Heat and Mass Transfer, 3rd Edition
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Chapter 9
NATURAL CONVECTION

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Objectives

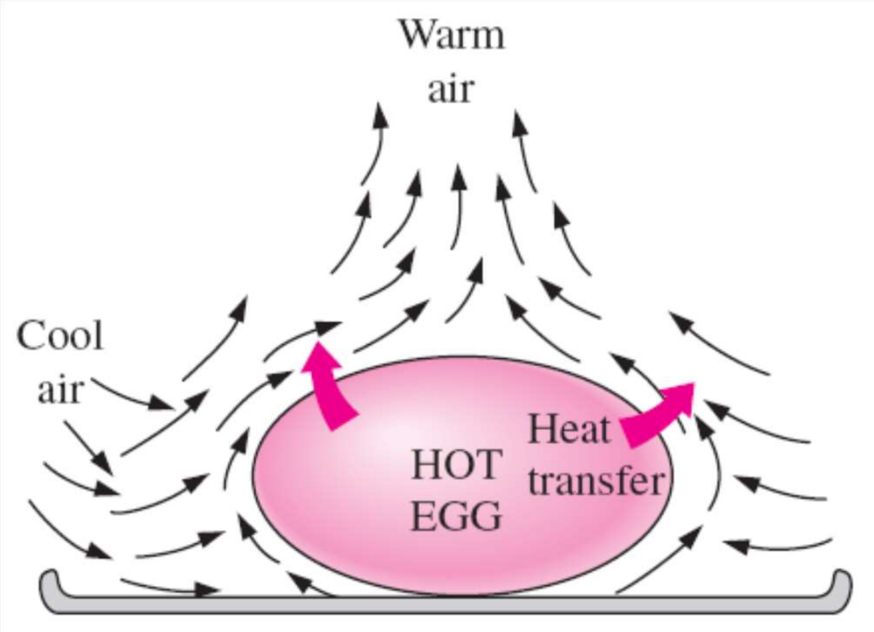
- Understand the physical mechanism of natural convection
- Derive the governing equations of natural convection, and obtain the dimensionless Grashof number by nondimensionalizing them
- Evaluate the Nusselt number for natural convection associated with vertical, horizontal, and inclined plates as well as cylinders and spheres
- Examine natural convection from finned surfaces, and determine the optimum fin spacing
- Analyze natural convection inside enclosures such as double-pane windows.
- Consider combined natural and forced convection, and assess the relative importance of each mode.

PHYSICAL MECHANISM OF NATURAL CONVECTION

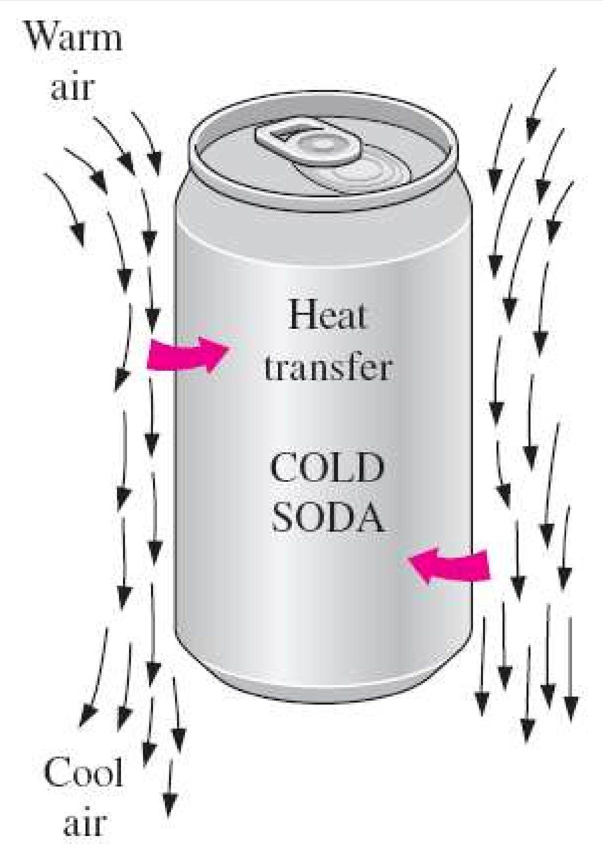
Many familiar heat transfer applications involve natural convection as the primary mechanism of heat transfer. Examples?

Natural convection in gases is usually accompanied by radiation of comparable magnitude except for low-emissivity surfaces.

The motion that results from the continual replacement of the heated air in the vicinity of the egg by the cooler air nearby is called a natural convection current, and the heat transfer that is enhanced as a result of this current is called natural convection heat transfer.



The cooling of a boiled egg in a cooler environment by natural convection.



The warming up of a cold drink in a warmer environment by natural convection.

Buoyancy force: The upward force exerted by a fluid on a body completely or partially immersed in it in a gravitational field. The magnitude of the buoyancy force is equal to the weight of the *fluid displaced* by the body.

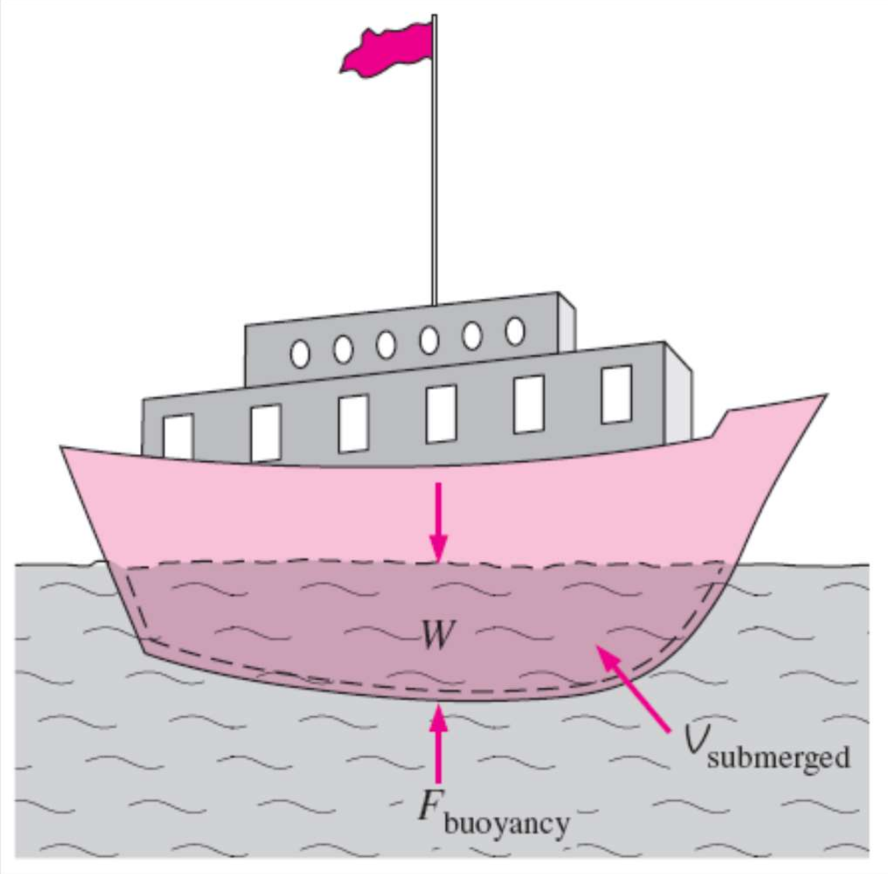
$$F_{\text{buoyancy}} = \rho_{\text{fluid}}\, g V_{\text{body}}$$

