Fluid Mechanics: Fundamentals and Applications, 2nd Edition
Yunus A. Cengel, John M. Cimbala
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Chapter 8
INTERNAL FLOW

Prof. Dr. Ali PINARBAŞI
Yildiz Technical University
Mechanical Engineering Department
Yildiz, ISTANBUL

INTERNAL FLOW

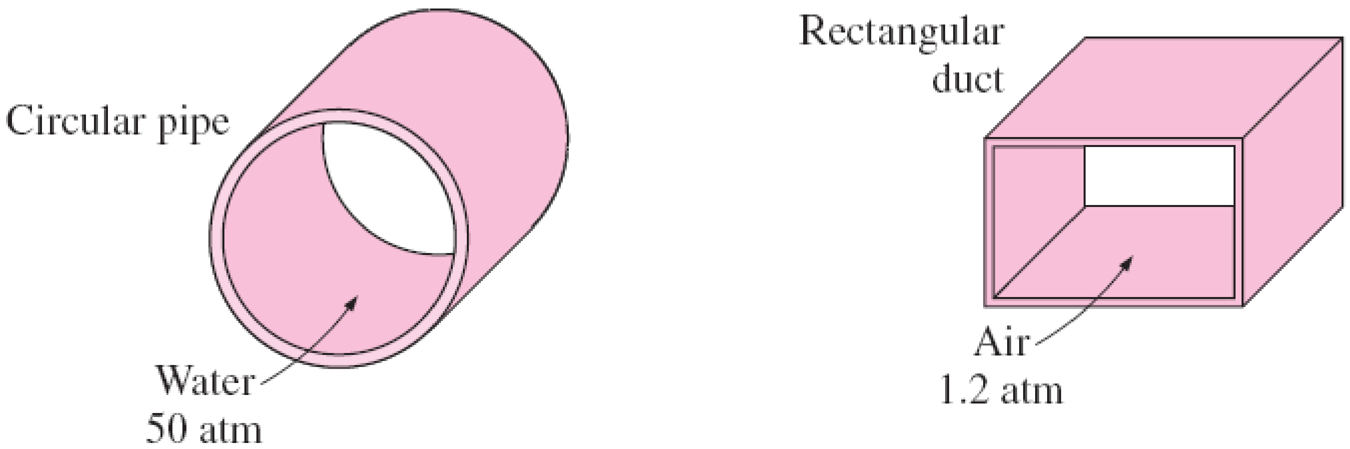
Internal flows through pipes, elbows, tees, valves, etc., as in this oil refinery, are found in nearly every industry.

Objectives

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
- Understand various velocity and flow rate measurement techniques and learn their advantages and disadvantages

8–1 INTRODUCTION

- Liquid or gas flow through ***pipes*** or ***ducts*** is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to ***friction***, which is directly related to the ***pressure drop*** and ***head loss*** during flow through pipes and ducts.
- The pressure drop is then used to determine the ***pumping power requirement***.



Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

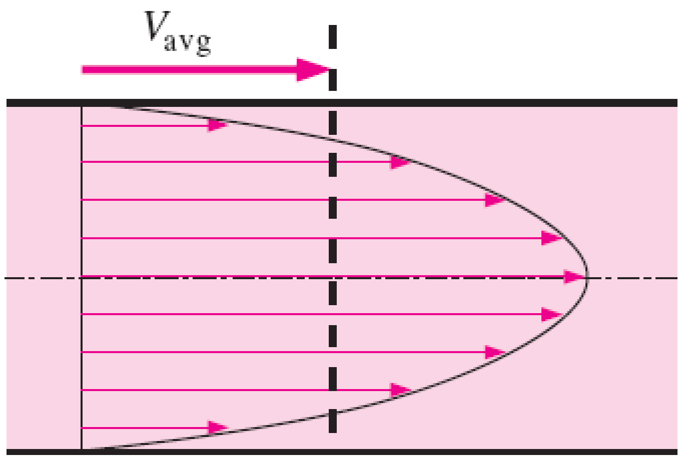
Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.

Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.

The value of the average velocity V_{avg} at some streamwise cross-section is determined from the requirement that the ***conservation of mass*** principle be satisfied

$$\dot{m} = \rho V_{\text{avg}} A_c = \int_{A_c} \rho u(r)\, dA_c$$

$$V_{\text{avg}} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r\, dr$$

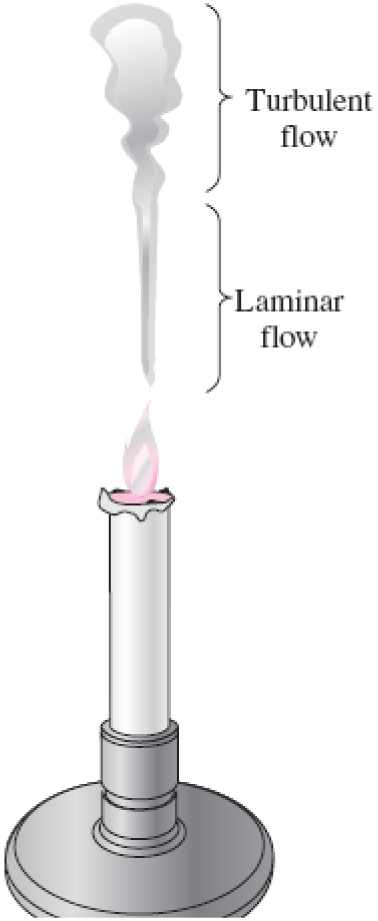


The average velocity for incompressible flow in a circular pipe of radius *R*

Average velocity V_{avg} is defined as the average speed through a cross section. For fully developed laminar pipe flow, V_{avg} is half of the maximum velocity.

8–2 LAMINAR AND TURBULENT FLOWS

Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



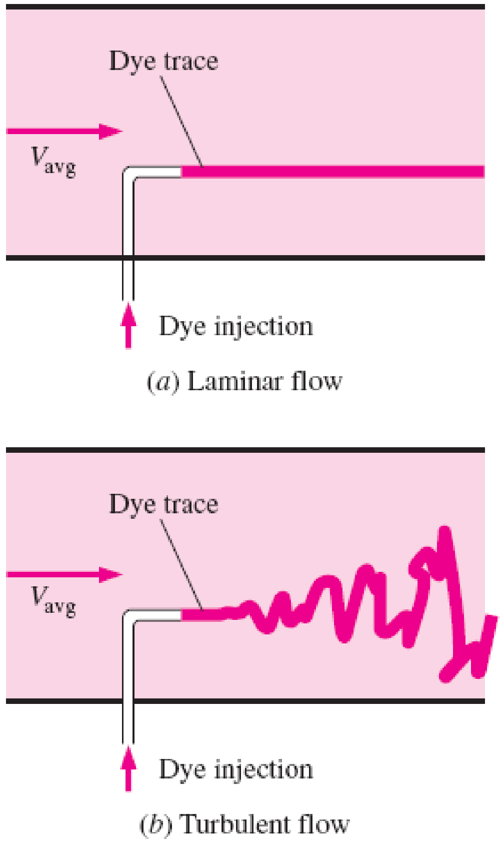
Laminar and turbulent flow regimes of candle smoke.

Laminar: Smooth streamlines and highly ordered motion.

Turbulent: Velocity fluctuations and highly disordered motion.

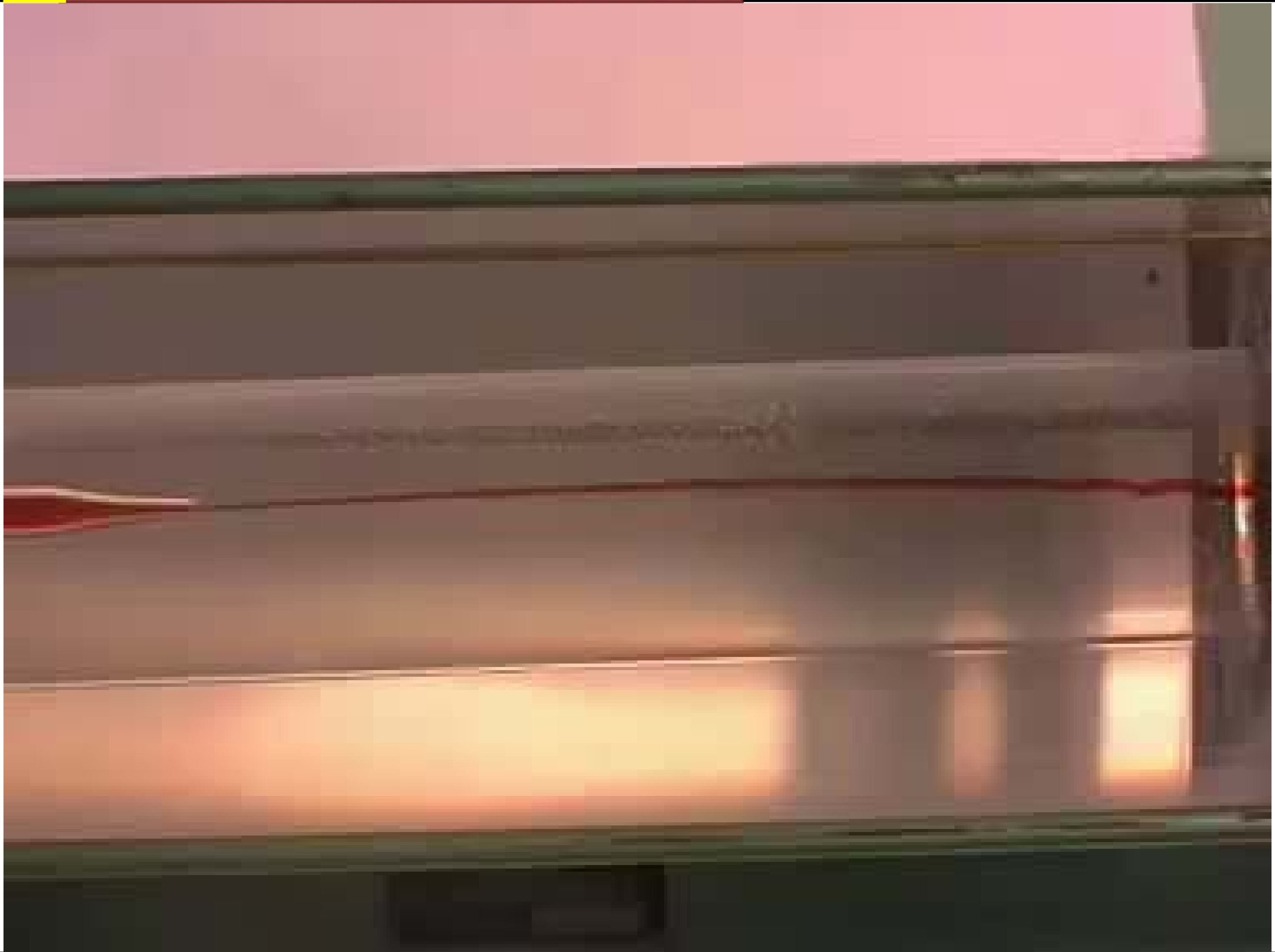
Transition: The flow fluctuates between laminar and turbulent flows.

Most flows encountered in practice are turbulent.



The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

Laminar versus Turbulent Flow

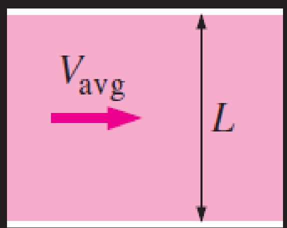


Reynolds Number

The transition from laminar to turbulent flow depends on the ***geometry, surface roughness, flow velocity, surface temperature***, and ***type of fluid***.

The flow regime depends mainly on the ratio of ***inertial forces*** to ***viscous forces*** **(Reynolds number).**

$$\mathrm{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg} D}{\nu} = \frac{\rho V_{avg} D}{\mu}$$



$$\mathbf{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{\rho V_{avg}^2 L^2}{\mu V_{avg} L} = \frac{\rho V_{avg} L}{\mu} = \frac{V_{avg} L}{\nu}$$

The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent).

At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid “in line” (laminar).

Critical Reynolds number, Re_{cr}: The Reynolds number at which the flow becomes turbulent.

The value of the critical Reynolds number is different for different geometries and flow conditions.

Reynolds Number

[redacted]

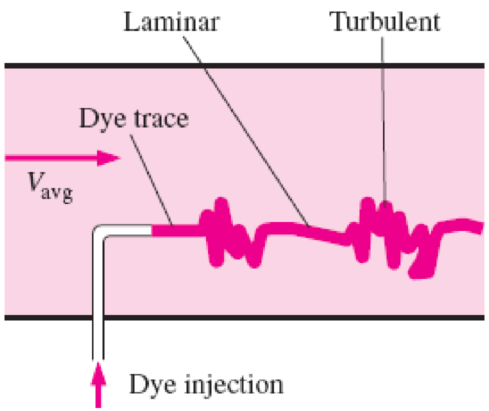
For flow through noncircular pipes, the Reynolds number is based on the **hydraulic diameter**

$$D_h = \frac{4A_c}{p}$$

where A_c is the cross-sectional area of the pipe and p is its wetted perimeter

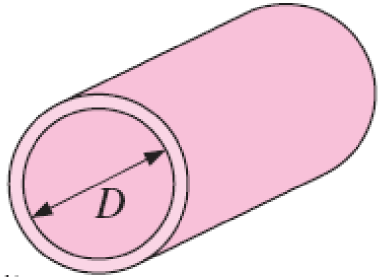
For flow in a circular pipe:

$\mathrm{Re} \lesssim 2300$	laminar flow
$2300 \lesssim \mathrm{Re} \lesssim 4000$	transitional flow
$\mathrm{Re} \gtrsim 4000$	turbulent flow



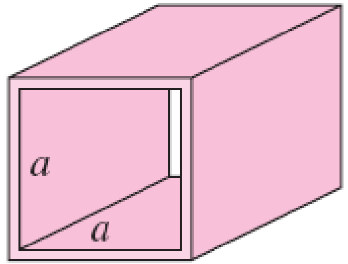
the transitional flow region of $2300 \le \mathrm{Re} \le 10{,}000$, the flow.

Circular tube:



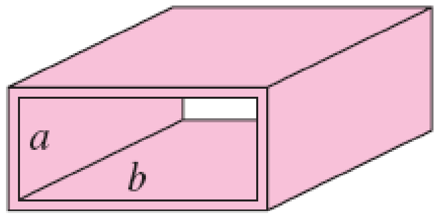
$$D_h = \frac{4(\pi D^2/4)}{\pi D} = D$$

Square duct:



$$D_h = \frac{4a^2}{4a} = a$$

Rectangular duct:

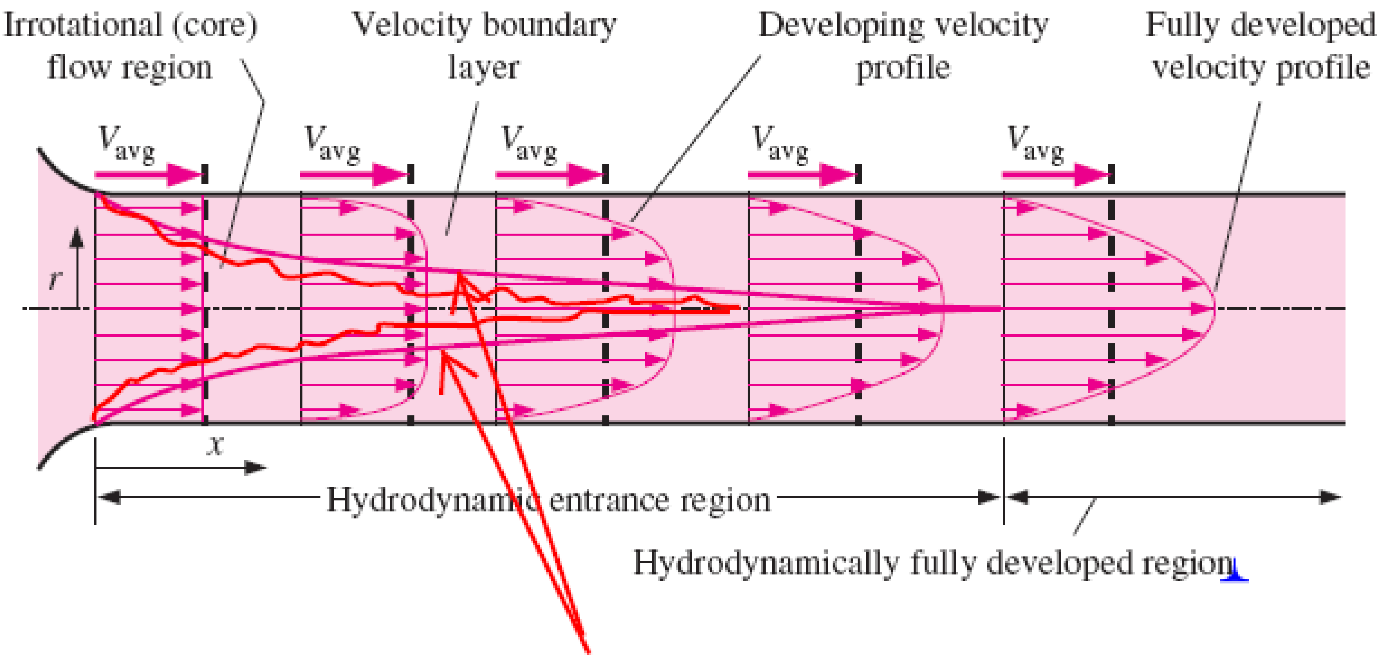


$$D_h = \frac{4ab}{2(a+b)} = \frac{2ab}{a+b}$$

The hydraulic diameter $D_h = 4A_c/p$ is defined such that it reduces to ordinary diameter for circular tubes.

8–3 THE ENTRANCE REGION

Velocity boundary layer: The region of the flow in which the effects of the viscous shearing forces caused by fluid viscosity are felt.
Boundary layer region: The viscous effects and the velocity changes are significant.
Irrotational (core) flow region: The frictional effects are negligible and the velocity remains essentially constant in the radial direction.



The development of the velocity boundary layer in a pipe. The developed average velocity profile is parabolic in laminar flow, but somewhat flatter or fuller in turbulent flow.

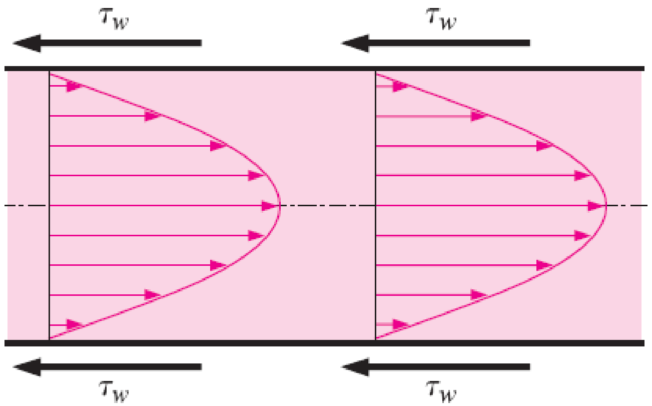
Hydrodynamic entrance region: The region from the pipe inlet to the point at which the boundary layer merges at the centerline.

Hydrodynamic entry length L_h: The length of this region.

Hydrodynamically developing flow: Flow in the entrance region. This is the region where the velocity profile develops.

Hydrodynamically fully developed region: The region beyond the entrance region in which the velocity profile is fully developed and remains unchanged.

Fully developed: When both the velocity profile the normalized temperature profile remain unchanged.



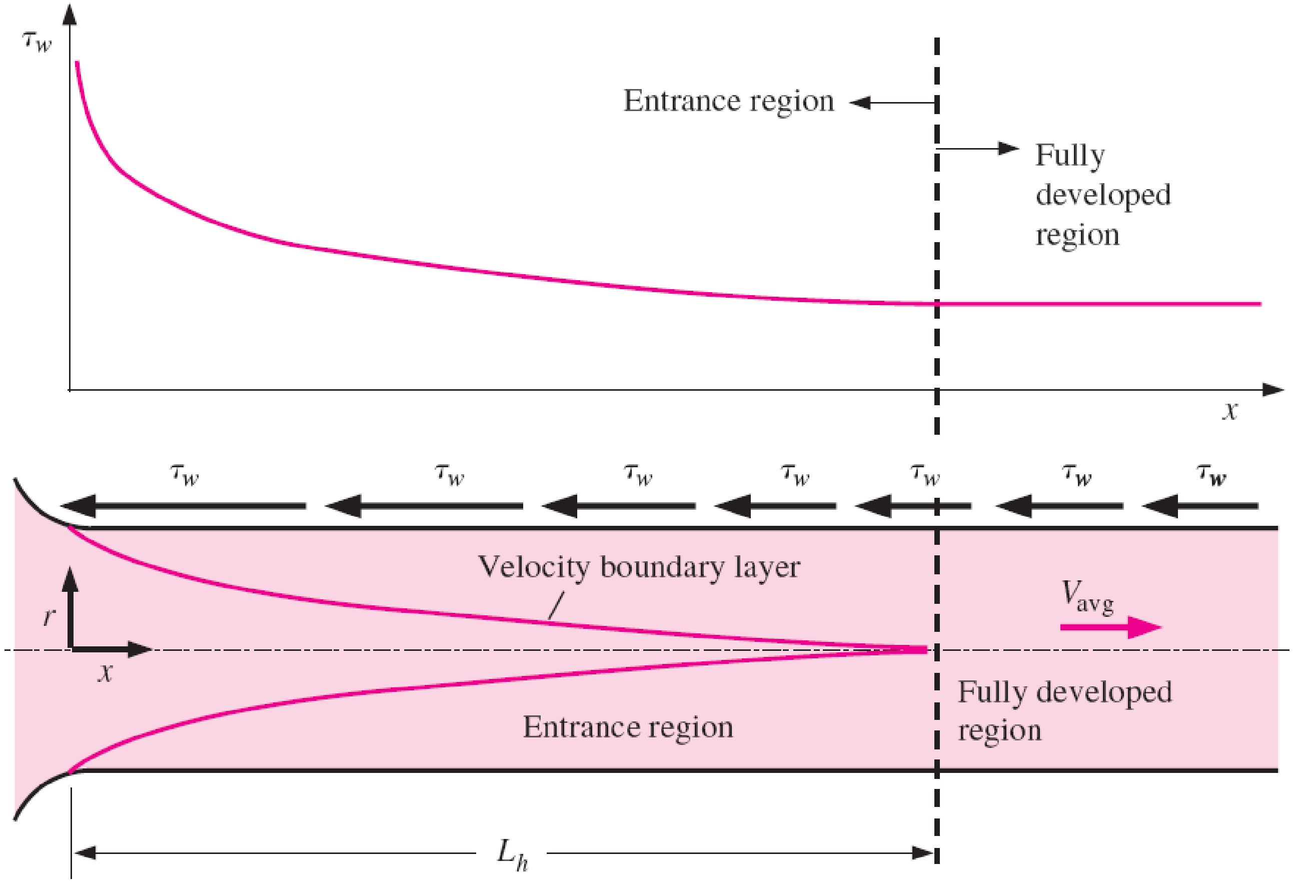
In the fully developed flow region of a pipe, the velocity profile does not change downstream, and thus the wall shear stress remains constant as well.

Hydrodynamically fully developed

$$\frac{\partial u(r, x)}{\partial x} = 0 \quad \rightarrow \quad u = u(r)$$

[redacted]

The pressure drop is ***higher*** in the entrance regions of a pipe, and the effect of the entrance region is always to *increase* the average friction factor for the entire pipe.



The variation of wall shear stress in the flow direction for flow in a pipe from the entrance region into the fully developed region.

Entry Lengths

The hydrodynamic entry length is usually taken to be the distance from the pipe entrance to where the wall shear stress (and thus the friction factor) reaches within about 2 percent of the fully developed value.

hydrodynamic entry length for laminar flow

$$L_{h,\,\text{laminar}} \cong 0.05\text{Re}D$$

hydrodynamic entry length for turbulent flow

$$L_{h,\,\text{turbulent}} = 1.359D\text{Re}_D^{1/4}$$

hydrodynamic entry length for turbulent flow, an approximation

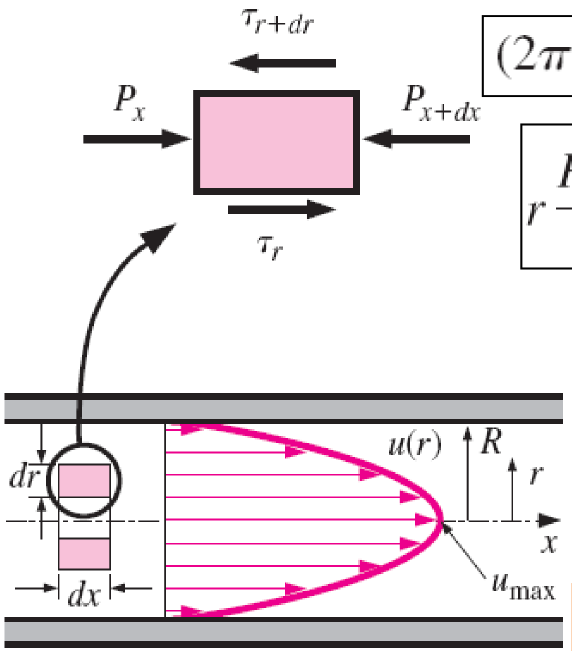
$$L_{h,\,\text{turbulent}} \approx 10D$$

The pipes used in practice are usually several times the length of the entrance region, and thus the flow through the pipes is often assumed to be fully developed for the entire length of the pipe.

This simplistic approach gives *reasonable* results for long pipes but sometimes poor results for short ones since it under predicts the wall shear stress and thus the friction factor.

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile *u*(*r*) remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.



$$(2\pi r\,dr\,P)_x - (2\pi r\,dr\,P)_{x+dx} + (2\pi r\,dx\,\tau)_r - (2\pi r\,dx\,\tau)_{r+dr} = 0$$

$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0$$

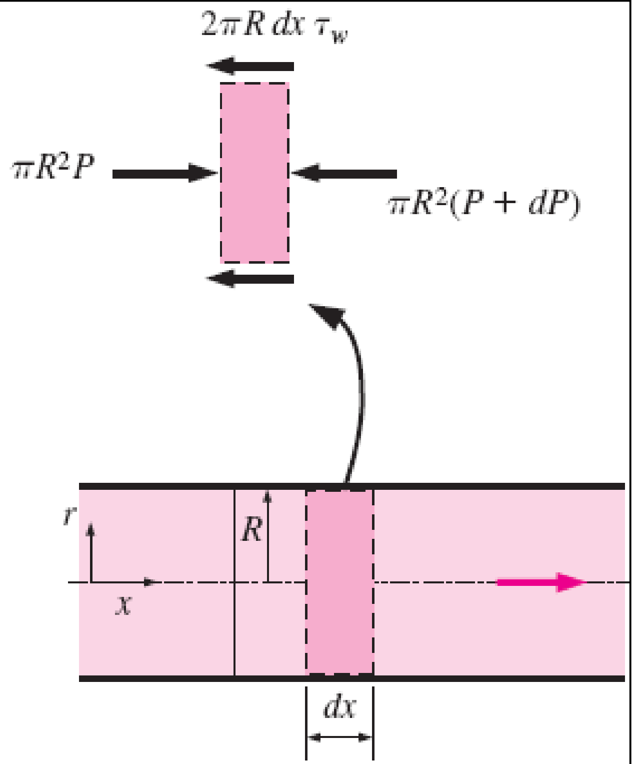
$$\tau = -\mu(du/dr)$$

$$\mu = \text{constant}$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$

Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with a horizontal pipe in fully developed laminar flow.

[redacted]



Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\, dx\, \tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Free-body diagram of a fluid disk element in fully developed laminar flow.

[redacted]

τ_w is constant since the viscosity and the velocity profile are constants in the fully developed region. Therefore, *dP/dx*= constant.

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

Boundary conditions

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$$\partial u/\partial r = 0 \text{ at } r = 0$$
$$u = 0 \text{ at } \quad r = R$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Average velocity

$$V_{\text{avg}} = \frac{2}{R^2}\int_0^R u(r) r\, dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\, dr$$

$$= -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

Velocity profile

$$u(r) = 2V_{\text{avg}}\left(1 - \frac{r^2}{R^2}\right)$$

Maximim velocity at centerline

$$u_{\max} = 2V_{\text{avg}}$$

Pressure Drop and Head Loss

The ***pressure drop P*** directly related to the power requirements of the fan or pump to maintain flow.

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

Laminar flow:

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called **pressure loss ΔP_L.**

$$\boxed{\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}}$$

Circular pipe, laminar

$$\boxed{f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{Re}}$$

Darcy friction factor

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

Head loss

dynamic pressure

$$\rho V_{avg}^2/2$$

$$\boxed{h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}}$$

In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

[redacted]

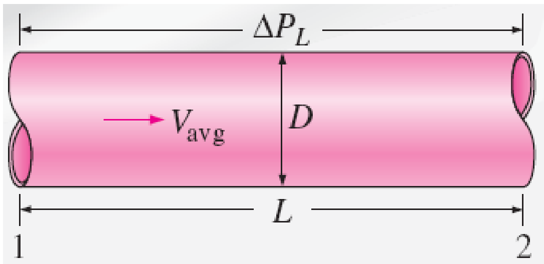
$$\dot{W}_{\text{pump},\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L$$

$$V_{\text{avg}} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$

Poiseuille's law

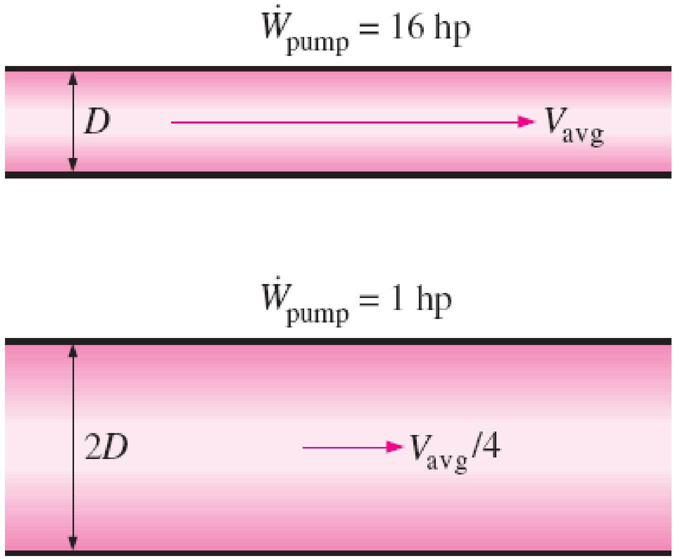
$$\dot{V} = V_{\text{avg}} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$

For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.



