Fluid Mechanics: Fundamentals and Applications, 2nd Edition
Yunus A. Cengel, John M. Cimbala
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Chapter 3
PRESSURE AND FLUID STATICS

Prof. Dr. Ali PINARBAŞI
Yildiz Technical University
Mechanical Engineering Department
Yildiz, ISTANBUL

PRESSURE AND FLUID STATICS

John Ninomiya flying a cluster of 72 helium-filled balloons over Temecula, California in April of 2003. The helium balloons displace approximately 230 m^3 of air, providing the necessary buoyant force.

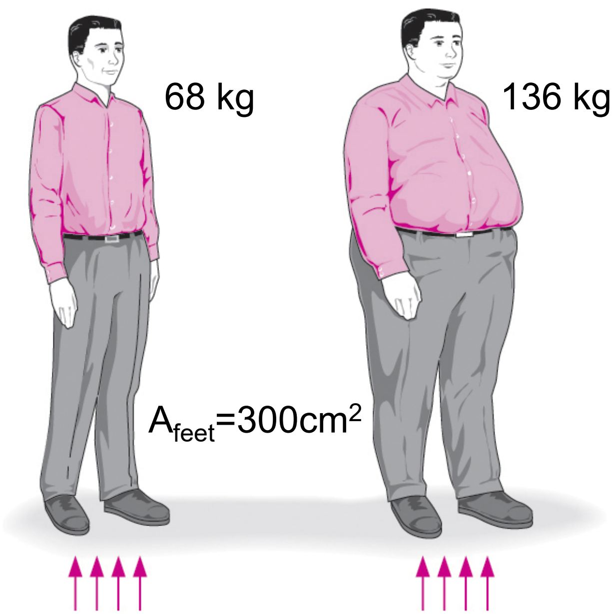
Don’t try this at home!

Objectives

- Determine the variation of pressure in a fluid at rest
- Calculate pressure using various kinds of manometers, the barometer, and pressure measurement devices.
- Calculate the forces exerted by a fluid at rest on plane or curved submerged surfaces.
- Analyze the stability of floating and submerged bodies.
- Analyze the rigid-body motion of fluids in containers during linear acceleration or rotation.

3–1 PRESSURE

Pressure: A normal force exerted by a fluid per unit area.
Pressure only deal with a gas or a liquid.
The counterpart of pressure in solids is normal stress.



P=68/300=0.23 kgf/cm²

The normal stress (or "pressure") on the feet of a chubby person is much greater than on the feet of a slim person

$$1 \text{ Pa} = 1 \text{ N/m}^2$$

Some basic pressure gages.

$$1 \text{ bar} = 10^5 \text{ Pa} = 0.1 \text{ MPa} = 100 \text{ kPa}$$

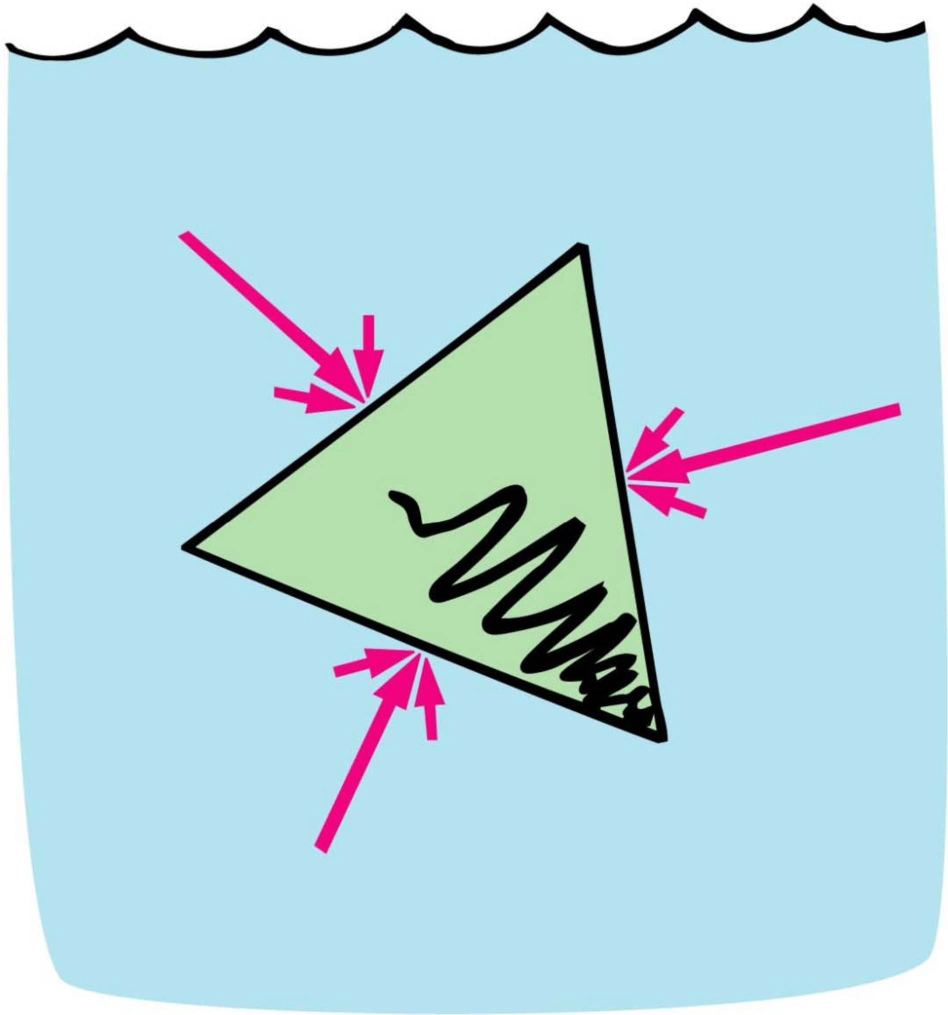
$$1 \text{ atm} = 101{,}325 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ bars}$$

$$\begin{aligned} 1 \text{ kgf/cm}^2 &= 9.807 \text{ N/cm}^2 = 9.807 \times 10^4 \text{ N/m}^2 = 9.807 \times 10^4 \text{ Pa} \\ &= 0.9807 \text{ bar} \\ &= 0.9679 \text{ atm} \end{aligned}$$

Pressure in a Fluid

- The pressure is just the weight of all the fluid above you Atmospheric pressure is just the weight of all the air above on area on the surface of the earth
- In a swimming pool the pressure on your body surface is just the weight of the water above you (plus the air pressure above the water)
- So, the only thing that counts in fluid pressure is the gravitational force acting on the mass ABOVE you
- The deeper you go, the more weight above you and the more pressure
- Go to a mountain top and the air pressure is lower

Pressure in a Fluid



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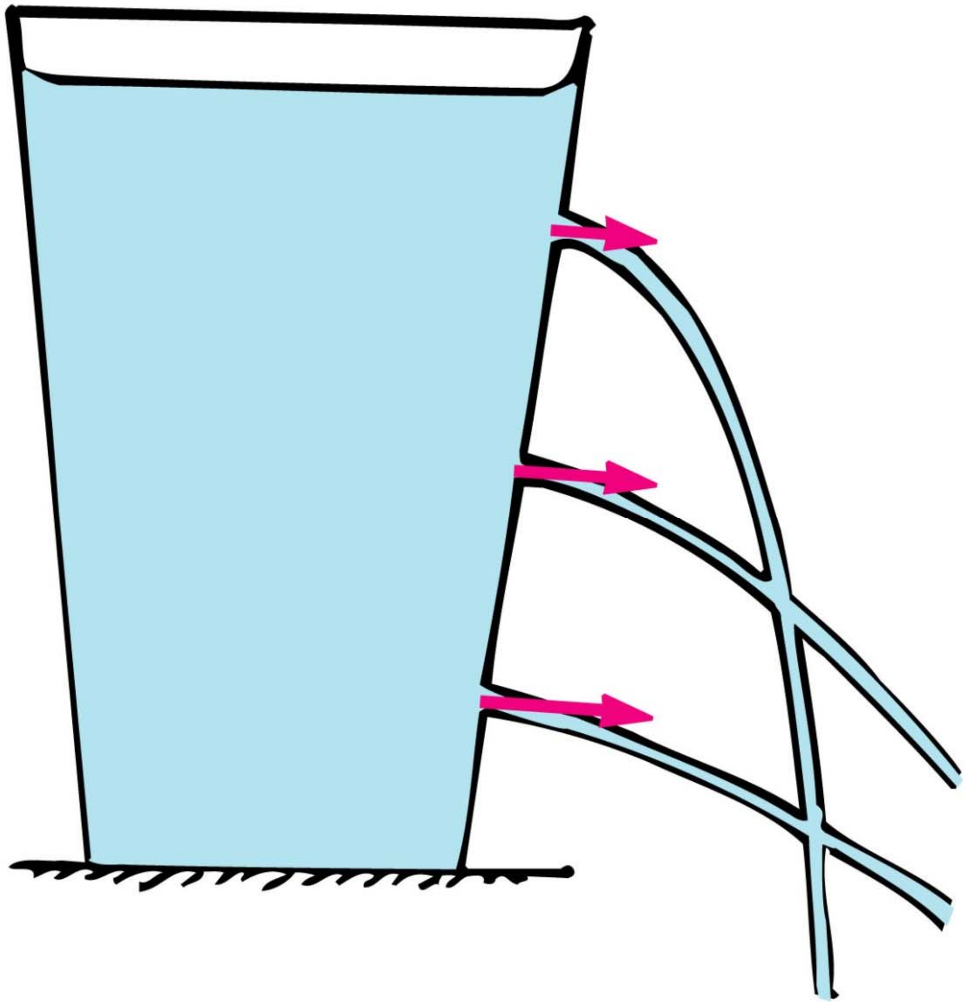
Pressure acts perpendicular to the surface and increases at greater depth.

Pressure in a Fluid



Absolute pressure: The actual pressure at a given position. It is measured relative to absolute vacuum (i.e., absolute zero pressure).

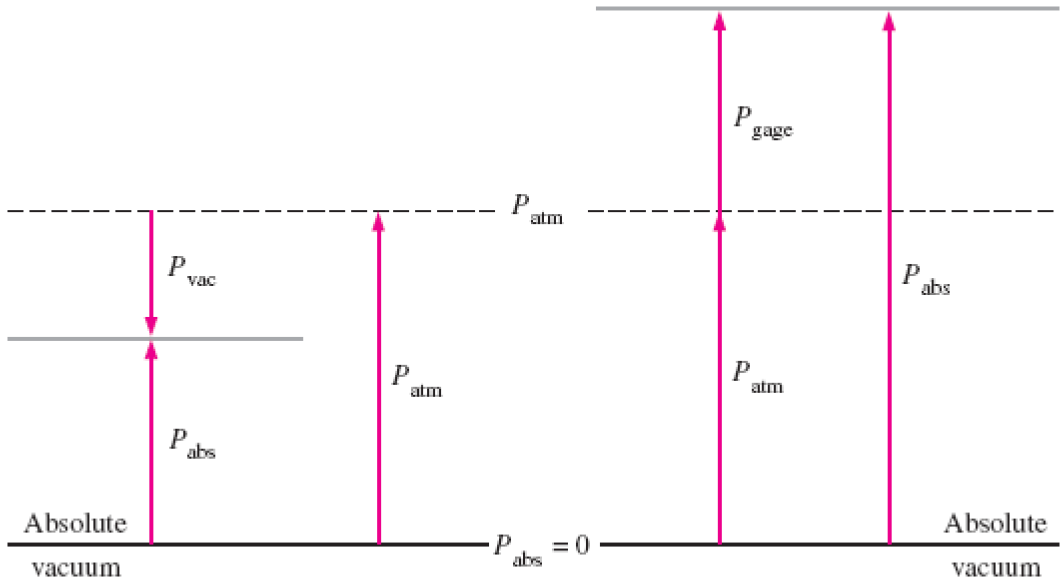
Gage pressure: The difference between the absolute pressure and the local atmospheric pressure. Most pressure-measuring devices are calibrated to read zero in the atmosphere, and so they indicate gage pressure.

Vacuum pressures: Pressures below atmospheric pressure.

Throughout this text, the pressure ***P*** will denote ***absolute pressure*** unless specified otherwise.

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$



EXAMPLE 3–1

A vacuum gage connected to a chamber reads 99.97 kPa at a location where the atmospheric pressure is 39.8 kPa. Determine the absolute pressure in the chamber.

Solution: The gage pressure of a vacuum chamber is given. The absolute pressure in the chamber is to be determined.

Analysis The absolute pressure is easily determined from;

$$P_{gage} = P_{abs} - P_{atm}$$

$$P_{vac} = P_{atm} - P_{abs}$$

$$P_{abs} = P_{atm} = P_{vac} = 99.97\text{-}39.8 = 60.17 \text{ kPa}$$

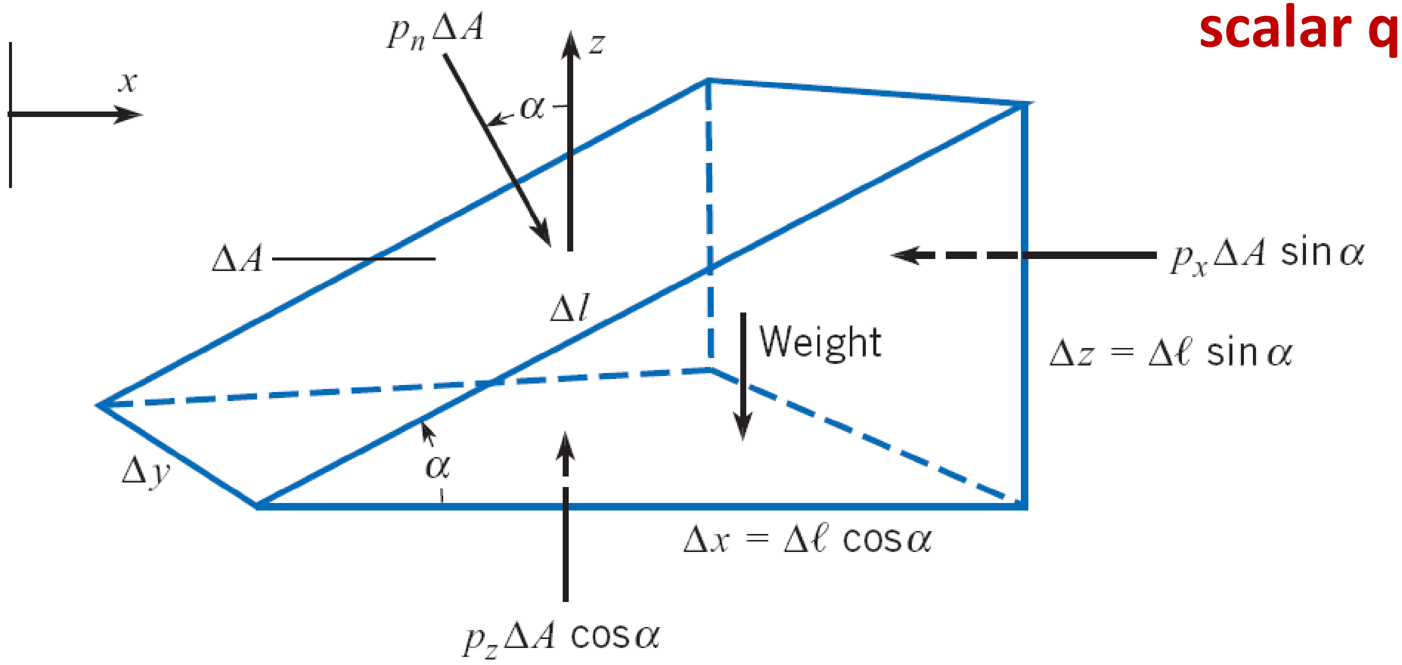
Discussion Note that the *local* value of the atmospheric pressure is used when determining the absolute pressure.