The net vertical force acting on a body

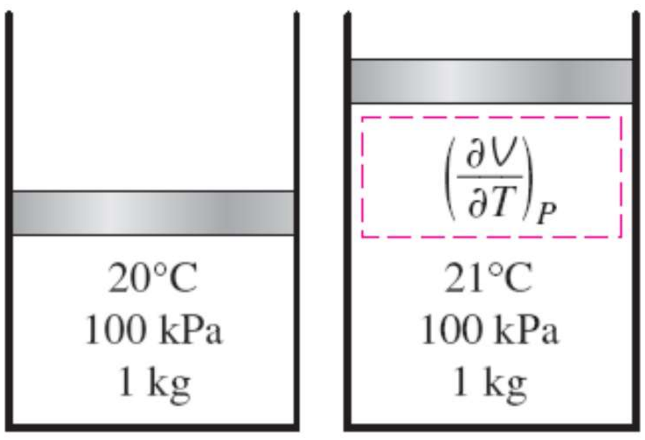
$$\begin{aligned} F_{\text{net}} &= W - F_{\text{buoyancy}} \\ &= \rho_{\text{body}}\, g V_{\text{body}} - \rho_{\text{fluid}}\, g V_{\text{body}} \\ &= (\rho_{\text{body}} - \rho_{\text{fluid}})\, g V_{\text{body}} \end{aligned}$$

Archimedes' principle: A body immersed in a fluid will experience a "weight loss" in an amount equal to the weight of the fluid it displaces.

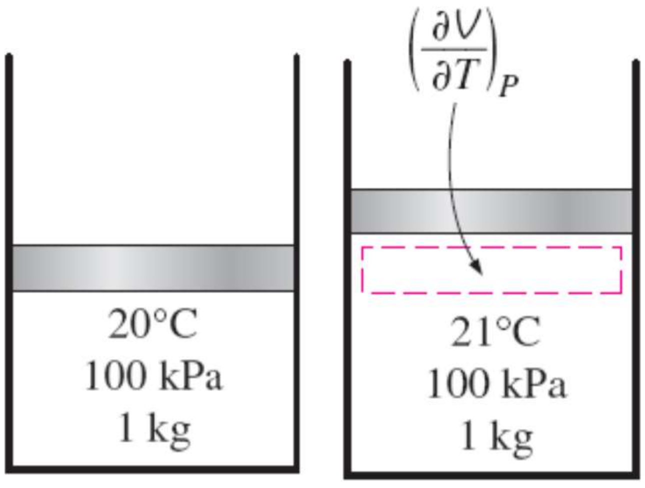
The "chimney effect" that induces the upward flow of hot combustion gases through a chimney is due to the buoyancy effect.



It is the buoyancy force that keeps the ships afloat in water ($W = F_{\text{buoyancy}}$ for floating objects).



(*a*) A substance with a large β



(*b*) A substance with a small β

The coefficient of volume expansion is a measure of the change in volume of a substance with temperature at constant pressure.

Volume expansion coefficient: Variation of the density of a fluid with temperature at constant pressure.

$$\beta = \frac{1}{v}\left(\frac{\partial v}{\partial T}\right)_P = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_P \qquad (1/\mathrm{K})$$

$$\beta \approx -\frac{1}{\rho}\frac{\Delta\rho}{\Delta T} = -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T} \qquad (\text{at constant } P)$$

$$\rho_\infty - \rho = \rho\beta(T - T_\infty) \qquad (\text{at constant } P)$$

$$\beta_{\text{ideal gas}} = \frac{1}{T} \qquad (1/\mathrm{K})$$

ideal gas

$(P = \rho RT)$

The larger the temperature difference between the fluid adjacent to a hot (or cold) surface and the fluid away from it, the *larger* the buoyancy force and the *stronger* the natural convection currents, and thus the *higher* the heat transfer rate.

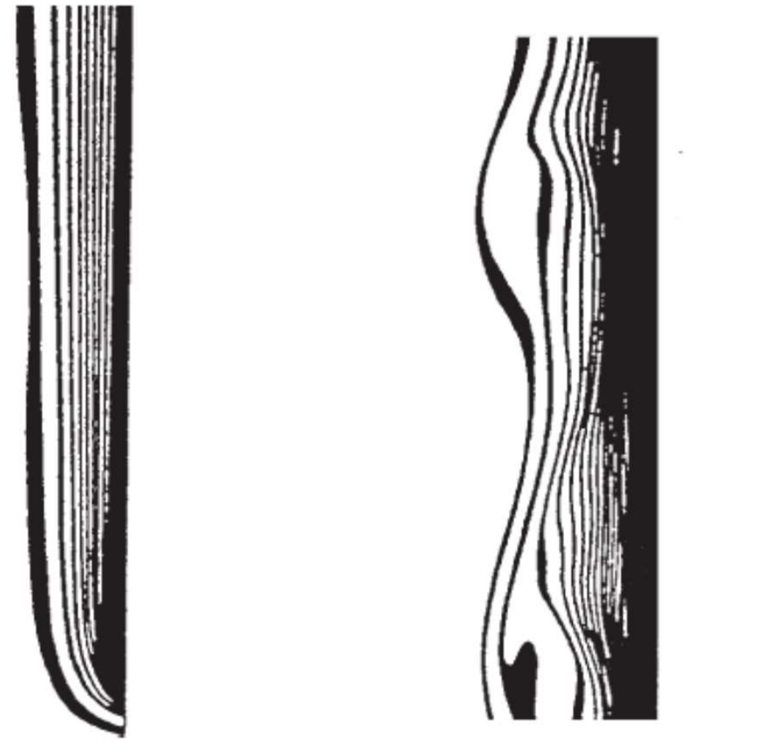
In natural convection, no blowers are used, and therefore the flow rate cannot be controlled externally.