Pressure loss: $\Delta P_L = f\,\dfrac{L}{D}\,\dfrac{\rho V_{\text{avg}}^2}{2}$

Head loss: $h_L = \dfrac{\Delta P_L}{\rho g} = f\,\dfrac{L}{D}\,\dfrac{V_{\text{avg}}^2}{2g}$



The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

[redacted]

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\boxed{\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, }u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, }e} + h_L}$$

$$\boxed{P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine, }e} - h_{\text{pump, }u} + h_L]}$$

Effect of Gravity on Velocity and Flow Rate in Laminar Flow

$$W_x = W \sin\theta = \rho g V_{\text{element}} \sin\theta = \rho g(2\pi r\, dr\, dx) \sin\theta$$

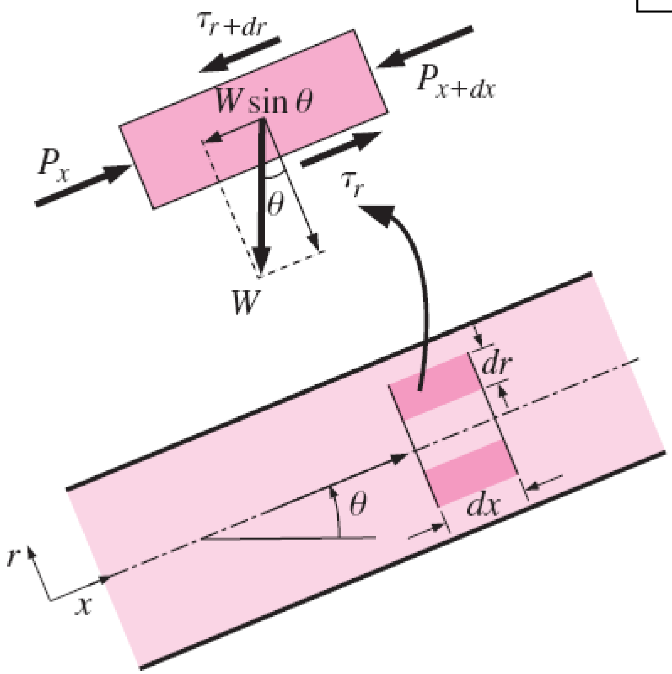
$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$



Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

$$\textit{Horizontal pipe: } \dot{V} = \frac{\Delta P\ \pi D^4}{128\mu L}$$

$$\textit{Inclined pipe: } \dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

Uphill flow: $\theta > 0$ and $\sin\ \theta > 0$

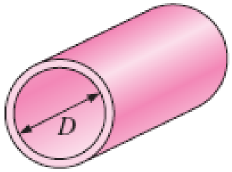
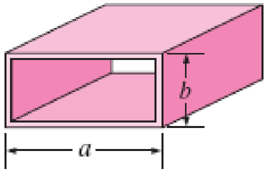
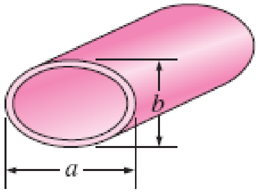
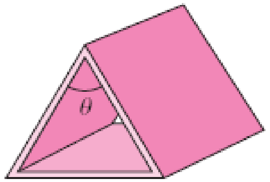
Downhill flow: $\theta\ < 0$ and $\sin\theta\ < 0$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with ΔP=ρgL sinθ.

Laminar Flow in Noncircular Pipes

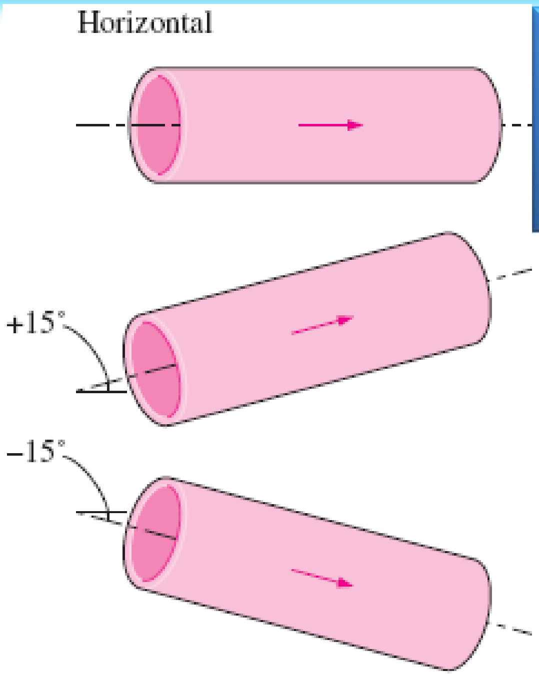
The friction factor f relations are given in Table for ***fully developed laminar flow*** in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{avg} D_h/\nu$)

Tube Geometry	a/b or $\theta°$	Friction Factor f
Circle 	—	64.00/Re
Rectangle 	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse 	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle 	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

EXAMPLE 8–4

Oil at 20°C (ρ = 888 kg/m^3 and μ =0.800 kg/m·s) is flowing steadily through a 5-cm-diameter 40-m-long pipe. The pressure at the pipe inlet and outlet are measured to be 745 and 97 kPa, respectively. Determine the flow rate of oil through the pipe assuming the pipe is (*a*) horizontal, (*b*) inclined 15° upward, (*c*) inclined 15° downward. Also verify that the flow through the pipe is laminar.



Assumptions **1** The flow is steady and incompressible.
2 The entrance effects are negligible, and flow is fully developed.
3 The pipe involves no components.
4 no work devices such as a pump or a turbine.

$$\Delta P = P_1 - P_2 = 745 - 97 = 648 \text{ kPa}$$

$$A_c = \pi D^2/4 = \pi(0.05 \text{ m})^2/4 = 0.001963 \text{ m}^2$$

(*a*) The flow rate for all three cases can be determined from

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

For the horizontal case, $\theta = 0$ and thus $\sin\theta = 0$.

$$\dot{V}_{\text{horiz}} = \frac{\Delta P\, \pi D^4}{128\mu L} = \frac{(648\text{ kPa})\pi(0.05\text{ m})^4}{128(0.800\text{ kg/m}\cdot\text{s})(40\text{ m})}\left(\frac{1000\text{ N/m}^2}{1\text{ kPa}}\right)\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ N}}\right) = \mathbf{0.00311\ m^3/s}$$

(*b*) For uphill flow with an inclination of 15°, we have $\theta = \mp 15°$, and

$$\dot{V}_{\text{uphill}} = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[648{,}000\text{ Pa} - (888\text{ kg/m}^3)(9.81\text{ m/s}^2)(40\text{ m})\sin 15°]\pi(0.05\text{ m})^4}{128(0.800\text{ kg/m}\cdot\text{s})(40\text{ m})}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{0.00267\ m^3/s}$$

(*b*) For downhill flow with an inclination of 15°, we have $\theta = -15°$, and

$$\dot{V}_{\text{downhill}} = \frac{(\Delta P - \rho g L\sin\theta)\pi D^4}{128\mu L}$$

$$= \frac{[648{,}000\text{ Pa} - (888\text{ kg/m}^3)(9.81\text{ m/s}^2)(40\text{ m})\sin(-15°)]\pi(0.05\text{ m})^4}{128(0.800\text{ kg/m}\cdot\text{s})(40\text{ m})}\left(\frac{1\text{ kg}\cdot\text{m/s}^2}{1\text{ Pa}\cdot\text{m}^2}\right)$$

$$= \mathbf{0.00354\ m^3/s}$$

[redacted]

The flow rate is the highest for the downhill flow case, as expected. The average fluid velocity and the Reynolds number in this case are

$$V_{avg} = \frac{\dot{V}}{A_c} = \frac{0.00354 \text{ m}^3/\text{s}}{0.001963 \text{ m}^2} = 1.80 \text{ m/s}$$

$$\text{Re} = \frac{\rho V_{avg} D}{\mu} = \frac{(888 \text{ kg/m}^3)(1.80 \text{ m/s})(0.05 \text{ m})}{0.800 \text{ kg/m} \cdot \text{s}} = 100$$

which is much less than 2300. Therefore, the flow is *laminar* for all three cases and the analysis is valid.

Gravity has no effect on the flow rate in the horizontal case.
Downhill flow can occur even in the absence of an applied pressure difference. For the case of $P_1 = P_2 = 97$ kPa , the pressure throughout the entire pipe would remain constant at 97 Pa, and the fluid would flow through the pipe at a rate of 0.00043 m^3/s under the influence of gravity.

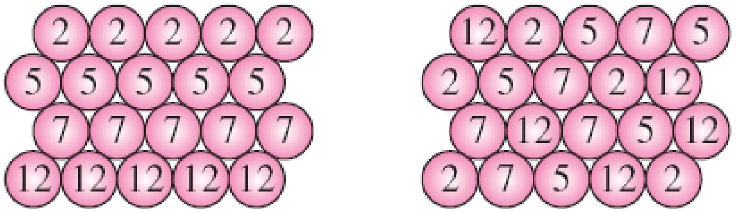
The flow rate increases as the tilt angle of the pipe from the horizontal is increased in the negative direction and would reach its maximum value when the pipe is vertical.

8–5 TURBULENT FLOW IN PIPES

Most flows encountered in engineering practice are turbulent affects wall shear stress.

Turbulent flow is a complex mechanism dominated by fluctuations, and it is still not fully understood.

We must rely on experiments and the empirical or semi-empirical correlations developed for various situations.



(*a*) Before turbulence (*b*) After turbulence

The intense mixing in turbulent flow brings fluid particles at different momentums into close contact and thus enhances momentum transfer.

Turbulent flow is characterized by disorderly and rapid fluctuations of swirling regions of fluid, called eddies, throughout the flow.

These fluctuations provide an additional mechanism for momentum and energy transfer.

In turbulent flow, the swirling eddies transport mass, momentum, and energy to other regions of flow much more rapidly than molecular diffusion, greatly enhancing mass, momentum, and heat transfer.

As a result, turbulent flow is associated with much higher values of friction, heat transfer, and mass transfer coefficients

[redacted]

the sum of an *average value* ū and a *fluctuating component u'*,

$$u = \overline{u} + u'$$

$$v = \overline{v} + v', \; P = \overline{P} + P'$$

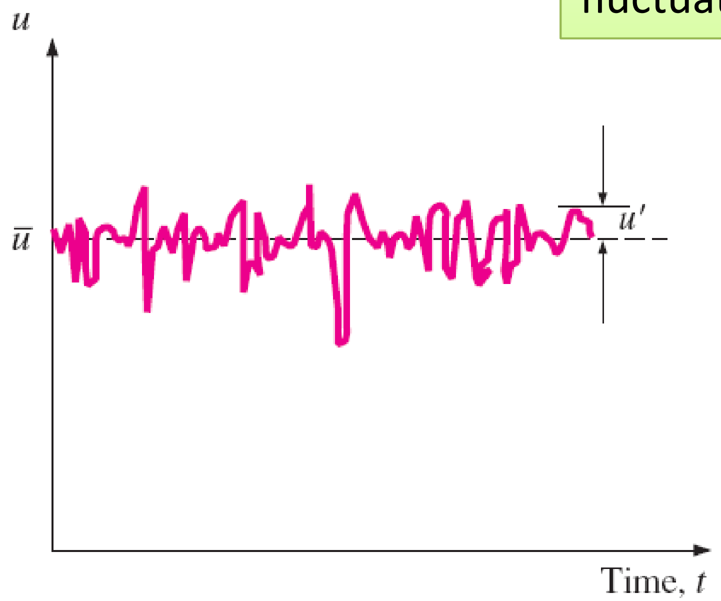
$$T = \overline{T} + T'$$

$$\tau_{\text{total}} = \tau_{\text{lam}} + \tau_{\text{turb}}$$

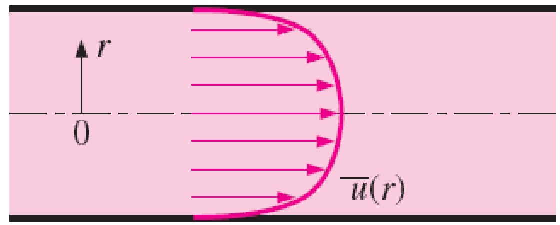
$$\tau_{\text{lam}} = -\mu \; d\overline{u}/dr)$$

The *laminar component:* accounts for the friction between layers in the flow direction

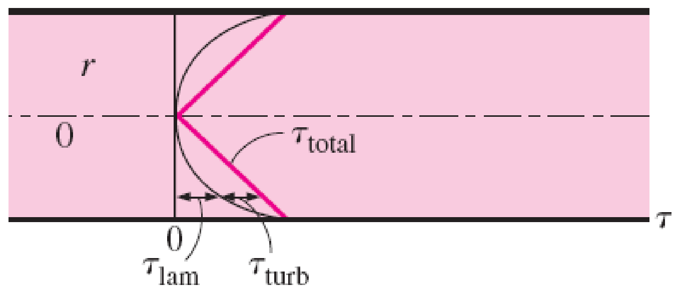
The *turbulent component*: accounts for the friction between the fluctuating fluid particles and the fluid body (related to the fluctuation components of velocity).



Fluctuations of the velocity component *u* with time at a specified location in turbulent flow.



The velocity profile and the variation of shear stress for turbulent flow in a pipe.



Turbulent Shear Stress

$$\tau_{\text{turb}} = -\rho\overline{u'v'}$$

turbulent shear stress

$$-\rho\overline{u'v'} \quad \text{or} \quad -\rho\overline{u'^2}$$

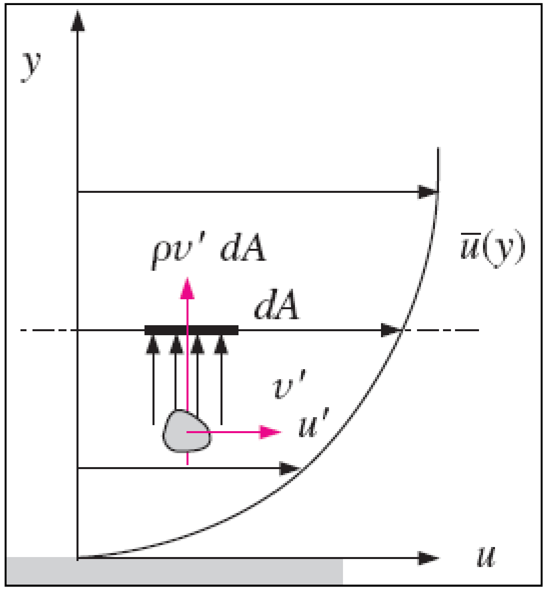
are called **Reynolds stresses** or **turbulent stresses**

$$\tau_{\text{turb}} = -\rho\overline{u'v'} = \mu_t \frac{\partial \overline{u}}{\partial y}$$

Turbulent shear stress

μ_t

eddy viscosity or **turbulent viscosity**: accounts for momentum transport by turbulent eddies.



Fluid particle moving upward through a differential area *dA* as a result of the velocity fluctuation *v*.

$$\tau_{\text{total}} = (\mu + \mu_t)\frac{\partial \overline{u}}{\partial y} = \rho(\nu + \nu_t)\frac{\partial \overline{u}}{\partial y}$$

Total shear stress

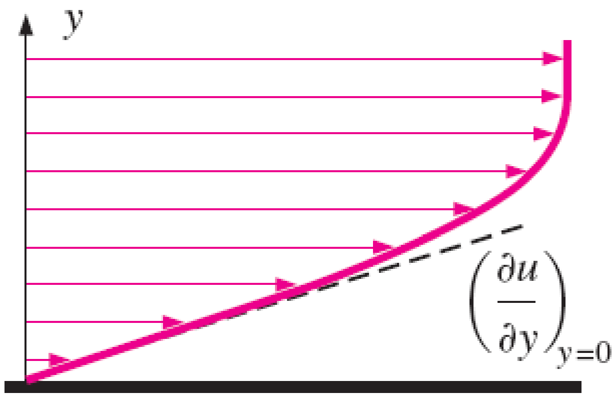
$$\nu_t = \mu_t/\rho$$

kinematic eddy viscosity or **kinematic turbulent viscosity** (also called the ***eddy diffusivity of momentum***).

[redacted]

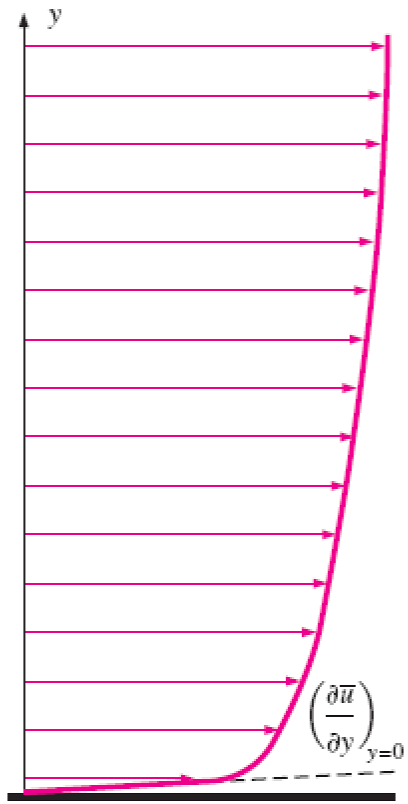
$$\tau_{\text{turb}} = \mu_t \frac{\partial \overline{u}}{\partial y} = \rho l_m^2 \left(\frac{\partial \overline{u}}{\partial y} \right)^2$$

Mixing length l_m: related to the average size of the eddies that are primarily responsible for mixing



Laminar flow

The velocity gradients at the wall, and thus the wall shear stress, are much larger for turbulent flow than they are for laminar flow, even though the turbulent boundary layer is thicker than the laminar one for the same value of free-stream velocity.



Turbulent flow

Molecular diffusivity of momentum ν is a fluid property.

Eddy diffusivity ν_t is *not* a fluid property, and its value depends on flow conditions.

Eddy diffusivity $\boldsymbol{\mu_t}$ decreases toward the wall, becoming zero at the wall.

Turbulent Velocity Profile

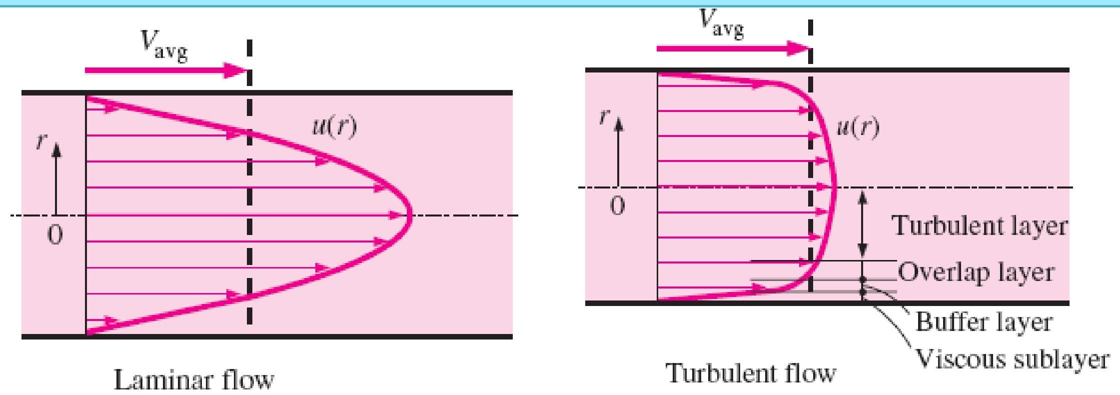
The very thin layer next to the wall where viscous effects are dominant is the viscous (or laminar or linear or wall) sublayer.

The velocity profile in this layer is very nearly *linear*, and the flow is streamlined.

Next to the viscous sublayer is the buffer layer, in which turbulent effects are becoming significant, but the flow is still dominated by viscous effects.

Above the buffer layer is the overlap (or transition) layer, also called the inertial sublayer, in which the turbulent effects are much more significant, but still not dominant.

Above that is the outer (or turbulent) layer in the remaining part of the flow in which turbulent effects dominate over molecular diffusion (viscous) effects.



The velocity profile in fully developed pipe flow is parabolic in laminar flow, but much fuller in turbulent flow. Note that $u(r)$ in the turbulent case is the *time-averaged* velocity component in the axial direction (the overbar on u has been dropped for simplicity).

[redacted]

$$\tau_w = \mu \frac{u}{y} = \rho\nu \frac{u}{y} \qquad \text{or} \qquad \frac{\tau_w}{\rho} = \frac{\nu u}{y}$$

$$u_* = \sqrt{\tau_w/\rho}$$

friction velocity

law of the wall

$$\textit{Viscous sublayer:} \qquad \frac{u}{u_*} = \frac{yu_*}{\nu}$$

Eq. 8.42

$$\textit{Thickness of viscous sublayer:} \qquad y = \delta_{\text{sublayer}} = \frac{5\nu}{u_*} = \frac{25\nu}{u_\delta}$$

The thickness of the viscous sublayer is proportional to the kinematic viscosity and inversely proportional to the average flow velocity.

$$\nu/u_*$$

Viscous length; it is used to nondimensionalize the distance *y* from the surface.

$$\textit{Nondimensionalized variables:} \qquad y^+ = \frac{yu_*}{\nu} \qquad \text{and} \qquad u^+ = \frac{u}{u_*}$$

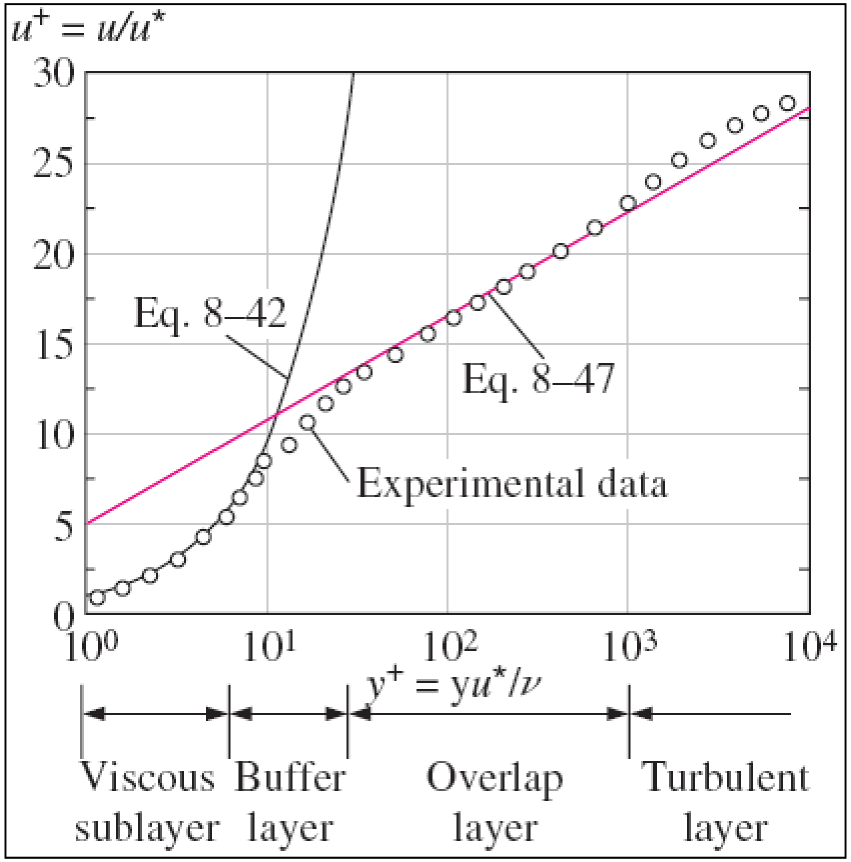
$$\textit{Normalized law of the wall:} \qquad u^+ = y^+$$

[redacted]

The logarithmic law: $$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y u_*}{\nu} + B$$

Overlap layer: $$\frac{u}{u_*} = 2.5 \ln \frac{y u_*}{\nu} + 5.0 \quad \text{or} \quad u^+ = 2.5 \ln y^+ + 5.0$$

Eq. 8.47



Comparison of the law of the wall and the logarithmic-law velocity profiles with experimental data for fully developed turbulent flow in a pipe.

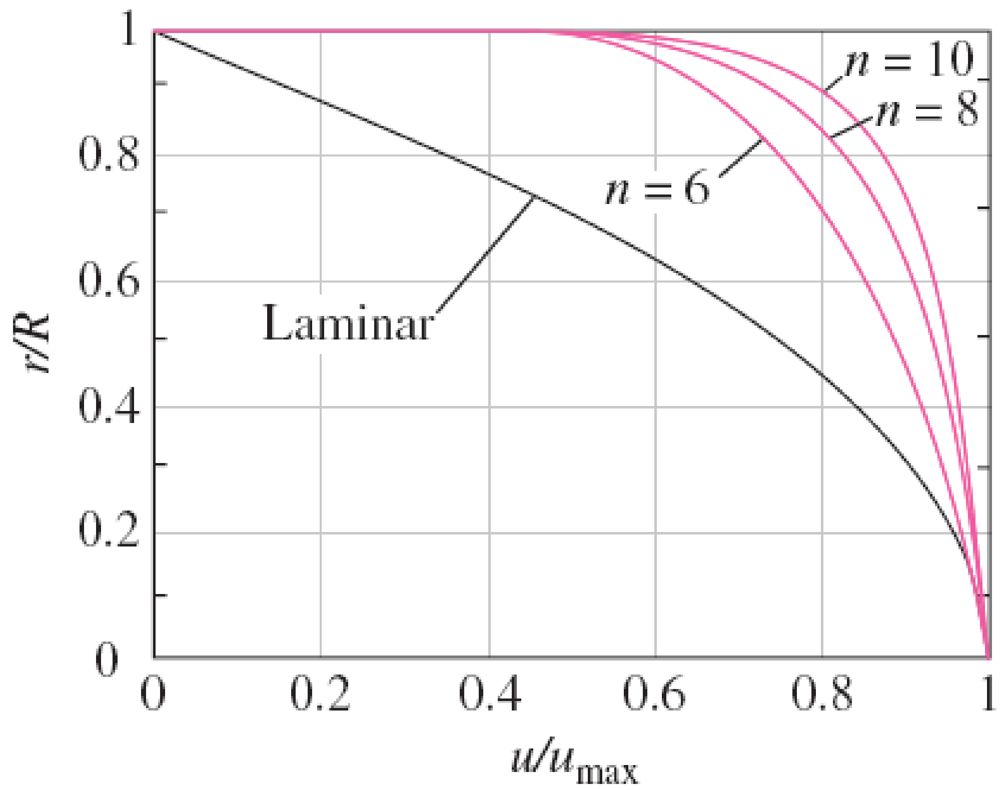
[redacted]

Outer turbulent layer: $$\frac{u_{max} - u}{u_*} = 2.5 \ln \frac{R}{R - r}$$

Velocity defect law

The deviation of velocity from the centerline value u_{max}-u is called the **velocity defect.**

Power-law velocity profile: $$\frac{u}{u_{max}} = \left(\frac{y}{R}\right)^{1/n} \quad \text{or} \quad \frac{u}{u_{max}} = \left(1 - \frac{r}{R}\right)^{1/n}$$



The value n = 7 generally approximates many flows in practice, giving rise to the term ***one-seventh power-law velocity profile*.**

Power-law velocity profiles for fully developed turbulent flow in a pipe for different exponents, and its comparison with the laminar velocity profile.

The Moody Chart and the Colebrook Equation

Colebrook equation (for smooth and rough pipes)

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right) \qquad \text{(turbulent flow)}$$

The friction factor in fully developed turbulent pipe flow depends on the Reynolds number and the relative roughness ε/D.

Relative Roughness, ε/D	Friction Factor, f
0.0*	0.0119
0.00001	0.0119
0.0001	0.0134
0.0005	0.0172
0.001	0.0199
0.005	0.0305
0.01	0.0380
0.05	0.0716

The friction factor is minimum for a smooth pipe and increases with roughness.