Pressure at a Point

From Newton's second law, a force balance in the *x*- and *z* directions, gives

Forces acting on a wedge-shaped fluid element in equilibrium.

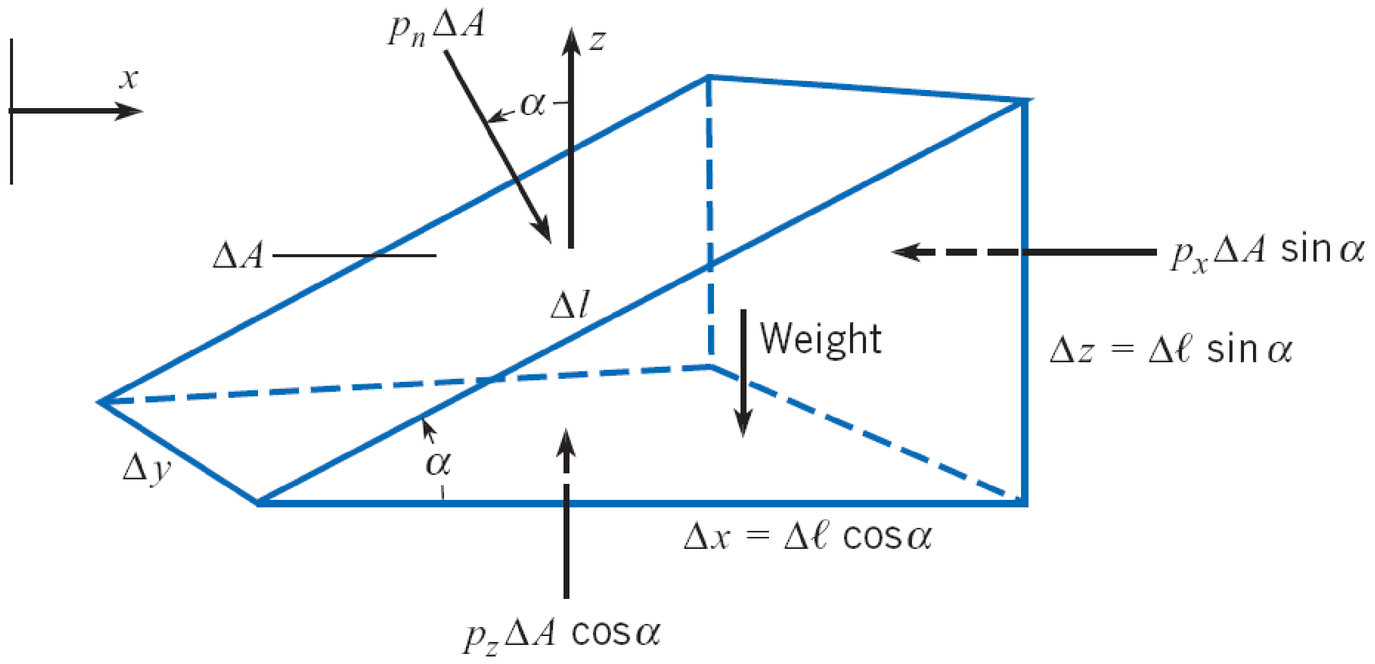
Pressure is a scalar quantity



Force balance in the x-direction:

$$(p_n \,\Delta y\, \Delta\ell) \sin\alpha = p_x (\Delta y\, \Delta\ell\, \sin\alpha)$$

$$\Rightarrow \qquad p_n = p_x$$



Pressure at any point in a fluid is the same in all directions.

Pressure is the *compressive force* per unit area but it is not a vector.

Pressure has magnitude but not a specific direction, and thus it is a scalar quantity.

Force balance in the z-direction:

$$-\underbrace{(p_n\ \Delta y\ \Delta\ell)\cos\alpha}_{\text{Vertical force on } \Delta A} + \underbrace{p_z(\Delta y\ \Delta\ell\cos\alpha)}_{\text{Vertical force on lower boundary}} - \underbrace{\frac{1}{2}\gamma\ \Delta\ell\ \cos\alpha\ \Delta\ell\ \sin\alpha\ \Delta y}_{\text{Total weight of wedge element}} = 0$$

$\gamma = \rho g$ = specific weight

$$-(p_n\ \Delta y\ \Delta \ell)\cos\alpha + p_z(\Delta y\ \Delta \ell \cos\alpha) - \frac{1}{2}\gamma\ \Delta \ell\ \cos\alpha\ \Delta \ell\ \sin\alpha\ \Delta y = 0$$

Divide through by $\Delta \ell\ \Delta y\ \cos\alpha$

$$-p_n + p_z - \frac{1}{2}\gamma \Delta \ell = 0$$

Now shrink the element to a point: $\Delta \ell \to 0$

$$\Rightarrow \qquad p_n = p_z$$

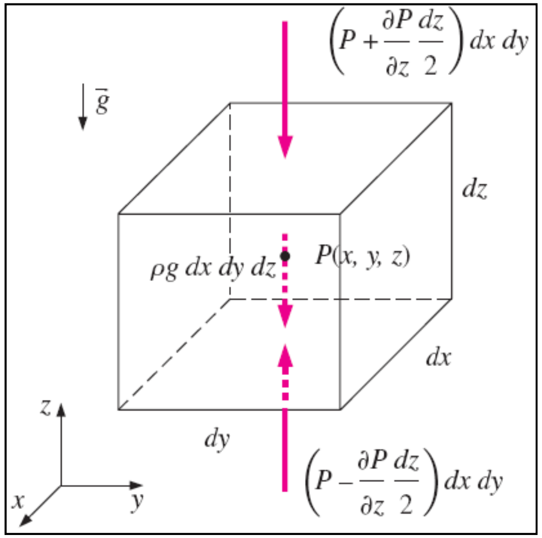
This can be done for any orientation α, so

$$\boxed{p_n = p_x = p_y = p_z}$$

FLUIDS IN RIGID-BODY MOTION

Pressure at a given point has the same magnitude in all directions, and thus it is a *scalar* function.

In this section we obtain relations for the variation of pressure in fluids moving like a solid body with or without acceleration in the absence of any shear stresses (i.e., no motion between fluid layers relative to each other).



$$\delta \vec{F} = \delta m \cdot \vec{a}$$

$$\delta m = \rho\, dV = \rho\, dx\, dy\, dz$$

$$\delta F_{S,z} = \left(P - \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy - \left(P + \frac{\partial P}{\partial z}\frac{dz}{2}\right) dx\, dy = -\frac{\partial P}{\partial z}\, dx\, dy\, dz$$

$$\delta F_{S,x} = -\frac{\partial P}{\partial x}\, dx\, dy\, dz \quad \text{and} \quad \delta F_{S,y} = -\frac{\partial P}{\partial y}\, dx\, dy\, dz$$

$$\begin{aligned}\delta \vec{F}_S &= \delta F_{S,x}\vec{i} + \delta F_{S,y}\vec{j} + \delta F_{S,z}\vec{k} \\ &= -\left(\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}\right) dx\, dy\, dz = -\vec{\nabla} P\, dx\, dy\, dz\end{aligned}$$

$$\vec{\nabla} P = \frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k}$$

$$\delta \vec{F}_{B,z} = -g\delta m\vec{k} = -\rho g\, dx\, dy\, dz\vec{k}$$

$$\delta \vec{F} = \delta \vec{F}_S + \delta \vec{F}_B = -(\vec{\nabla} P + \rho g\vec{k})\, dx\, dy\, dz$$

Substituting into Newton's second law of motion

$$\delta \vec{F} = \delta m \cdot \vec{a} = \rho\, dx\, dy\, dz \cdot \vec{a}$$

The general **equation of motion** for a fluid that acts as a rigid body (no shear stresses) is determined to be;

Rigid-body motion of fluids: $\vec{\nabla} P + \rho g\vec{k} = -\rho\vec{a}$

$$\frac{\partial P}{\partial x}\vec{i} + \frac{\partial P}{\partial y}\vec{j} + \frac{\partial P}{\partial z}\vec{k} + \rho g\vec{k} = -\rho(a_x\vec{i} + a_y\vec{j} + a_z\vec{k})$$

Accelerating fluids: $\frac{\partial P}{\partial x} = -\rho a_x,\quad \frac{\partial P}{\partial y} = -\rho a_y,\quad \text{and}\quad \frac{\partial P}{\partial z} = -\rho(g + a_z)$

Special Case 1: Fluids at Rest

For fluids at rest or moving on a straight path at constant velocity, all components of acceleration are zero, and the relations reduce to

$$\textit{Fluids at rest:} \qquad \frac{\partial P}{\partial x} = 0, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{dP}{dz} = -\rho g$$

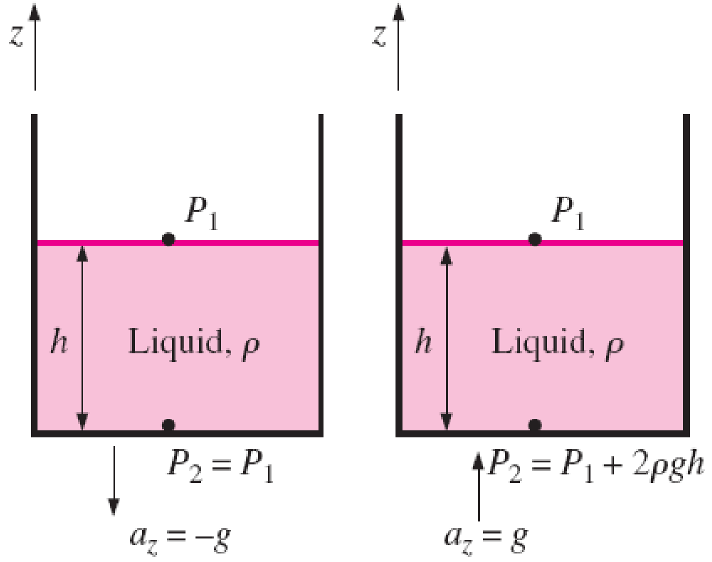
The pressure remains constant in any horizontal direction (*P* is independent of *x* and *y*) and varies only in the vertical direction as a result of gravity [and thus $P=P(z)$]. These relations are applicable for both compressible and incompressible fluids.

Special Case 2: Free Fall of a Fluid Body

A freely falling body accelerates under the influence of gravity. When the air resistance is negligible, the acceleration of the body equals the gravitational acceleration, and acceleration in any horizontal direction is zero.
Therefore, $a_x = a_y = 0$ and $a_z = -g$.

Free-falling fluids: $$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial y} = \frac{\partial P}{\partial z} = 0 \quad \rightarrow \quad P = \text{constant}$$

In a frame of reference moving with the fluid, it behaves like it is in an environment with zero gravity. Also, the gage pressure in a drop of liquid in free fall is zero throughout.



(*a*) Free fall of a liquid

(*b*) Upward acceleration of a liquid with $a_z = +g$

The effect of acceleration on the pressure of a liquid during free fall and upward acceleration.

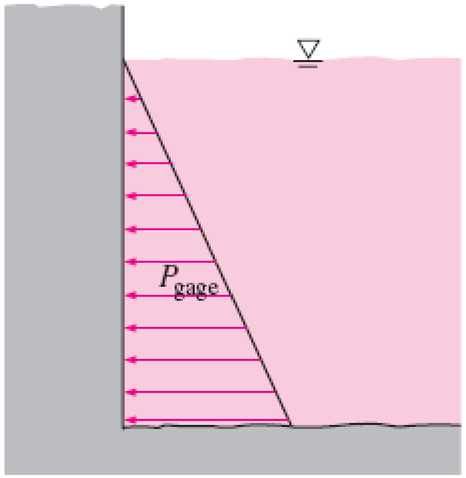
Assuming the density of the fluid to be constant, a force balance in the vertical *z*-direction gives;

$$\sum F_z = ma_z = 0: \qquad P_2\,\Delta x - P_1\,\Delta x - \rho g\,\Delta x\,\Delta z = 0$$

$$W = mg = \rho g\,\Delta x\,\Delta z$$

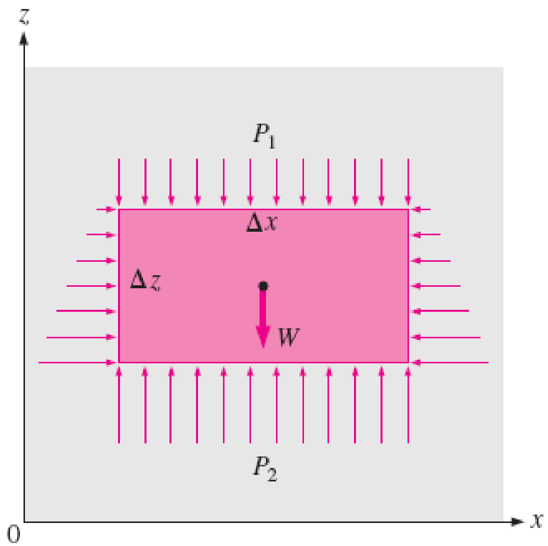
$$\Delta P = P_2 - P_1 = \rho g\,\Delta z = \gamma_s\,\Delta z$$

$$P = P_{\text{atm}} + \rho g h \qquad \text{or} \qquad P_{\text{gage}} = \rho g h$$



$$\frac{dP}{dz} = -\rho g$$

$$\Delta P = P_2 - P_1 = -\int_1^2 \rho g\, dz$$



The pressure of a fluid at rest increases with depth (as a result of added weight).

Free-body diagram of a rectangular fluid element in equilibrium.

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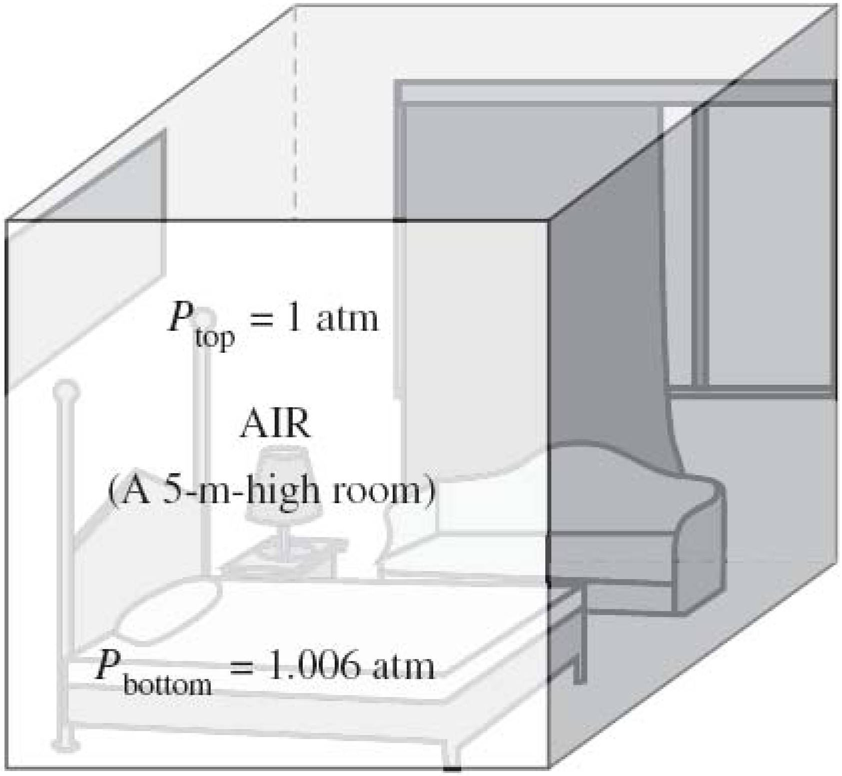
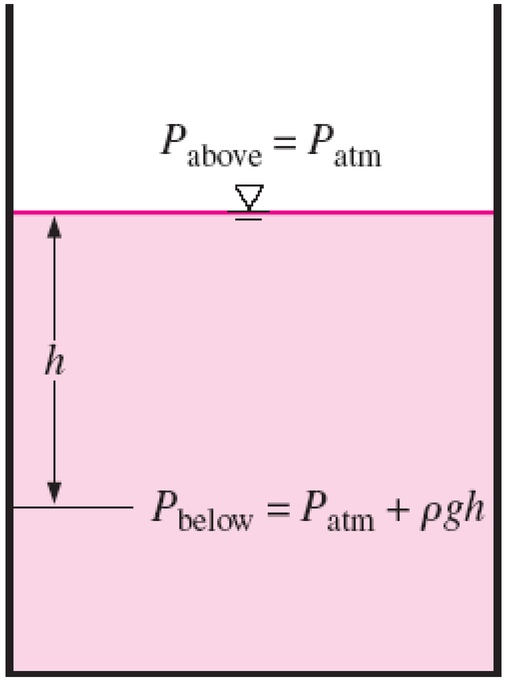


In a room filled with a gas, the variation of pressure with height is negligible.