The flow rate in this case is established by the dynamic balance of *buoyancy* and *friction.*

An interferometer produces a map of interference fringes, which can be interpreted as lines of *constant temperature.*

The smooth and parallel lines in (*a*) indicate that the flow is *laminar,* whereas the eddies and irregularities in (*b*) indicate that the flow is *turbulent.*

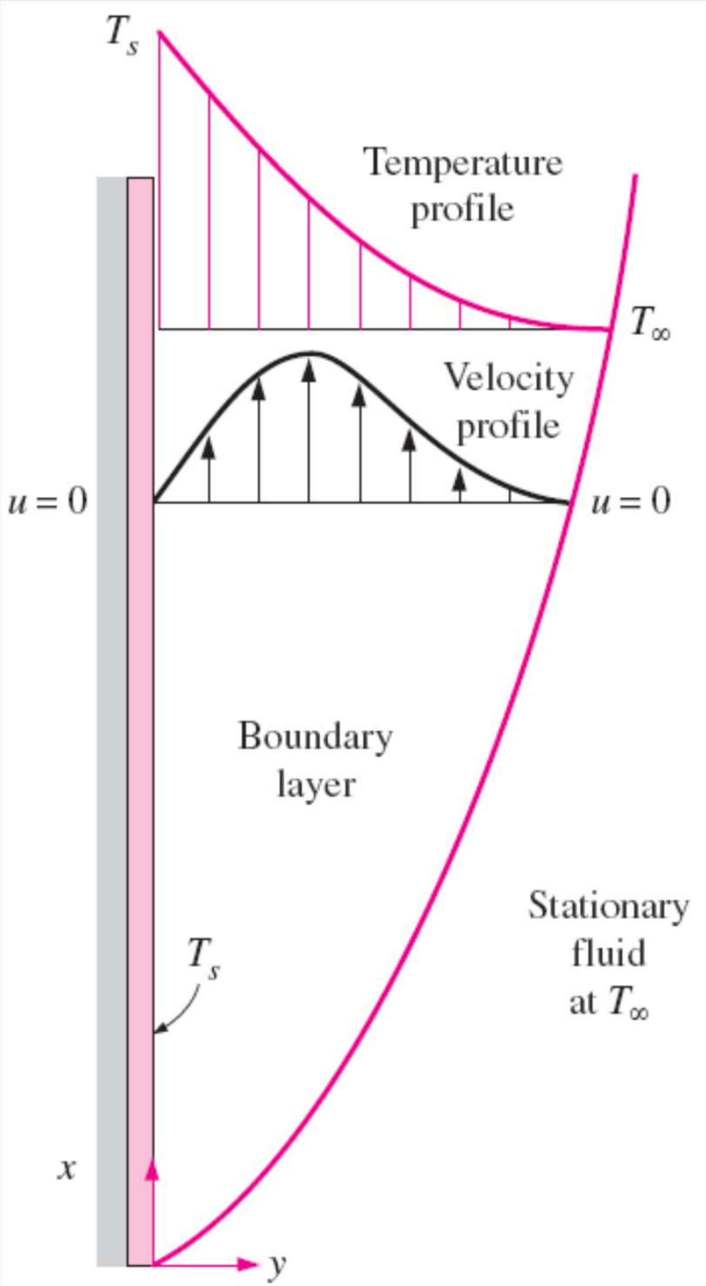
The lines are closest near the surface, indicating a *higher temperature gradient.*



(*a*) Laminar flow (*b*) Turbulent flow

Isotherms in natural convection over a hot plate in air.

EQUATION OF MOTION AND THE GRASHOF NUMBER



The thickness of the boundary layer increases in the flow direction.

Unlike forced convection, the fluid velocity is *zero* at the outer edge of the velocity boundary layer as well as at the surface of the plate.

At the surface, the fluid temperature is equal to the plate temperature, and gradually decreases to the temperature of the surrounding fluid at a distance sufficiently far from the surface.

In the case of *cold surfaces,* the shape of the velocity and temperature profiles remains the same but their direction is reversed.

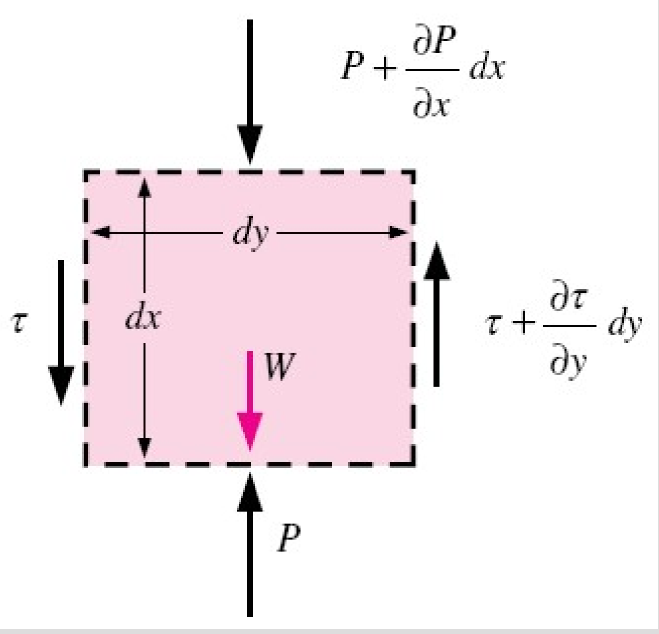
Typical velocity and temperature profiles for natural convection flow over a hot vertical plate at temperature T_s inserted in a fluid at temperature T_∞.