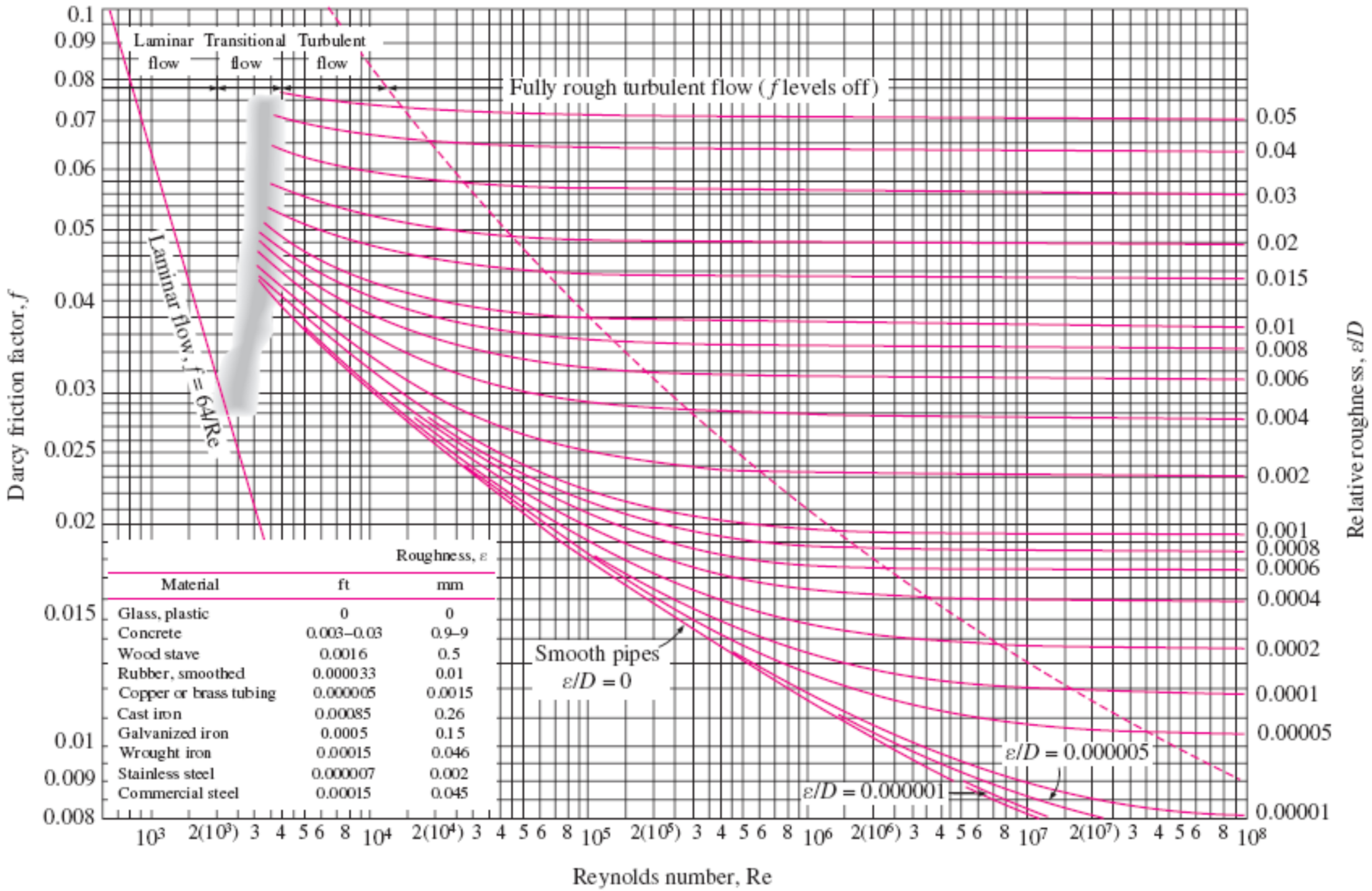
Explicit Haaland equation

$$\frac{1}{\sqrt{f}} \cong -1.8 \log\left[\frac{6.9}{\mathrm{Re}} + \left(\frac{\varepsilon/D}{3.7}\right)^{1.11}\right]$$

Equivalent roughness values for new commercial pipes*

Material	Roughness, ε ft	Roughness, ε mm
Glass, plastic	0 (smooth)	
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

The Moody Chart



Material	Roughness, ε: ft	Roughness, ε: mm
Glass, plastic	0	0
Concrete	0.003–0.03	0.9–9
Wood stave	0.0016	0.5
Rubber, smoothed	0.000033	0.01
Copper or brass tubing	0.000005	0.0015
Cast iron	0.00085	0.26
Galvanized iron	0.0005	0.15
Wrought iron	0.00015	0.046
Stainless steel	0.000007	0.002
Commercial steel	0.00015	0.045

Observations from the Moody chart

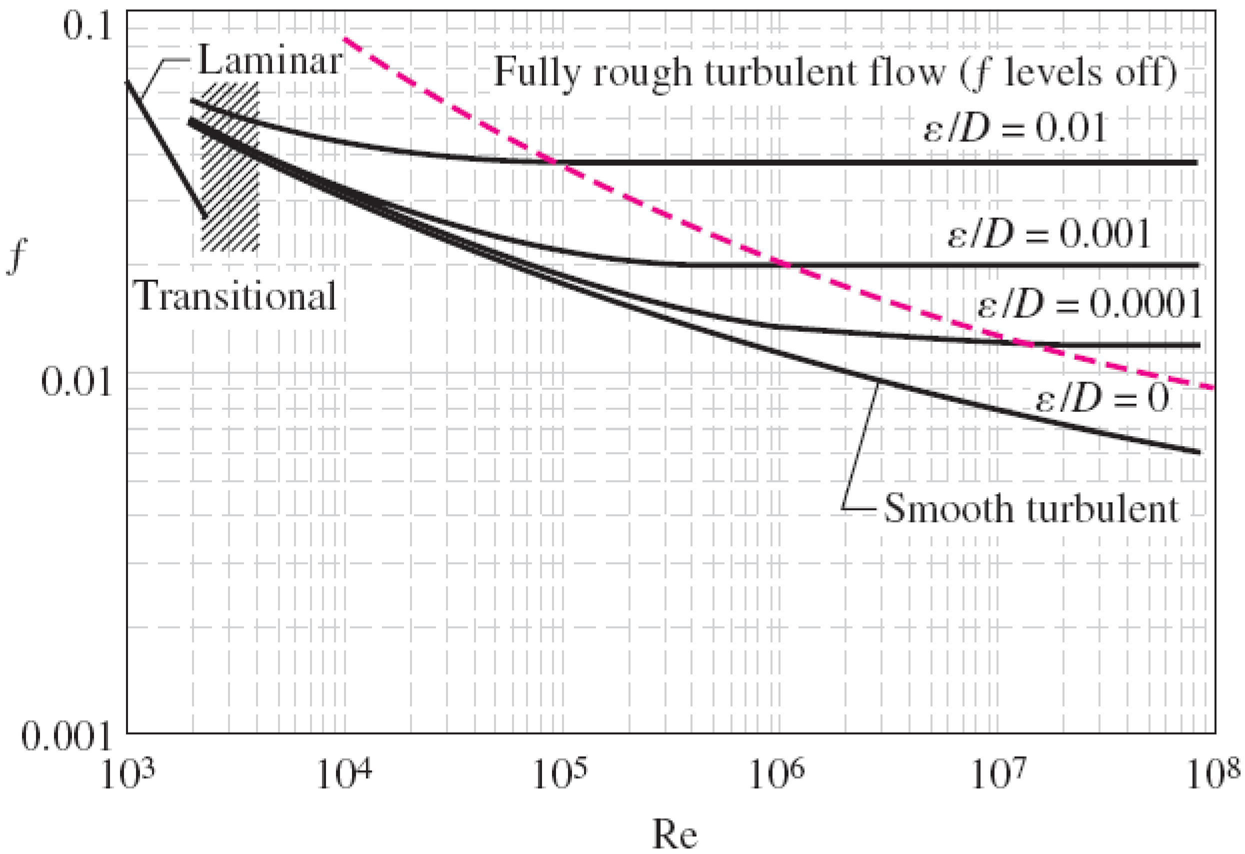
- For laminar flow, the friction factor decreases with increasing Reynolds number, and it is independent of surface roughness.
- The friction factor is a minimum for a smooth pipe and increases with roughness. The Colebrook equation in this case ($\varepsilon = 0$) reduces to the Prandtl equation.

$$1/\sqrt{f}=2.0 \log (\mathrm{Re}\sqrt{f}) - 0.8$$

- The transition region from the laminar to turbulent regime is indicated by the shaded area in the Moody chart. At small relative roughnesses, the friction factor increases in the transition region and approaches the value for smooth pipes.
- At very large Reynolds numbers (to the right of the dashed line on the Moody chart) the friction factor curves corresponding to specified relative roughness curves are nearly horizontal, and thus the friction factors are independent of the Reynolds number. The flow in that region is called ***fully rough turbulent flow*** or just *fully rough flow* because the thickness of the viscous sublayer decreases with increasing Reynolds number, and it becomes so thin that it is negligibly small compared to the surface roughness height. The Colebrook equation in the *fully rough* zone reduces to the von Kármán equation.

$$1/\sqrt{f}=-2.0 \log \left[\left(\frac{\varepsilon}{D}\right)/3.7\right]$$

[redacted]



Standard sizes for Schedule 40 steel pipes

Nominal Size, in	Actual Inside Diameter, in
$\frac{1}{8}$	0.269
$\frac{1}{4}$	0.364
$\frac{3}{8}$	0.493
$\frac{1}{2}$	0.622
$\frac{3}{4}$	0.824
1	1.049
$1\frac{1}{2}$	1.610
2	2.067
$2\frac{1}{2}$	2.469
3	3.068
5	5.047
10	10.02

At very large Reynolds numbers, the friction factor curves on the Moody chart are nearly horizontal, and thus the friction factors are independent of the Reynolds number. See Fig. A–12 for a full-page moody chart.

In calculations, we should make sure that we use the actual internal diameter of the pipe, which may be different than the nominal diameter.

Types of Fluid Flow Problems

1. Determining the **pressure drop** (or head loss) when the pipe length and diameter are given for a specified flow rate (or velocity)
2. Determining the **flow rate** when the pipe length and diameter are given for a specified pressure drop (or head loss)
3. Determining the **pipe diameter** when the pipe length and flow rate are given for a specified pressure drop (or head loss)

Problem type	Given	Find
1	$L, D, \dot{V}$	ΔP (or h_L)
2	$L, D, \Delta P$	$\dot{V}$
3	$L, \Delta P, \dot{V}$	D

The three types of problems encountered in pipe flow.

To avoid tedious iterations in head loss, flow rate, and diameter calculations, these explicit relations that are accurate to within 2 percent of the Moody chart may be used.

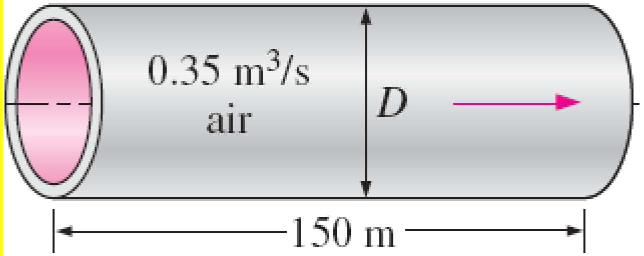
$$h_L = 1.07\frac{\dot{V}^2 L}{gD^5}\left\{\ln\left[\frac{\varepsilon}{3.7D} + 4.62\left(\frac{\nu D}{\dot{V}}\right)^{0.9}\right]\right\}^{-2} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 3000 < \text{Re} < 3\times 10^{8} \end{matrix}$$

$$\dot{V} = -0.965\left(\frac{gD^5 h_L}{L}\right)^{0.5}\ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17\nu^2 L}{gD^3 h_L}\right)^{0.5}\right] \qquad \text{Re} > 2000$$

$$D = 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^2}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \qquad \begin{matrix} 10^{-6} < \varepsilon/D < 10^{-2} \\ 5000 < \text{Re} < 3\times 10^{8} \end{matrix}$$

EXAMPLE 8–4

Heated air at 1 atm and 35°C is to be transported in a 150-m-long circular plastic duct at a rate of 0.35 m^3/s. If the head loss in the pipe is not to exceed 20 m, determine the minimum diameter of the duct.



Assumptions **1** The flow is steady and incompressible.
2 The flow is fully developed.
3 The duct involves no components such as bends, valves, and connectors.
4 Air is an ideal gas.
5 The duct is smooth since it is made of plastic.
6 The flow is turbulent (to be verified).

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.35 \text{ m}^3/\text{s}}{\pi D^2/4}$$

$$\text{Re} = \frac{VD}{\nu} = \frac{VD}{1.655 \times 10^{-5} \text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g}$$

$$20 = f\frac{150 \text{ m}}{D}\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

$$D = \mathbf{0.267 \text{ m}}$$

$$V = 6.24 \text{ m/s}$$

$$f = 0.0180$$

$$\text{Re} = 100{,}800$$

Therefore, the diameter of the duct should be more than 26.7 cm if the head loss is not to exceed 20 m. Note that Re>4000, and thus the turbulent flow assumption is verified.

[redacted]

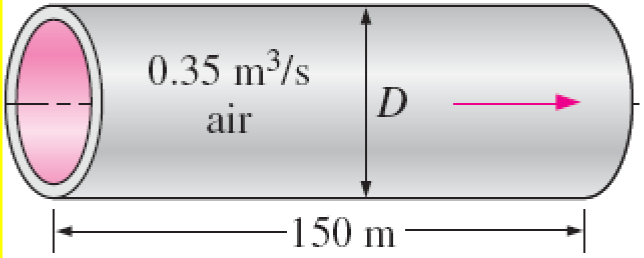
The diameter can also be determined directly from the third Swamee–Jain formula to be

$$
\begin{aligned}
D &= 0.66\left[\varepsilon^{1.25}\left(\frac{L\dot{V}^{2}}{gh_L}\right)^{4.75} + \nu\dot{V}^{9.4}\left(\frac{L}{gh_L}\right)^{5.2}\right]^{0.04} \\
&= 0.66\left[0 + (1.655 \times 10^{-5}\ \mathrm{m^2/s})(0.35\ \mathrm{m^3/s})^{9.4}\left(\frac{150\ \mathrm{m}}{(9.81\ \mathrm{m/s^2})(20\ \mathrm{m})}\right)^{5.2}\right]^{0.04} \\
&= \mathbf{0.271\ m}
\end{aligned}
$$

Note that the difference between the two results is less than 2%. Therefore, the simple Swamee–Jain relation can be used with confidence. Finally, the first (iterative) approach requires an initial guess for *D*. If we use the Swamee–Jain result as our initial guess, the diameter converges to *D*=0.267 m in short order.

EXAMPLE 8–5

Reconsider, Now the duct length is doubled while its diameter is maintained constant. If the total head loss is to remain constant, determine the drop in the flow rate through the duct.



$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi (0.267\text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.267\text{ m})}{1.655 \times 10^{-5}\text{ m}^2/\text{s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$h_L = f\frac{L}{D}\frac{V^2}{2g} \quad \rightarrow \quad 20 = f\frac{300\text{ m}}{0.267\text{ m}}\frac{V^2}{2(9.81\text{ m/s}^2)}$$

[redacted]

$\dot{V} = 0.24 \text{ m}^3/\text{s}, \qquad f = 0.0195,$ $V = 4.23 \text{ m/s}, \qquad \text{and} \qquad \text{Re} = 68{,}300$

$$\dot{V}_{\text{drop}} = \dot{V}_{\text{old}} - \dot{V}_{\text{new}} = 0.35 - 0.24 = \mathbf{0.11 \ m^3/s}$$

(a drop of 31 percent)

Therefore, for a specified head loss (or available head or fan pumping power), the flow rate drops by about 31 percent from 0.35 to 0.24 m^3/s when the duct length doubles.

Alternative Solution **If a computer is not available (as in an exam situation), another option is to set up a *manual iteration loop*. We have found that the best convergence is usually realized by first guessing the friction factor *f*, and then solving for the velocity *V*. The equation for *V* as a function of *f* is**

Average velocity through the pipe: $$V = \sqrt{\frac{2gh_L}{fL/D}}$$

Iteration	f (guess)	V, m/s	Re	Corrected f
1	0.04	2.955	4.724×10^4	0.0212
2	0.0212	4.059	6.489×10^4	0.01973
3	0.01973	4.207	6.727×10^4	0.01957
4	0.01957	4.224	6.754×10^4	0.01956
5	0.01956	4.225	6.756×10^4	0.01956

The new flow rate can also be determined directly from the second Swamee–Jain formula to be

$$\dot{V} = -0.965\left(\frac{gD^5h_L}{L}\right)^{0.5} \ln\left[\frac{\varepsilon}{3.7D} + \left(\frac{3.17v^2L}{gD^3h_L}\right)^{0.5}\right]$$

$$= -0.965\left(\frac{(9.81 \text{ m/s}^2)(0.267 \text{ m})^5(20 \text{ m})}{300 \text{ m}}\right)^{0.5}$$

$$\times \ln\left[0 + \left(\frac{3.17(1.655 \times 10^{-5} \text{ m}^2/\text{s})^2(300 \text{ m})}{(9.81 \text{ m/s}^2)(0.267 \text{ m})^3(20 \text{ m})}\right)^{0.5}\right]$$

$$= 0.24 \text{ m}^3/\text{s}$$

Note that the result from the Swamee–Jain relation is the same (to two significant digits) as that obtained with the Colebrook equation using EES or using our manual iteration technique. Therefore, the simple Swamee–Jain relation can be used with confidence.

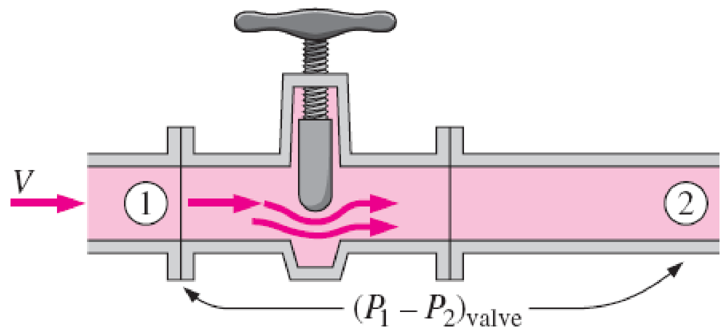
The fluid in a typical piping system passes through various fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions in addition to the pipes.

These components interrupt the smooth flow of the fluid and cause additional losses because of the flow separation and mixing they induce.

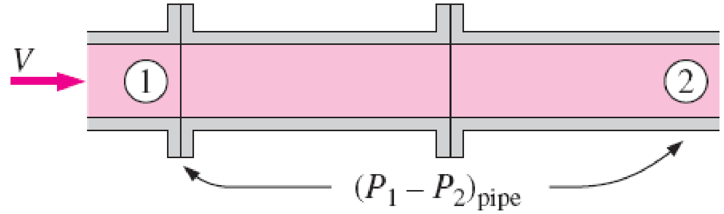
In a typical system with long pipes, these losses are minor compared to the total head loss in the pipes (the major losses) and are called minor losses.

Minor losses are usually expressed in terms of the loss coefficient K_L.

Pipe section with valve:



Pipe section without valve:



$$\Delta P_L = (P_1 - P_2)_{\text{valve}} - (P_1 - P_2)_{\text{pipe}}$$

Head loss due to component

$$h_L = \Delta P_L / \rho g$$

$$\boxed{K_L = \frac{h_L}{V^2/(2g)}}$$

For a constant-diameter section of a pipe with a minor loss component, the loss coefficient of the component (such as the gate valve shown) is determined by measuring the additional pressure loss it causes and dividing it by the dynamic pressure in the pipe.

[redacted]

When the inlet diameter equals outlet diameter, the loss coefficient of a component can also be determined by measuring the pressure loss across the component and dividing it by the dynamic pressure:

$$K_L = \Delta P_L / (\rho V^2/2)$$

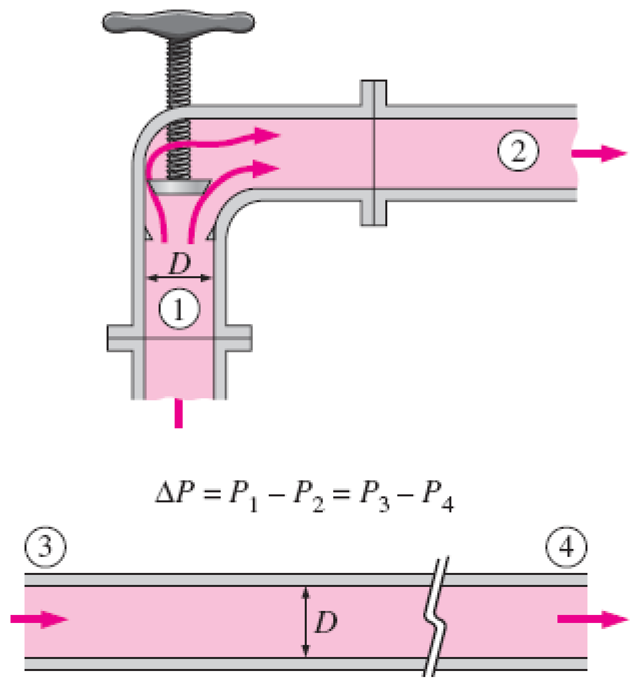
When the loss coefficient for a component is available, the head loss for that component is

$$h_L = K_L \frac{V^2}{2g}$$

Minor loss

Minor losses are also expressed in terms of the **equivalent length L_{equiv}**.

$$h_L = K_L \frac{V^2}{2g} = f \frac{L_{\text{equiv}}}{D} \frac{V^2}{2g} \quad \rightarrow \quad L_{\text{equiv}} = \frac{D}{f} K_L$$



The head loss caused by a component (such as the angle valve shown) is equivalent to the head loss caused by a section of the pipe whose length is the equivalent length.

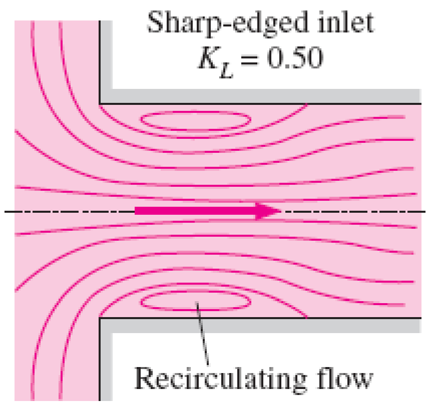
[redacted]

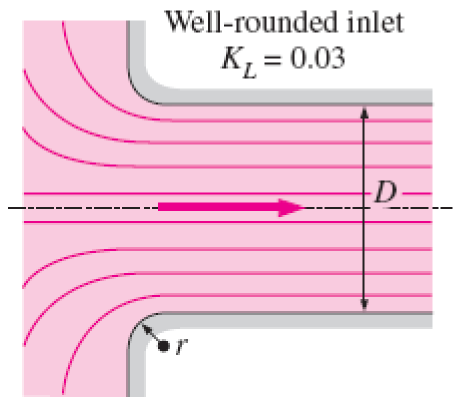
Total head loss (general)

$$h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}}$$
$$= \sum_i f_i \frac{L_i}{D_i}\frac{V_i^2}{2g} + \sum_j K_{L,\,j}\frac{V_j^2}{2g}$$

Total head loss (D = constant)

$$h_{L,\,\text{total}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$



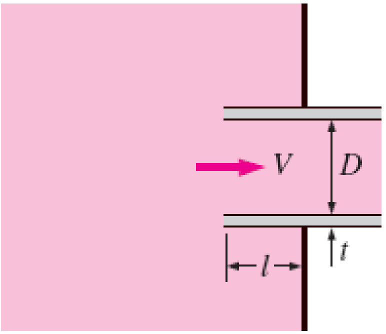


The head loss at the inlet of a pipe is almost negligible for well-rounded inlets (K_L = 0.03 for r/D > 0.2) but increases to about 0.50 for sharp-edged inlets.

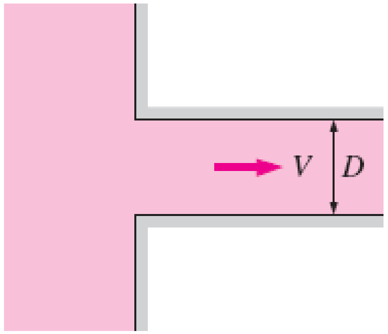
Loss coefficients K_L of various pipe components for turbulent flow (for use in the relation $h_L = K_L V^2/(2g)$, where V is the average velocity in the pipe that contains the component)*

Pipe Inlet

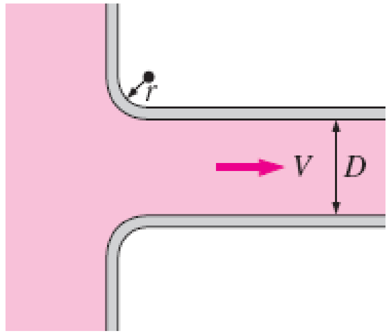
Reentrant: $K_L = 0.80$
($t \ll D$ and $l \approx 0.1D$)



Sharp-edged: $K_L = 0.50$

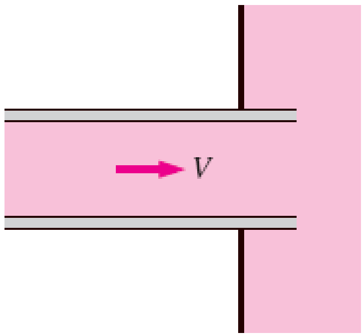


Well-rounded ($r/D > 0.2$): $K_L = 0.03$
Slightly rounded ($r/D = 0.1$): $K_L = 0.12$
(see Fig. 8–36)

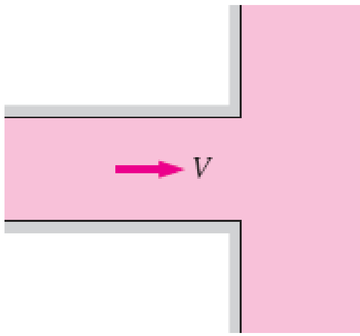


Pipe Exit

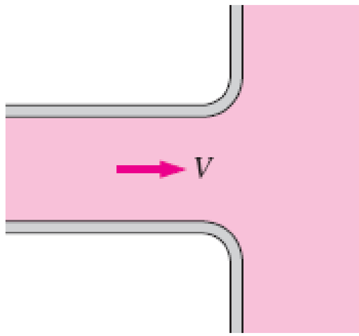
Reentrant: $K_L = \alpha$



Sharp-edged: $K_L = \alpha$



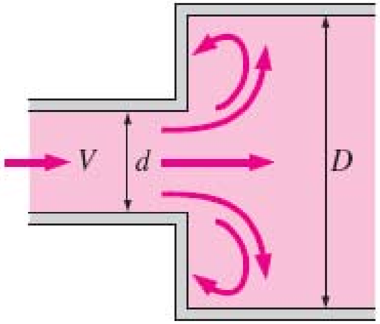
Rounded: $K_L = \alpha$



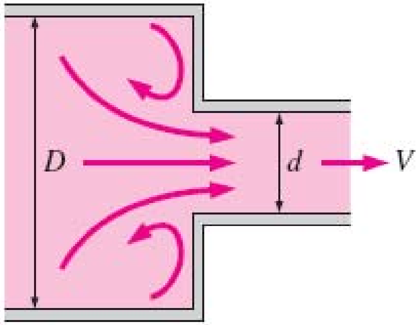
Note: The kinetic energy correction factor is $\alpha = 2$ for fully developed laminar flow, and $\alpha \approx 1.05$ for fully developed turbulent flow.

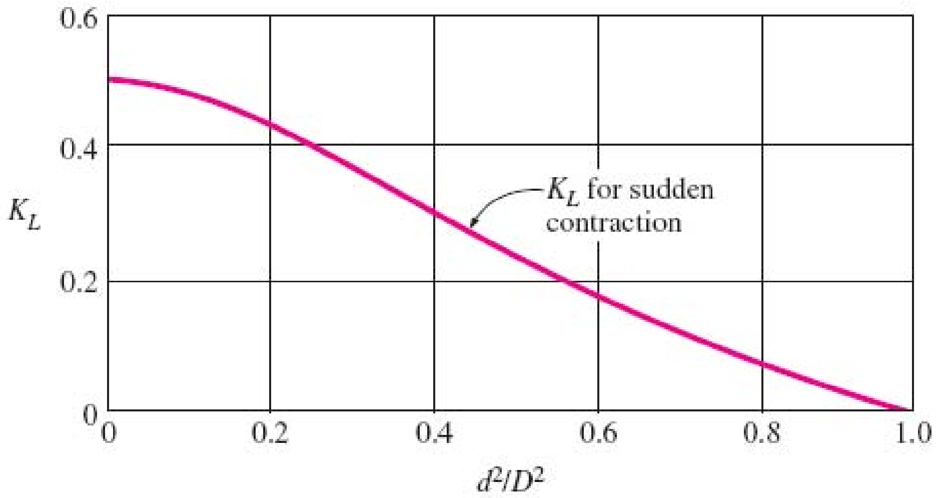
Sudden Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

Sudden expansion: $K_L = \alpha\left(1 - \frac{d^2}{D^2}\right)^2$



Sudden contraction: See chart.