Pressure in a liquid at rest increases linearly with distance from the free surface.

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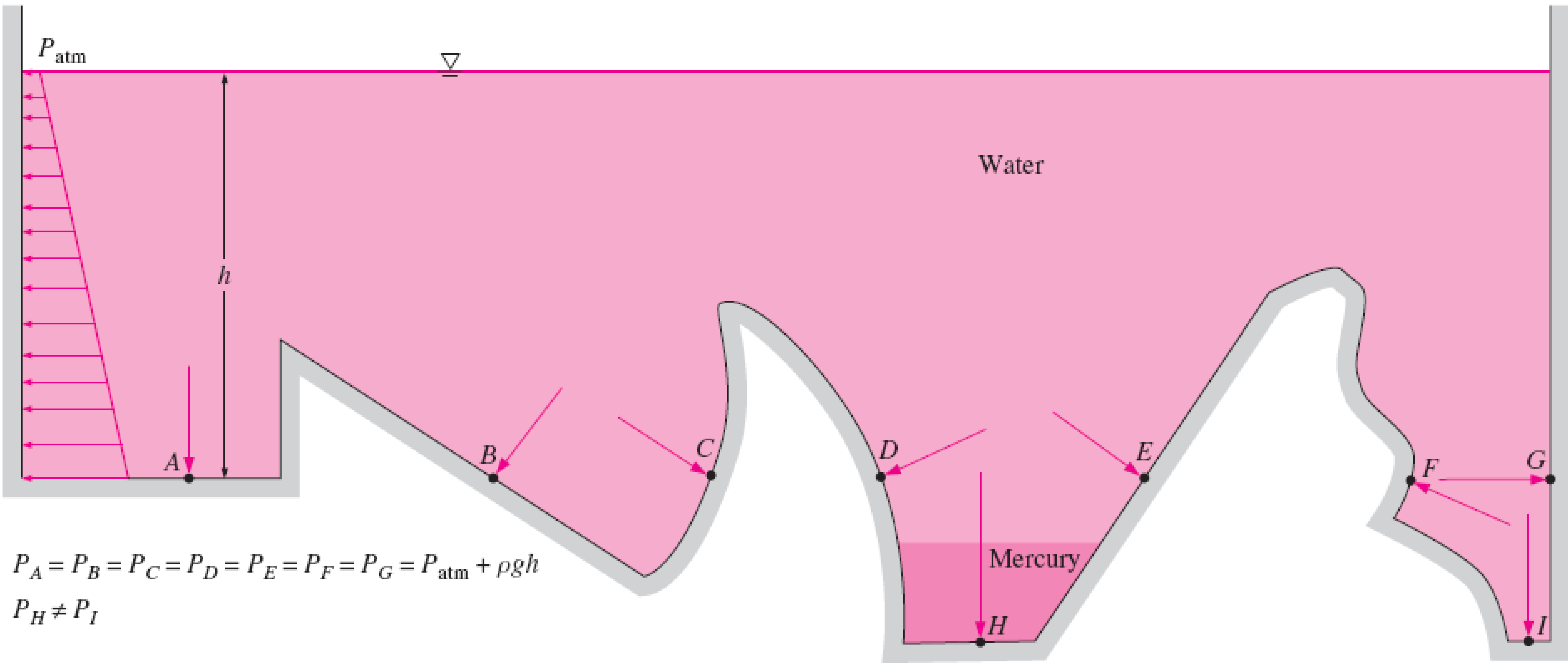


The pressure is the same at all points on a horizontal plane in a given fluid regardless of geometry, provided that the points are interconnected by the same fluid.

[redacted]

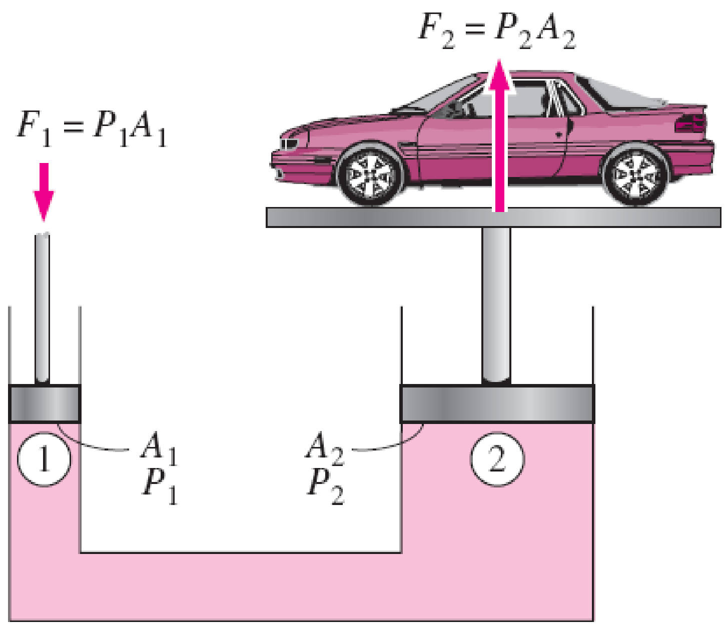
Pascal's law: The pressure applied to a confined fluid increases the pressure throughout by the same amount.

$$P_1 = P_2 \quad \rightarrow \quad \frac{F_1}{A_1} = \frac{F_2}{A_2} \quad \rightarrow \quad \frac{F_2}{F_1} = \frac{A_2}{A_1}$$

The area ratio A_2/A_1 is called the *ideal mechanical advantage* of the hydraulic lift.

Using a hydraulic car jack with a piston area ratio of $A_2 / A_1 = 10$,

For example, a person can lift a 1000-kg car by applying a force of just 100 kgf (=908 N)



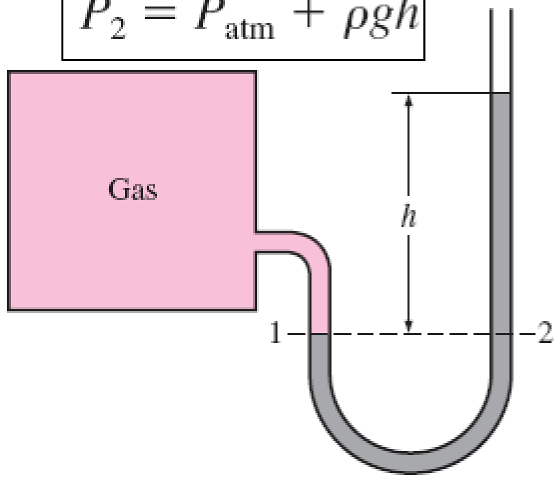
Lifting of a large weight by a small force by the application of Pascal's law.

3–2 PRESSURE MEASUREMENT DEVICES

The Manometer

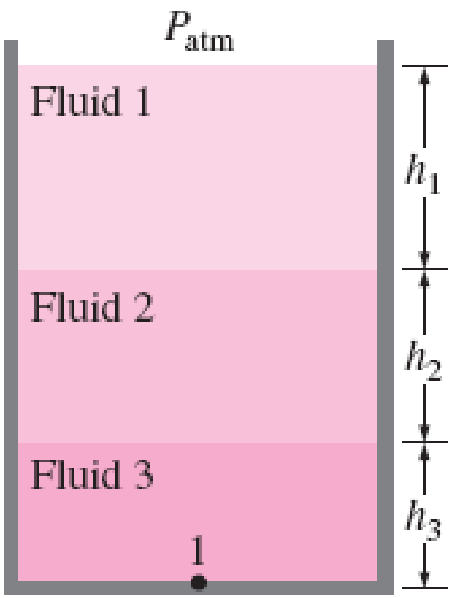
It is commonly used to measure small and moderate pressure differences. A manometer contains one or more fluids such as mercury, water, alcohol, or oil.

$$P_2 = P_{\text{atm}} + \rho g h$$

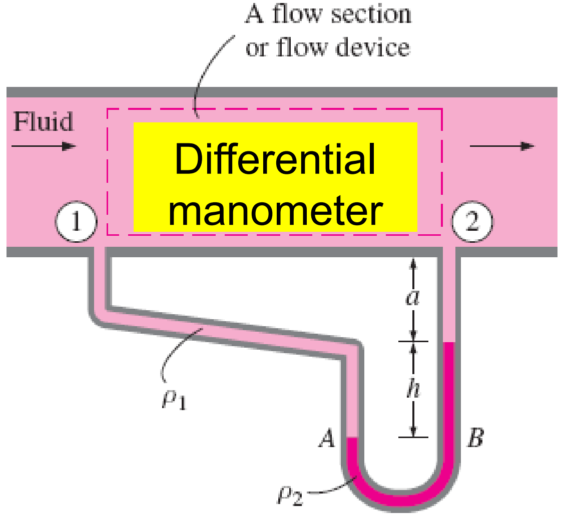


The basic manometer.

$$P_{\text{atm}} + \rho_1 g h_1 + \rho_2 g h_2 + \rho_3 g h_3 = P_1$$



In stacked-up fluid layers, the pressure change across a fluid layer of density ρ and height h is $\rho g h$.



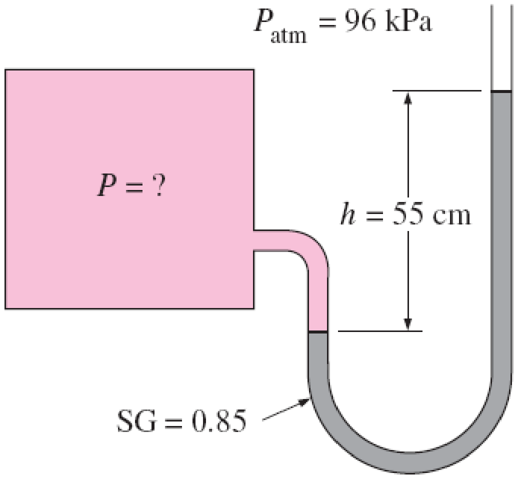
Measuring the pressure drop across a flow section or a flow device

$$P_1 + \rho_1 g(a + h) - \rho_2 g h - \rho_1 g a = P_2$$

$$P_1 - P_2 = (\rho_2 - \rho_1) g h$$

EXAMPLE 3-2

A manometer is used to measure the pressure in a tank. The fluid used has a specific gravity of 0.85, and the manometer column height is 55 cm. If the local atmospheric pressure is 96 kPa, determine the absolute pressure within the tank.



Solution The reading of a manometer attached to a tank and the atmospheric pressure are given. The absolute pressure in the tank is to be determined.

Assumptions The fluid in the tank is a gas whose density is much lower than the density of manometer fluid.

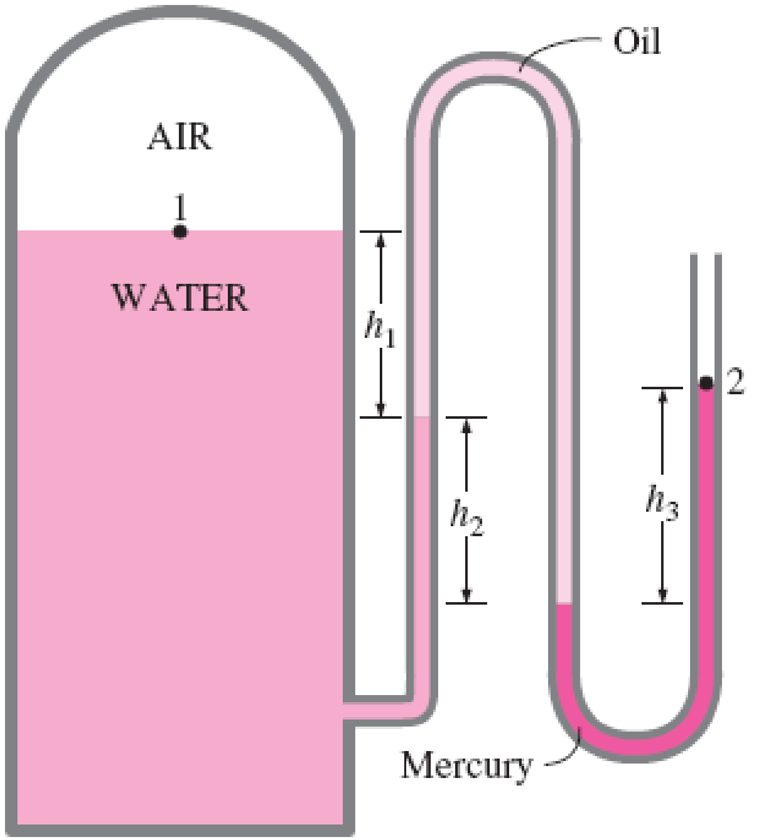
$$\rho = \text{SG}\,(\rho_{H_2O}) = (0.85)(1000\ \text{kg/m}^3) = 850\ \text{kg/m}^3$$

$$\begin{aligned} P &= P_{atm} + \rho g h \\ &= 96\ \text{kPa} + (850\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.55\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{100.6\ kPa} \end{aligned}$$

Discussion Note that the gage pressure in the tank is 4.6 kPa.

EXAMPLE 3–3

The water in a tank is pressurized by air, and the pressure is measured by a multi fluid manometer. The tank is located on a mountain at an altitude of 1400 m where the atmospheric pressure is 85.6 kPa. Determine the air pressure in the tank if h_1=0.1 m, h_2=0.2 m, and h_3=0.35 m. Take the densities of water, oil, and mercury to be 1000 kg/m^3, 850 kg/m^3 and 13,600 kg/m^3 respectively.



[redacted]

Solution The pressure in a pressurized water tank is measured by a multifluid manometer.
Assumption The air pressure in the tank is uniform (its variation with elevation is negligible due to its low density), and thus we can determine the pressure at the air–water interface.

$$P_1 + \rho_{\text{water}} g h_1 + \rho_{\text{oil}} g h_2 - \rho_{\text{mercury}} g h_3 = P_{\text{atm}}$$

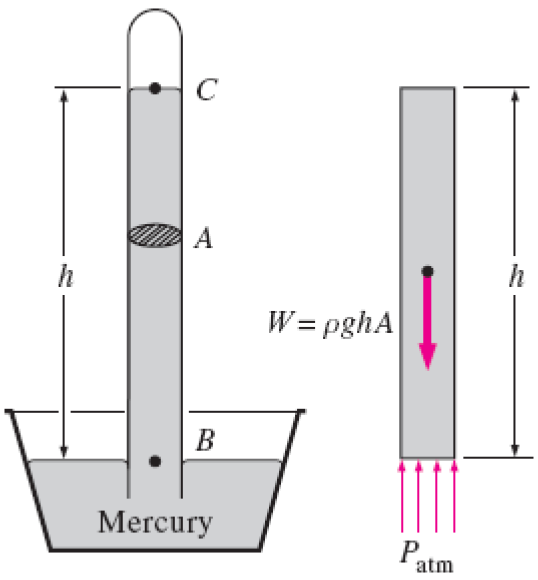
$$\begin{aligned} P_1 &= P_{\text{atm}} - \rho_{\text{water}} g h_1 - \rho_{\text{oil}} g h_2 + \rho_{\text{mercury}} g h_3 \\ &= P_{\text{atm}} + g(\rho_{\text{mercury}} h_3 - \rho_{\text{water}} h_1 - \rho_{\text{oil}} h_2) \\ &= 85.6 \text{ kPa} + (9.81 \text{ m/s}^2)[(13{,}600 \text{ kg/m}^3)(0.35 \text{ m}) - (1000 \text{ kg/m}^3)(0.1 \text{ m}) \\ &\quad - (850 \text{ kg/m}^3)(0.2 \text{ m})]\left(\frac{1 \text{ N}}{1 \text{ kg} \cdot \text{m/s}^2}\right)\left(\frac{1 \text{ kPa}}{1000 \text{ N/m}^2}\right) \\ &= \mathbf{130 \text{ kPa}} \end{aligned}$$

Discussion Note that jumping horizontally from one tube to the next and realizing that pressure remains the same in the same fluid simplifies the analysis considerably.