Derivation of the equation of motion that governs the natural convection flow in laminar boundary layer

$$\delta m \cdot a_x = F_x$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$F_x = \left(\frac{\partial \tau}{\partial y}dy\right)(dx \cdot 1) - \left(\frac{\partial P}{\partial x}dx\right)(dy \cdot 1) - \rho g(dx \cdot dy \cdot 1)$$
$$= \left(\mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g\right)(dx \cdot dy \cdot 1)$$



Forces acting on a differential volume element in the natural convection boundary layer over a vertical flat plate.

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} - \frac{\partial P}{\partial x} - \rho g$$

$$\frac{\partial P_\infty}{\partial x} = -\rho_\infty g$$

$$\rho\left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = \mu\frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta(T - T_\infty)$$

This is the equation that governs the fluid motion in the boundary layer due to the effect of buoyancy. The momentum equation involves the temperature, and thus the momentum and energy equations must be solved simultaneously.

The Grashof Number

The governing equations of natural convection and the boundary conditions can be nondimensionalized by dividing all dependent and independent variables by suitable constant quantities:

$$x^* = \frac{x}{L_c} \qquad y^* = \frac{y}{L_c} \qquad u^* = \frac{u}{V} \qquad v^* = \frac{v}{V} \qquad \text{and} \qquad T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

Substituting them into the momentum equation and simplifying give

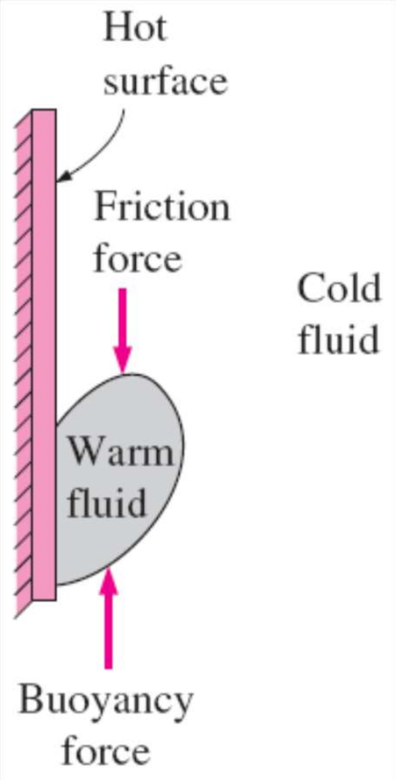
$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \left[\frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\right] \frac{T^*}{\mathrm{Re}_L^2} + \frac{1}{\mathrm{Re}_L} \frac{\partial^2 u^*}{\partial y^{*2}}$$

$$\mathrm{Gr}_L = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}$$

Grashof number: Represents the natural convection effects in momentum equation

g = gravitational acceleration, $\mathrm{m/s^2}$
β = coefficient of volume expansion, 1/K ($\beta = 1/T$ for ideal gases)
T_s = temperature of the surface, °C
T_∞ = temperature of the fluid sufficiently far from the surface, °C
L_c = characteristic length of the geometry, m
ν = kinematic viscosity of the fluid, $\mathrm{m^2/s}$

- The Grashof number provides the main criterion in determining whether the fluid flow is laminar or turbulent in natural convection.
- For vertical plates, the critical Grashof number is observed to be about 10^9.



The Grashof number Gr is a measure of the relative magnitudes of the *buoyancy force* and the opposing *viscous force* acting on the fluid.

When a surface is subjected to external flow, the problem involves both natural and forced convection.

The relative importance of each mode of heat transfer is determined by the value of the coefficient Gr/Re^2:

- Natural convection effects are negligible if $Gr/Re^2 << 1$.
- Free convection dominates and the forced convection effects are negligible if $Gr/Re^2 >> 1$.
- Both effects are significant and must be considered if Gr/Re2 $\approx$ 1.

NATURAL CONVECTION OVER SURFACES

Natural convection heat transfer on a surface depends on the geometry of the surface as well as its orientation. It also depends on the variation of temperature on the surface and the thermophysical properties of the fluid involved.

With the exception of some simple cases, Heat transfer relations in natural convection are based on experimental studies.

$$\text{Nu} = \frac{hL_c}{k} = C(\text{Gr}_L\,\text{Pr})^n = C\,\text{Ra}_L^n$$

$$\text{Ra}_L = \text{Gr}_L\,\text{Pr} = \frac{g\beta(T_s - T_\infty)L_c^3}{\nu^2}\,\text{Pr}$$

Rayleigh number

The constants *C* and *n* depend on the *geometry* of the surface and the *flow regime,* which is characterized by the range of the Rayleigh number.

The value of *n* is1/4 usually for laminar flow and 1/3 for turbulent flow.

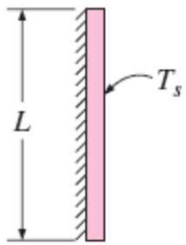
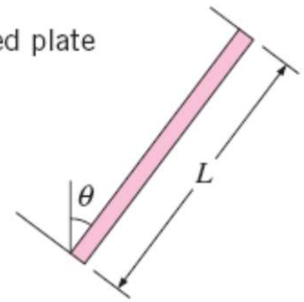
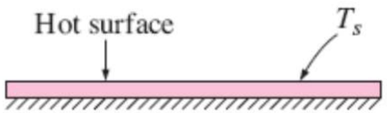
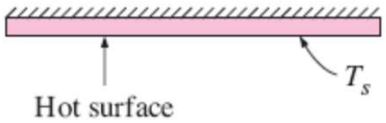
All fluid properties are to be evaluated at the film temperature $T_f = (T_s + T_\infty)/2$.

$$\dot{Q}_{\text{conv}} = hA_s(T_s - T_\infty) \qquad (\text{W})$$

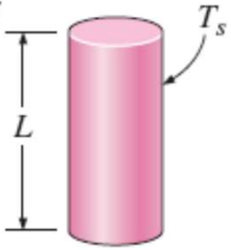
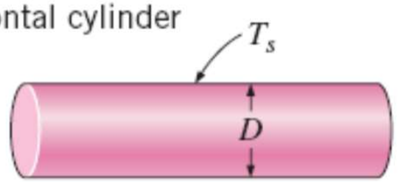
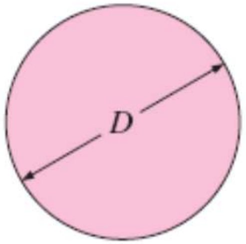
$$\underbrace{\text{Nu}}_{\text{Nusselt number}} = \overbrace{C}^{\text{Constant coefficient}}\,\underbrace{\text{Ra}_L^{\overbrace{n}^{\text{Constant exponent}}}}_{\text{Rayleigh number}}$$

Natural convection heat transfer correlations are usually expressed in terms of the Rayleigh number raised to a constant *n* multiplied by another constant *C,* both of which are determined experimentally.