Gradual Expansion and Contraction (based on the velocity in the smaller-diameter pipe)

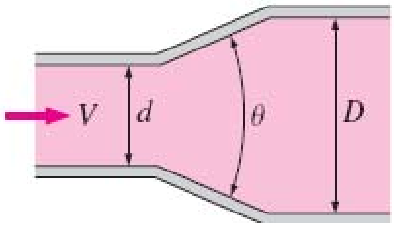
Expansion (for $\theta = 20°$):

$K_L = 0.30$ for $d/D = 0.2$

$K_L = 0.25$ for $d/D = 0.4$

$K_L = 0.15$ for $d/D = 0.6$

$K_L = 0.10$ for $d/D = 0.8$

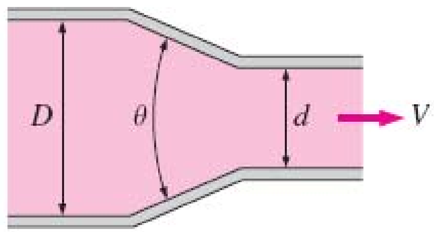


Contraction:

$K_L = 0.02$ for $\theta = 30°$

$K_L = 0.04$ for $\theta = 45°$

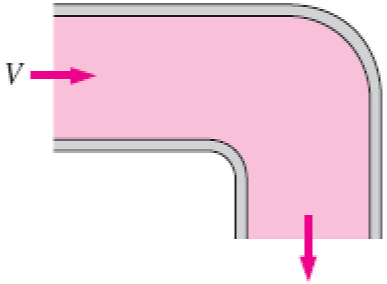
$K_L = 0.07$ for $\theta = 60°$



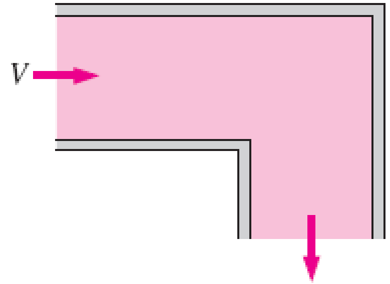
[redacted]

Bends and Branches

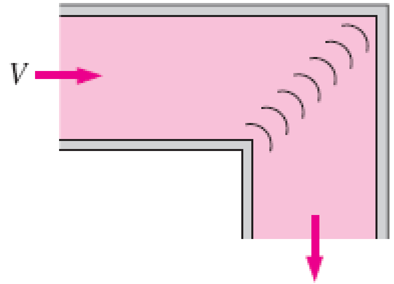
90° smooth bend:
Flanged: $K_L = 0.3$
Threaded: $K_L = 0.9$



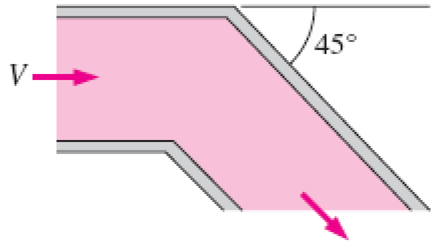
90° miter bend
(without vanes): $K_L = 1.1$



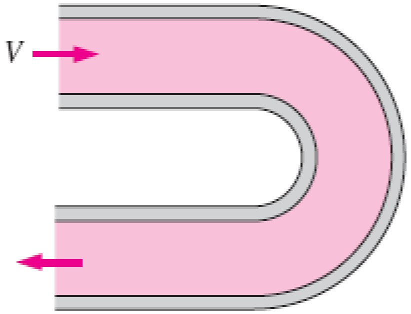
90° miter bend
(with vanes): $K_L = 0.2$



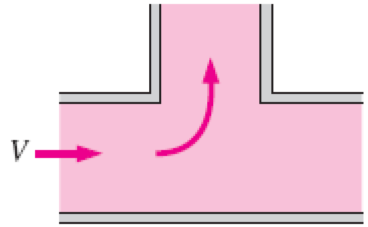
45° threaded elbow:
$K_L = 0.4$



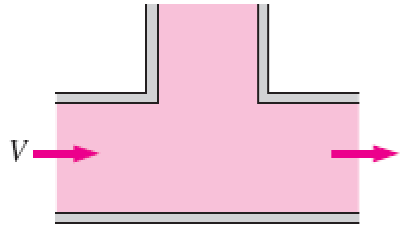
180° return bend:
Flanged: $K_L = 0.2$
Threaded: $K_L = 1.5$



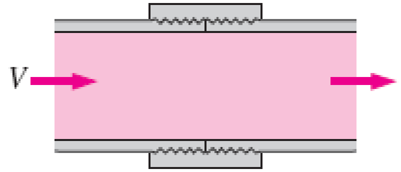
Tee (branch flow):
Flanged: $K_L = 1.0$
Threaded: $K_L = 2.0$



Tee (line flow):
Flanged: $K_L = 0.2$
Threaded: $K_L = 0.9$



Threaded union:
$K_L = 0.08$

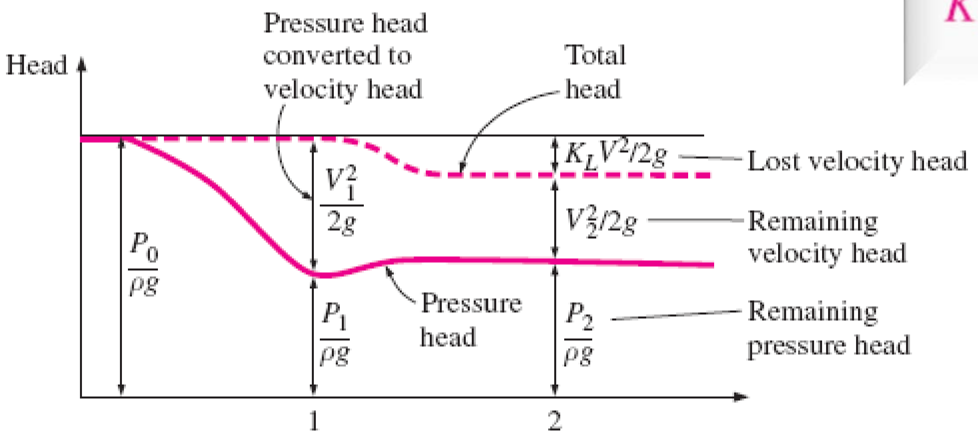


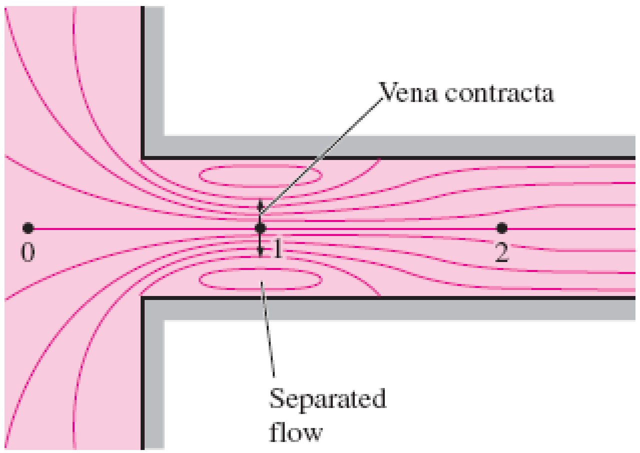
Valves

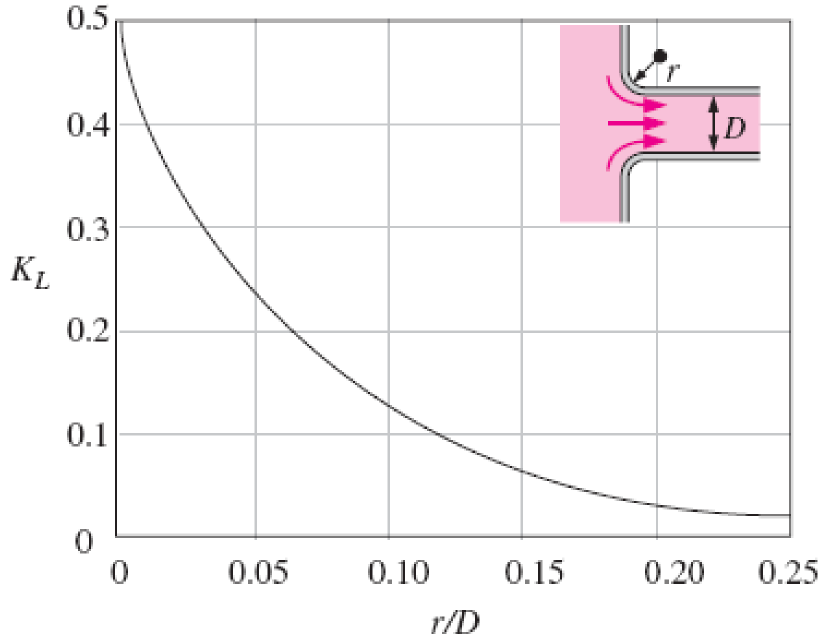
Globe valve, fully open: $K_L = 10$
Angle valve, fully open: $K_L = 5$
Ball valve, fully open: $K_L = 0.05$
Swing check valve: $K_L = 2$

Gate valve, fully open: $K_L = 0.2$
$\frac{1}{4}$ closed: $K_L = 0.3$
$\frac{1}{2}$ closed: $K_L = 2.1$
$\frac{3}{4}$ closed: $K_L = 17$

$$K_L = \alpha\left(1 - \frac{A_{\text{small}}}{A_{\text{large}}}\right)^2 \qquad \text{(sudden expansion)}$$



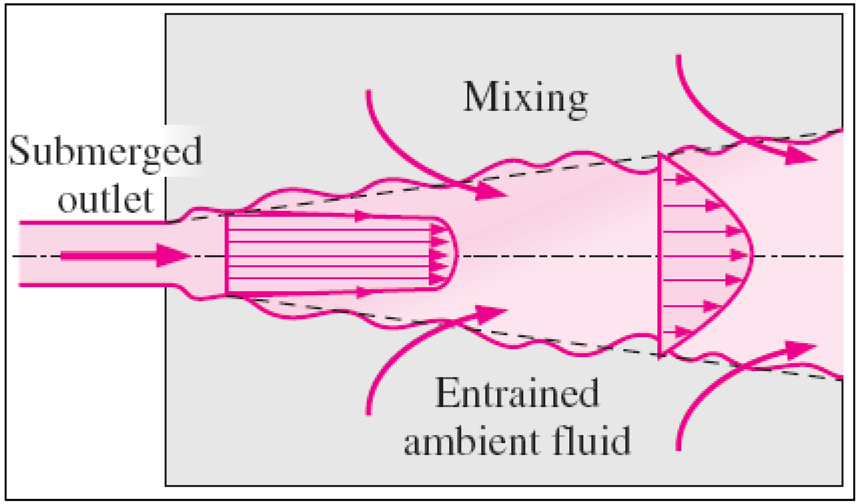




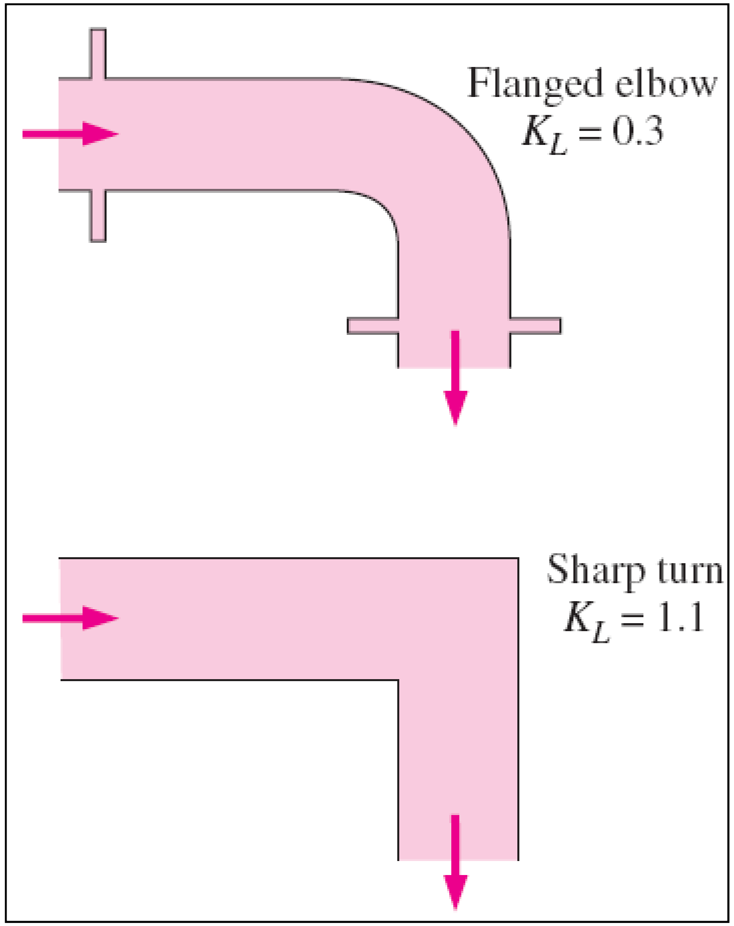
Graphical representation of flow contraction and the associated head loss at a sharp-edged pipe inlet.

The effect of rounding of a pipe inlet on the loss coefficient.

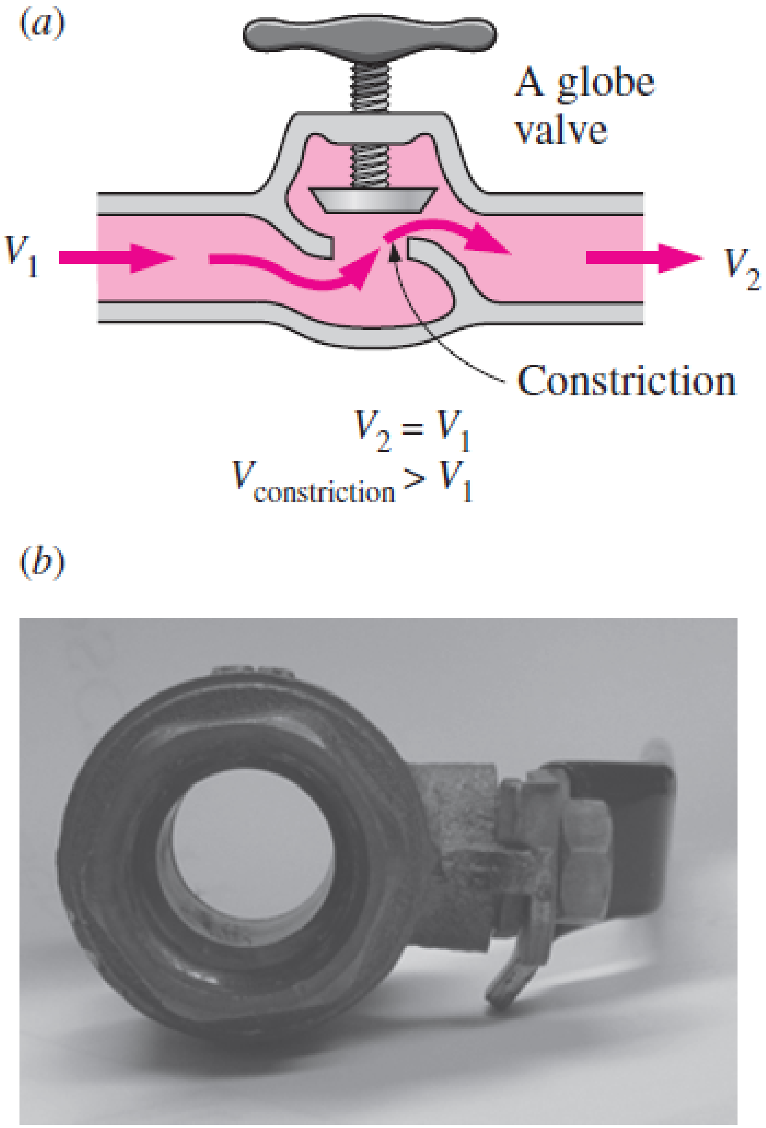
[redacted]



All the kinetic energy of the flow is "lost" (turned into thermal energy) through friction as the jet decelerates and mixes with ambient fluid downstream of a submerged outlet.



The losses during changes of direction can be minimized by making the turn "easy" on the fluid by using circular arcs instead of sharp turns.

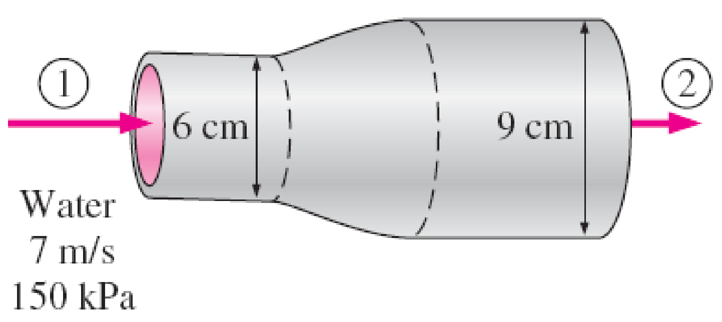


The large head loss in a partially closed valve is due to irreversible deceleration, flow separation, and mixing of high-velocity fluid coming from the narrow valve passage.

The head loss through a fully-open ball valve, on the other hand, is quite small.

EXAMPLE 8–6

A 6-cm-diameter horizontal water pipe expands gradually to a 9-cm-diameter pipe. The walls of the expansion section are angled 10° from the axis. The average velocity and pressure of water before the expansion section are 7 m/s and 150 kPa, respectively. Determine the head loss in the expansion section and the pressure in the larger-diameter pipe.



Assumptions **1** The flow is steady and incompressible. **2** The flow at sections 1 and 2 is fully developed and turbulent with $\alpha_1 = \alpha_2 = 1.06$.

The loss coefficient for gradual expansion of μ= 60° total included angle is K_L =0.133.

$$\dot{m}_1 = \dot{m}_2 \quad \rightarrow \quad \rho V_1 A_1 = \rho V_2 A_2 \quad \rightarrow \quad V_2 = \frac{A_1}{A_2} V_1 = \frac{D_1^2}{D_2^2} V_1$$

$$V_2 = \frac{(0.06 \text{ m})^2}{(0.09 \text{ m})^2}(7 \text{ m/s}) = 3.11 \text{ m/s}$$

$$h_L = K_L \frac{V_1^2}{2g} = (0.133)\frac{(7 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = \mathbf{0.333 \text{ m}}$$

[redacted]

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + \cancel{z_1} + \cancelto{0}{h_{\text{pump}, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + \cancel{z_2} + \cancelto{0}{h_{\text{turbine}, e}} + h_L$$

$$\rightarrow \frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + h_L$$

$$P_2 = P_1 + \rho\left\{\frac{\alpha_1 V_1^2 - \alpha_2 V_2^2}{2} - g h_L\right\} = (150 \text{ kPa}) + (1000 \text{ kg/m}^3)$$

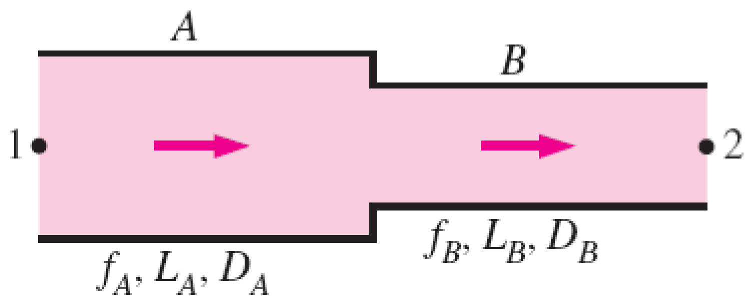
$$\times \left\{\frac{1.06(7 \text{ m/s})^2 - 1.06(3.11 \text{ m/s})^2}{2} - (9.81 \text{ m/s}^2)(0.333 \text{ m})\right\}$$

$$\times \left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1 \text{ kN/m}^2}\right)$$

$$= \mathbf{168 \text{ kPa}}$$

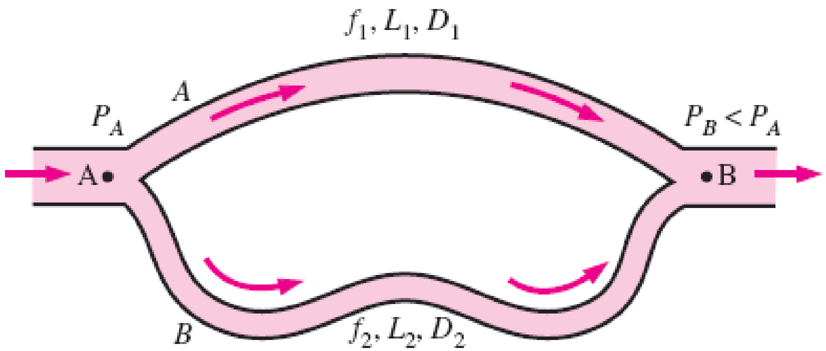
Higher pressure upstream is necessary to cause flow, and it may come as a surprise to you that the downstream pressure has *increased* after the expansion, despite the loss. This is because the flow is driven by the sum of the three heads that comprise the total head. During flow expansion, the higher velocity head upstream is converted to pressure head downstream, and this increase outweighs the nonrecoverable head loss. Also, you may be tempted to solve this problem using the Bernoulli equation. Such a solution would ignore the head loss and result in an incorrect higher pressure for the fluid downstream.

A piping network in an industrial facility.



$$\dot{V}_A = \dot{V}_B$$
$$h_{L,\,1\text{-}2} = h_{L,\,A} + h_{L,\,B}$$

For pipes *in series*, the flow rate is the same in each pipe, and the total head loss is the sum of the head losses in individual pipes.



$$h_{L,\,1} = h_{L,\,2}$$
$$\dot{V}_A = \dot{V}_1 + \dot{V}_2 = \dot{V}_B$$

For pipes *in parallel*, the head loss is the same in each pipe, and the total flow rate is the sum of the flow rates in individual pipes.

[redacted]

The relative flow rates in parallel pipes are established from the requirement that the head loss in each pipe be the same.

$$h_{L,1} = h_{L,2} \quad \rightarrow \quad f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g}$$

$$\frac{V_1}{V_2} = \left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2} \quad \text{and} \quad \frac{\dot{V}_1}{\dot{V}_2} = \frac{A_{c,1}V_1}{A_{c,2}V_2} = \frac{D_1^2}{D_2^2}\left(\frac{f_2}{f_1}\frac{L_2}{L_1}\frac{D_1}{D_2}\right)^{1/2}$$

The flow rate in one of the parallel branches is proportional to its diameter to the power 5/2 and is inversely proportional to the square root of its length and friction factor.

The analysis of piping networks is based on two simple principles:

1. *Conservation of mass throughout the system must be satisfied*. This is done by requiring the total flow into a junction to be equal to the total flow out of the junction for all junctions in the system.
2. *Pressure drop (and thus head loss) between two junctions must be the same for all paths between the two junctions*. This is because pressure is a point function and it cannot have two values at a specified point. In practice this rule is used by requiring that the algebraic sum of head losses in a loop (for all loops) be equal to zero.

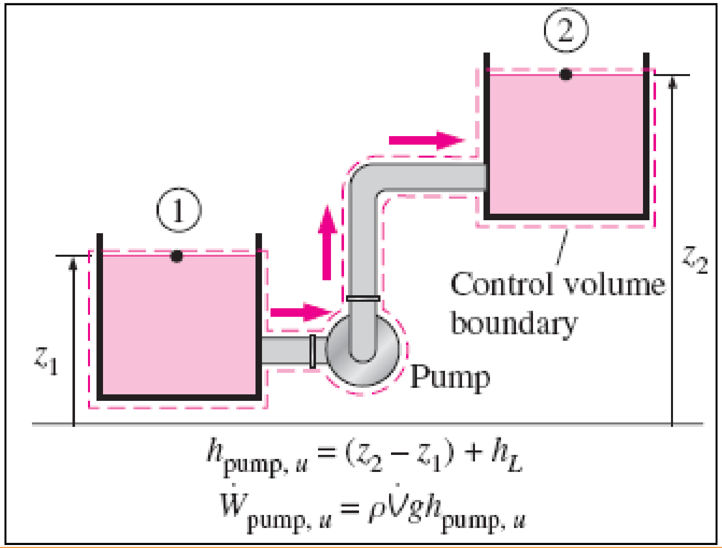
Piping Systems with Pumps and Turbines

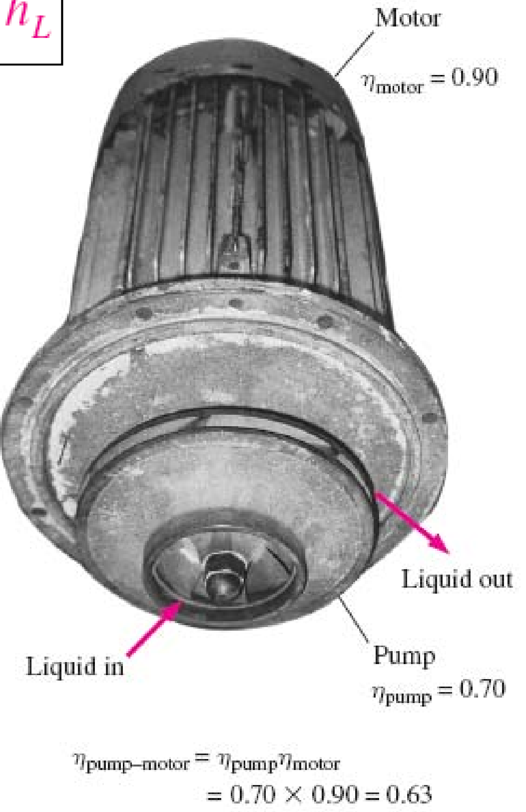
$$\frac{P_1}{\rho} + \alpha_1 \frac{V_1^2}{2} + gz_1 + w_{\text{pump}, u} = \frac{P_2}{\rho} + \alpha_2 \frac{V_2^2}{2} + gz_2 + w_{\text{turbine}, e} + gh_L$$

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

$$h_{\text{pump}, u} = w_{\text{pump}, u}/g \qquad h_{\text{pump}, u} = (z_2 - z_1) + h_L$$

$$\dot{W}_{\text{pump, shaft}} = \frac{\rho \dot{V} g h_{\text{pump}, u}}{\eta_{\text{pump}}} \quad \text{and} \quad \dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump}, u}}{\eta_{\text{pump–motor}}}$$

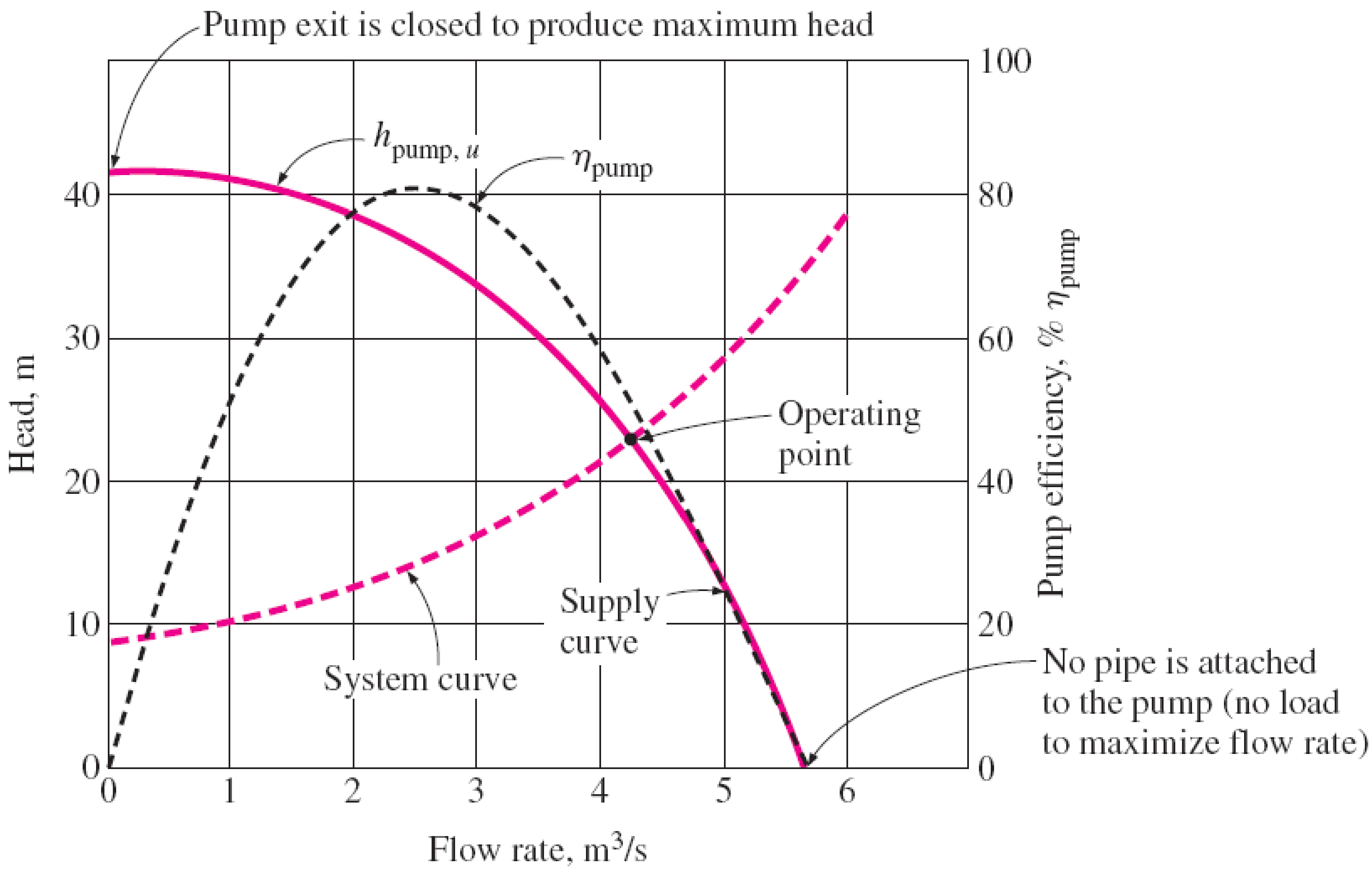




When a pump moves a fluid from one reservoir to another, the useful pump head requirement is equal to the elevation difference between the two reservoirs plus the head loss.

The efficiency of the pump–motor combination is the product of the pump and the motor efficiencies.

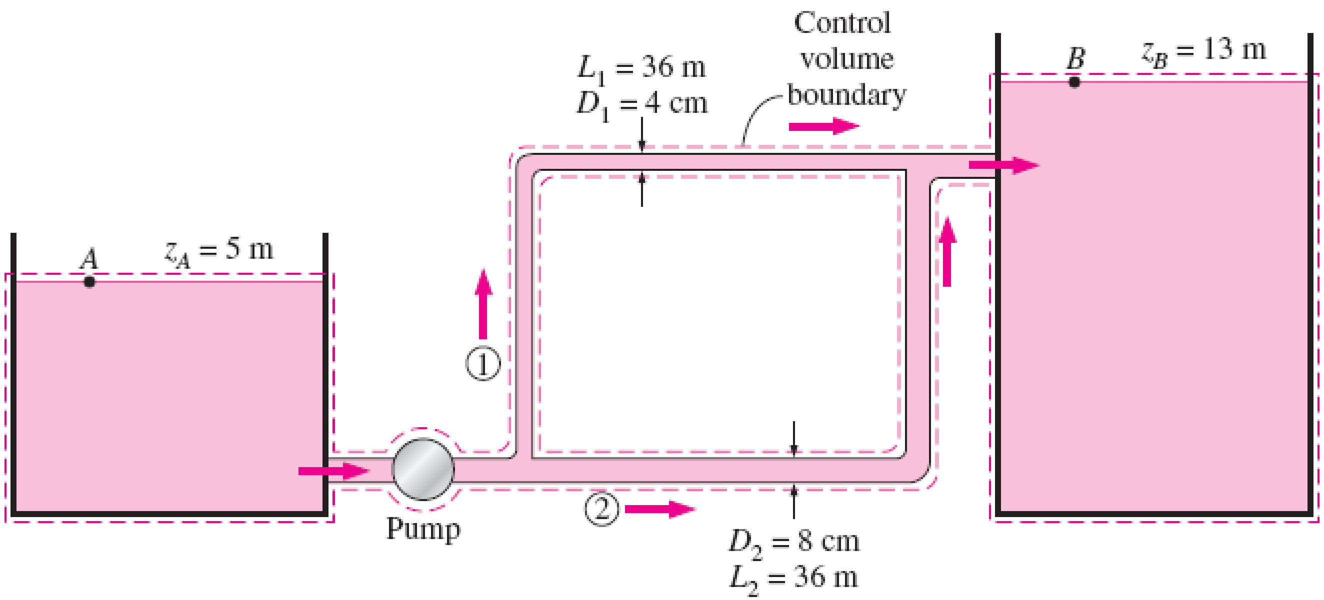
[redacted]



Characteristic pump curves for centrifugal pumps, the system curve for a piping system, and the operating point.