The Barometer

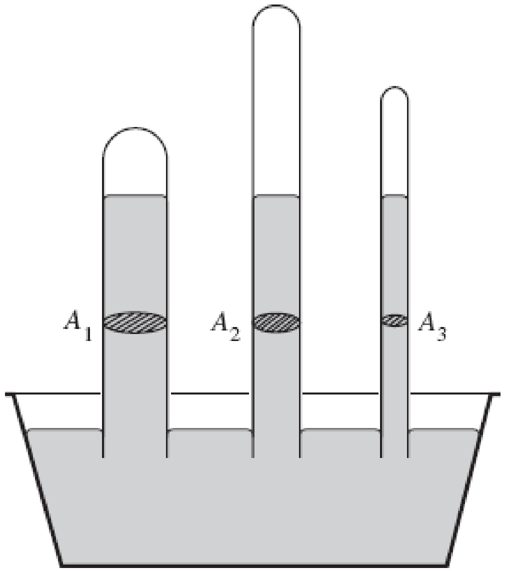
Atmospheric pressure is measured by a device called a barometer; thus, the atmospheric pressure is often referred to as the *barometric pressure*.

A frequently used pressure unit is the *standard atmosphere*, which is defined as the pressure produced by a column of mercury 760 mm in height at 0°C (ρ_{Hg}=13,595 kg/m^3) under standard gravitational acceleration (g = 9.807 m/s^2).



The basic barometer.

$$P_{atm} = \rho g h$$



The length or the cross-sectional area of the tube has no effect on the height of the fluid column of a barometer, provided that the tube diameter is large enough to avoid surface tension (capillary) effects.

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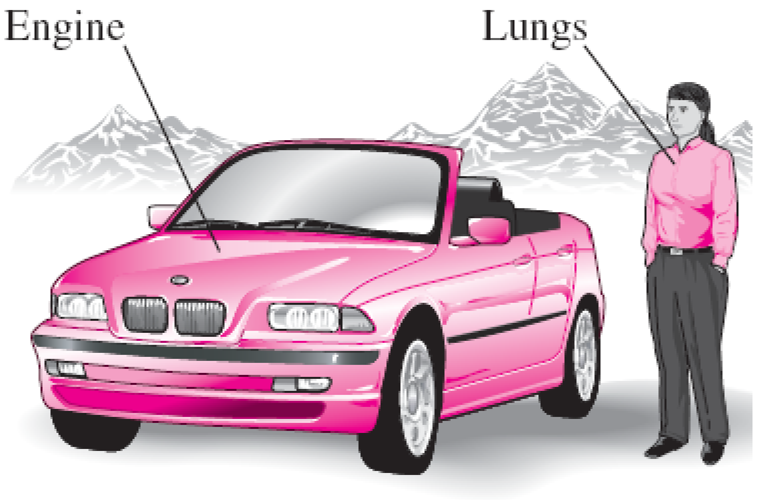


At high altitudes, a car engine generates less power and a person gets less oxygen because of the lower density of air.

A fan or compressor will displace 15 percent less air at that altitude for the same volume displacement rate. Therefore, larger cooling fans may need to be selected for operation at high altitudes to ensure the specified mass flow rate.

The lower pressure and thus lower density also affects lift and drag: airplanes need a longer runway at high altitudes to develop the required lift, and they climb to very high altitudes for cruising for reduced drag and thus better fuel efficiency.

EXAMPLE 3-5

Determine the atmospheric pressure at a location where the barometric reading is 740 mm Hg and the gravitational acceleration is g=9.81 m/s^2. Assume the temperature of mercury to be 10°C, at which its density is 13,570 kg/m^3.

Solution: The barometric reading at a location in height of mercury column is given. The atmospheric pressure is to be determined.

Assumptions: The temperature of mercury is assumed to be 10°C.

Properties: The density of mercury is given to be 13,570 kg/m^3.

$$\begin{aligned} P_{\text{atm}} &= \rho g h \\ &= (13{,}570\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.74\ \text{m})\left(\frac{1\ \text{N}}{1\ \text{kg}\cdot\text{m/s}^2}\right)\left(\frac{1\ \text{kPa}}{1000\ \text{N/m}^2}\right) \\ &= \mathbf{98.5\ kPa} \end{aligned}$$

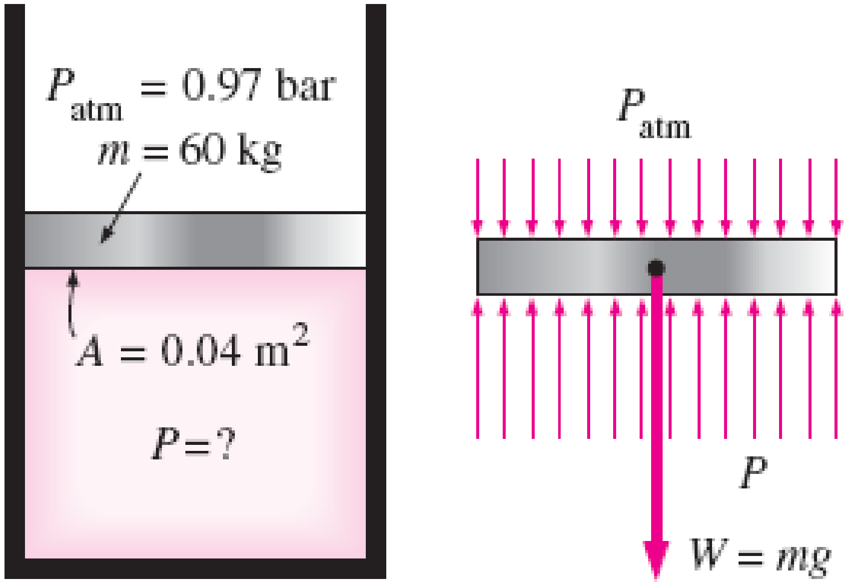
Discussion Note that density changes with temperature, and thus this effect should be considered in calculations.

EXAMPLE 3–6

The piston of a vertical piston–cylinder device vontaining a gas has a mass of 60 kg and a cross-sectional area of 0.04 m^2. The local atmospheric pressure is 0.97 bar, and the gravitational acceleration is 9.81 m/s^2.

(a) Determine the pressure inside the cylinder.

(b)If some heat is transferred to the gas and its volume is doubled do you expect the pressure inside the cylinder to change.



[redacted]

Solution A gas is contained in a vertical cylinder with a heavy piston. The pressure inside the cylinder and the effect of volume change on pressure are to be determined.

Assumptions Friction between the piston and the cylinder is negligible.

Analysis (*a*) The gas pressure in the piston–cylinder device depends on the atmospheric pressure and the weight of the piston.

$$PA = P_{\text{atm}}A + W$$

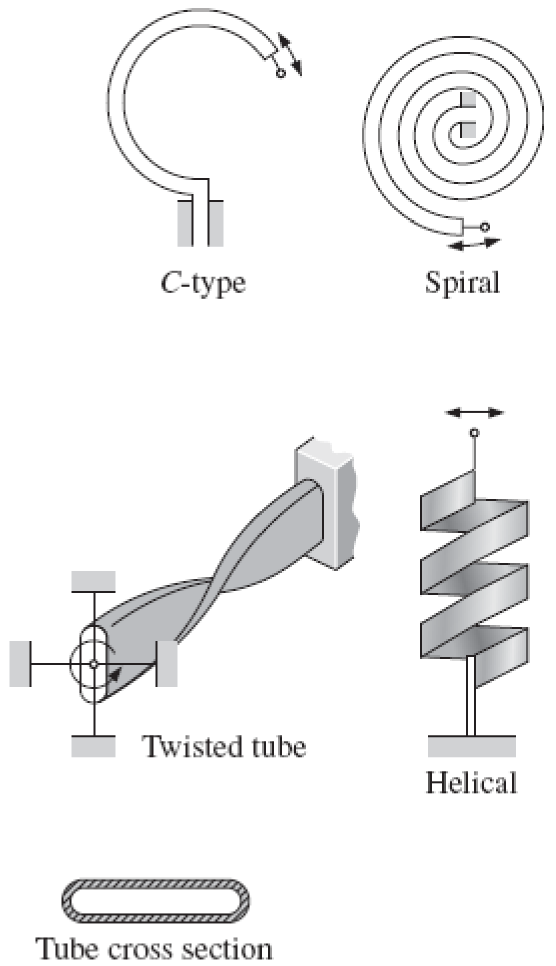
$$\begin{aligned}
P &= P_{\text{atm}} + \frac{mg}{A} \\
&= 0.97 \text{ bar} + \frac{(60 \text{ kg})(9.81 \text{ m/s}^2)}{0.04 \text{ m}^2}\left(\frac{1 \text{ N}}{1 \text{ kg}\cdot\text{m/s}^2}\right)\left(\frac{1 \text{ bar}}{10^5 \text{ N/m}^2}\right) \\
&= \mathbf{1.12 \text{ bars}}
\end{aligned}$$

(*b*) The volume change will have no effect on the free-body diagram and therefore the pressure inside the cylinder will remain the same.

Discussion If the gas behaves as an ideal gas, the absolute temperature doubles when the volume is doubled at constant pressure.

Other Pressure Measurement Devices

- **Bourdon tube:** Consists of a hollow metal tube bent like a hook whose end is closed and connected to a dial indicator needle.
- **Pressure transducers**: Use various techniques to convert the pressure effect to an electrical effect such as a change in voltage, resistance, or capacitance.
- Pressure transducers are smaller and faster, and they can be more sensitive, reliable, and precise than their mechanical counterparts.
- **Strain-gage pressure transducers:** Work by having a diaphragm deflect between two chambers open to the pressure inputs.
- **Piezoelectric transducers:** Also called solid-state pressure transducers, work on the principle that an electric potential is generated in a crystalline substance when it is subjected to mechanical pressure.



Various types of Bourdon tubes used to measure pressure.

INTRODUCTION TO FLUID STATICS

Fluid statics: Deals with problems associated with fluids at rest.

The fluid can be either gaseous or liquid.

Hydrostatics: When the fluid is a liquid.

Aerostatics: When the fluid is a gas.

In fluid statics, there is no relative motion between adjacent fluid layers, and thus there are no shear (tangential) stresses in the fluid trying to deform it.

The only stress we deal with in fluid statics is the ***normal stress***, which is the pressure, and the variation of pressure is due only to the weight of the fluid.

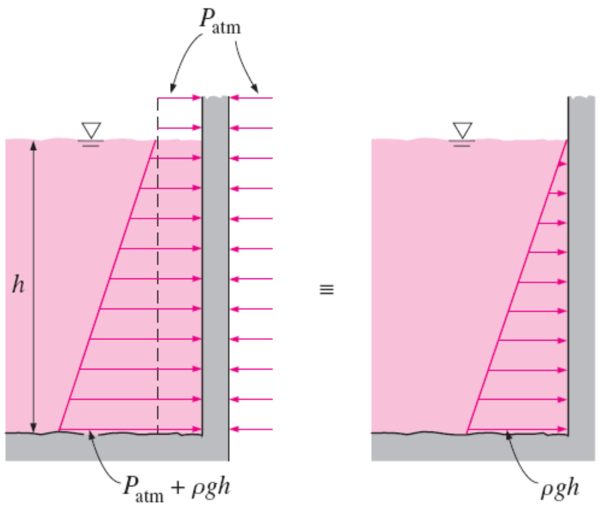
The topic of fluid statics has significance only in gravity fields.

The design of many engineering systems such as water dams and liquid storage tanks requires the determination of the forces acting on the surfaces using fluid statics.

HYDROSTATIC FORCES ON SUBMERGED PLANE SURFACES

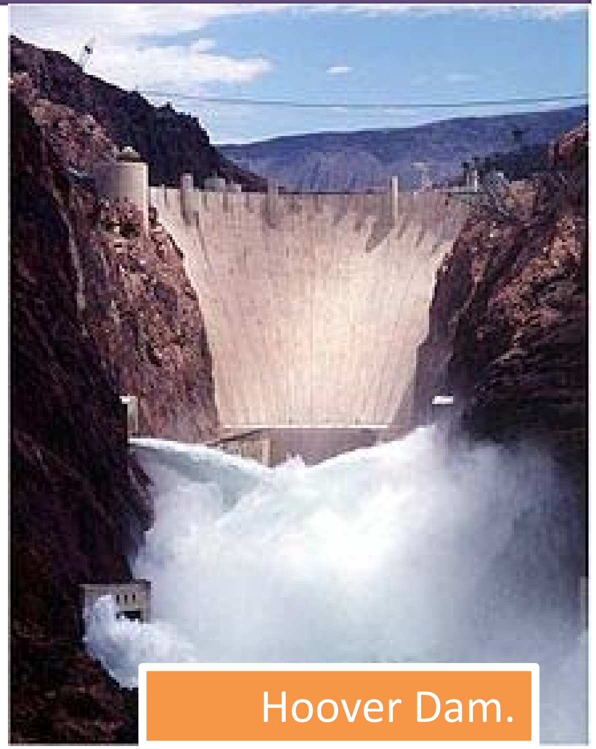
A plate, such as a gate valve in a dam, the wall of a liquid storage tank, or the hull of a ship at rest, is subjected to fluid pressure distributed over its surface when exposed to a liquid.

On a ***plane*** surface, the hydrostatic forces form a system of parallel forces, and we often need to determine the *magnitude* of the force and its *point of application*, which is called the **center of pressure**.



(*a*) P_{atm} considered (*b*) P_{atm} subtracted

When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.



Hoover Dam.

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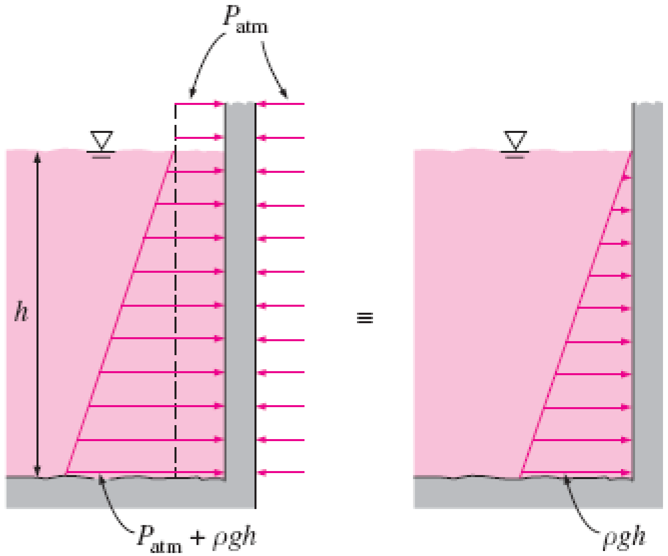


(*a*) P_{atm} considered (*b*) P_{atm} subtracted

When analyzing hydrostatic forces on submerged surfaces, the atmospheric pressure can be subtracted for simplicity when it acts on both sides of the structure.