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ $Nu = 0.1Ra_L^{1/3}$ $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for Ra $< 10^9$
Horizontal plate (Surface area A and perimeter p) (*a*) Upper surface of a hot plate (or lower surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11}	$Nu = 0.54Ra_L^{1/4}$ $Nu = 0.15Ra_L^{1/3}$
(*b*) Lower surface of a hot plate (or upper surface of a cold plate) 		10^5–10^{11}	$Nu = 0.27Ra_L^{1/4}$

Empirical correlations for the average Nusselt number for natural convection over surfaces

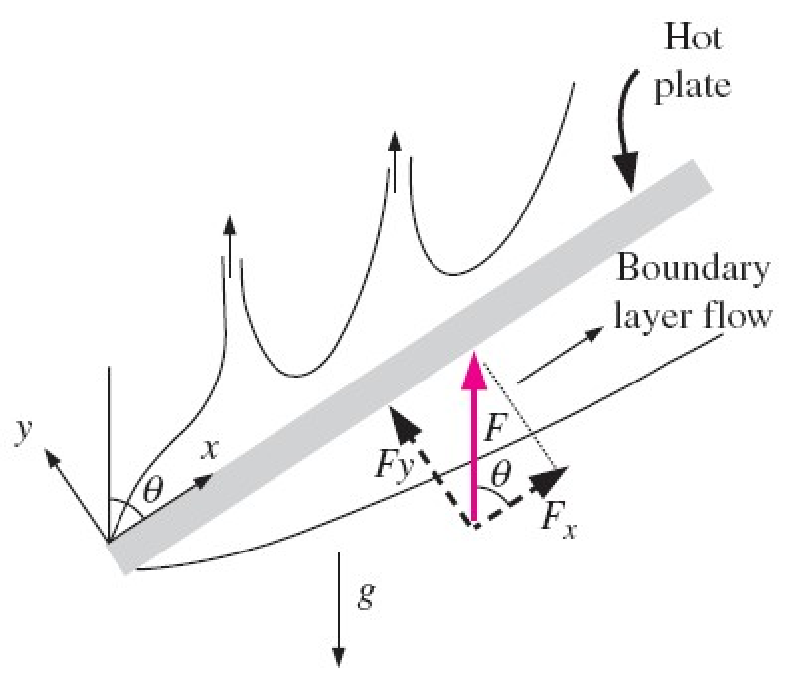
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \dfrac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \dfrac{0.387 Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$
Sphere 	D	$Ra_D \leq 10^{11}$ ($Pr \geq 0.7$)	$Nu = 2 + \dfrac{0.589 Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$

Vertical Plates (q_s = constant)

The relations for isothermal plates in the table can also be used for plates subjected to uniform heat flux, provided that the plate midpoint temperature $T_{L/2}$ is used for T_s in the evaluation of the film temperature, Rayleigh number, and the Nusselt number.

$$\text{Nu} = \frac{hL}{k} = \frac{\dot{q}_s L}{k(T_{L/2} - T_\infty)} \qquad \dot{Q} = \dot{q}_s A_s$$

Inclined Plates



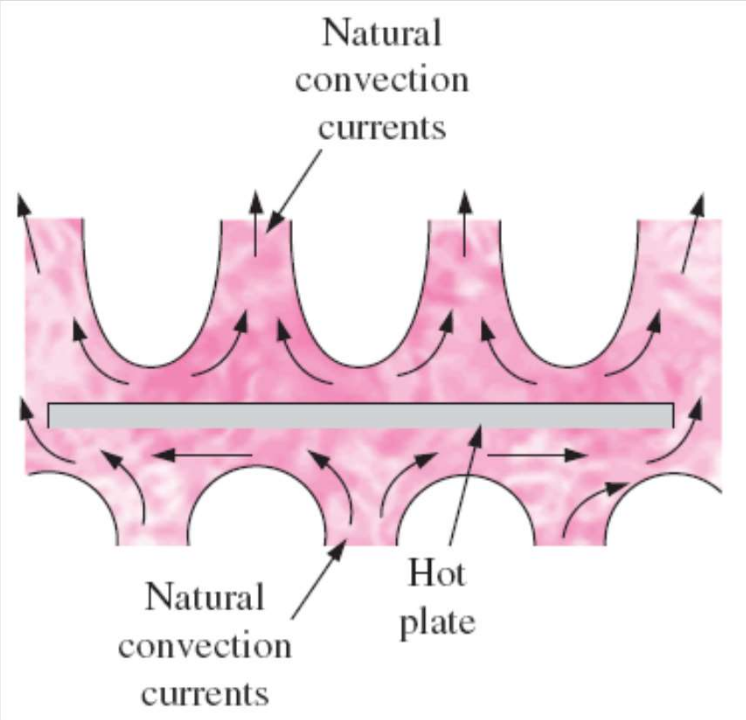
Natural convection flows on the upper and lower surfaces of an inclined hot plate.

In a hot plate in a cooler environment for the lower surface of a hot plate, the convection currents are weaker, and the rate of heat transfer is lower relative to the vertical plate case.

On the upper surface of a hot plate, the thickness of the boundary layer and thus the resistance to heat transfer decreases, and the rate of heat transfer increases relative to the vertical orientation.

In the case of a cold plate in a warmer environment, the opposite occurs.

Horizontal Plates



Natural convection flows on the upper and lower surfaces of a horizontal hot plate.

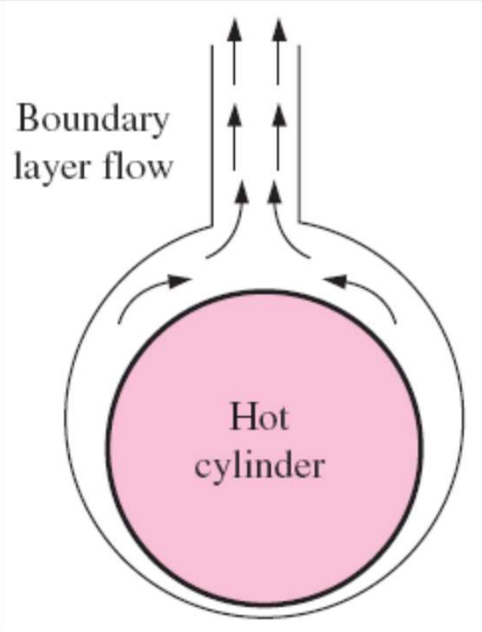
For a hot surface in a cooler environment, the net force acts upward, forcing the heated fluid to rise.