EXAMPLE 8-7

Water at 20°C is to be pumped from a reservoir (z_A =5 m) to another reservoir at a higher elevation (z_B =13 m) through two 36-m-long pipes connected in parallel. The pipes are made of commercial steel, and the diameters of the two pipes are 4 and 8 cm. Water is to be pumped by a 70% efficient motor–pump combination that draws 8 kW of electric power during operation. The minor losses and the head loss in pipes that connect the parallel pipes to the two reservoirs are considered to be negligible. Determine the total flow rate between the reservoirs and the flow rate through each of the parallel pipes.



[redacted]

Assumptions **1** The flow is steady and incompressible. **2** The entrance effects are negligible, and thus the flow is fully developed. **3** The elevations of the reservoirs remain constant. **4** The minor losses and the head loss in pipes other than the parallel pipes are said to be negligible. **5** Flows through both pipes are turbulent (to be verified).

$$\dot{W}_{\text{elect}} = \frac{\rho \dot{V} g h_{\text{pump},\,u}}{\eta_{\text{pump}-\text{motor}}} \quad \rightarrow \quad 8000 \text{ W} = \frac{(998 \text{ kg/m}^3)\dot{V}(9.81 \text{ m/s}^2)h_{\text{pump},\,u}}{0.70}$$

$$\cancel{\frac{P_A}{\rho g}} + \alpha_A \cancelto{0}{\frac{V_A^2}{2g}} + z_A + h_{\text{pump},\,u} = \cancel{\frac{P_B}{\rho g}} + \alpha_B \cancelto{0}{\frac{V_B^2}{2g}} + z_B + h_L \rightarrow h_{\text{pump},\,u}$$
$$= (z_B - z_A) + h_L$$

$$h_{\text{pump},\,u} = (13 - 5) + h_L \qquad h_L = h_{L,\,1} = h_{L,\,2}$$

$$V_1 = \frac{\dot{V}_1}{A_{c,\,1}} = \frac{\dot{V}_1}{\pi D_1^2/4} \quad \rightarrow \quad V_1 = \frac{\dot{V}_1}{\pi (0.04 \text{ m})^2/4}$$

$$V_2 = \frac{\dot{V}_2}{A_{c,\,2}} = \frac{\dot{V}_2}{\pi D_2^2/4} \quad \rightarrow \quad V_2 = \frac{\dot{V}_2}{\pi (0.08 \text{ m})^2/4}$$

[redacted]

$$\mathrm{Re}_1 = \frac{\rho V_1 D_1}{\mu} \quad \rightarrow \quad \mathrm{Re}_1 = \frac{(998\ \mathrm{kg/m^3})V_1(0.04\ \mathrm{m})}{1.002 \times 10^{-3}\ \mathrm{kg/m \cdot s}}$$

$$\mathrm{Re}_2 = \frac{\rho V_2 D_2}{\mu} \quad \rightarrow \quad \mathrm{Re}_2 = \frac{(998\ \mathrm{kg/m^3})V_2(0.08\ \mathrm{m})}{1.002 \times 10^{-3}\ \mathrm{kg/m \cdot s}}$$

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{\varepsilon/D_1}{3.7} + \frac{2.51}{\mathrm{Re}_1\sqrt{f_1}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.04} + \frac{2.51}{\mathrm{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{\varepsilon/D_2}{3.7} + \frac{2.51}{\mathrm{Re}_2\sqrt{f_2}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{0.000045}{3.7 \times 0.08} + \frac{2.51}{\mathrm{Re}_2\sqrt{f_2}}\right)$$

$$h_{L,1} = f_1 \frac{L_1}{D_1}\frac{V_1^2}{2g} \quad \rightarrow \quad h_{L,1} = f_1 \frac{36\ \mathrm{m}}{0.04\ \mathrm{m}}\frac{V_1^2}{2(9.81\ \mathrm{m/s^2})}$$

[redacted]

$$h_{L,2} = f_2 \frac{L_2}{D_2}\frac{V_2^2}{2g} \quad \rightarrow \quad h_{L,2} = f_2 \frac{36 \text{ m}}{0.08 \text{ m}} \frac{V_2^2}{2(9.81 \text{ m/s}^2)}$$

$$\dot{V} = \dot{V}_1 + \dot{V}_2$$

$$\dot{V} = \mathbf{0.0300\ m^3/s}, \quad \dot{V}_1 = \mathbf{0.00415\ m^3/s}, \quad \dot{V}_2 = \mathbf{0.0259\ m^3/s}$$

$$V_1 = 3.30 \text{ m/s}, \quad V_2 = 5.15 \text{ m/s}, \quad h_L = h_{L,1} = h_{L,2} = 11.1 \text{ m}, \quad h_{\text{pump}} = 19.1 \text{ m}$$

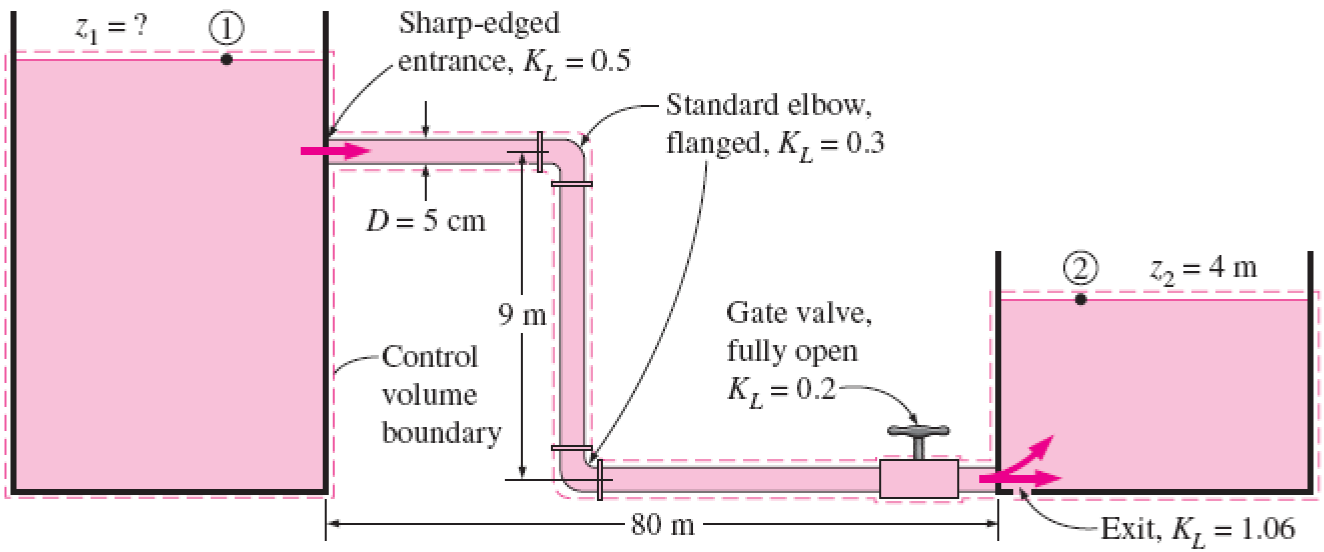
$$\text{Re}_1 = 131{,}600, \quad \text{Re}_2 = 410{,}000, \quad f_1 = 0.0221, \quad f_2 = 0.0182$$

Note that Re>4000 for both pipes, and thus the assumption of turbulent flow is verified.

The two parallel pipes are identical, except the diameter of the first pipe is half the diameter of the second one. But only 14% of the water flows through the first pipe. This shows the strong dependence of the flow rate (and the head loss) on diameter. Also, it can be shown that if the free surfaces of the two reservoirs were at the same elevation (and thus $z_A = z_B$), the flow rate would increase by 20 % from 0.0300 to 0.0361 m^3/s. Alternately, if the reservoirs were as given but the irreversible head losses were negligible, the flow rate would become 0.0715 m3/s (an increase of 138%).

EXAMPLE 8-8

Water at 10°C flows from a large reservoir to a smaller one through a 5-m diameter cast iron piping system. Determine the elevation $z1$ for a flow rate of 6 L/s.



Assumptions **1** The flow is steady and incompressible. **2** The elevations of the reservoirs remain constant. **3** There are no pumps or turbines in the line.

$$\cancel{\frac{P_1}{\rho g}} + \alpha_1 \frac{V_1^2}{2g}^{\nearrow 0} + z_1 = \cancel{\frac{P_2}{\rho g}} + \alpha_2 \frac{V_2^2}{2g}^{\nearrow 0} + z_2 + h_L$$

$$z_1 = z_2 + h_L$$

[redacted]

$$h_L = h_{L,\,\text{total}} = h_{L,\,\text{major}} + h_{L,\,\text{minor}} = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g}$$

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{0.006\ \text{m}^3/\text{s}}{\pi(0.05\ \text{m})^2/4} = 3.06\ \text{m/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(999.7\ \text{kg/m}^3)(3.06\ \text{m/s})(0.05\ \text{m})}{1.307 \times 10^{-3}\ \text{kg/m}\cdot\text{s}} = 117{,}000$$

$$\frac{1}{\sqrt{f}} = -2.0\log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right) \quad \rightarrow \quad \frac{1}{\sqrt{f}} = -2.0\log\left(\frac{0.0052}{3.7} + \frac{2.51}{117{,}000\sqrt{f}}\right)$$

It gives *f* =0.0315. The sum of the loss coefficients is

$$\sum K_L = K_{L,\,\text{entrance}} + 2K_{L,\,\text{elbow}} + K_{L,\,\text{valve}} + K_{L,\,\text{exit}}$$
$$= 0.5 + 2 \times 0.3 + 0.2 + 1.06 = 2.36$$

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} = \left(0.0315\frac{89\ \text{m}}{0.05\ \text{m}} + 2.36\right)\frac{(3.06\ \text{m/s})^2}{2(9.81\ \text{m/s}^2)} = 27.9\ \text{m}$$

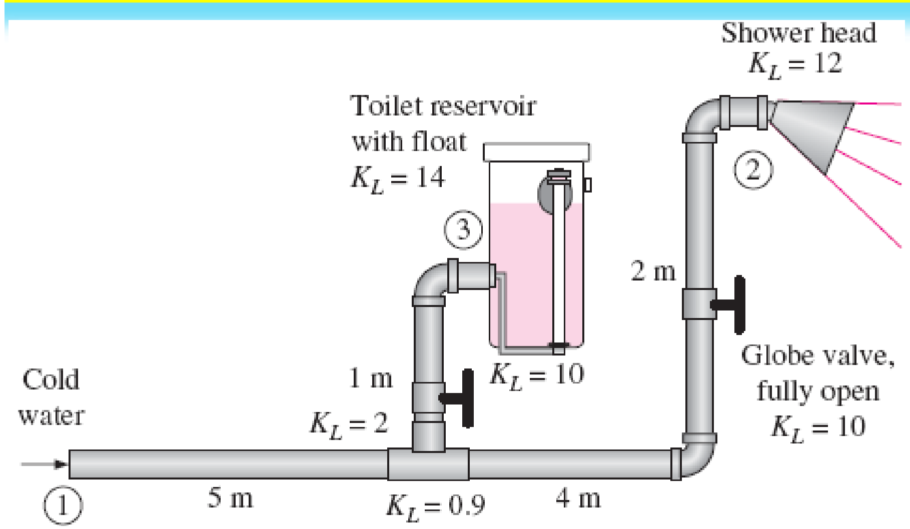
$$z_1 = z_2 + h_L = 4 + 27.9 = \mathbf{31.9\ m}$$

Therefore, the free surface of the first reservoir must be 31.9 m above the ground level to ensure water flow between the two reservoirs at the specified rate.

Note that fL/D=56.1 in this case, which is about 24 times the total minor loss coefficient. Therefore, ignoring the sources of minor losses in this case would result in about 4 percent error. It can be shown that the total head loss would be 35.9 m (instead of 27.9 m) if the valve were three-fourths closed, and it would drop to 24.8 m if the pipe between the two reservoirs were straight at the ground level (thus eliminating the elbows and the vertical section of the pipe). The head loss could be reduced further (from 24.8 to 24.6 m) by rounding the entrance. The head loss can be reduced significantly (from 27.9 to 16.0 m) by replacing the cast iron pipes by smooth pipes such as those made of plastic.

EXAMPLE 8–9

The bathroom plumbing of a building consists of 1.5-cm-diameter copper pipes with threaded connectors. (*a*) If the gage pressure at the inlet of the system is 200 kPa during a shower and the toilet reservoir is full (no flow in that branch), determine the flow rate of water through the shower head. (*b*) Determine the effect of flushing of the toilet on the flow rate through the shower head. Take the loss coefficients of the shower head and the reservoir to be 12 and 14, respectively.



Assumptions **1** steady & incompressible.
2 The flow is turbulent and fully developed.
3 The reservoir is open to the atmosphere
4 The velocity heads are negligible

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump}, u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine}, e} + h_L$$

$$\rightarrow \quad \frac{P_{1,\text{ gage}}}{\rho g} = (z_2 - z_1) + h_L$$

[redacted]

$$h_L = \frac{200{,}000 \text{ N/m}^2}{(998 \text{ kg/m}^3)(9.81 \text{ m/s}^2)} - 2 \text{ m} = 18.4 \text{ m}$$

$$h_L = \left(f\frac{L}{D} + \sum K_L\right)\frac{V^2}{2g} \quad \rightarrow \quad 18.4 = \left(f\frac{11 \text{ m}}{0.015 \text{ m}} + 24.7\right)\frac{V^2}{2(9.81 \text{ m/s}^2)}$$

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} \quad \rightarrow \quad V = \frac{\dot{V}}{\pi(0.015 \text{ m})^2/4}$$

$$\text{Re} = \frac{VD}{\nu} \quad \rightarrow \quad \text{Re} = \frac{V(0.015 \text{ m})}{1.004 \times 10^{-6} \text{ m}^2\text{/s}}$$

$$\frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{\varepsilon/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

$$\rightarrow \quad \frac{1}{\sqrt{f}} = -2.0 \log\left(\frac{1.5 \times 10^{-6} \text{ m}}{3.7(0.015 \text{ m})} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

This is a set of four equations with four unknowns, and solving them with EES gives

$$\dot{V} = 0.00053 \text{ m}^3\text{/s}, \quad f = 0.0218, \quad V = 2.98 \text{ m/s}, \quad \text{and} \quad \text{Re} = 44{,}550$$

Therefore, the flow rate of water through the shower head is **0.53 L/s.**

(*b*) When the toilet is flushed, the float moves and opens the valve. The discharged water starts to refill the reservoir, resulting in parallel flow after the tee connection. The head loss and minor loss coefficients for the shower branch were determined in (*a*) to be $h_{L,2}$= 18.4 m and $\sum K_{L,2}$= 24.7, respectively. The corresponding quantities for the reservoir branch can be determined similarly to be

$$h_{L,3} = \frac{200{,}000\ \text{N/m}^2}{(998\ \text{kg/m}^3)(9.81\ \text{m/s}^2)} - 1\ \text{m} = 19.4\ \text{m}$$

$$\Sigma K_{L,3} = 2 + 10 + 0.9 + 14 = 26.9$$

$$\dot{V}_1 = \dot{V}_2 + \dot{V}_3$$

$$h_{L,2} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_2 \frac{6\ \text{m}}{0.015\ \text{m}} + 24.7\right) \frac{V_2^2}{2(9.81\ \text{m/s}^2)} = 18.4$$

$$h_{L,3} = f_1 \frac{5\ \text{m}}{0.015\ \text{m}} \frac{V_1^2}{2(9.81\ \text{m/s}^2)} + \left(f_3 \frac{1\ \text{m}}{0.015\ \text{m}} + 26.9\right) \frac{V_3^2}{2(9.81\ \text{m/s}^2)} = 19.4$$

$$V_1 = \frac{\dot{V}_1}{\pi(0.015\ \text{m})^2/4}, \quad V_2 = \frac{\dot{V}_2}{\pi(0.015\ \text{m})^2/4}, \quad V_3 = \frac{\dot{V}_3}{\pi(0.015\ \text{m})^2/4}$$

$$\text{Re}_1 = \frac{V_1(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}, \quad \text{Re}_2 = \frac{V_2(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}, \quad \text{Re}_3 = \frac{V_3(0.015\ \text{m})}{1.004 \times 10^{-6}\text{m}^2/\text{s}}$$

[redacted]

$$\frac{1}{\sqrt{f_1}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_1\sqrt{f_1}}\right)$$

$$\frac{1}{\sqrt{f_2}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_2\sqrt{f_2}}\right)$$

$$\frac{1}{\sqrt{f_3}} = -2.0 \log\left(\frac{1.5 \times 10^{-6}\ \text{m}}{3.7(0.015\ \text{m})} + \frac{2.51}{\text{Re}_3\sqrt{f_3}}\right)$$

Flow rate of cold water through a shower may be affected significantly by the flushing of a nearby toilet.

$$\dot{V}_1 = 0.00090\ \text{m}^3/\text{s}, \quad \dot{V}_2 = 0.00042\ \text{m}^3/\text{s}, \quad \text{and} \quad \dot{V}_3 = 0.00048\ \text{m}^3/\text{s}$$

Therefore, the flushing of the toilet **reduces the flow rate of cold water through the shower by 21 percent** from 0.53 to 0.42 L/s, causing the shower water to suddenly get very hot

If the velocity heads were considered, the flow rate through the shower would be 0.43 instead of 0.42 L/s. Therefore, the assumption of negligible velocity heads is reasonable in this case. Note that a leak in a piping system will cause the same effect, and thus an unexplained drop in flow rate at an end point may signal a leak in the system.

8–8 FLOW RATE AND VELOCITY MEASUREMENT

A major application area of fluid mechanics is the determination of the flow rate of fluids, and numerous devices have been developed over the years for the purpose of flow metering.

Flowmeters range widely in their level of sophistication, size, cost, accuracy, versatility, capacity, pressure drop, and the operating principle.

We give an overview of the meters commonly used to measure the flow rate of liquids and gases flowing through pipes or ducts.

We limit our consideration to incompressible flow.

A primitive (but fairly accurate) way of measuring the flow rate of water through a garden hose involves collecting water in a bucket and recording the collection time.

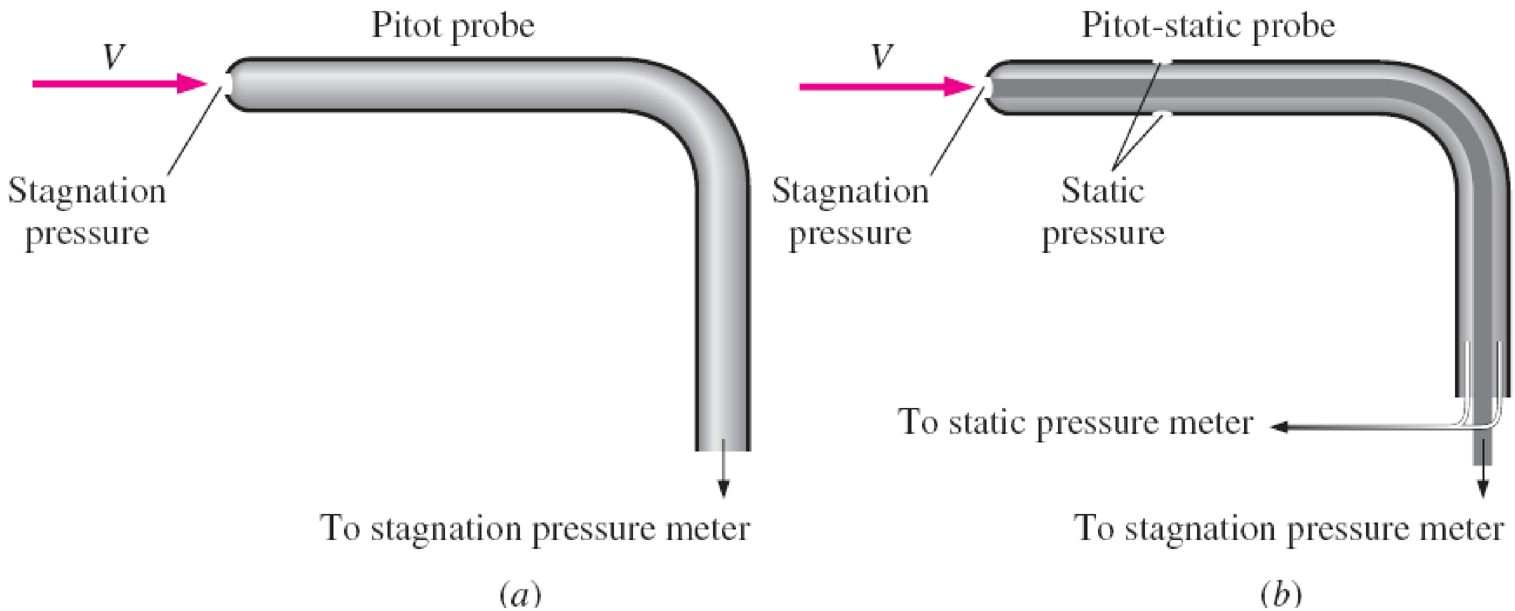
$$\dot{V} = VA_c$$

Measuring the flow rate is usually done by measuring flow velocity, and many flowmeters are simply velocimeters used for the purpose of metering flow.

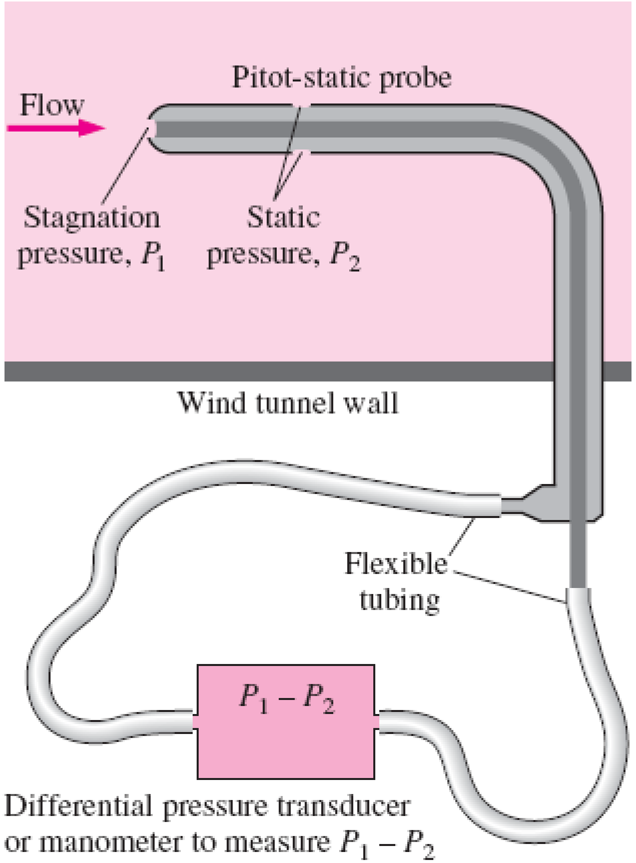
Pitot and Pitot-Static Probes

Pitot probes (also called *Pitot tubes*) and **Pitot-static probes** are widely used for flow speed measurement.

A Pitot probe is just a tube with a pressure tap at the stagnation point that measures stagnation pressure, while a Pitot-static probe has both a stagnation pressure tap and several circumferential static pressure taps and it measures both stagnation and static pressures



(*a*) A Pitot probe measures stagnation pressure at the nose of the probe, while (*b*) a Pitot-static probe measures both stagnation pressure and static pressure, from which the flow speed is calculated.

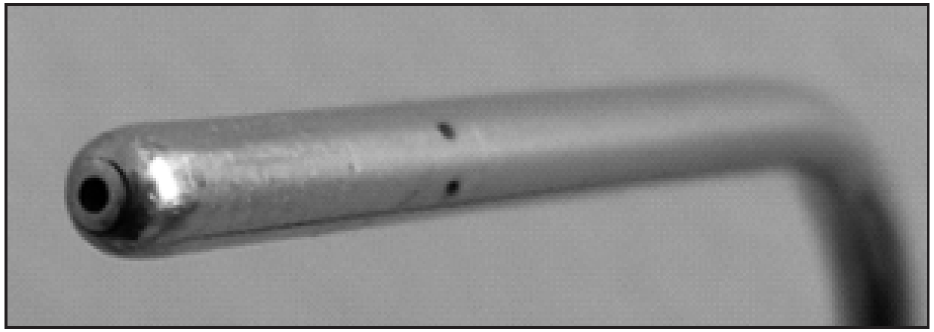


$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} + z_1 = \frac{P_2}{\rho g} + \frac{V_2^2}{2g} + z_2$$

Pitot formula: $$V = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

$$\dot{V} = VA_c$$

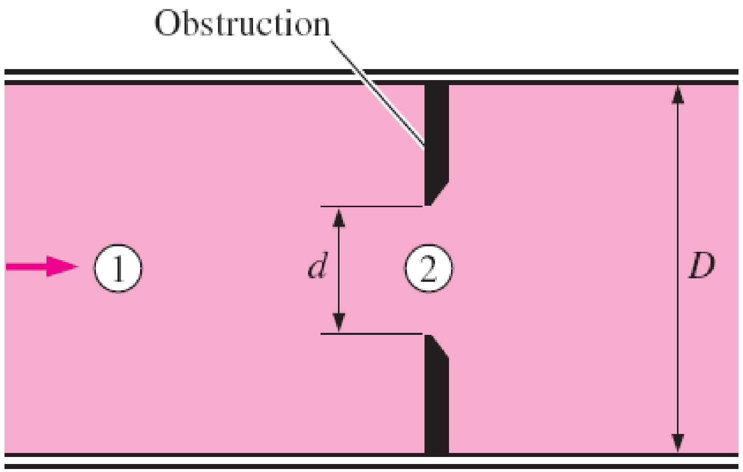
Measuring flow velocity with a Pitotstatic probe. (A manometer may be used in place of the differential pressure transducer.)



Close-up of a Pitot-static probe, showing the stagnation pressure hole and two of the five static circumferential pressure holes.

Obstruction Flowmeters: Orifice, Venturi, and Nozzle Meters

Flowmeters based on this principle are called **obstruction flowmeters** and are widely used to measure flow rates of gases and liquids.



Flow through a constriction in a pipe.

Mass balance:

$$\dot{V} = A_1V_1 = A_2V_2 \quad \rightarrow \quad V_1 = (A_2/A_1)V_2 = (d/D)^2V_2$$

Bernoulli equation ($z_1 = z_2$):

$$\frac{P_1}{\rho g} + \frac{V_1^2}{2g} = \frac{P_2}{\rho g} + \frac{V_2^2}{2g}$$

Obstruction (with no loss): $$V_2 = \sqrt{\frac{2(P_1 - P_2)}{\rho(1 - \beta^4)}}$$

$$\beta = d/D \qquad \dot{V} = A_2V_2 = (\pi d^2/4)V_2$$

Orifismeter

[redacted]

The losses can be accounted for by incorporating a correction factor called the discharge coefficient C_d whose value (which is less than 1) is determined experimentally.

Obstruction flowmeters: $$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}}$$

The value of C_d depends on both b and the Reynolds number, and charts and curve-fit correlations for C_d are available for various types of obstruction meters.

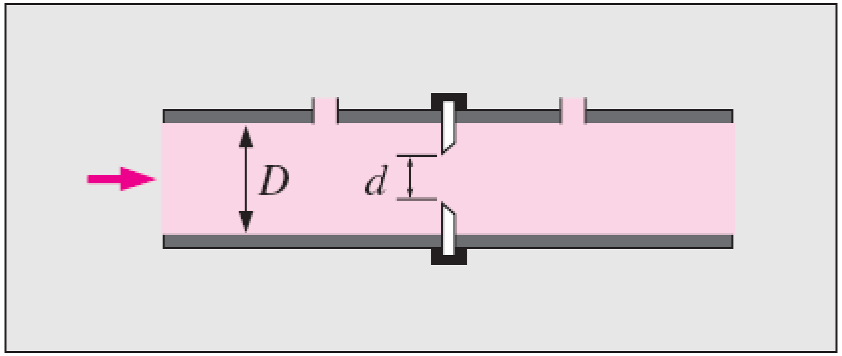
Orifice meters: $$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\mathrm{Re}^{0.75}}$$

Nozzle meters: $$C_d = 0.9975 - \frac{6.53\beta^{0.5}}{\mathrm{Re}^{0.5}}$$

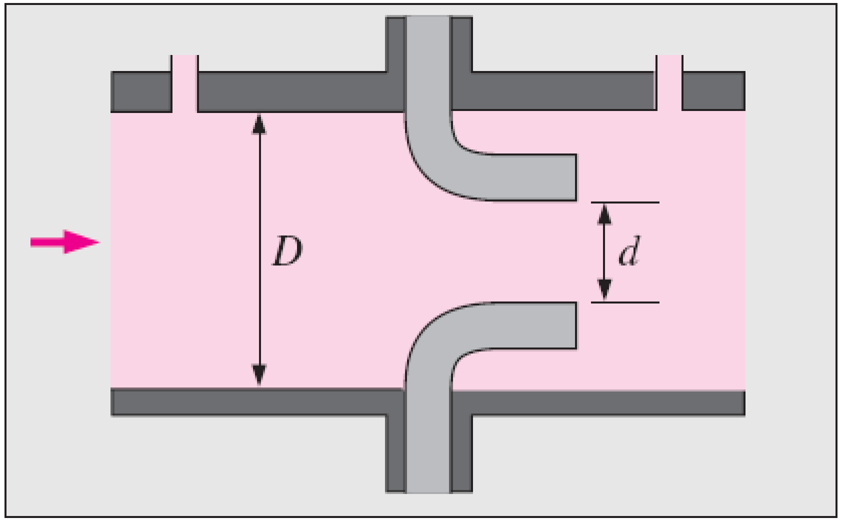
$$0.25 < \beta < 0.75 \text{ and } 10^4 < \mathrm{Re} < 10^7$$

For flows with high Reynolds numbers (Re > 30,000), the value of C_d can be taken to be 0.96 for flow nozzles and 0.61 for orifices.

