium



$\theta(x,y,z,t) =$
$\theta_{semi\text{-}inf}(x,t)\,\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$

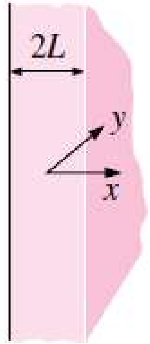
Corner region of a large medium

Multidimensional solutions expressed as products of one-dimensional solutions for bodies that are initially at a uniform temperature T_i and exposed to convection from all surfaces to a medium at T_∞



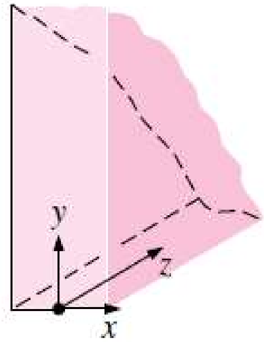
$\theta(x,t) = \theta_{wall}(x,t)$

Infinite plate (or plane wall)



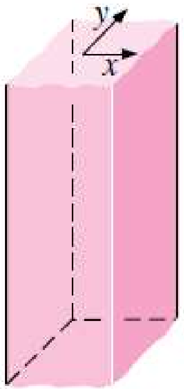
$\theta(x,y,t) = \theta_{wall}(x,t)\,\theta_{semi\text{-}inf}(y,t)$

Semi-infinite plate



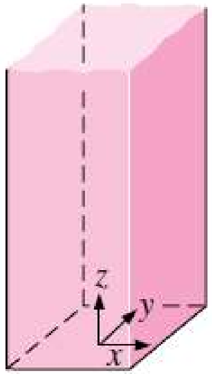
$\theta(x,y,z,t) =$
$\theta_{wall}(x,t)\,\theta_{semi\text{-}inf}(y,t)\,\theta_{semi\text{-}inf}(z,t)$

Quarter-infinite plate



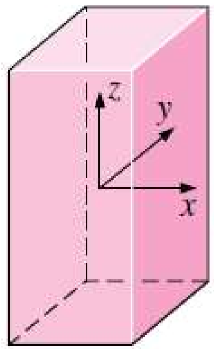
$\theta(x,y,t) = \theta_{wall}(x,t)\theta_{wall}(y,t)$

Infinite rectangular bar



$\theta(x,y,z,t) =$
$\theta_{wall}(x,t)\,\theta_{wall}(y,t)\,\theta_{semi\text{-}inf}(z,t)$

Semi-infinite rectangular bar



$\theta(x,y,z,t) =$
$\theta_{wall}(x,t)\theta_{wall}(y,t)\,\theta_{wall}(z,t)$

Rectangular parallelepiped