If the hot surface is facing upward, the heated fluid rises freely, inducing strong natural convection currents and thus effective heat transfer.

But if the hot surface is facing downward, the plate blocks the heated fluid that tends to rise, impeding heat transfer.

The opposite is true for a cold plate in a warmer environment since the net force (weight minus buoyancy force) in this case acts downward, and the cooled fluid near the plate tends to descend.

Horizontal Cylinders and Spheres



Natural convection flow over a horizontal hot cylinder.

The boundary layer over a hot horizontal cylinder starts to develop at the bottom, increasing in thickness along the circumference, and forming a rising plume at the top.

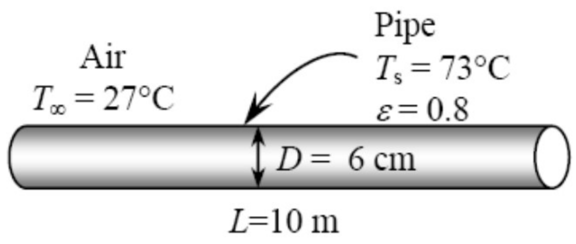
Therefore, the local Nusselt number is highest at the bottom, and lowest at the top of the cylinder when the boundary layer flow remains laminar.

The opposite is true in the case of a cold horizontal cylinder in a warmer medium, and the boundary layer in this case starts to develop at the top of the cylinder and ending with a descending plume at the bottom.

9–19 A 10-m-long section of a 6-cm-diameter horizontal hot-water pipe passes through a large room whose temperature is 27°C. If the temperature and the emissivity of the outer surface of the pipe are 73°C and 0.8, respectively, determine the rate of heat loss from the pipe by (*a*) natural convection and (*b*) radiation.

Properties The properties of air at 1 atm and the film temperature of $(T_s+T_\infty)/2 = (73+27)/2 = 50$°C are (Table A-15)

$$k = 0.02735 \text{ W/m.°C}$$
$$\nu = 1.798\times10^{-5} \text{ m}^2\text{/s}$$
$$\Pr = 0.7228$$
$$\beta = \frac{1}{T_f} = \frac{1}{(50+273)\text{K}} = 0.003096 \text{ K}^{-1}$$



Analysis (*a*) The characteristic length in this case is the outer diameter of the pipe, $L_c = D = 0.06$ m. Then,

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2}\Pr = \frac{(9.81 \text{ m/s}^2)(0.003096 \text{ K}^{-1})(73-27 \text{ K})(0.06 \text{ m})^3}{(1.798\times10^{-5} \text{ m}^2\text{/s})^2}(0.7228) = 6.747\times10^5$$

$$Nu = \left\{0.6 + \frac{0.387Ra^{1/6}}{\left[1+(0.559/\Pr)^{9/16}\right]^{8/27}}\right\}^2 = \left\{0.6 + \frac{0.387(6.747\times10^5)^{1/6}}{\left[1+(0.559/0.7228)^{9/16}\right]^{8/27}}\right\}^2 = 13.05$$

$$h = \frac{k}{D}Nu = \frac{0.02735 \text{ W/m.°C}}{0.06 \text{ m}}(13.05) = 5.950 \text{ W/m}^2\text{.°C}$$

$$A_s = \pi DL = \pi(0.06 \text{ m})(10 \text{ m}) = 1.885 \text{ m}^2$$

$$\dot{Q} = hA_s(T_s - T_\infty) = (5.950 \text{ W/m}^2\text{.°C})(1.885 \text{ m}^2)(73-27)\text{°C} = \mathbf{516 \text{ W}}$$

(*b*) The radiation heat loss from the pipe is

$$\dot{Q}_{rad} = \varepsilon A_s\sigma(T_s^{\ 4} - T_{surr}^{\ \ 4})$$
$$= (0.8)(1.885 \text{ m}^2)(5.67\times10^{-8} \text{ W/m}^2\text{.K}^4)\left[(73+273 \text{ K})^4 - (27+273 \text{ K})^4\right] = \mathbf{533 \text{ W}}$$

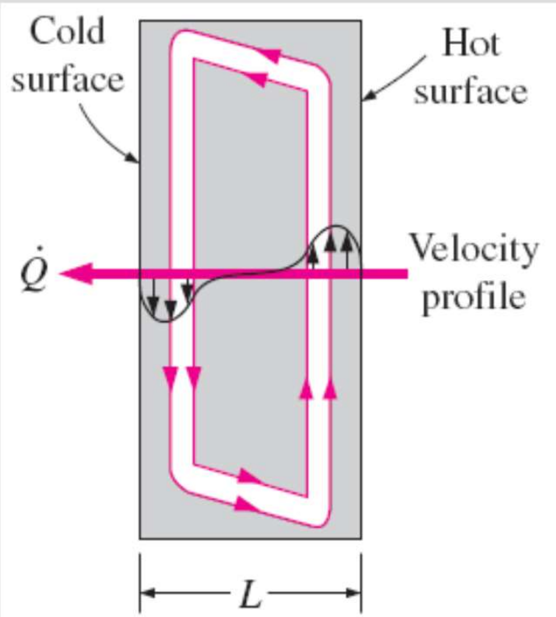
NATURAL CONVECTION INSIDE ENCLOSURES

Enclosures are frequently encountered in practice, and heat transfer through them is of practical interest. In a vertical enclosure, the fluid adjacent to the hotter surface rises and the fluid adjacent to the cooler one falls, setting off a rotationary motion within the enclosure that enhances heat transfer through the enclosure.

$$\mathrm{Ra}_L = \frac{g\beta(T_1 - T_2)L_c^3}{\nu^2}\,\mathrm{Pr}$$

L_c charecteristic length: the distance between the hot and cold surfaces,

T_1 and ***T_2***: the temperatures of the hot and cold surfaces



Convective currents in a vertical rectangular enclosure.