Hydrostatic pressure increases along y;

$$\boxed{P = P_0 + \rho gh = P_0 + \rho gy \sin\theta}$$

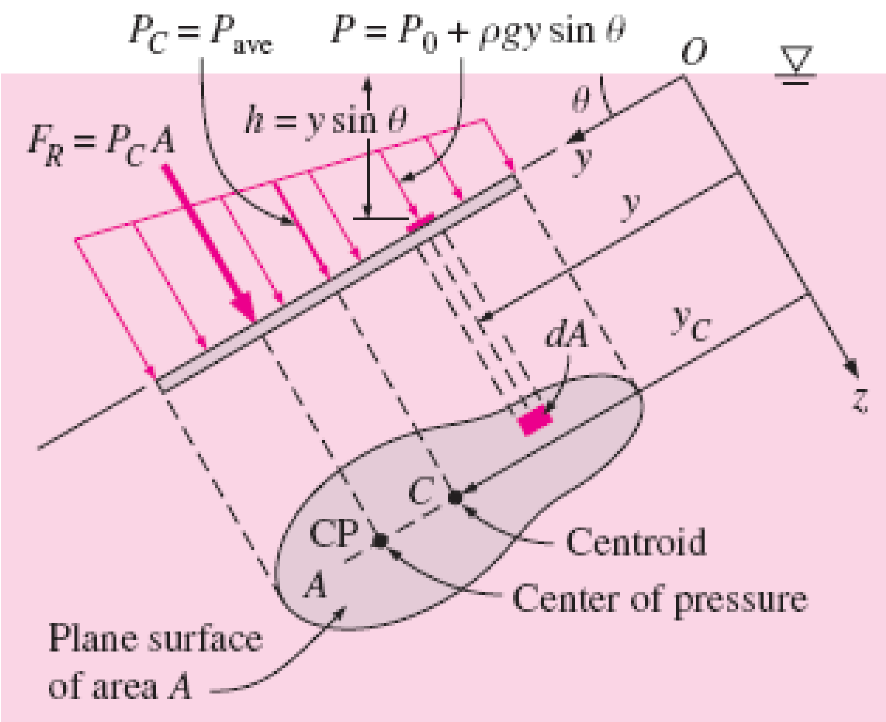
$$\boxed{F_R = \int_A P\, dA = \int_A (P_0 + \rho gy \sin\theta)\, dA = P_0 A + \rho g \sin\theta \int_A y\, dA}$$

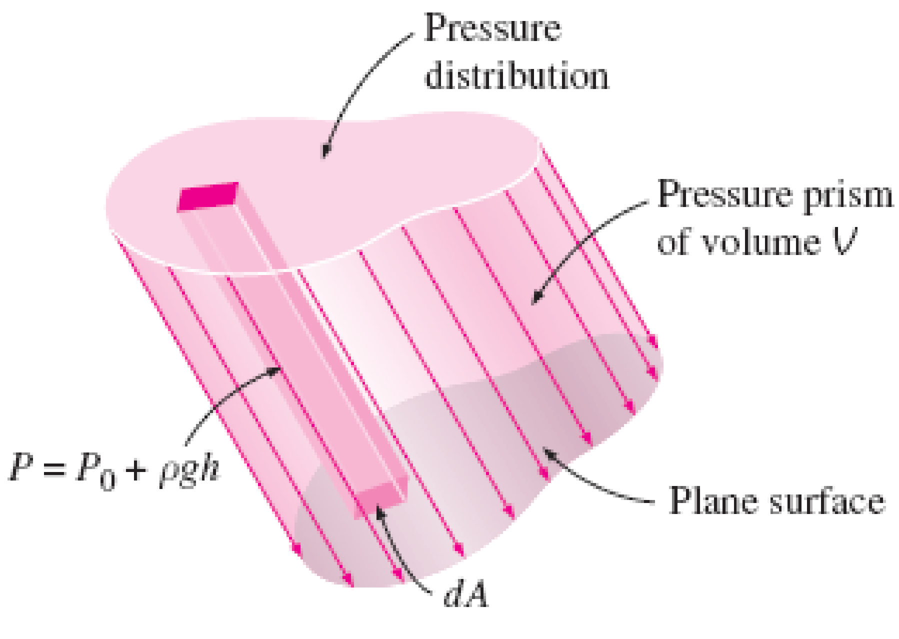
First moment of the area is defined as;

$$\boxed{y_C = \frac{1}{A}\int_A y\, dA}$$

Resultant force

$$F_R = (P_0 + \rho g y_C \sin\theta)A = (P_0 + \rho g h_C)A = P_C A = P_{ave} A$$





$$V = \int dV = \int P\, dA = F_R$$

Hydrostatic force on an inclined plane surface completely submerged in a liquid.

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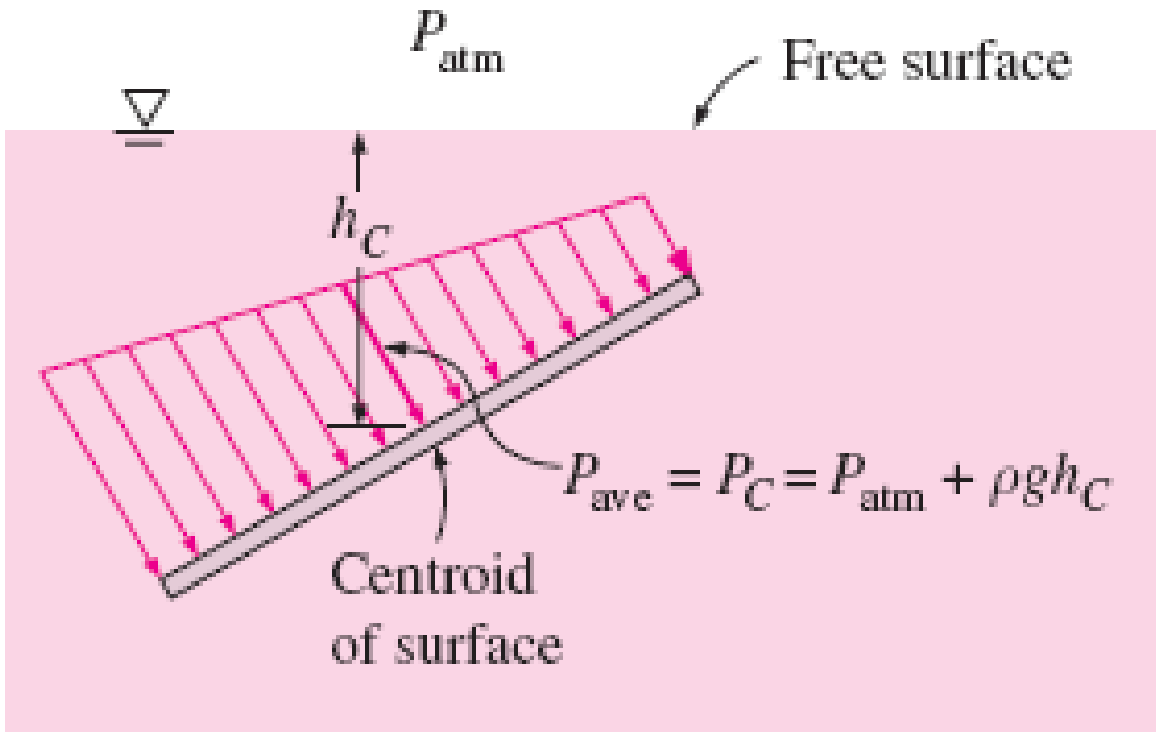
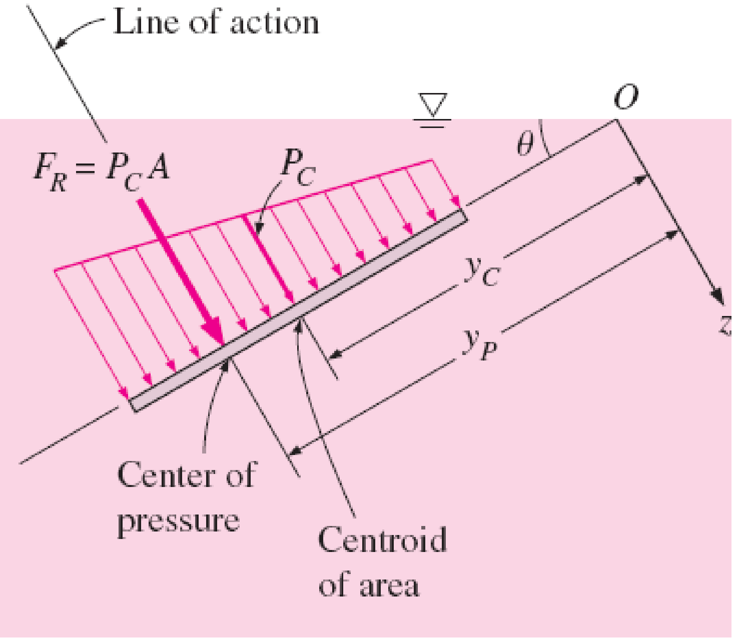


The pressure at the centroid of a surface is equivalent to the *average* pressure on the surface.

[redacted]

The magnitude of the resultant force acting on a plane surface of a completely submerged plate in a homogeneous (constant density) fluid is equal to the product of the pressure *Pc* at the centroid of the surface and the area *A* of the surface

$$y_P F_R = \int_A yP\,dA = \int_A y(P_0 + \rho g y \sin\theta)\,dA = P_0 \int_A y\,dA + \rho g \sin\theta \int_A y^2\,dA$$



$$y_P F_R = P_0 y_C A + \rho g \sin\theta\, I_{xx,O}$$

The resultant force acting on a plane surface is equal to the product of the pressure at the centroid of the surface and the surface area, and its line of action passes through the center of pressure.

[redacted]

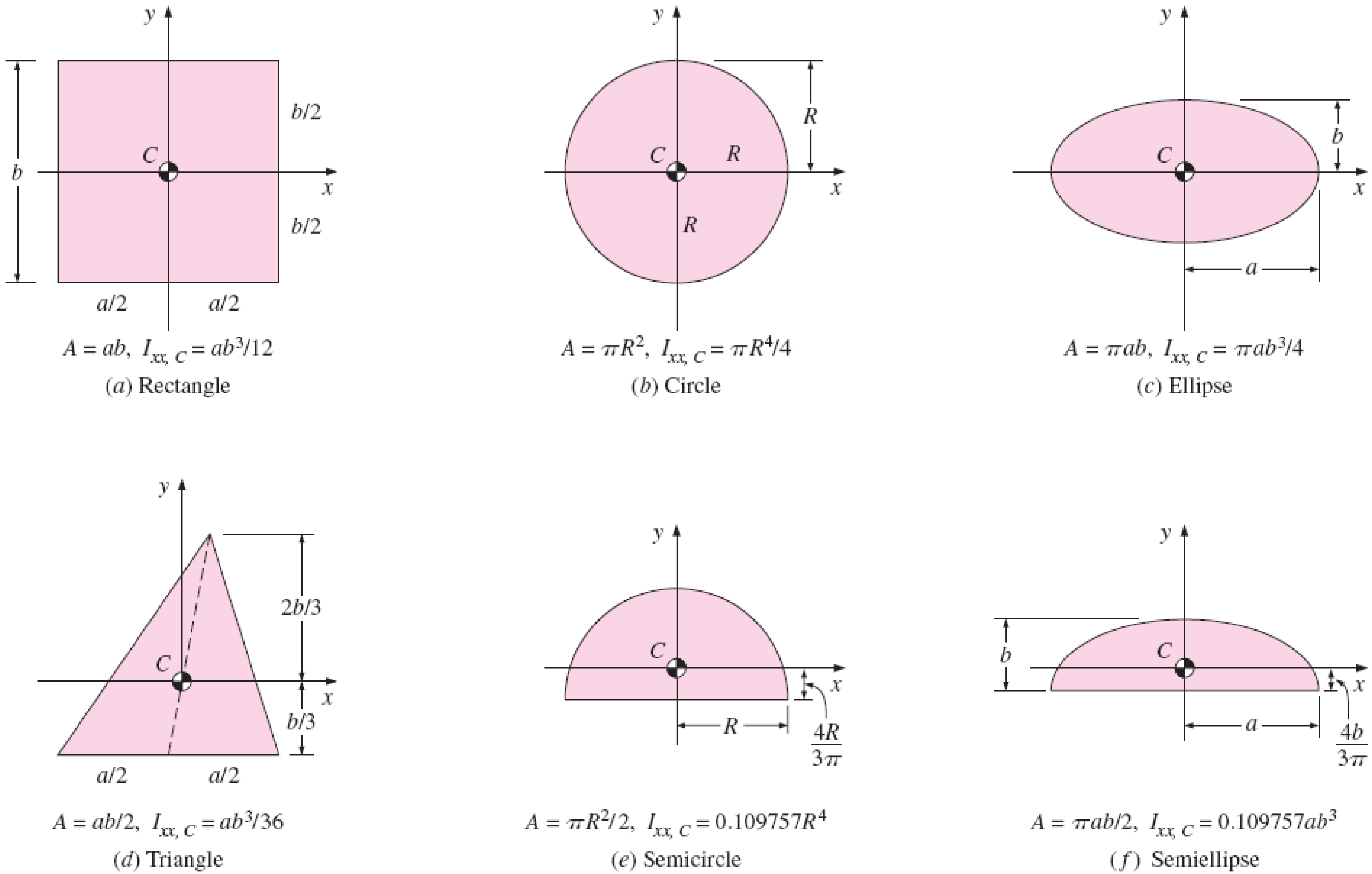
$$I_{xx,\,O} = \int_A y^2\, dA \text{ is the } \textit{second moment of area}$$

$$I_{xx,\,O} = I_{xx,\,C} + y_C^2 A$$

$$y_P = y_C + \frac{I_{xx,\,C}}{[y_C + P_0/(\rho g \sin\theta)]A}$$

$$y_P = y_C + \frac{I_{xx,\,C}}{y_C A}$$

[redacted]



The centroid and the centroidal moments of inertia for some common geometries.

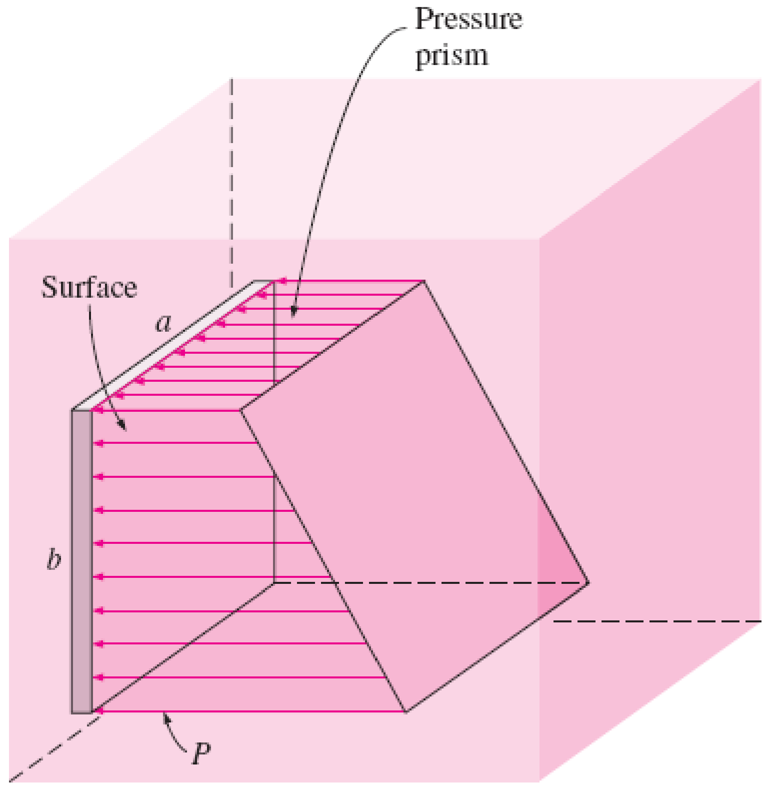
[redacted]

Pressure acts normal to the surface, and the hydrostatic forces acting on a flat plate of any shape form a volume whose base is the plate area and whose length is the linearly varying pressure.