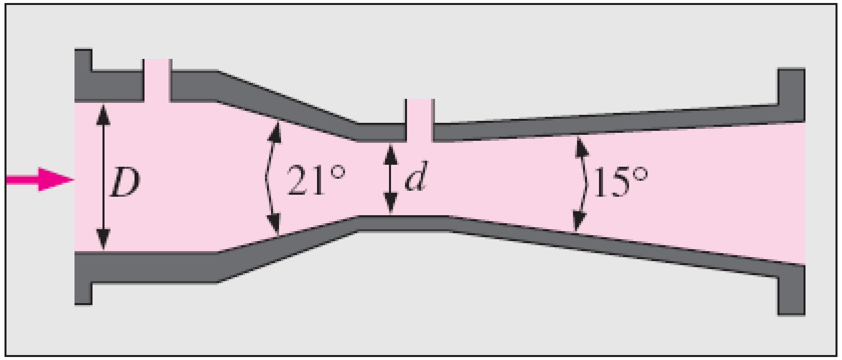
[redacted]



(*a*) Orifice meter

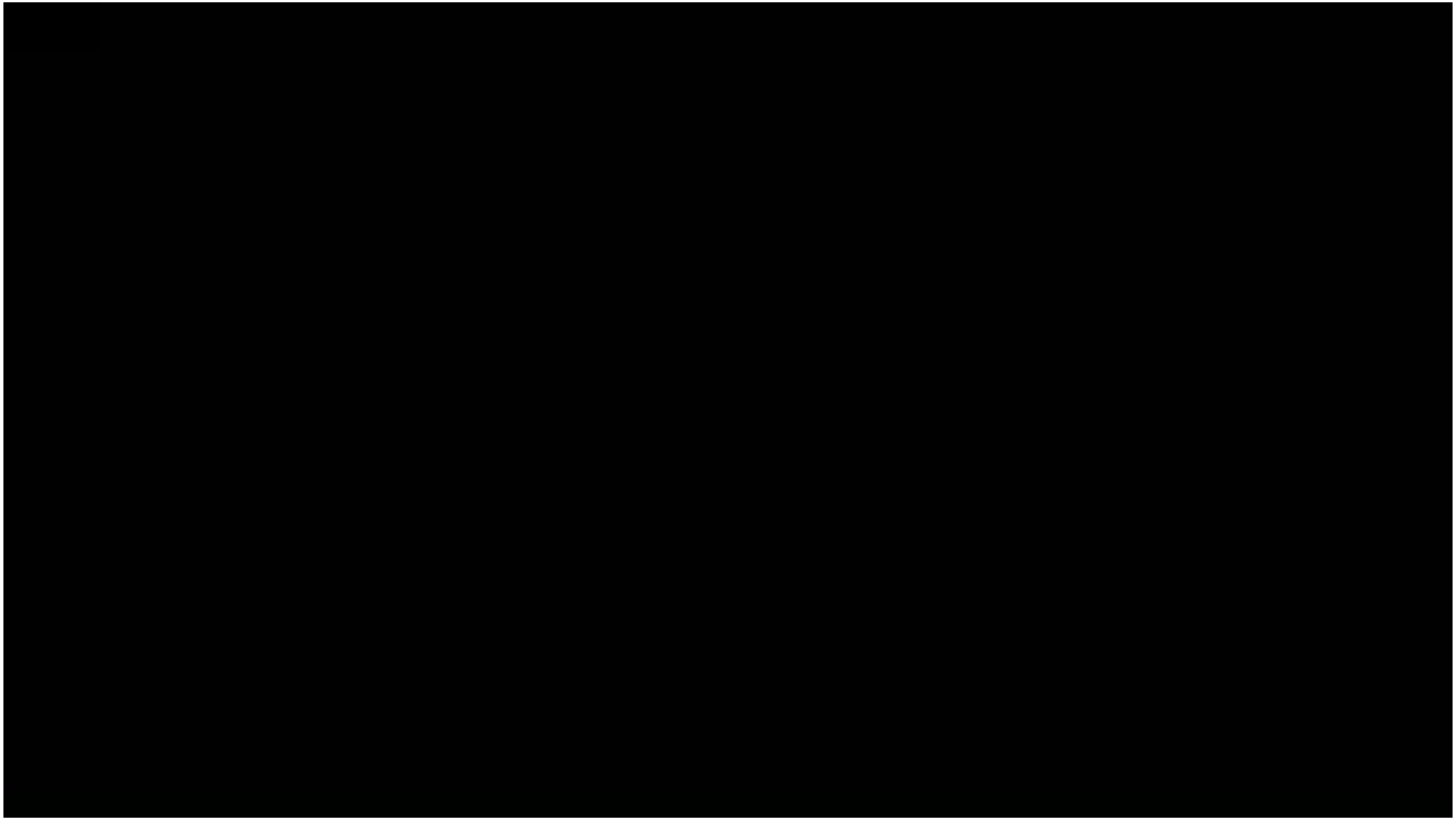


(*b*) Flow nozzle



(*c*) Venturi meter

Common types of obstruction meters.



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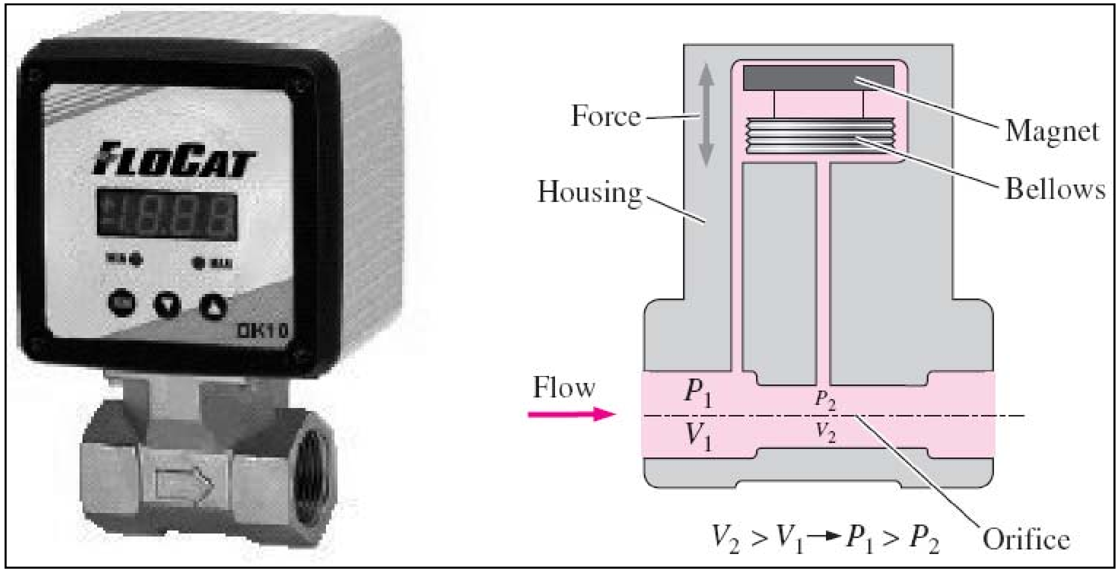
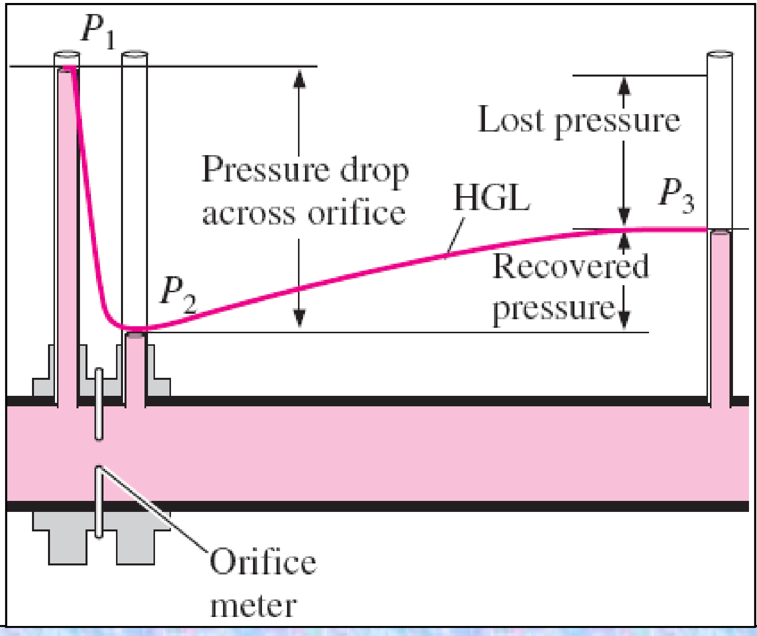
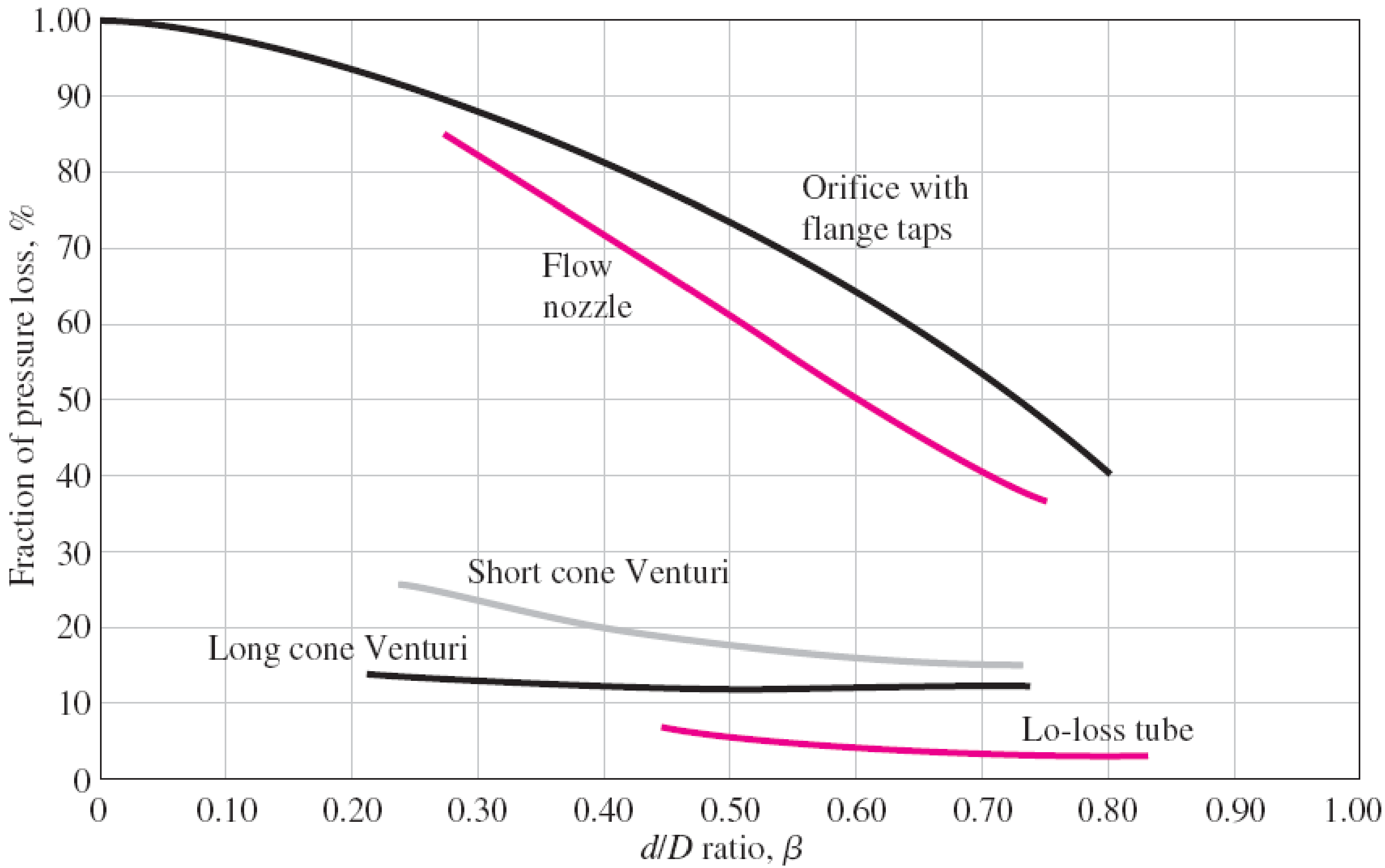


An orifice meter and schematic showing its built-in pressure transducer and digital readout.



The variation of pressure along a flow section with an orifice meter as measured with piezometer tubes; the lost pressure and the pressure recovery are shown.

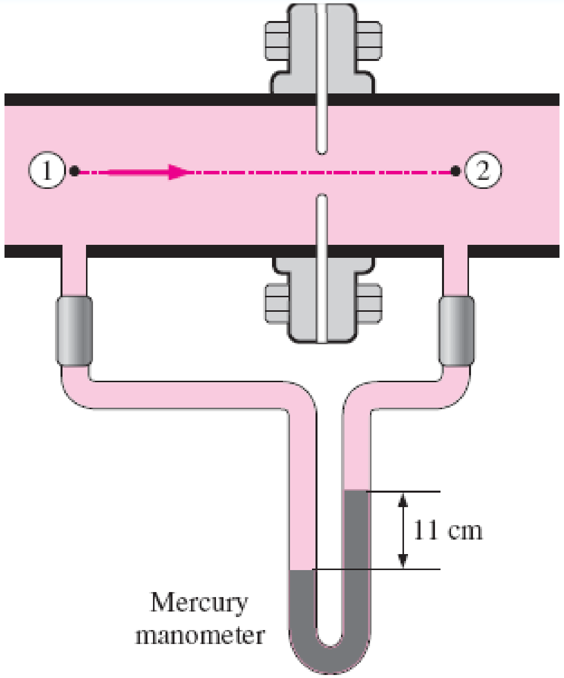


The fraction of pressure (or head) loss for various obstruction meters.

EXAMPLE 8-10

The flow rate of methanol at 20°C (ρ =788.4 kg/m^3 and μ =5.857x10^{-4} kg/m·s) through a 4-cm-diameter pipe is to be measured with a 3-cm-diameter orifice meter equipped with a mercury manometer across the orifice place. If the differential height of the manometer is read to be 11 cm, determine the flow rate of methanol through the pipe and the average flow velocity.

Solution The flow rate of methanol is to be measured with an orifice meter. For a given pressure drop across the orifice plate, the flow rate and the average flow velocity are to be determined.
Assumptions **1** The flow is steady and incompressible. **2** The discharge coefficient of the orifice meter is C_d =0.61.



$$\beta = \frac{d}{D} = \frac{3}{4} = 0.75$$

$$A_0 = \frac{\pi d^2}{4} = \frac{\pi(0.03\text{ m})^2}{4} = 7.069 \times 10^{-4}\text{ m}^2$$

$$\Delta P = P_1 - P_2 = (\rho_{\text{Hg}} - \rho_{\text{met}})gh$$

$$\dot{V} = A_0 C_d \sqrt{\frac{2(P_1 - P_2)}{\rho(1-\beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{\text{Hg}} - \rho_{\text{met}})gh}{\rho_{\text{met}}(1-\beta^4)}} = A_0 C_d \sqrt{\frac{2(\rho_{\text{Hg}}/\rho_{\text{met}} - 1)gh}{1-\beta^4}}$$

$$\dot{V} = (7.069 \times 10^{-4}\ \text{m}^2)(0.61)\sqrt{\frac{2(13{,}600/788.4 - 1)(9.81\ \text{m/s}^2)(0.11\ \text{m})}{1 - 0.75^4}}$$

$$= \mathbf{3.09 \times 10^{-3}\ m^3/s}$$

$$V = \frac{\dot{V}}{A_c} = \frac{\dot{V}}{\pi D^2/4} = \frac{3.09 \times 10^{-3}\ \text{m}^3/\text{s}}{\pi (0.04\ \text{m})^2/4} = \mathbf{2.46\ m/s}$$

The Reynolds number of flow through the pipe is

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(788.4\ \text{kg/m}^3)(2.46\ \text{m/s})(0.04\ \text{m})}{5.857 \times 10^{-4}\ \text{kg/m} \cdot \text{s}} = 1.32 \times 10^5$$

$\beta = 0.75$ and $\text{Re} = 1.32 \times 10^5$

$$C_d = 0.5959 + 0.0312\beta^{2.1} - 0.184\beta^8 + \frac{91.71\beta^{2.5}}{\text{Re}^{0.75}}$$

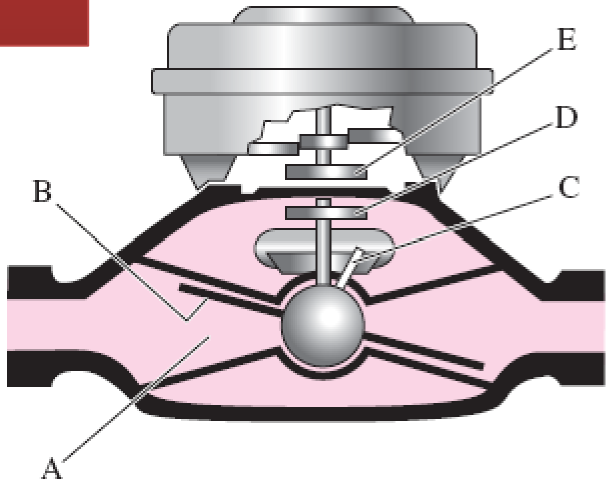
gives C_d =0.601, which is very close to the assumed value of 0.61. Using this refined value of C_d, the flow rate becomes 3.04 L/s, which differs from our original result by only 1.6 percent. Therefore, it is convenient to analyze orifice meters using the recommended value of C_d=0.61 for the discharge coefficient, and then to verify the assumed value. If the problem is solved using an equation solver such as EES, then the problem can be formulated using the curve-fit formula for C_d, and all equations can be solved simultaneously by letting the equation solver perform the iterations as necessary.

Positive Displacement Flowmeters

The total amount of mass or volume of a fluid that passes through a cross section of a pipe over a certain period of time is measured by **positive displacement flowmeters**.

There are numerous types of displacement meters, and they are based on continuous filling and discharging of the measuring chamber. They operate by trapping a certain amount of incoming fluid, displacing it to the discharge side of the meter, and counting the number of such discharge–recharge cycles to determine the total amount of fluid displaced.

A positive displacement flowmeter with double helical three-lobe impeller design.



A nutating disk flowmeter.

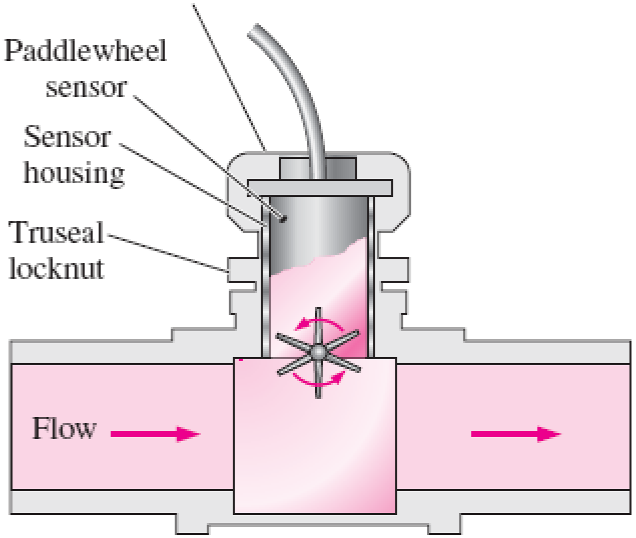
Turbine Flowmeters

(*a*) An in-line turbine flowmeter to measure liquid flow, with flow from left to right, (*b*) a cutaway view of the turbine blades inside the flowmeter, and
(*c*) a handheld turbine flowmeter to measure wind speed, measuring no flow at the time the photo was taken so that the turbine blades are visible. The flowmeter in (*c*) also measures the air termperature for convenience.

Paddlewheel Flowmeters

Paddlewheel flowmeters are low-cost alternatives to turbine flowmeters for flows where very high accuracy is not required.

The paddlewheel (the rotor and the blades) is perpendicular to the flow rather than parallel as was the case with turbine flowmeters.



Paddlewheel flowmeter to measure liquid flow, with flow from left to right, and a schematic diagram of its operation.

Variable-Area Flowmeters (Rotameters)

A simple, reliable, inexpensive, and easy-to-install flowmeter with reasonably low pressure drop and no electrical connections that gives a direct reading of flow rate for a wide range of liquids and gases is the **variable-area flowmeter**, also called a **rotameter** or **floatmeter**.

A variable-area flowmeter consists of a vertical tapered conical transparent tube made of glass or plastic with a float inside that is free to move.

As fluid flows through the tapered tube, the float rises within the tube to a location where the float weight, drag force, and buoyancy force balance each other and the net force acting on the float is zero.

The flow rate is determined by simply matching the position of the float against the graduated flow scale outside the tapered transparent tube.

The float itself is typically either a sphere or a loose-fitting piston-like cylinder.

Two types of variable-area flowmeters: (*a*) an ordinary gravity-based meter and (*b*) a spring-opposed meter.

Variable Area Flowmeters

Ultrasonic Flowmeters

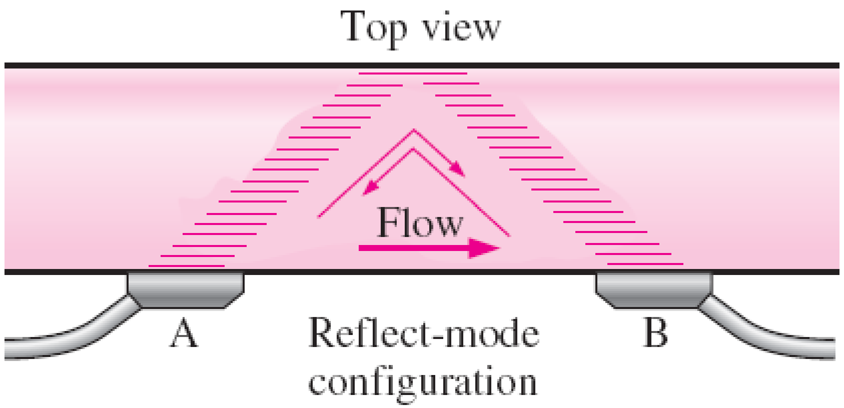
Ultrasonic flowmeters operate using sound waves in the ultrasonic range (beyond human hearing ability, typically at a frequency of 1 MHz).

Ultrasonic (or acoustic) flowmeters operate by generating sound waves with a transducer and measuring the propagation of those waves through a flowing fluid.

There are two basic kinds of ultrasonic flowmeters: ***transit time*** and ***Doppler-effect*** **(or** ***frequency shift***) flowmeters.

L is the distance between the transducers and *K* is a constant

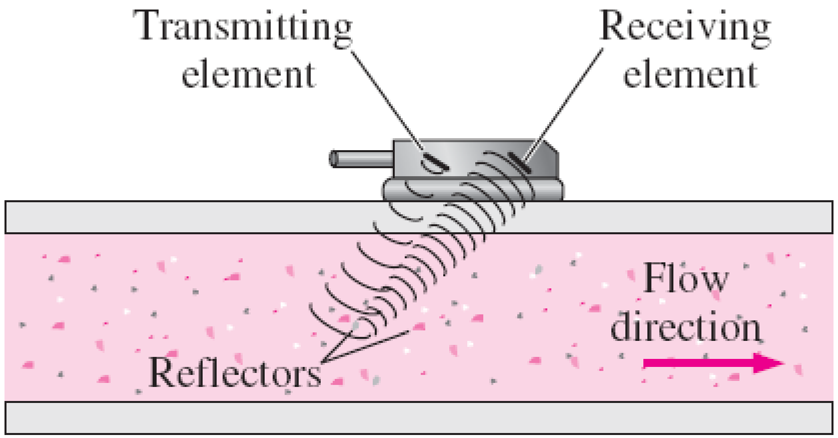
$$V = KL\,\Delta t$$



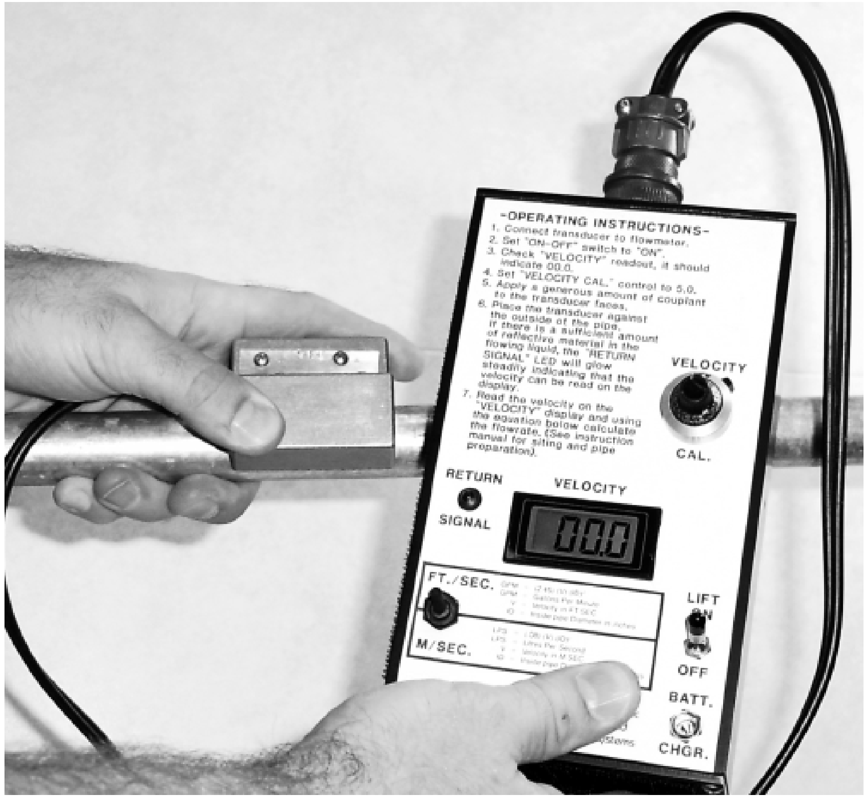
The operation of a transit time ultrasonic flowmeter equipped with two transducers.

Doppler-Effect Ultrasonic Flowmeters

Doppler-effect ultrasonic flowmeters measure the average flow velocity along the sonic path.



The operation of a Doppler-effect ultrasonic flowmeter equipped with a transducer pressed on the outer surface of a pipe.

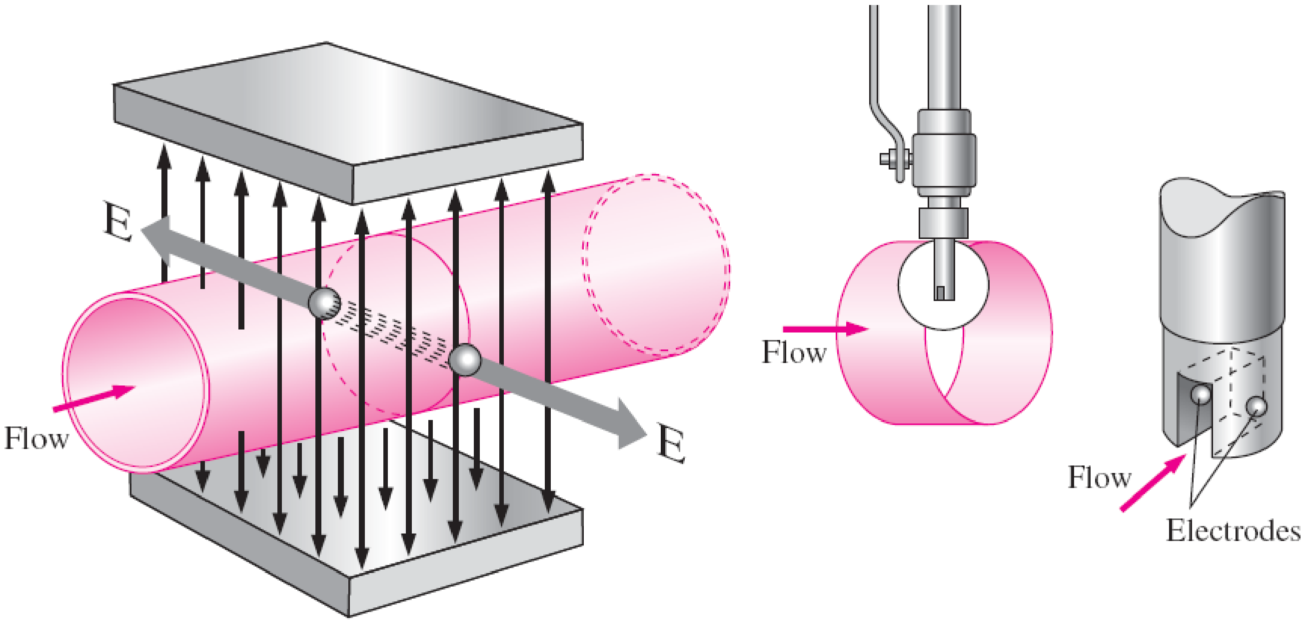


Ultrasonic clamp-on flowmeters enable one to measure flow velocity without even contacting (or disturbing) the fluid by simply pressing a transducer on the outer surface of the pipe.