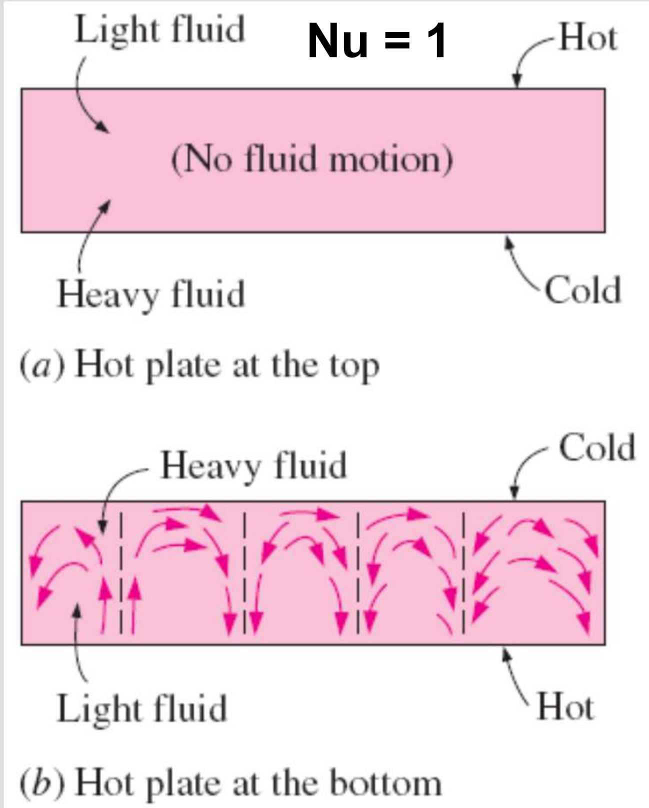
Ra > 1708, natural convection currents

Ra > 3×10^5, turbulent fluid motion

Fluid properties at

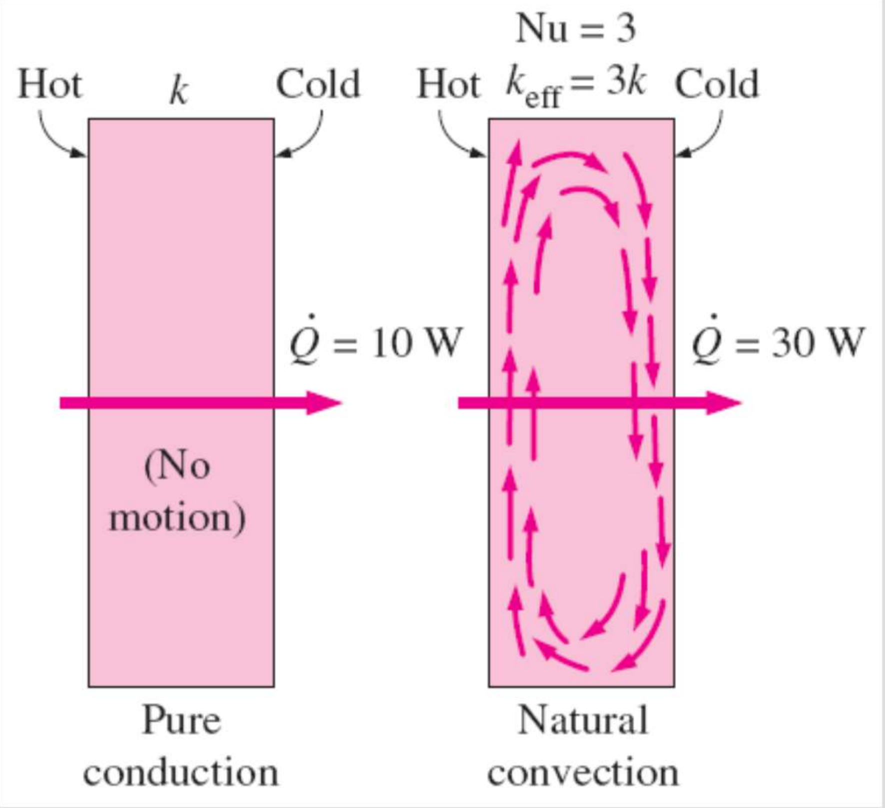
$$T_{avg} = (T_1 + T_2)/2.$$

Convective currents in a horizontal enclosure with (*a*) hot plate at the top and (*b*) hot plate at the bottom.



Effective Thermal Conductivity

$$\dot{Q} = hA_s(T_1 - T_2) = k\text{Nu}A_s \frac{T_1 - T_2}{L_c}$$ $$h = k\text{Nu}/L_c$$ $$\dot{Q}_{\text{cond}} = kA_s \frac{T_1 - T_2}{L_c}$$



A Nusselt number of 3 for an enclosure indicates that heat transfer through the enclosure by *natural convection* is three times that by *pure conduction.*

$$k_{\text{eff}} = k\text{Nu}$$ effective thermal conductivity

The fluid in an enclosure behaves like a fluid whose thermal conductivity is kNu as a result of convection currents.

Nu = 1, the effective thermal conductivity of the enclosure is equal to the conductivity of the fluid. This case corresponds to pure conduction.

Numerous correlations for the Nusselt number exist. Simple power-law type relations in the form of $\text{Nu} = C\text{Ra}^n$, where C and n are constants, are sufficiently accurate, but they are usually applicable to a narrow range of Prandtl and Rayleigh numbers and aspect ratios.

Horizontal Rectangular Enclosures

$$\mathrm{Nu} = 0.195\mathrm{Ra}_L^{1/4} \qquad 10^4 < \mathrm{Ra}_L < 4 \times 10^5$$

$$\mathrm{Nu} = 0.068\mathrm{Ra}_L^{1/3} \qquad 4 \times 10^5 < \mathrm{Ra}_L < 10^7$$

For horizontal enclosures that contain air, These relations can also be used for other gases with $0.5 < \mathrm{Pr} < 2$.