This virtual pressure prism has an interesting physical interpretation: its *volume* is equal to the *magnitude* of the resultant hydrostatic force acting on the plate since $F_R = \int PdA$, and the line of action of this force passes through the *centroid* of this homogeneous prism.

The projection of the centroid on the plate is the *pressure center.*

Therefore, with the concept of pressure prism, the problem of describing the resultant hydrostatic force on a plane surface reduces to finding the volume and the two coordinates of the centroid of this pressure prism.



The hydrostatic forces acting on a plane surface form a pressure prism whose base (left face) is the surface and whose length is the pressure.

Special Case: Submerged Rectangular Plate

Tilted rectangular plate:

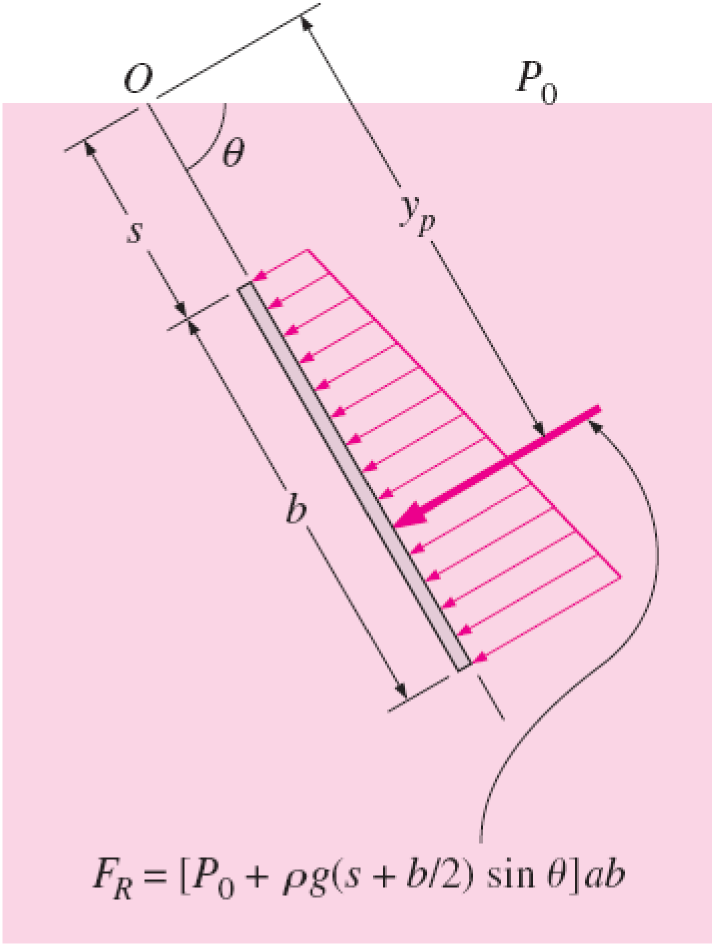
$$\boxed{F_R = P_C A = [P_0 + \rho g(s + b/2)\sin\theta]ab}$$

$$h_P = y_P \sin\theta$$

$$\boxed{\begin{aligned} y_P &= s + \frac{b}{2} + \frac{ab^3/12}{[s + b/2 + P_0/(\rho g \sin\theta)]ab} \\ &= s + \frac{b}{2} + \frac{b^2}{12[s + b/2 + P_0/(\rho g \sin\theta)]} \end{aligned}}$$

Tilted rectangular plate ($s = 0$):

$$\boxed{F_R = [P_0 + \rho g(b \sin\theta)/2]ab}$$



(*a*) Tilted plate

Hydrostatic force acting on the top surface of a submerged tilted rectangular plate.

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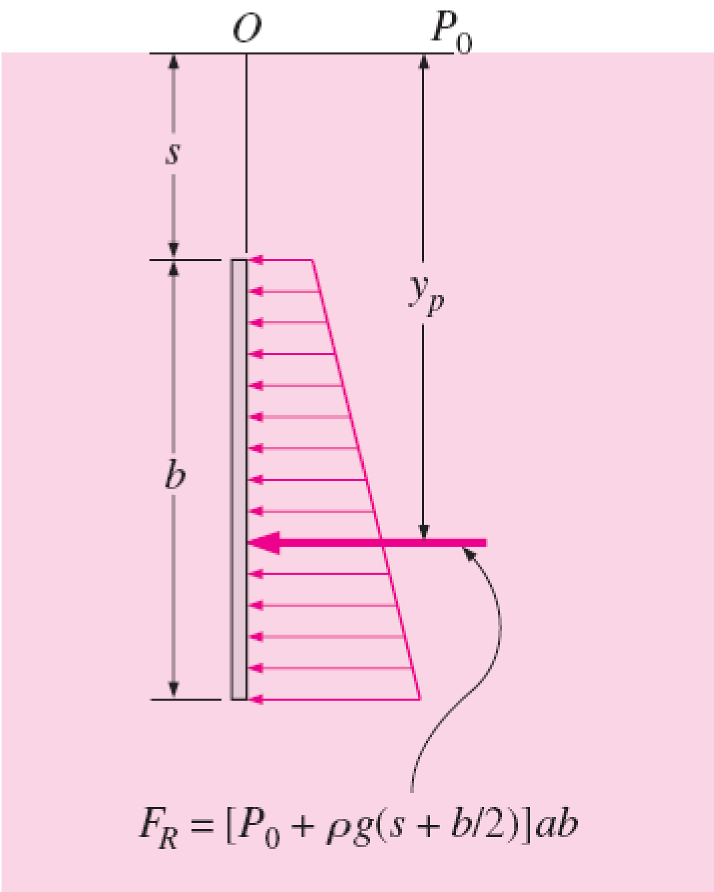


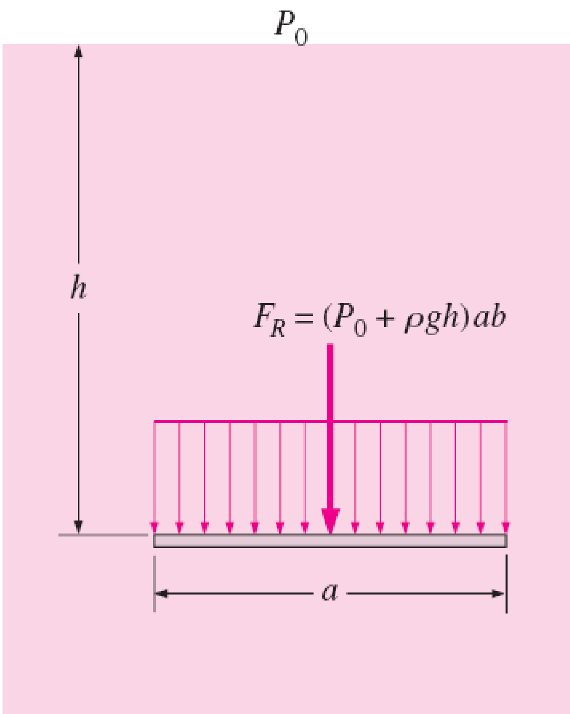
(*b*) Vertical plate

Hydrostatic force acting on the top surface of a submerged vertical rectangular plate.

Vertical rectangular plate: $F_R = [P_0 + \rho g(s + b/2)]ab$

Vertical rectangular plate ($s = 0$): $F_R = (P_0 + \rho g b/2)ab$

[redacted]



(c) Horizontal plate

Hydrostatic force acting on the top surface of a submerged horizontal rectangular plate.

$$\textit{Horizontal rectangular plate:} \qquad F_R = (P_0 + \rho gh)ab$$

EXAMPLE 3–8

A heavy car plunges into a lake during an accident and lands at the bottom of the lake on its wheels. The door is 1.2 m high and 1 m wide, and the top edge of the door is 8 m below the free surface of the water. Determine the hydrostatic force on the door and the location of the pressure center, and discuss if the driver can open the door.

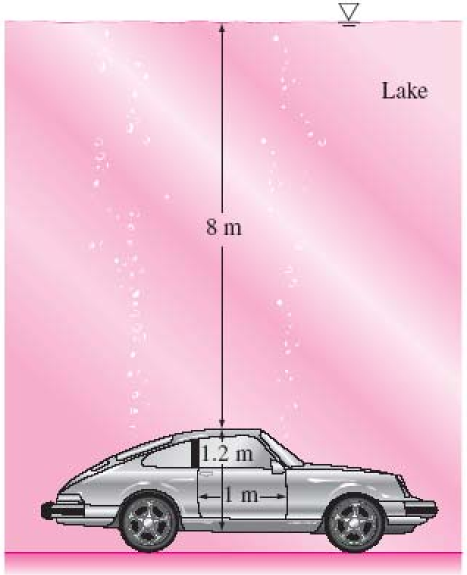
Solution A car is submerged in water. The hydrostatic force on the door is to be determined, and the likelihood of the driver opening the door is to be assessed.

Assumptions **1** The bottom surface of the lake is horizontal **2** The passenger cabin is well-sealed so that no water leaks inside. **3** The door can be approximated as a vertical rectangular plate. **4** The pressure in the passenger cabin remains at atmospheric value since there is no water leaking in, and thus no compression of the air inside. **5** The weight of the car is larger than the buoyant force acting on it.

$$P_{ave} = P_C = \rho g h_C = \rho g(s + b/2)$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(8 + 1.2/2\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg}\cdot\text{m/s}^2}\right)$$

$$= \mathbf{84.4\ kN/m^2}$$



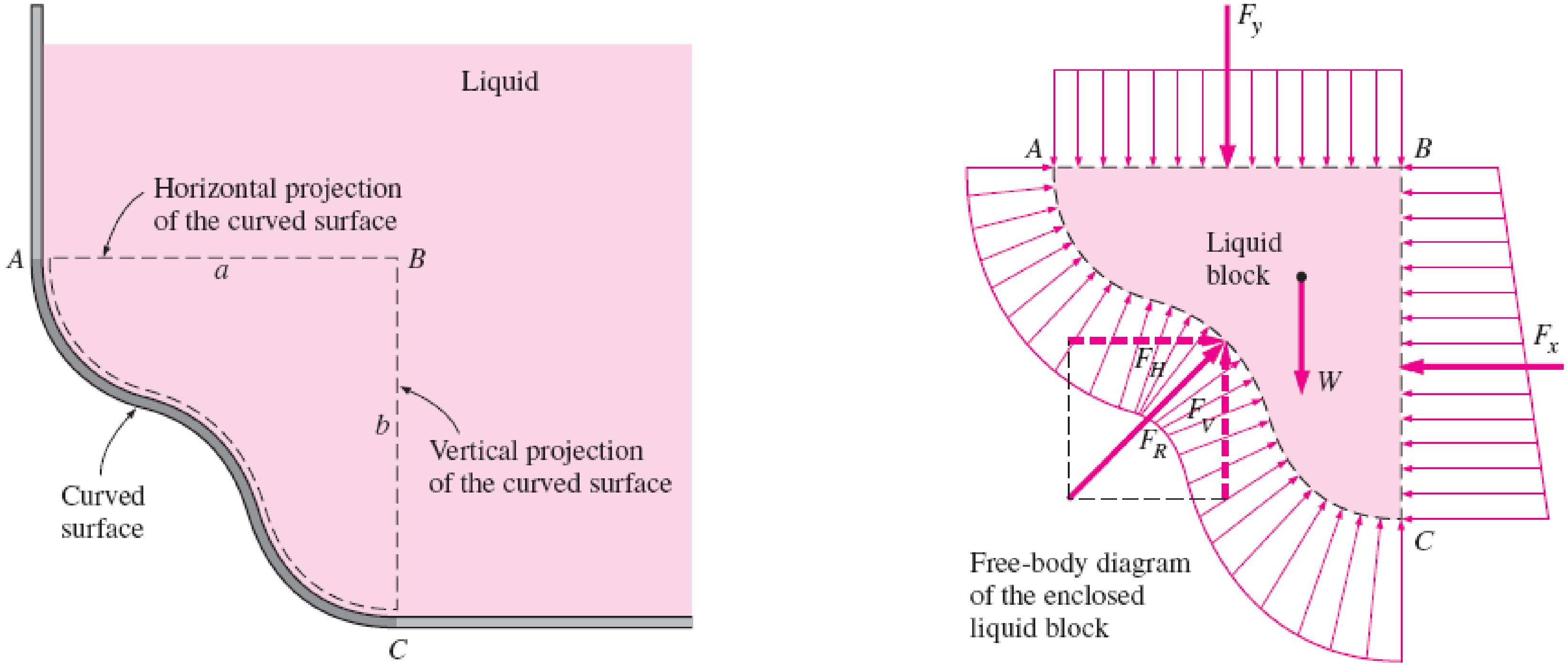
[redacted]

the resultant hydrostatic force on the door becomes

$$F_R = P_{\text{ave}}A = (84.4\ \text{kN/m}^2)(1\ \text{m} \times 1.2\ \text{m}) = \mathbf{101.3\ kN}$$

$$y_P = s + \frac{b}{2} + \frac{b^2}{12(s + b/2)} = 8 + \frac{1.2}{2} + \frac{1.2^2}{12(8 + 1.2/2)} = \mathbf{8.61\ m}$$

Discussion A strong person can lift 100 kg, whose weight is 981 N or about 1 kN. Also, the person can apply the force at a point farthest from the hinges (1 m farther) for maximum effect and generate a moment of 1 kN · m. The resultant hydrostatic force acts under the midpoint of the door, and thus a distance of 0.5 m from the hinges. This generates a moment of 50.6 kN · m, which is about 50 times the moment the driver can possibly generate. Therefore, it is impossible for the driver to open the door of the car. The driver's best bet is to let some water in (by rolling the window down a little, for example) and to keep his or her head close to the ceiling. The driver should be able to open the door shortly before the car is filled with water since at that point the pressures on both sides of the door are nearly the same and opening the door in water is almost as easy as opening it in air.