Electromagnetic Flowmeters

A full-flow electromagnetic flowmeter is a nonintrusive device that consists of a magnetic coil that encircles the pipe, and two electrodes drilled into the pipe along a diameter flush with the inner surface of the pipe so that the electrodes are in contact with the fluid but do not interfere with the flow and thus do not cause any head loss.

Insertion electromagnetic flowmeters operate similarly, but the magnetic field is confined within a flow channel at the tip of a rod inserted into the flow.

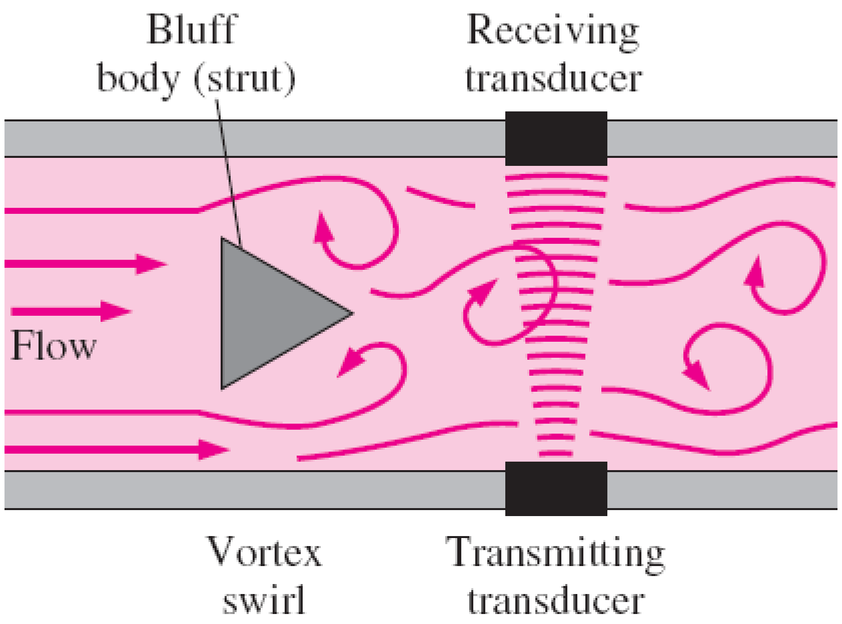


(*a*) Full-flow electromagnetic flowmeter (*b*) Insertion electromagnetic flowmeter

Vortex Flowmeters

This suggests that the flow rate can be determined by generating vortices in the flow by placing an obstruction in the flow and measuring the shedding frequency. The flow measurement devices that work on this principle are called **vortex flowmeters.**

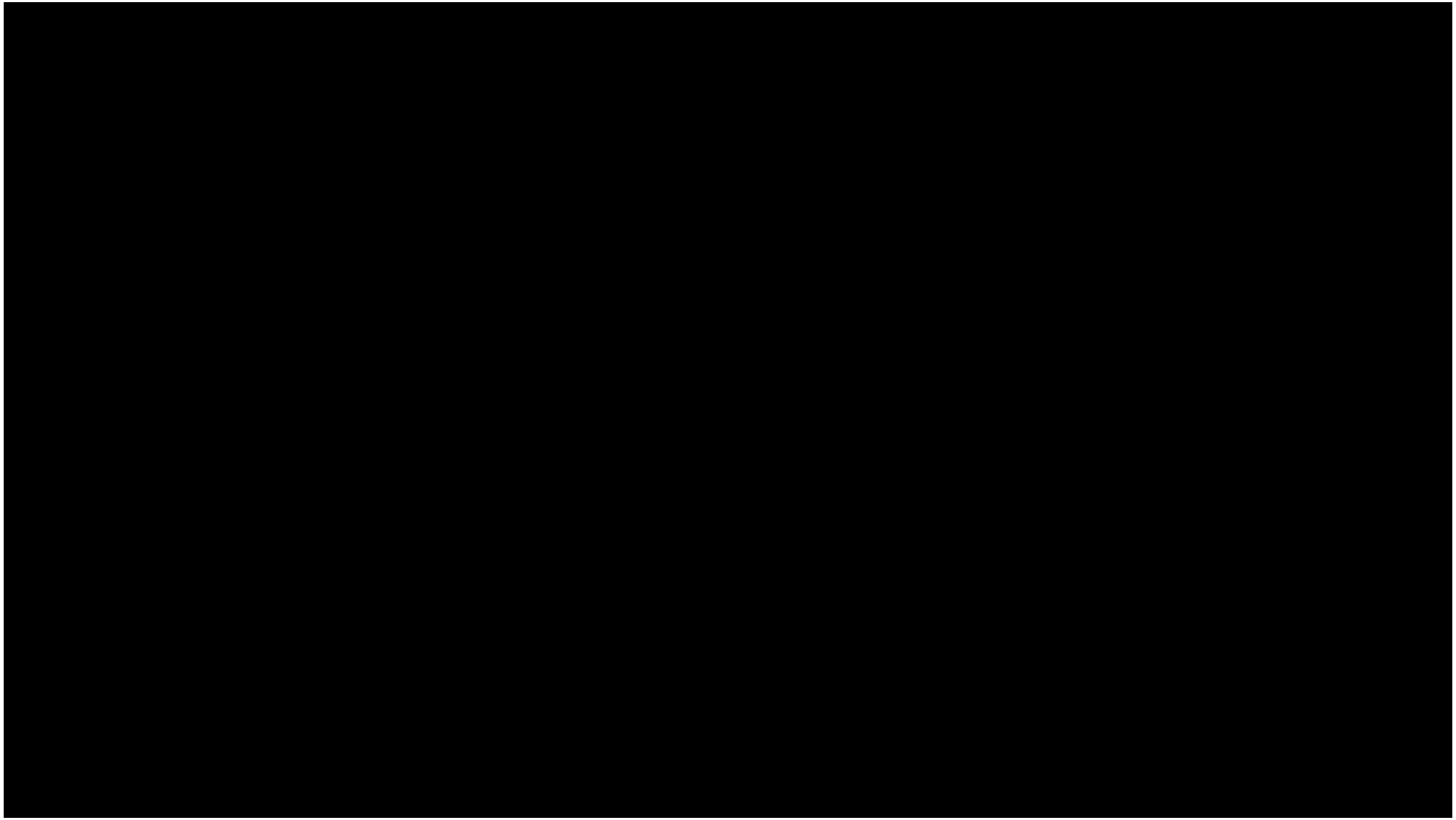
The *Strouhal number*, defined as St = fd/V, where f is the vortex shedding frequency, d is the characteristic diameter or width of the obstruction, and V is the velocity of the flow impinging on the obstruction, also remains constant in this case, provided that the flow velocity is high enough.



The operation of a vortex flowmeter.

The vortex flowmeter has the advantage that it has no moving parts and thus is inherently reliable, versatile, and very ccurate (usually 1 percent over a wide range of flow rates), but it obstructs the flow and thus causes considerable head loss.

Vortex Flowmeters

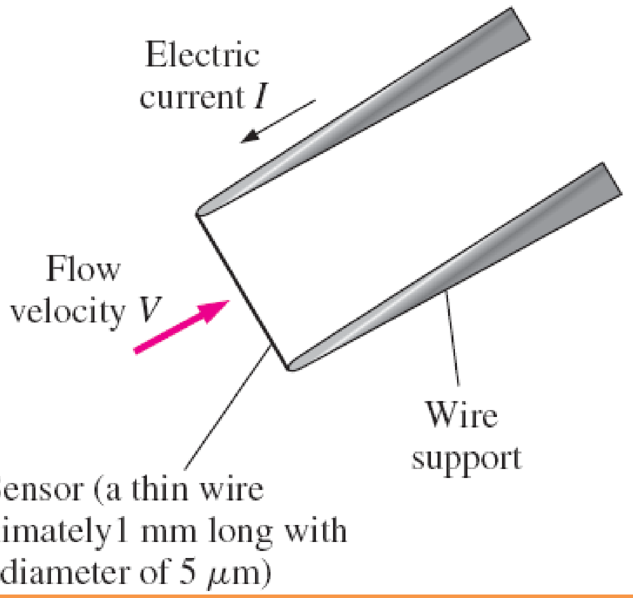


Thermal (Hot-Wire and Hot-Film) Anemometers

Thermal anemometers involve an electrically heated sensor and utilize a thermal effect to measure flow velocity.

Thermal anemometers have extremely small sensors, and thus they can be used to measure the instantaneous velocity at any point in the flow without appreciably disturbing the flow.

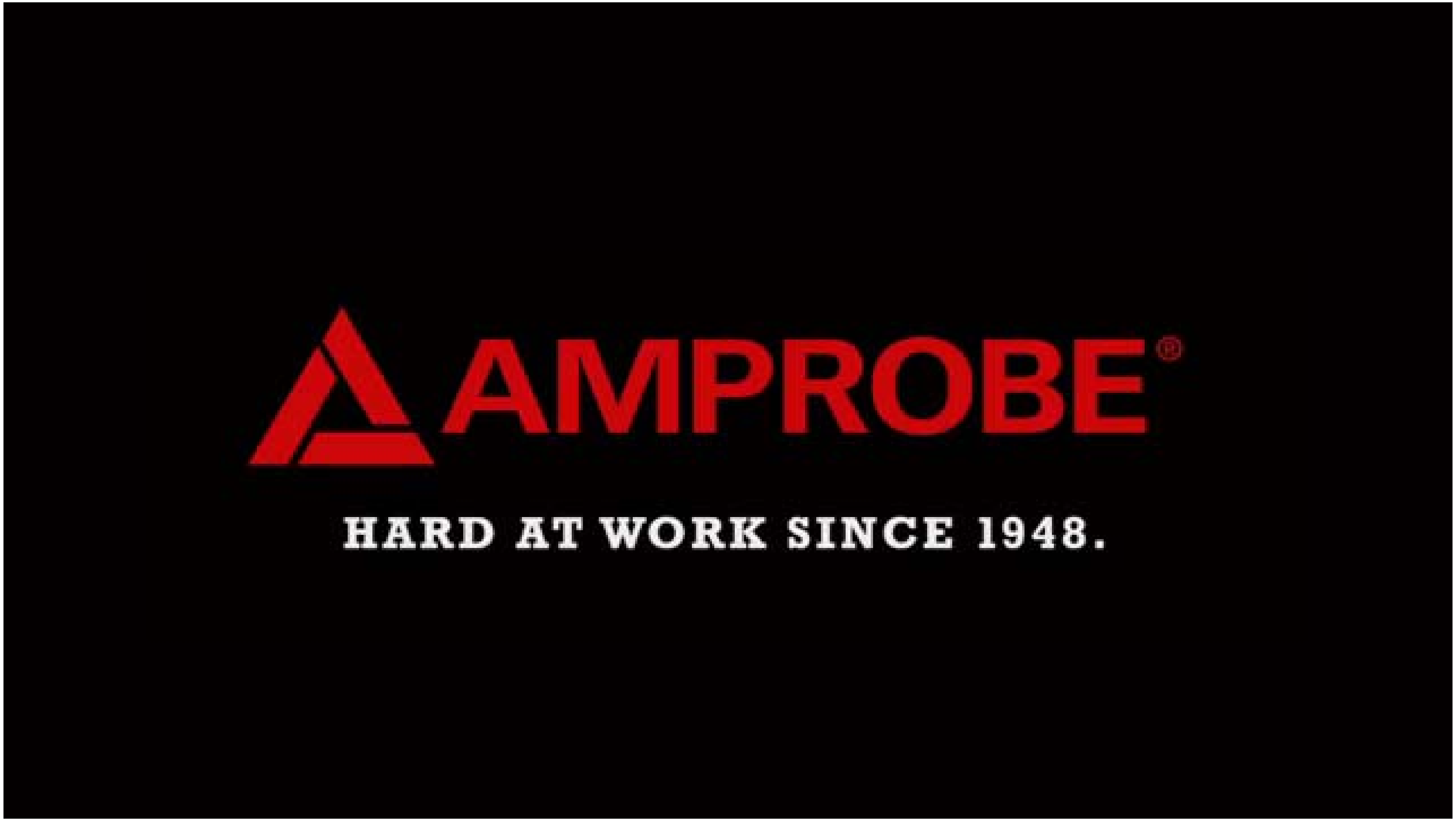
They can measure velocities in liquids and gases accurately over a wide range—from a few centimeters to over a hundred meters per second.



The electrically heated sensor and its support, components of a hot-wire probe.

A thermal anemometer is called a hot-wire anemometer if the sensing element is a wire, and a hot-film anemometer if the sensor is a thin metallic film (less than 0.1 μm thick) mounted usually on a relatively thick ceramic support having a diameter of about 50 μm.

Hot wire Anemometer



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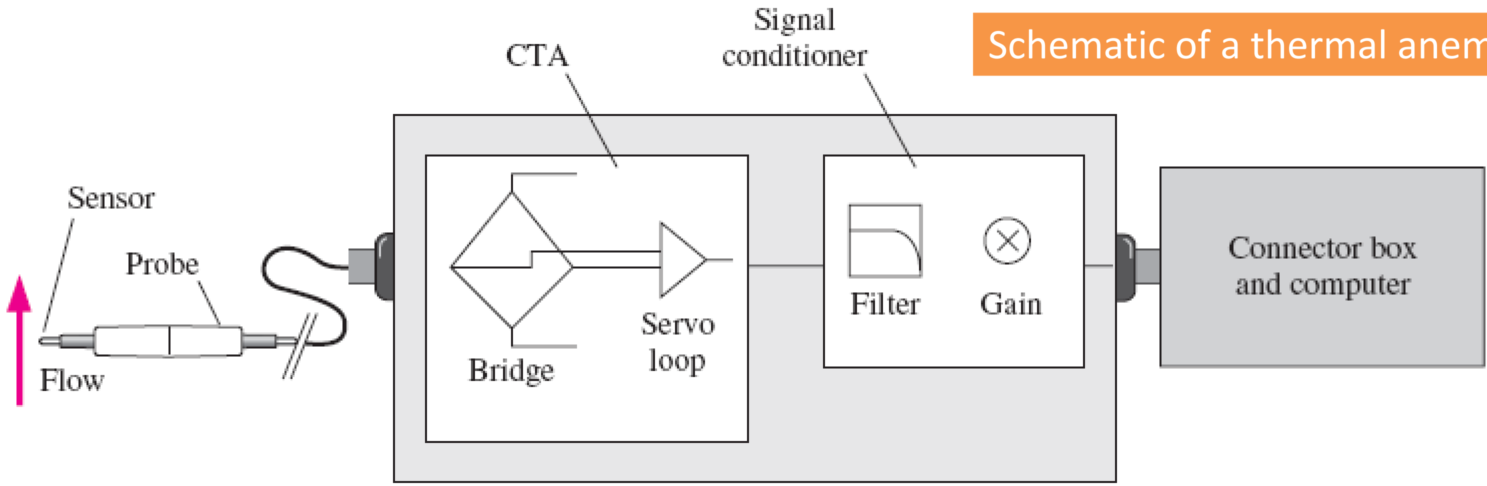
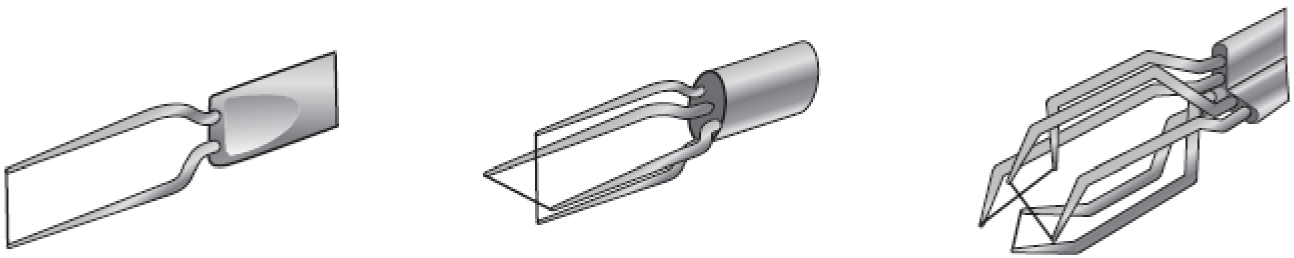


Schematic of a thermal anemometer system.

$$E^2 = a + bV^n$$

King's law

E is the voltage, and the values of the constants *a*, *b*, and *n* are calibrated for a given probe. Once the voltage is measured, this relation gives the flow velocity *V* directly.



(*a*) (*b*) (*c*)

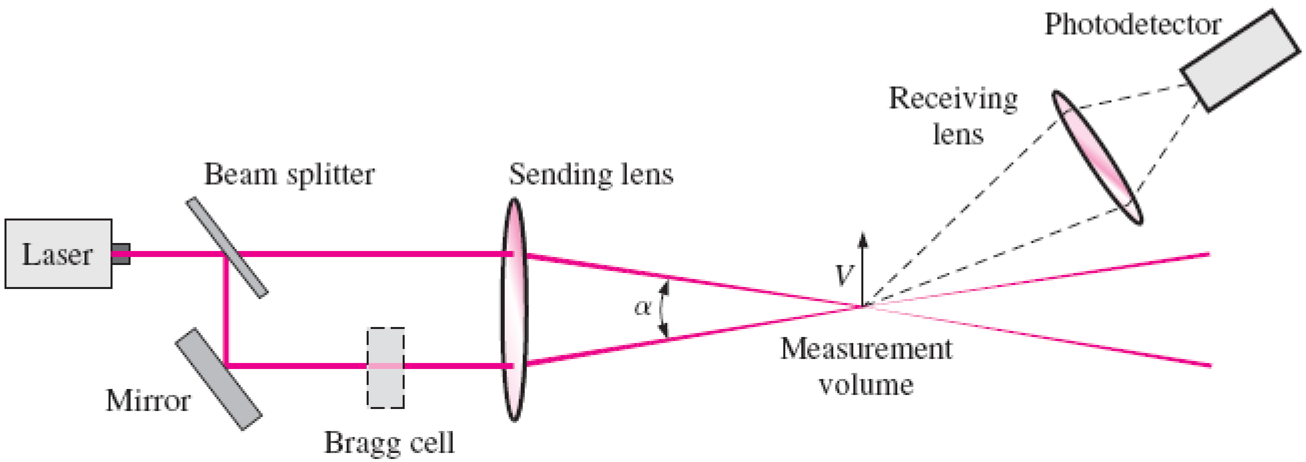
Thermal anemometer probes with single, double, and triple sensors to measure (*a*) one-, (*b*) two-,and (*c*) 3-D velocity components simultaneously.

Laser Doppler Velocimetry

Laser Doppler velocimetry (LDV), also **called laser velocimetry (LV) or laser Doppler anemometry (LDA)**, is an optical technique to measure flow velocity at any desired point without disturbing the flow.

Unlike thermal anemometry, LDV involves no probes or wires inserted into the flow, and thus it is a nonintrusive method.

Like thermal anemometry, it can accurately measure velocity at a very small volume and thus it can also be used to study the details of flow at a locality, including turbulent fluctuations, and it can be traversed through the entire flow field without intrusion.



A dual-beam LDV system in forward scatter mode.

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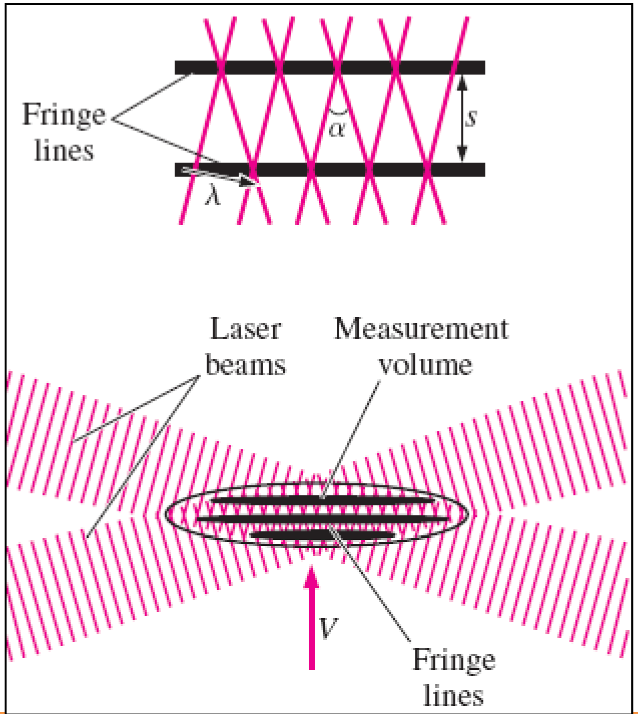
$$f = \frac{V}{s} = \frac{2V\,\sin(\alpha/2)}{\lambda}$$

LDV equation

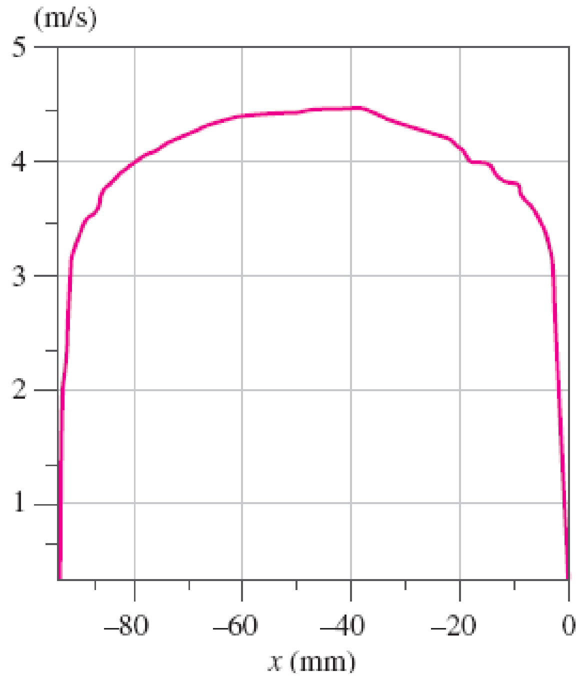
$$s = \lambda/[2\,\sin(\alpha/2)]$$

λ is the wavelength of the laser beam and α is the angle between the two laser beams

This fundamental relation shows the flow velocity to be proportional to the frequency.



Fringes that form as a result of the interference at the intersection of two laser beams of an LDV system (lines represent peaks of waves). The top diagram is a close-up view of two fringes.



A time-averaged velocity profile in turbulent pipe flow obtained by an LDV system.

Particle Image Velocimetry

Particle image velocimetry (PIV) is a double-pulsed laser technique used to measure the instantaneous velocity distribution in a plane of flow by photographically determining the displacement of particles in the plane during a very short time interval.

Unlike methods like hot-wire anemometry and LDV that measure velocity at a point, PIV provides velocity values simultaneously throughout an entire cross section, and thus it is a whole-field technique.

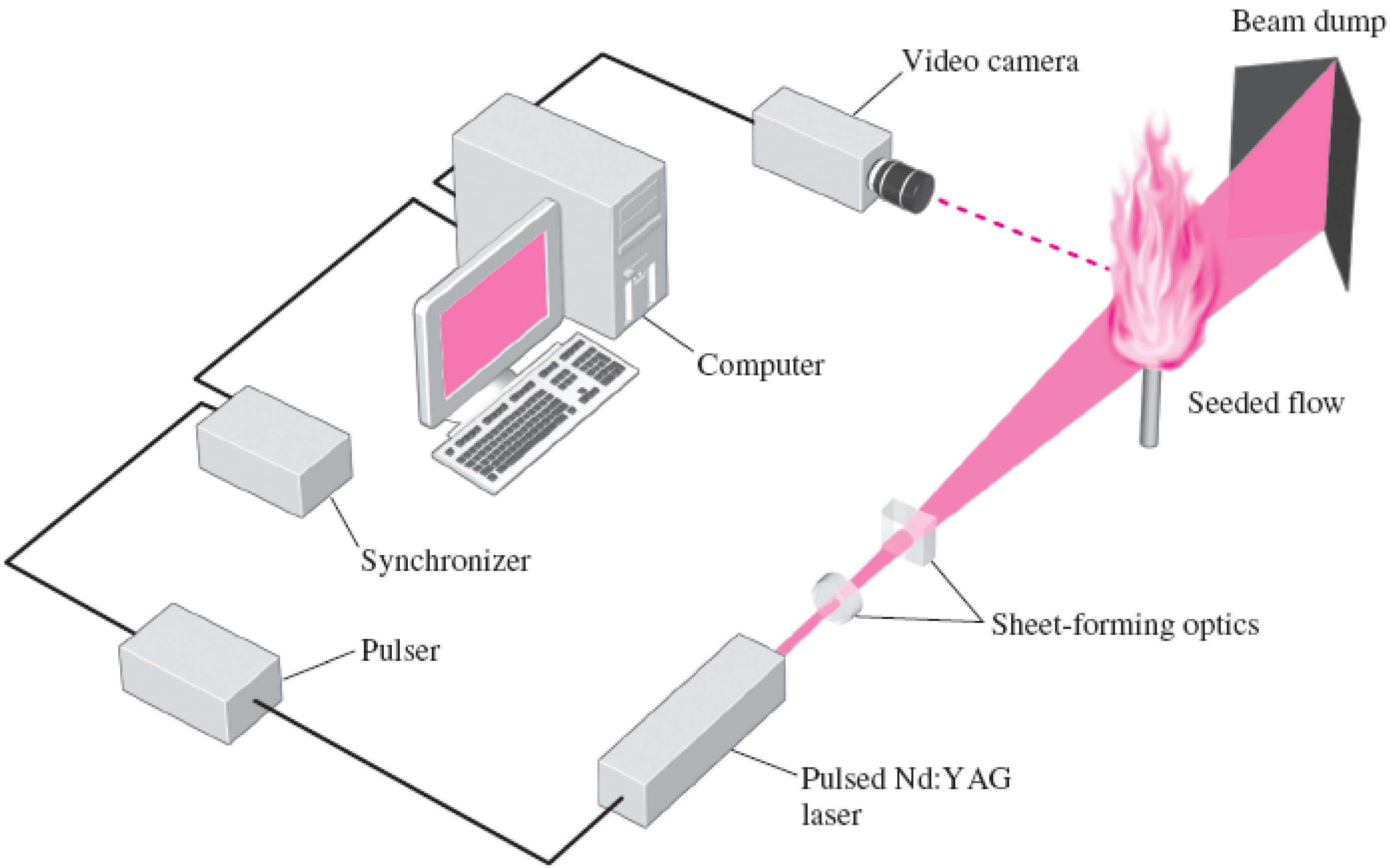
PIV combines the accuracy of LDV with the capability of flow visualization and provides instantaneous flow field mapping.

The entire instantaneous velocity profile at a cross section of pipe can be obtained with a single PIV measurement.

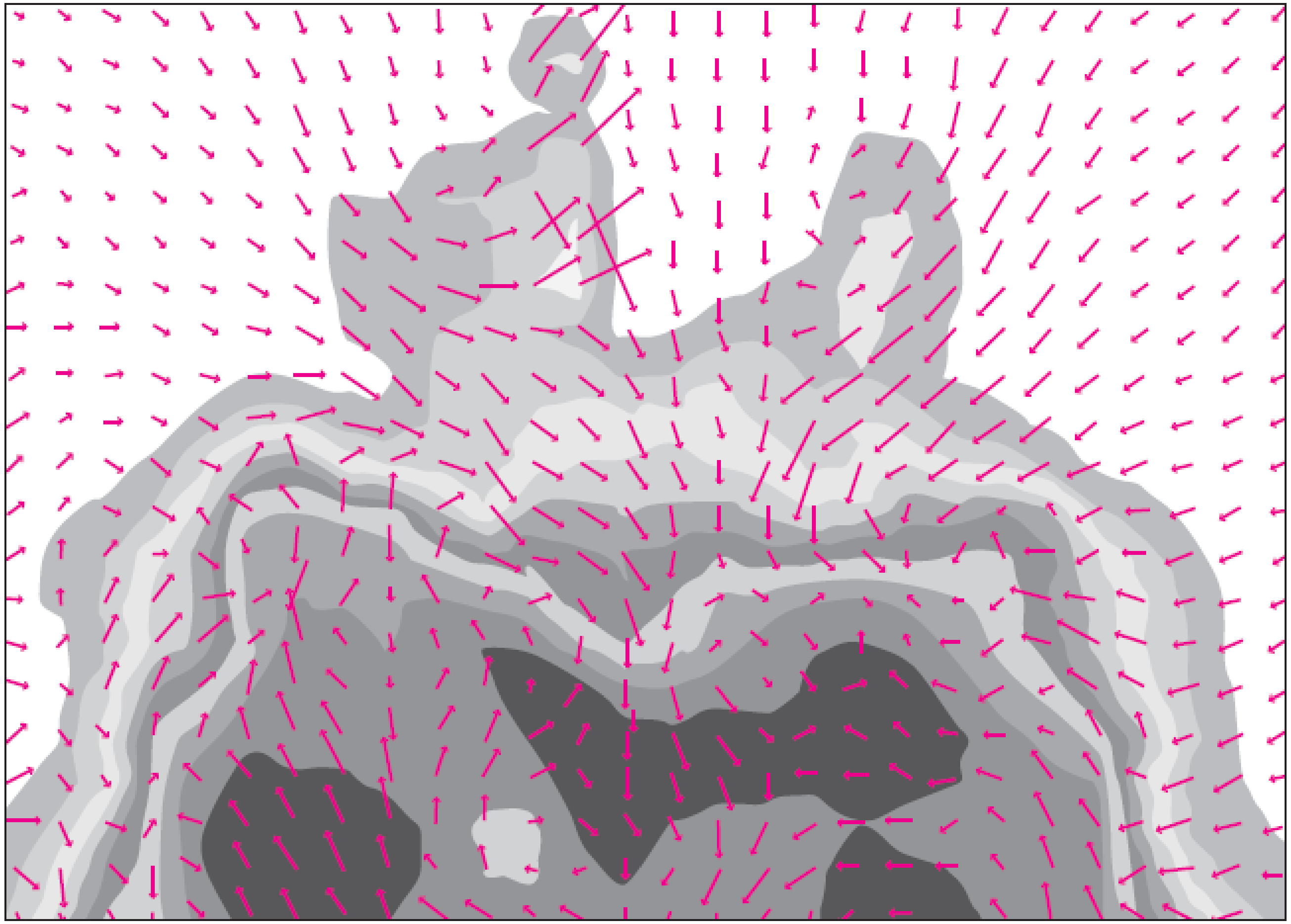
A PIV system can be viewed as a camera that can take a snapshot of velocity distribution at any desired plane in a flow.