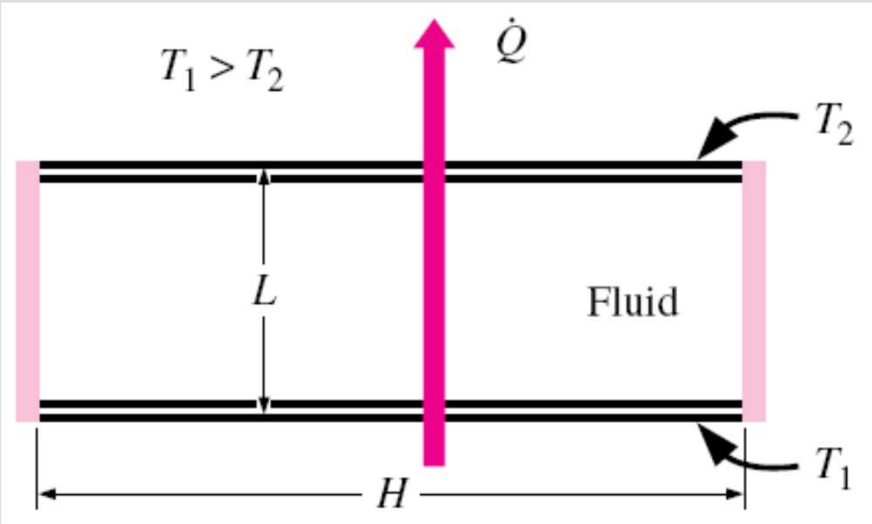
$$\mathrm{Nu} = 0.069\mathrm{Ra}_L^{1/3}\,\mathrm{Pr}^{0.074} \qquad 3 \times 10^5 < \mathrm{Ra}_L < 7 \times 10^9$$

For water, silicone oil, and mercury

$$\mathrm{Nu} = 1 + 1.44\left[1 - \frac{1708}{\mathrm{Ra}_L}\right]^+ + \left[\frac{\mathrm{Ra}_L^{1/3}}{18} - 1\right]^+ \qquad \mathrm{Ra}_L < 10^8$$

[ ]$^+$ only positive values to be used

Based on experiments with air. It may be used for liquids with moderate Prandtl numbers for $\mathrm{Ra}_L < 10^5$.



When the hotter plate is at the top, Nu = 1.

A horizontal rectangular enclosure with isothermal surfaces.

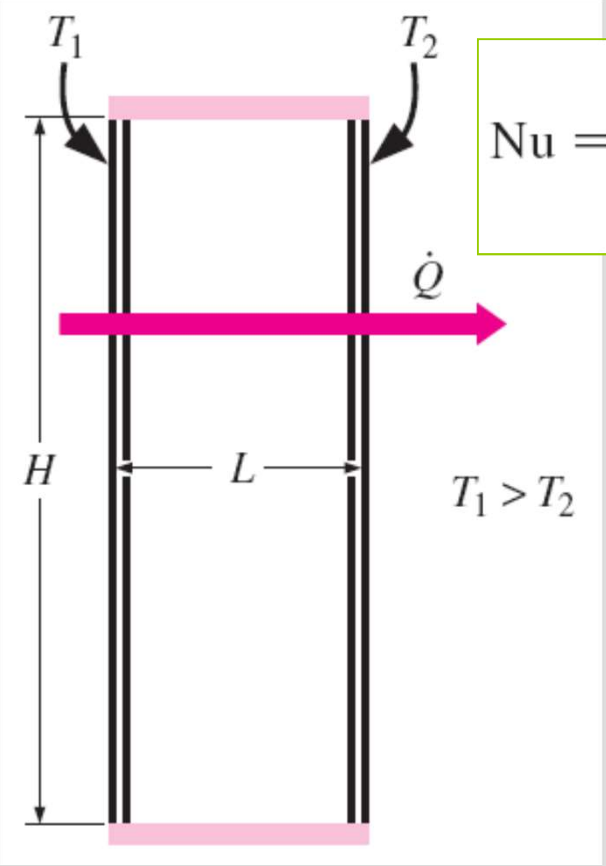
Vertical Rectangular Enclosures

$$\mathrm{Nu} = 0.18\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.29}$$

$1 < H/L < 2$
any Prandtl number
$\mathrm{Ra}_L\,\mathrm{Pr}/(0.2+\mathrm{Pr}) > 10^3$

$$\mathrm{Nu} = 0.22\left(\frac{\mathrm{Pr}}{0.2+\mathrm{Pr}}\mathrm{Ra}_L\right)^{0.28}\left(\frac{H}{L}\right)^{-1/4}$$

$2 < H/L < 10$
any Prandtl number
$\mathrm{Ra}_L < 10^{10}$



$$\mathrm{Nu} = 0.42\,\mathrm{Ra}_L^{1/4}\,\mathrm{Pr}^{0.012}\left(\frac{H}{L}\right)^{-0.3}$$

$10 < H/L < 40$
$1 < \mathrm{Pr} < 2\times 10^4$
$10^4 < \mathrm{Ra}_L < 10^7$

$$\mathrm{Nu} = 0.46\,\mathrm{Ra}_L^{1/3}$$

$1 < H/L < 40$
$1 < \mathrm{Pr} < 20$
$10^6 < \mathrm{Ra}_L < 10^9$

A vertical rectangular enclosure with isothermal surfaces.

Combined Natural Convection and Radiation

Gases are nearly transparent to radiation, and thus heat transfer through a gas layer is by simultaneous convection (or conduction) and radiation. Radiation is usually disregarded in forced convection problems, but it must be considered in natural convection problems that involve a gas. This is especially the case for surfaces with high emissivities.

$$\dot{Q}_{\text{total}} = \dot{Q}_{\text{conv}} + \dot{Q}_{\text{rad}}$$

Radiation heat transfer from a surface at temperature T_s surrounded by surfaces at a temperature T_{surr} is

$$\dot{Q}_{\text{rad}} = \varepsilon\sigma A_s(T_s^4 - T_{\text{surr}}^4) \qquad \text{(W)}$$

$\sigma = 5.67 \times 10^8$ W/m^2·K^4
Stefan–Boltzmann constant

Radiation heat transfer between two large parallel plates is

$$\dot{Q}_{\text{rad}} = \frac{\pi A_s(T_1^4 - T_2^4)}{1/\varepsilon_1 + 1/\varepsilon_2 - 1} = \varepsilon_{\text{effective}}\,\sigma A_s(T_1^4 - T_2^4) \qquad \text{(W)}$$

$$\varepsilon_{\text{effective}} = \frac{1}{1/\varepsilon_1 + 1/\varepsilon_2 - 1}$$

When $T_\infty < T_s$ and $T_{\text{surr}} > T_s$, convection and radiation heat transfers are in opposite directions and subtracted from each other.