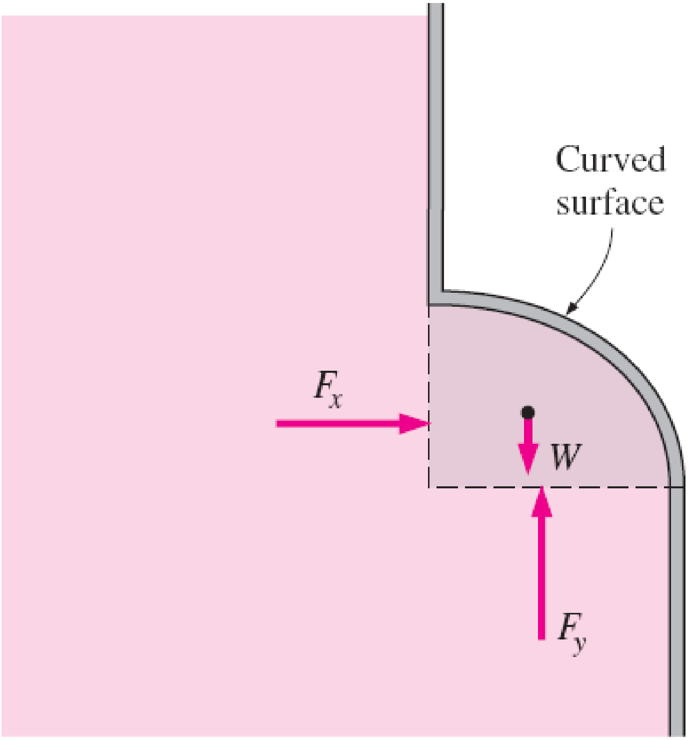
$$\alpha = F_V / F_H$$

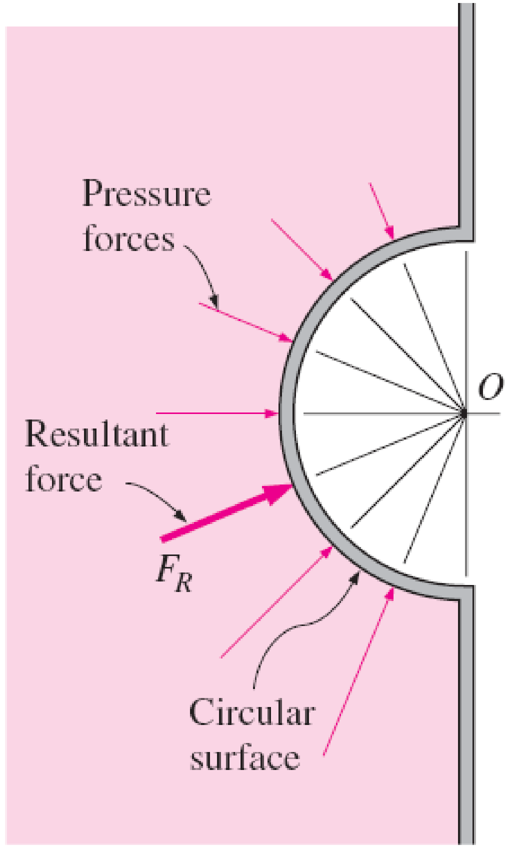
$$F_R = \sqrt{F_H^2 + F_V^2}$$

Determination of the hydrostatic force acting on a submerged curved surface.

Horizontal force component on curved surface:	$F_H = F_x$
Vertical force component on curved surface:	$F_V = F_y + W$

[redacted]





When a curved surface is above the liquid, the weight of the liquid and the vertical component of the hydrostatic force act in the opposite directions.

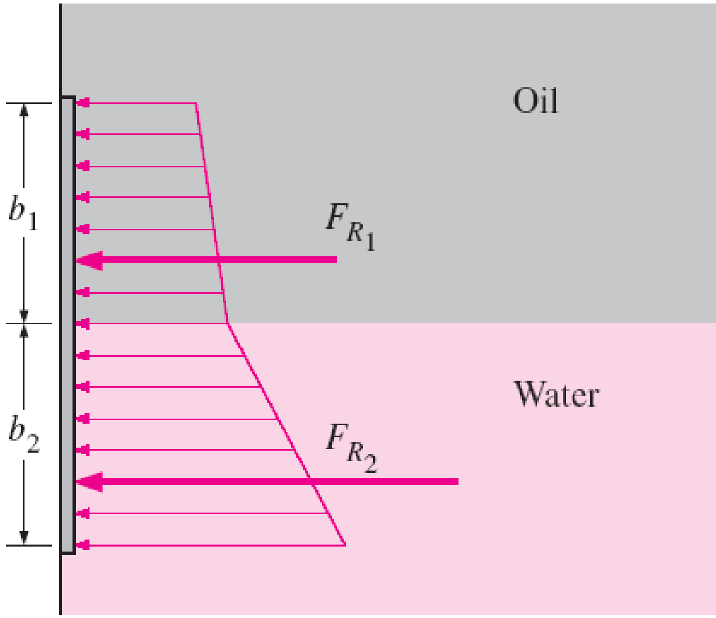
The hydrostatic force acting on a circular surface always passes through the center of the circle since the pressure forces are normal to the surface and they all pass through the center.

[redacted]

In a multilayered fluid of different densities can be determined by considering different parts of surfaces in different fluids as different surfaces, finding the force on each part, and then adding them using vector addition. For a plane surface, it can be expressed as

Plane surface in a multilayered fluid: $F_R = \sum F_{R,i} = \sum P_{C,i} A_i$

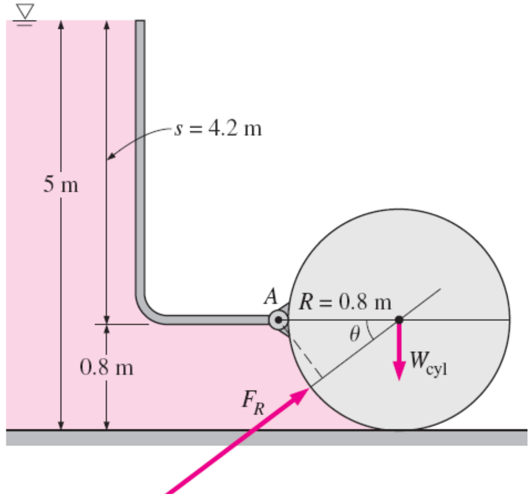
$$P_{C,\,i} = P_0 + \rho_i g h_{C,\,i}$$



The hydrostatic force on a surface submerged in a multilayered fluid can be determined by considering parts of the surface in different fluids as different surfaces.

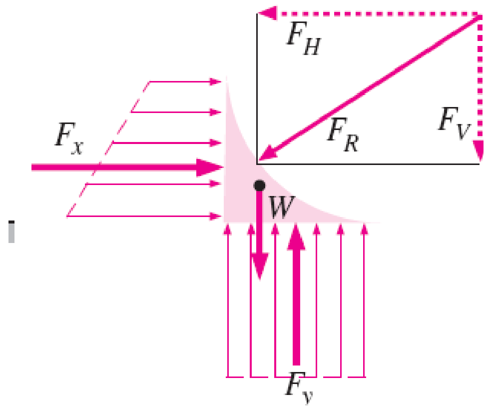
EXAMPLE 3-9

A long solid cylinder of radius 0.8 m hinged at point *A* is used as an automatic gate. When the water level reaches 5 m, the gate opens by turning about the hinge at point *A*. Determine (*a*) the hydrostatic force acting on the cylinder and its line of action when the gate opens and (*b*) the weight of the cylinder per m length of the cylinder.



Solution The height of a water reservoir is controlled by a cylindrical gate hinged to the reservoir. The hydrostatic force on the cylinder and the weight of the cylinder per m length are to be determined.

Assumptions **1** Friction at the hinge is negligible. **2** Atmospheric pressure acts on both sides of the gate, and thus it cancels out.



Horizontal force on vertical surface:

$$F_H = F_x = P_{\text{ave}}A = \rho g h_C A = \rho g(s + R/2)A$$

$$= (1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(4.2 + 0.8/2\ \text{m})(0.8\ \text{m} \times 1\ \text{m})\left(\frac{1\ \text{kN}}{1000\ \text{kg} \cdot \text{m/s}^2}\right)$$

$$= \mathbf{36.1\ kN}$$

[redacted]

Vertical force on horizontal surface (upward):

$$F_y = P_{\text{ave}}A = \rho g h_C A = \rho g h_{\text{bottom}} A$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(5 \text{ m})(0.8 \text{ m} \times 1 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 39.2 \text{ kN}$$

Weight of fluid block per m length (downward):

$$W = mg = \rho g V = \rho g(R^2 - \pi R^2/4)(1 \text{ m})$$

$$= (1000 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0.8 \text{ m})^2(1 - \pi/4)(1 \text{ m})\left(\frac{1 \text{ kN}}{1000 \text{ kg} \cdot \text{m/s}^2}\right)$$

$$= 1.3 \text{ kN}$$

the net upward vertical force is

$$F_V = F_y - W = 39.2 - 1.3 = 37.9 \text{ kN}$$

The magnitude and direction of the hydrostatic force

$$F_R = \sqrt{F_H^2 + F_V^2} = \sqrt{36.1^2 + 37.9^2} = \mathbf{52.3 \text{ kN}}$$

$$\tan\theta = F_V/F_H = 37.9/36.1 = 1.05 \rightarrow \theta = 46.4°$$

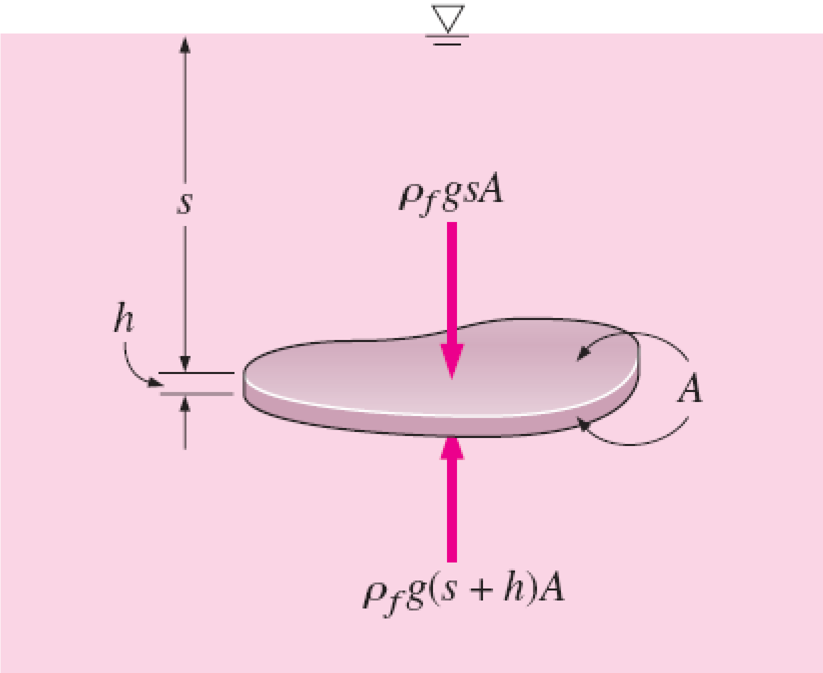
[redacted]

(*b*) When the water level is 5 m high, the gate is about to open and thus the reaction force at the bottom of the cylinder is zero. Then the forces other than those at the hinge acting on the cylinder are its weight, acting through the center, and the hydrostatic force exerted by water. Taking a moment about point *A* at the location of the hinge and equating it to zero gives

$$F_R R \sin\theta - W_{cyl} R = 0 \rightarrow W_{cyl} = F_R \sin\theta = (52.3\text{ kN}) \sin 46.4° = \mathbf{37.9\ kN}$$

Discussion The weight of the cylinder per m length is determined to be 37.9 kN. It can be shown that this corresponds to a mass of 3863 kg per m length and to a density of 1921 kg/m3 for the material of the cylinder.

Buoyant force: The upward force a fluid exerts on a body immersed in it. The buoyant force is caused by the increase of pressure with depth in a fluid.



A flat plate of uniform thickness *h* submerged in a liquid parallel to the free surface.

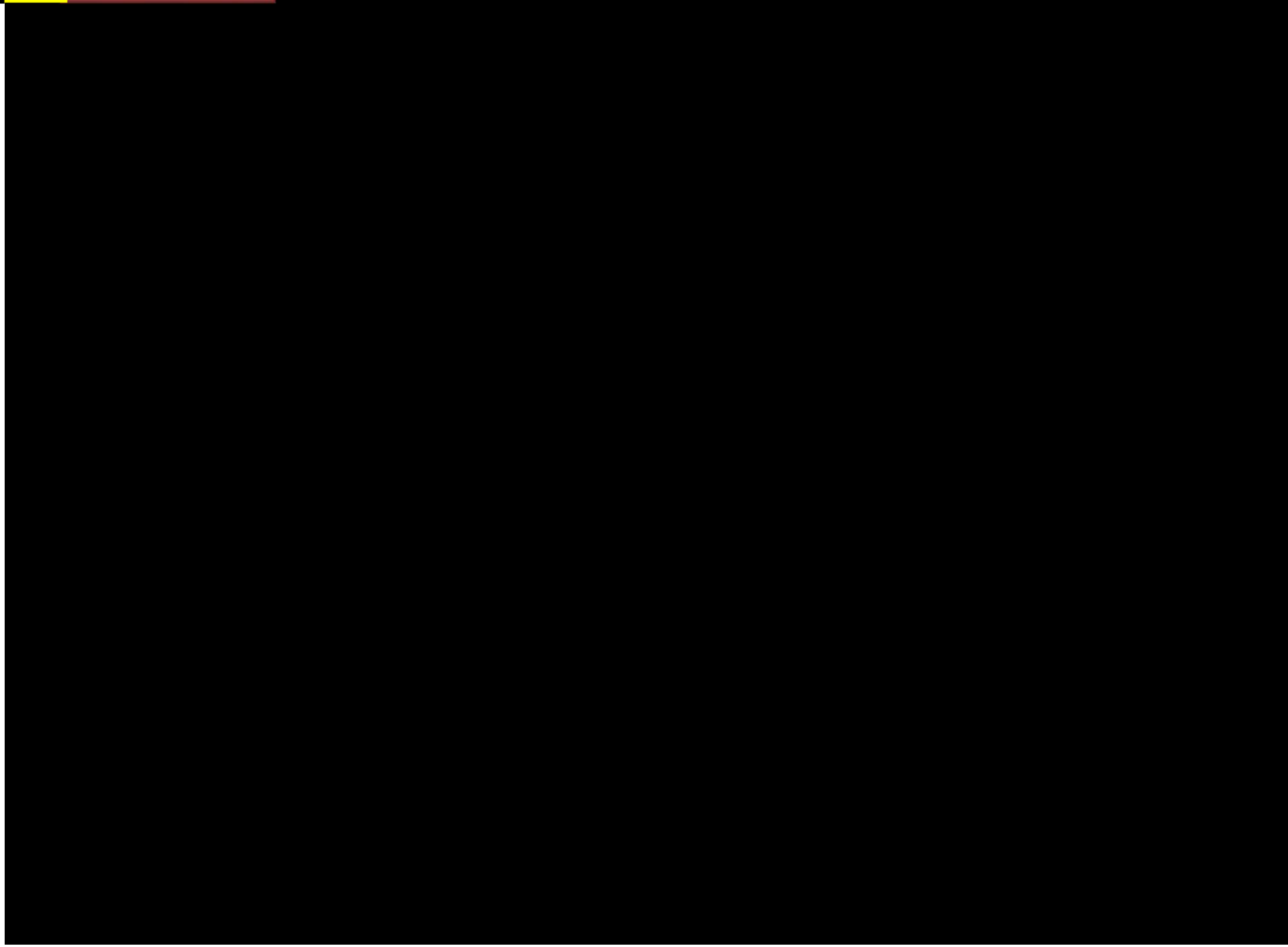
The buoyant force acting on the plate is equal to the weight of the liquid displaced by the plate.

For a fluid with constant density, the buoyant force is independent of the distance of the body from the free surface.