Ordinary flow visualization gives a qualitative picture of the details of flow.

PIV also provides an accurate *quantitative* description of various flow quantities such as the velocity field, and thus the capability to analyze the flow numerically using the velocity data provided.



A PIV system to study flame stabilization.



Instantaneous velocity field in the wake region of a car as measured by a PIV system in a wind tunnel. The velocity vectors are superimposed on a contour plot of pressure. The interface between two adjacent grayscale levels is an isobar.

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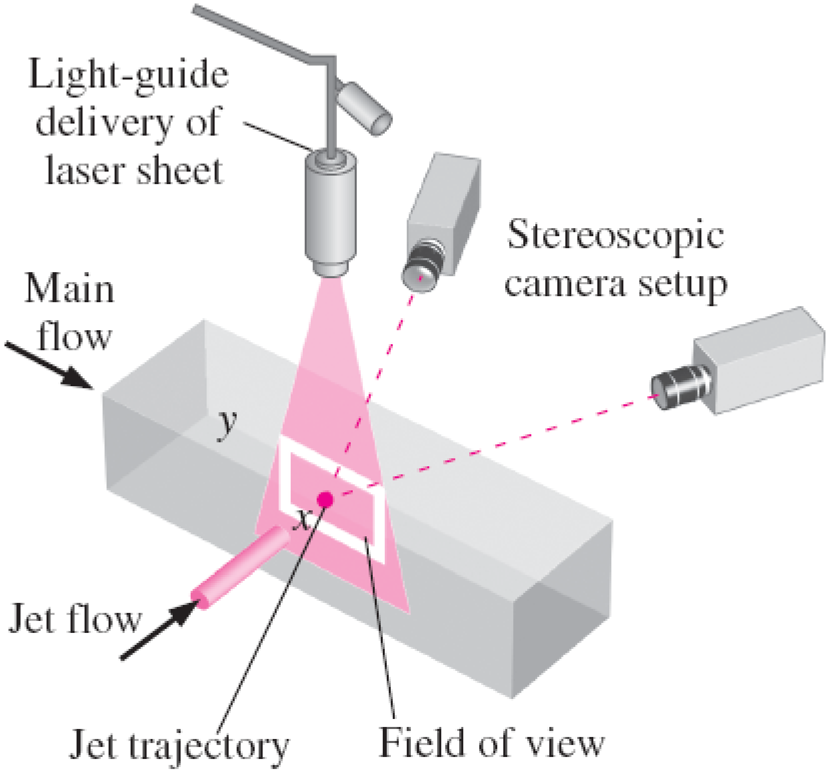

A three-dimensional PIV system set up to study the mixing of an air jet with cross duct flow.

A variety of laser light sources such as argon, copper vapor, and Nd:YAG can be used with PIV systems, depending on the requirements for pulse duration, power, and time between pulses.

Nd:YAG lasers are commonly used in PIV systems over a wide range of applications.

A beam delivery system such as a light arm or a fiber-optic system is used to generate and deliver a high-energy pulsed laser sheet at a specified thickness.

With PIV, other flow properties such as vorticity and strain rates can also be obtained, and the details of turbulence can be studied.

Summary

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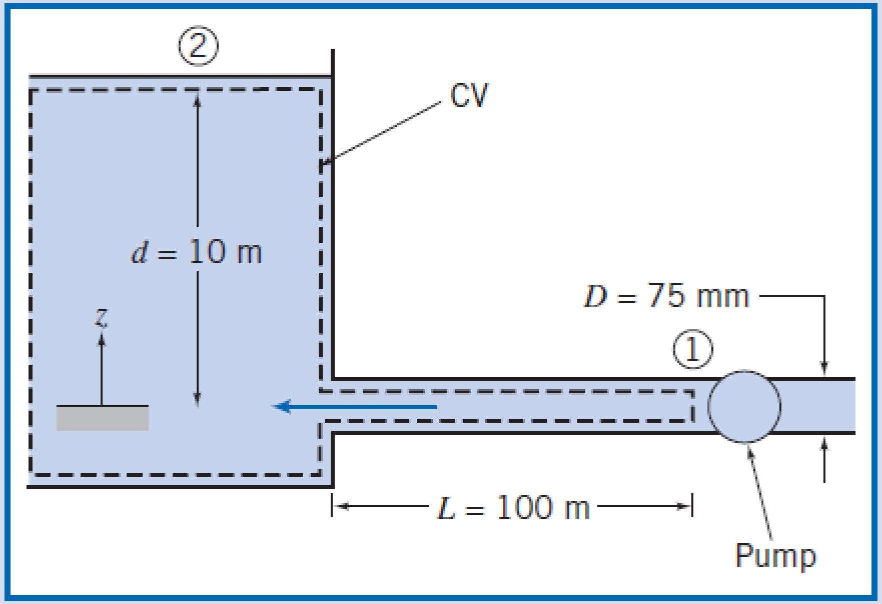
EXAMPLE 1

A 100-m length of smooth horizontal pipe is attached to a large reservoir. A pump is attached to the end of the pipe to pump water into the reservoir at a volume flow rate of 0.01 m^3/s. What pressure (gage) must the pump produce at the pipe to generate this flow rate? The inside diameter of the smooth pipe is 75 mm.

Water is pumped at 0.01 m^3/s through a 75-mm diameter smooth pipe, with L=100 m, into a constant-level reservoir of depth d=10 m.



$$\left(\frac{p_1}{\rho} + \alpha_1 \frac{\overline{V}_1^2}{2} + gz_1\right) - \left(\frac{p_2}{\rho} + \alpha_2 \frac{\overline{V}_2^2}{2} + gz_2\right) = h_{l_T} = h_l + h_{l_m}$$

$$h_l = f \frac{L}{D} \frac{\overline{V}^2}{2}$$

$$h_{l_m} = K \frac{\overline{V}^2}{2}$$

$p_1 = p_{\text{pump}}$ and $p_2 = 0$ (gage)

$\Delta p = p_1 - p_2 = p_{\text{pump}}$

$\overline{V}_1 = \overline{V}$, $\overline{V}_2 \approx 0$, K (exit loss) $= 1.0$,

$z_1 = 0$, then $z_2 = d$

$\alpha_1 \approx 1.0$

$$\frac{\Delta p}{\rho} + \frac{\overline{V}^2}{2} - gd = f \frac{L}{D} \frac{\overline{V}^2}{2} + \frac{\overline{V}^2}{2}$$

$$p_{\text{pump}} = \Delta p = \rho\left(gd + f\frac{L}{D}\frac{\overline{V}^2}{2}\right)$$

$$\overline{V} = \frac{Q}{A} = \frac{4Q}{\pi D^2} = \frac{4}{\pi} \times 0.01\,\frac{\text{m}^3}{\text{s}} \times \frac{1}{(0.075)^2\,\text{m}^2} = 2.26\,\text{m/s}$$

assuming water at 20°C, $\rho = 999\ \text{kg/m}^3$, and $\mu = 1.0 \times 10^{-3}\ \text{kg/(m} \cdot \text{s)}$

$$Re = \frac{\rho \overline{V} D}{\mu} = 999\,\frac{\text{kg}}{\text{m}^3} \times 2.26\,\frac{\text{m}}{\text{s}} \times 0.075\,\text{m} \times \frac{\text{m} \cdot \text{s}}{1.0 \times 10^{-3}\,\text{kg}} = 1.70 \times 10^5$$

For turbulent flow in a smooth pipe ($e = 0$), $f = 0.0162$

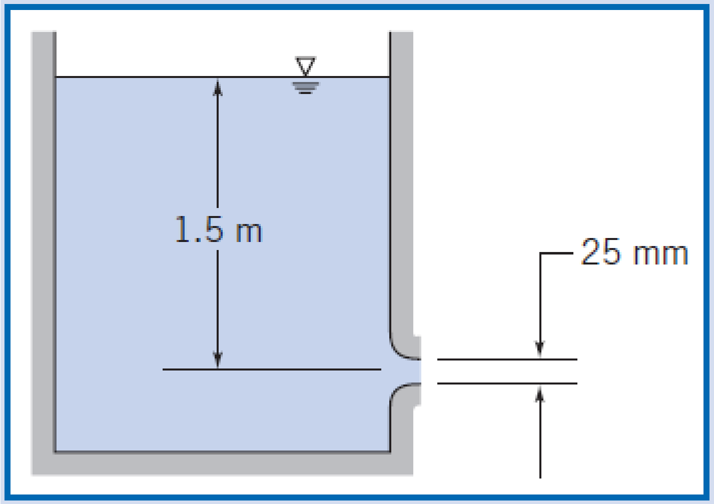
$$p_{\text{pump}} = \Delta p = \rho\left(gd + f\frac{L}{D}\frac{\overline{V}^2}{2}\right)$$

$$= 999\,\frac{\text{kg}}{\text{m}^3}\left(9.81\,\frac{\text{m}}{\text{s}^2} \times 10\ \text{m} + (0.0162) \times \frac{100\ \text{m}}{0.075\ \text{m}} \times \frac{(2.26)^2\,\text{m}^2}{2\ \text{s}^2}\right)$$

$$p_{\text{pump}} = 1.53 \times 10^5\ \text{N/m}^2\ (\text{gage}) \qquad p_{\text{pump}} = 153\,\text{kPa}\ (\text{gage})$$

EXAMPLE 2

Assume the static head available from the main is z_0=1.5 m and the nozzle exit diameter is D=25 mm. (The discharge is to atmospheric pressure.) Determine the increase in flow rate when a diffuser with N/R_1=3.0 and AR=2.0 is attached to the end of the nozzle.



Apply the energy equation for steady, incompressible pipe flow

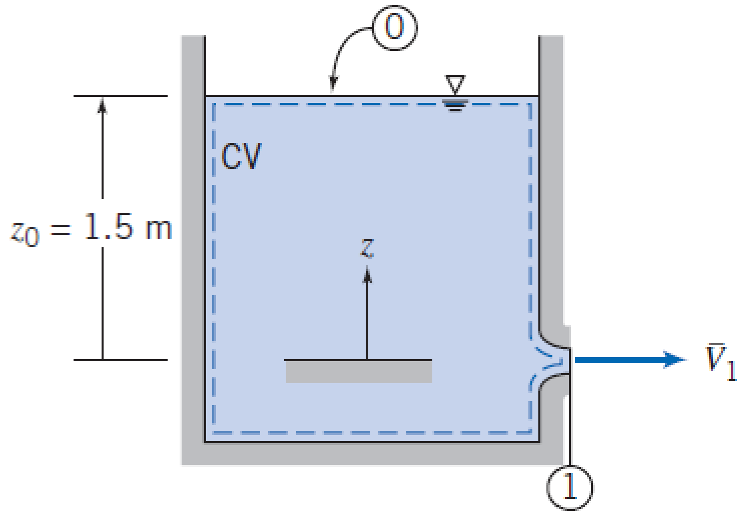
$$\frac{p_0}{\rho} + \alpha_0 \frac{\overline{V}_0^2}{2} + gz_0 = \frac{p_1}{\rho} + \alpha_1 \frac{\overline{V}_1^2}{2} + gz_1 + h_{l_T}$$

(1) $\overline{V}_0 \approx 0.$
(2) $\alpha_1 \approx 1.$

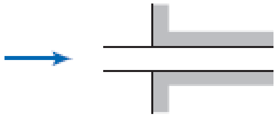
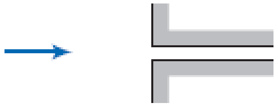
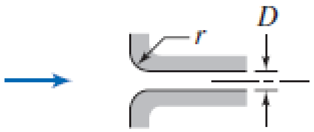
$$\cancel{\frac{p_0}{\rho}} + \alpha_0 \underbrace{\cancel{\frac{\overline{V}_0^2}{2}}}_{\approx 0(1)} + gz_0 = \cancel{\frac{p_1}{\rho}} + \underbrace{\cancel{\alpha_1}}_{\approx 1(2)} \frac{\overline{V}_1^2}{2} + \underbrace{\cancel{gz_1}}_{=0} + h_{l_T}$$

$$h_{l_T} = K_{\text{entrance}} \frac{\overline{V}_1^2}{2}$$

$$gz_0 = \frac{\overline{V}_1^2}{2} + K_{\text{entrance}} \frac{\overline{V}_1^2}{2} = (1 + K_{\text{entrance}}) \frac{\overline{V}_1^2}{2}$$



Minor Loss Coefficients for Pipe Entrances

Entrance Type		Minor Loss Coefficient, K[a]
Reentrant		0.78
Square-edged		0.5
Rounded		see below

r/D	0.02	0.06	≥ 0.15
K	0.28	0.15	0.04

[a] Based on $h_{l_m} = K(\overline{V}^2/2)$, where $\overline{V}$ is the mean velocity in the pipe.

$$K_{\text{entrance}} \approx 0.04$$

From Table

$$\overline{V}_1 = \sqrt{\frac{2gz_0}{1.04}} = \sqrt{\frac{2}{1.04} \times 9.81\,\frac{\text{m}}{\text{s}^2} \times 1.5\ \text{m}} = 5.32\ \text{m/s}$$

$$Q = \overline{V}_1 A_1 = \overline{V}_1 \frac{\pi D_1^2}{4} = 5.32\,\frac{\text{m}}{\text{s}} \times \frac{\pi}{4} \times (0.025)^2\ \text{m}^2 = 0.00261\ \text{m}^3/\text{s}$$

EXAMPLE 3

A fluid at 20°C flows at 850 cm^3/s through an 8-cm-diameter pipe. Determine the entrance length if the fluid is (a) hydrogen; (b) air; (c) gasoline; (d) water; (e) mercury; and (f) glycerin..

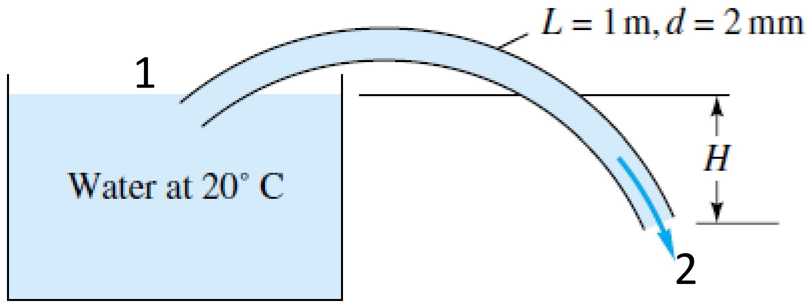
$$Laminar: \frac{L_{entrance}}{D} \approx 0.06 \mathrm{Re}_D; \quad Turbulent: \frac{L_{entrance}}{D} \approx 4.4 \mathrm{Re}_D^{1/6}$$

Fluid	ν, m^2/s	Re_D	Type of flow	L_{entr}/D	Entrance Length
(a) Hydrogen	1.08E–4	125	Laminar	7.5	0.6 m
(b) Air	1.5E–5	900	Laminar	54.0	4.32 m
(c) Gasoline	4.3E–7	31400	Turbulent	24.7	1.98 m
(d) Water	1.0E–6	13500	Turbulent	21.5	1.72 m
(e) Mercury	1.15E–7	117000	Turbulent	30.8	2.46 m
(f) Glycerin	1.18E–3	11.4	Laminar	0.68	0.055 m

EXAMPLE 4

Water at 20°C is to be siphoned through a tube 1 m long and 2 mm in diameter. Is there any height *H* for which the flow might not be laminar? What is the flow rate if *H*= 50 cm? Neglect the tube curvature.

For water at 20°C, take ρ = 998 kg/m3 and μ = 0.001 kg/m·s.



$$\frac{p_{atm}}{\rho g} + \frac{0^2}{2g} + z_1 = \frac{p_{atm}}{\rho g} + \frac{V_{tube}^2}{2g} + z_2 + h_f$$

$$H - \frac{V^2}{2g} = h_f = \frac{32\mu L}{\rho g d^2} V$$

if *H* = 50 cm $\quad 0.5 - \frac{V^2}{2(9.81)} = \frac{32(0.001)(1.0)V}{(998)(9.81)(0.002)^2} \quad V \approx 0.590\ \frac{m}{s}$

$$Q_{H=50\text{ cm}} = \frac{\pi}{4}(0.002)^2(0.590) = 1.85E{-6}\ \frac{m^3}{s} \approx \mathbf{0.0067}\ \frac{\mathbf{m^3}}{\mathbf{h}}$$

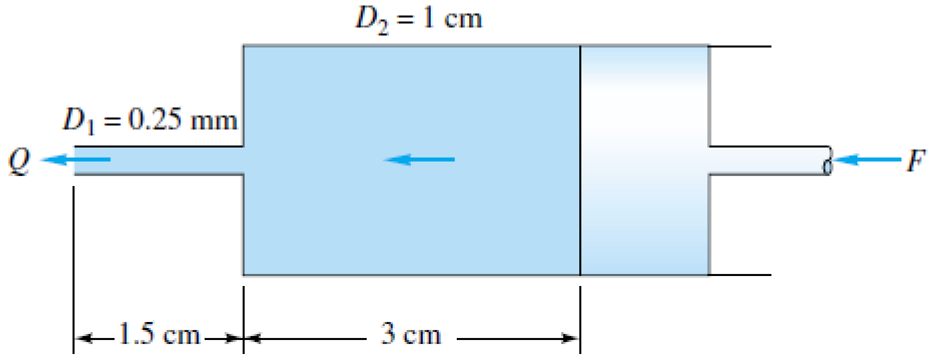
$$Re = (998)(0.590)(0.002)/(0.001) \approx 1180 \text{ (OK, laminar flow)}$$

$$\mathbf{Re = 2000 \text{ at } H \approx 0.87\ m.}$$

EXAMPLE 5

A steady push on the piston causes a flow rate Q = 0.15 cm^3/s through the needle. The fluid has ρ=900 kg/m^3 and μ= 0.002 kg/(m s). What force F is required to maintain the flow?.

$$V_1 = \frac{Q}{A} = \frac{0.15}{(\pi/4)(0.025)^2} = 306 \text{ cm/s}$$



$$\frac{p_2}{\rho g} + \frac{V_2^2}{2g} + z_2 = \frac{p_1}{\rho g} + \frac{V_1^2}{2g} + z_1 + h_{f1} + h_{f2}$$

$$z_1 = z_2,\ V_2 \approx 0,\ h_{f2} \approx 0$$

$$\frac{p_2 - p_1}{\rho g} = h_{f1} + \frac{V_1^2}{2g} = \frac{32(0.002)(0.015)(3.06)}{(900)(9.81)(0.00025)^2} + \frac{(3.06)^2}{2(9.81)} \approx 5.79 \text{ m}$$

$$F = \Delta p A_{piston} = (900)(9.81)(5.79)\frac{\pi}{4}(0.01)^2 \approx \mathbf{4.0\ N}$$

EXAMPLE 6

In Figure all pipes are 8-cm-diameter cast iron. Determine the flow rate from reservoir 1 if valve *C* is (*a*) closed and (*b*) open, K_{valve} =0.5.

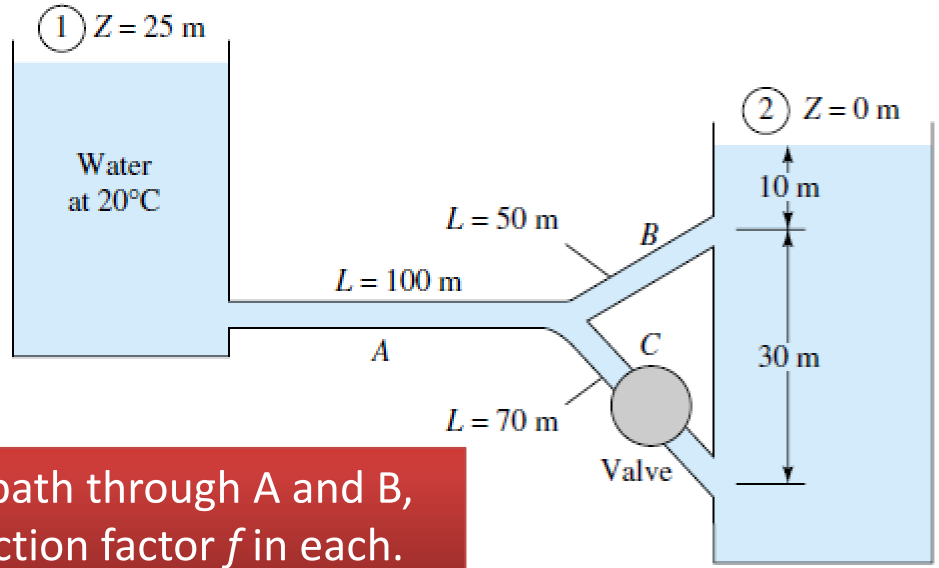
Solution: For water at 20°C, take $\rho = 998$ kg/m^3 and $\mu = 0.001$ kg/m·s. For cast iron, $\varepsilon \approx$ 0.26 mm, hence $\varepsilon/d = 0.26/80 \approx 0.00325$ for all three pipes. Note $p_1 = p_2$, $V_1 = V_2 \approx 0$. These are long pipes, but we might wish to account for minor losses anyway:

sharp entrance at A: $K_1 \approx 0.5$

line junction from A to B: $K_2 \approx 0.9$

branch junction from A to C: $K_3 \approx 1.3$

two submerged exits: $K_B = K_C \approx 1.0$



If valve C is closed, we have a straight *series* path through A and B, with the same flow rate Q, velocity V, and friction factor *f* in each.

$$z_1 - z_2 = h_{fA} + \sum h_{mA} + h_{fB} + \sum h_{mB}$$

$$f = \text{fcn}\left(\text{Re}, \frac{\varepsilon}{d}\right) \qquad 25 \text{ m} = \frac{V^2}{2(9.81)}\left[f\,\frac{100}{0.08} + 0.5 + 0.9 + f\,\frac{50}{0.08} + 1.0\right]$$

Guess $f \approx f_{fully\ rough} \approx 0.027$, then $V \approx 3.04$ m/s,

$Re \approx 998(3.04)(0.08)/(0.001) \approx \underline{\mathbf{243000}}$,

$\varepsilon/d = 0.00325$, then $f \approx 0.0273$ (converged).

Then the velocity through A and B is V =3.03 m/s,

and $Q = (\pi/4)(0.08)2(3.03) \approx$ **0.0152 m³/s**.

[redacted]

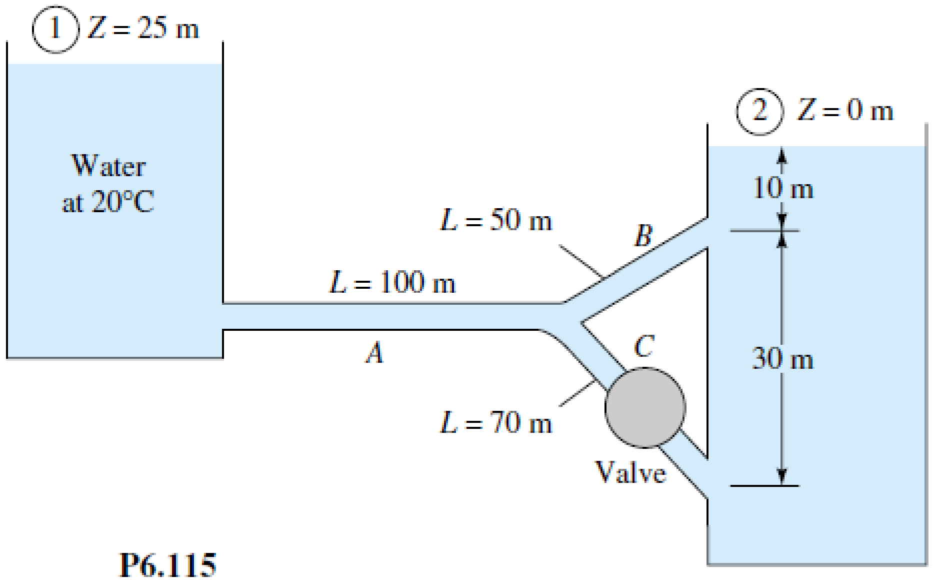
If valve C is open, we have parallel flow through B and C, with

$$Q_A = Q_B + Q_C$$

with *d* constant,

$$V_A = V_B + V_C.$$

The total head loss is the same for paths A-B and A-C:



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$$z_1 - z_2 = h_{fA} + \sum h_{mA\text{-}B} + h_{fB} + \sum h_{mB} = h_{fA} + \sum h_{mA\text{-}C} + h_{fC} + \sum h_{mC}$$

$$\text{or:} \quad 25 = \frac{V_A^2}{2(9.81)}\left[f_A \frac{100}{0.08} + 0.5 + 0.9\right] + \frac{V_B^2}{2(9.81)}\left[f_B \frac{50}{0.08} + 1.0\right]$$

$$= \frac{V_A^2}{2(9.81)}\left[f_A \frac{100}{0.08} + 0.5 + 1.3\right] + \frac{V_C^2}{2(9.81)}\left[f_C \frac{70}{0.08} + 1.0\right]$$

plus the additional relation $V_A = V_B + V_C$. Guess $f \approx f_{fully\ rough} \approx 0.027$ for all three pipes;

$$2g(25) = 490.5 = V_A^2(1250f_A + 1.4) + V_B^2(625f_B + 1) = V_A^2(1250f_A + 1.8) + V_C^2(875f_C + 1)$$

$$\text{If } f \approx 0.027, \ \text{solve (laboriously)} \ V_A \approx 3.48 \text{ m/s}, V_B \approx 1.91 \text{ m/s}, V_C \approx 1.57 \text{ m/s}.$$

$$\text{Compute} \ \ Re_A = 278000, \quad f_A \approx 0.0272, \quad Re_B = 153000, \quad f_B = 0.0276,$$
$$Re_C = 125000, \quad f_C = 0.0278$$

Repeat once for convergence: VA ≈ 3.46 m/s, VB ≈ 1.90 m/s, VC ≈ 1.56 m/s.

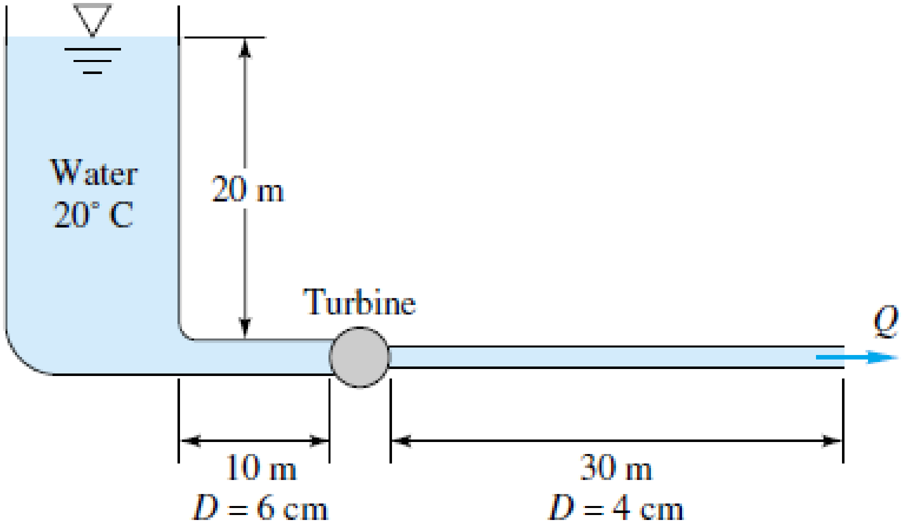
The flow rate from reservoir (1) is

Q_A = (π/4)(0.08)2(3.46) ≈ **0.0174 m³/s**. (14% more)

EXAMPLE 7

The small turbine extracts 400 W of power from the water flow. Both pipes are wrought iron. Compute the flow rate *Q* m^3/h. Sketch the EGL and HGL accurately.

For water, $\rho = 998\ kg/m3$
$\mu = 0.001\ kg/m \cdot s$.
For wrought iron,
$\varepsilon \approx 0.046$ mm,
$\varepsilon/d1 = 0.046/60 \approx 0.000767$
$\varepsilon/d2 = 0.046/40 \approx 0.00115$.
$V_1 \approx 0$ and $p_1 = p_2$



$$z_1 - z_2 = 20\ m = \frac{V_2^2}{2g} + h_{f2} + h_{f1} + h_{turbine}$$

$$h_{f1} = f_1 \frac{L_1}{d_1}\frac{V_1^2}{2g} \quad \text{and} \quad h_{f2} = f_2 \frac{L_2}{d_2}\frac{V_2^2}{2g}$$

$$h_{turbine} = \frac{P}{\rho g Q} = \frac{400\ W}{998(9.81)Q}$$

$$Q = \frac{\pi}{4} d_1^2 V_1 = \frac{\pi}{4} d_2^2 V_2$$

[redacted]

$$h_{turb} = \frac{400}{998(9.81)Q} = 20 - \frac{8f_1L_1Q^2}{\pi^2 g d_1^5} - \frac{8f_2L_2Q^2}{\pi^2 g d_2^5} - \frac{8Q^2}{\pi^2 g d_2^4}$$

$$\text{Guess} \quad Q = 0.003\ \frac{m^3}{s}, \quad \text{then} \ \ Re_1 = \frac{4\rho Q}{\pi\mu d_1} = 63500$$

$$f_{1,Moody} \approx 0.0226 \qquad\qquad Re_2 = 95300, \quad f_2 \approx 0.0228$$

$h_{turb} \approx 9.9$ meters. For $Q \approx 0.00413\ m^3/s \approx$ **15 m^3/h**