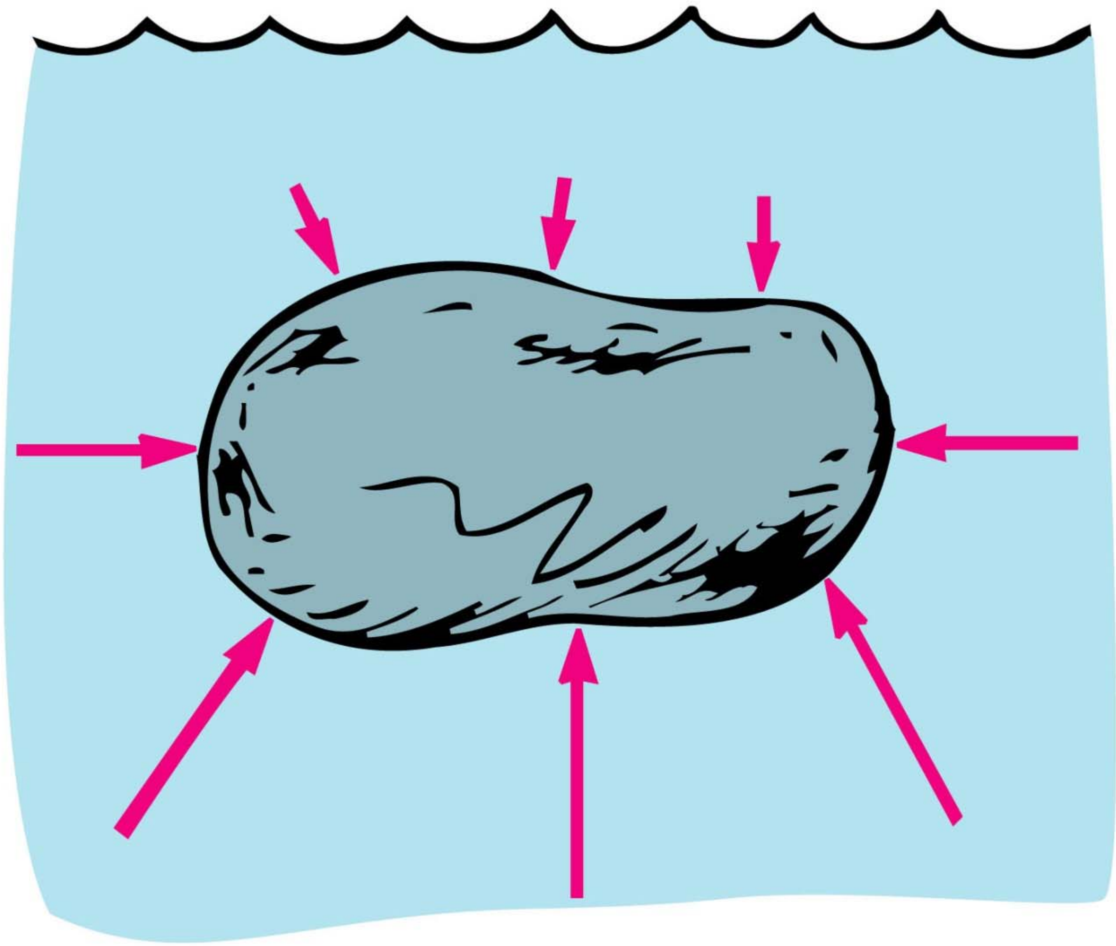
It is also independent of the density of the solid body.

$$F_B = F_{\text{bottom}} - F_{\text{top}} = \rho_f g(s+h)A - \rho_f gsA = \rho_f ghA = \rho_f gV$$

Stability



Buoyancy



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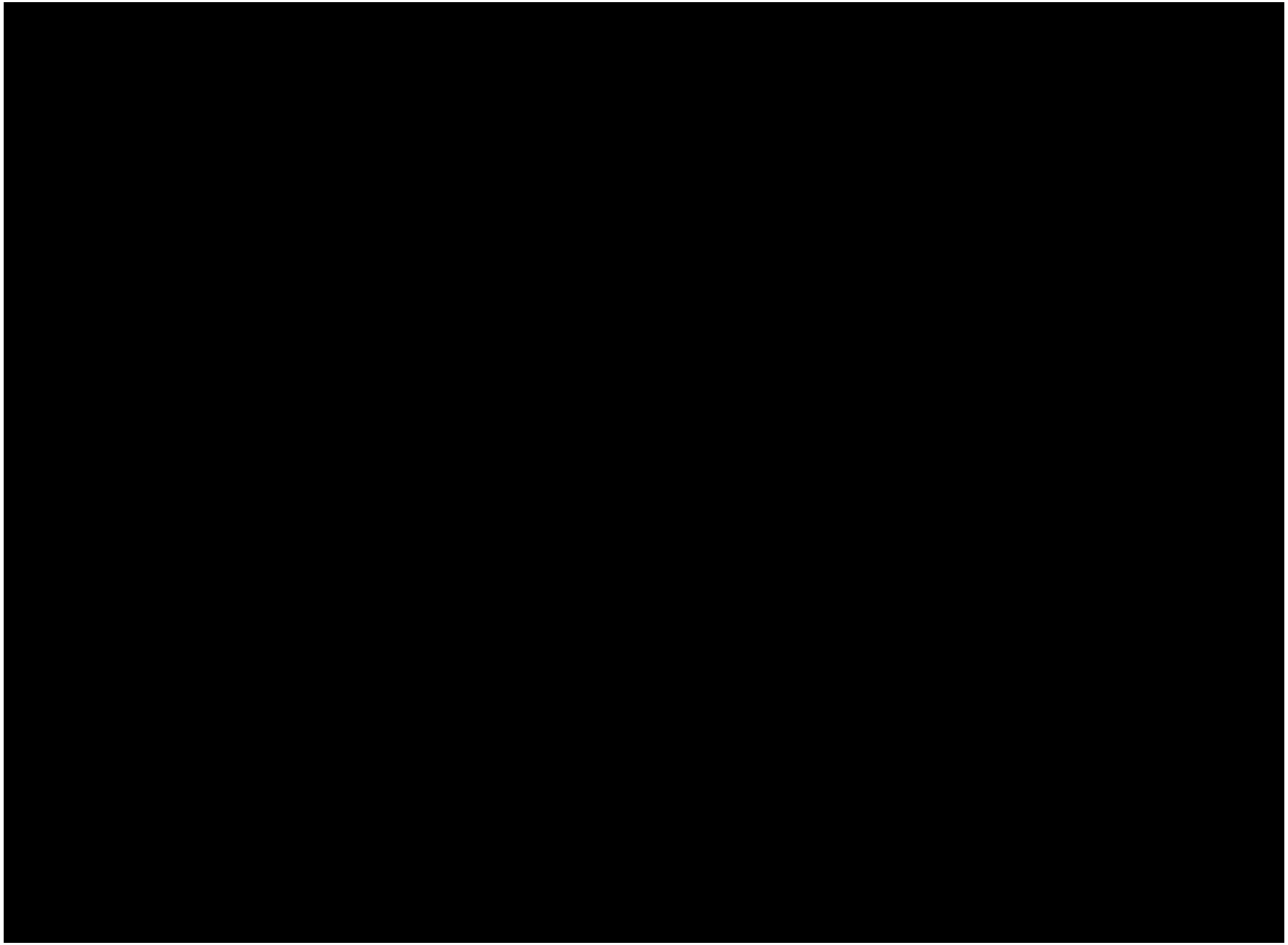
Net upward force is called the buoyant force!!!

Easier to lift a rock in water!!

Buoyancy



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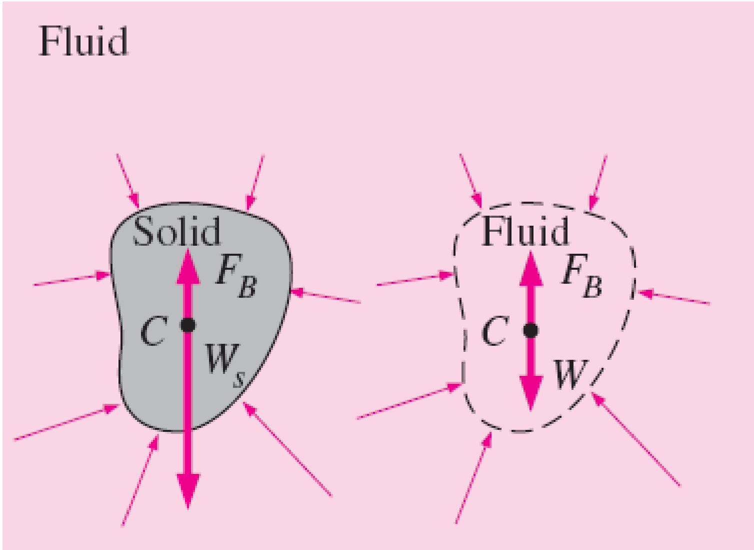


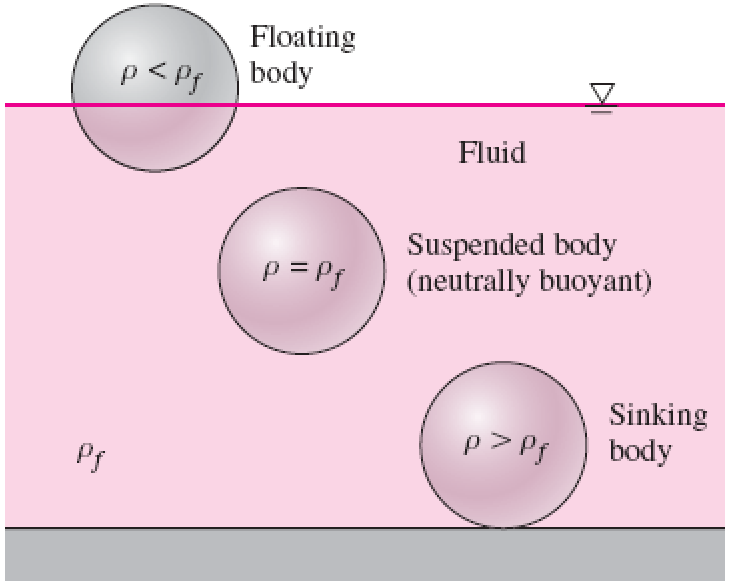
The buoyant forces acting on a solid body submerged in a fluid and on a fluid body of the same shape at the same depth are identical. The buoyant force F_B acts upward through the centroid C of the displaced volume and is equal in magnitude to the weight W of the displaced fluid, but is opposite in direction. For a solid of uniform density, its weight W_s also acts through the centroid, but its magnitude is not necessarily equal to that of the fluid it displaces. (Here $W_s > W$ and thus $W_s > F_B$; this solid body would sink.)

Archimedes' principle: The buoyant force acting on a body immersed in a fluid is equal to the weight of the fluid displaced by the body, and it acts upward through the centroid of the displaced volume.

[redacted]

For *floating* bodies, The weight of the entire body must be equal to the buoyant force, which is the weight of the fluid whose volume is equal to the volume of the submerged portion of the floating body:

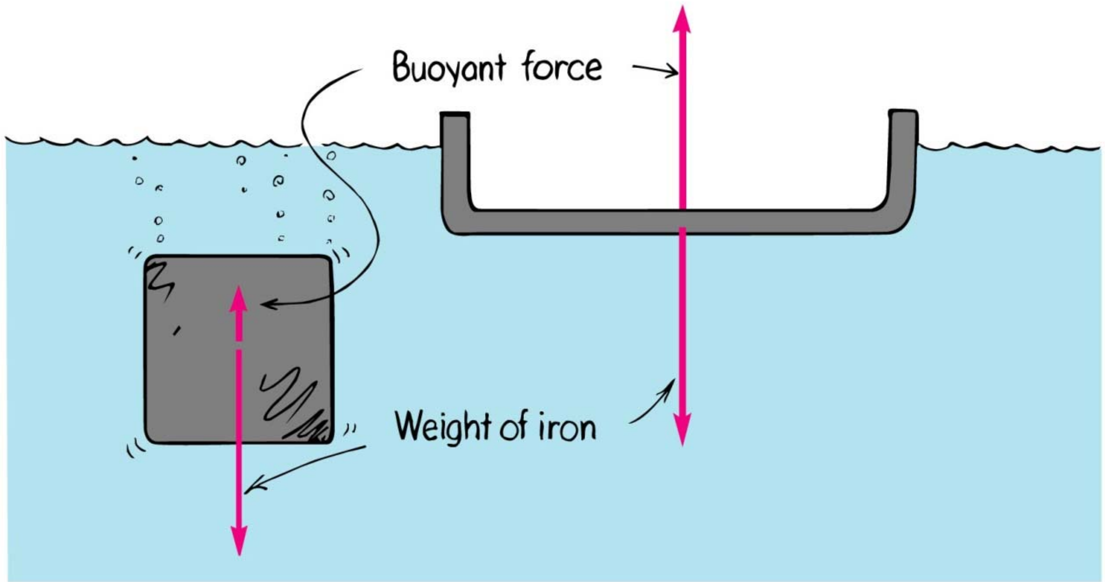
$$F_B = W \rightarrow \rho_f g V_{\text{sub}} = \rho_{\text{ave, body}} g V_{\text{total}} \rightarrow \frac{V_{\text{sub}}}{V_{\text{total}}} = \frac{\rho_{\text{ave, body}}}{\rho_f}$$



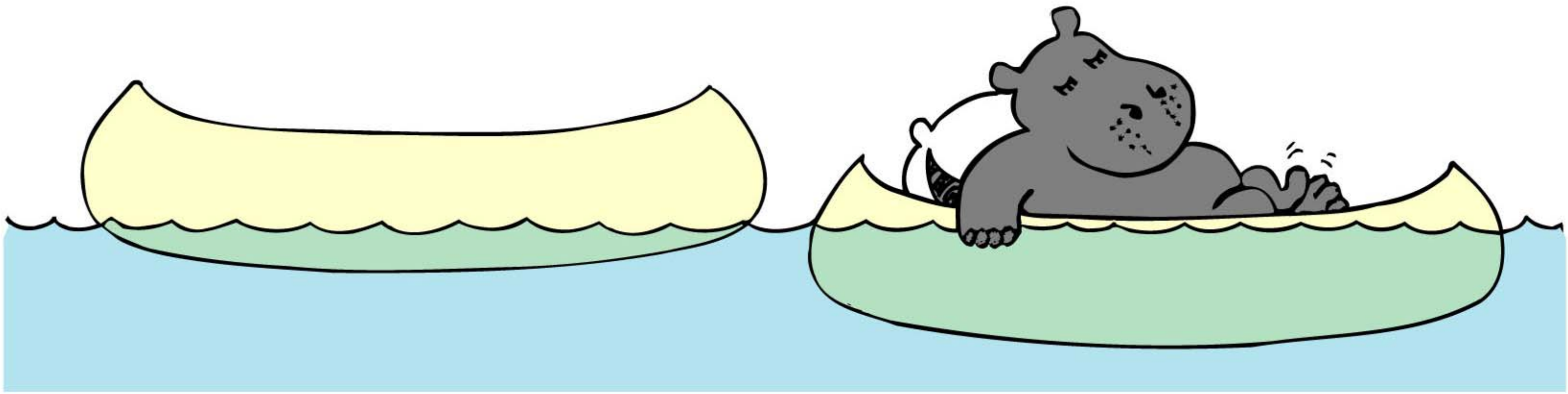
A solid body dropped into a fluid will sink, float, or remain at rest at any point in the fluid, depending on its average density relative to the density of the fluid.

Flotation

- *A floating object displaces a weight of fluid equal to its own weight.*



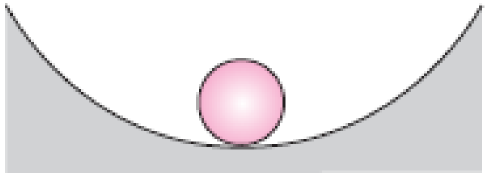
Flotation



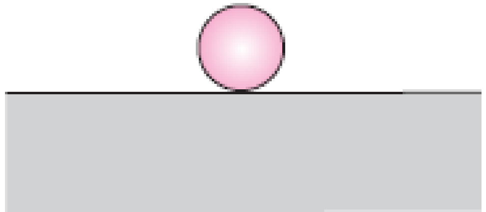
The altitude of a hot air balloon is controlled by the temperature difference between the air inside and outside the balloon, since warm air is less dense than cold air. When the balloon is neither rising nor falling, the upward buoyant force exactly balances the downward weight.

Stability of Immersed and Floating Bodies

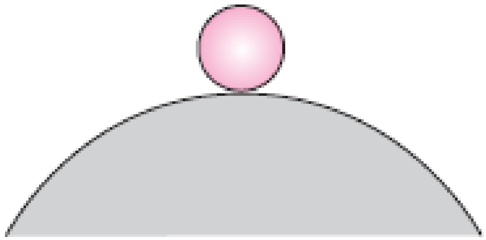
For floating bodies such as ships, stability is an important consideration for safety.



(*a*) Stable



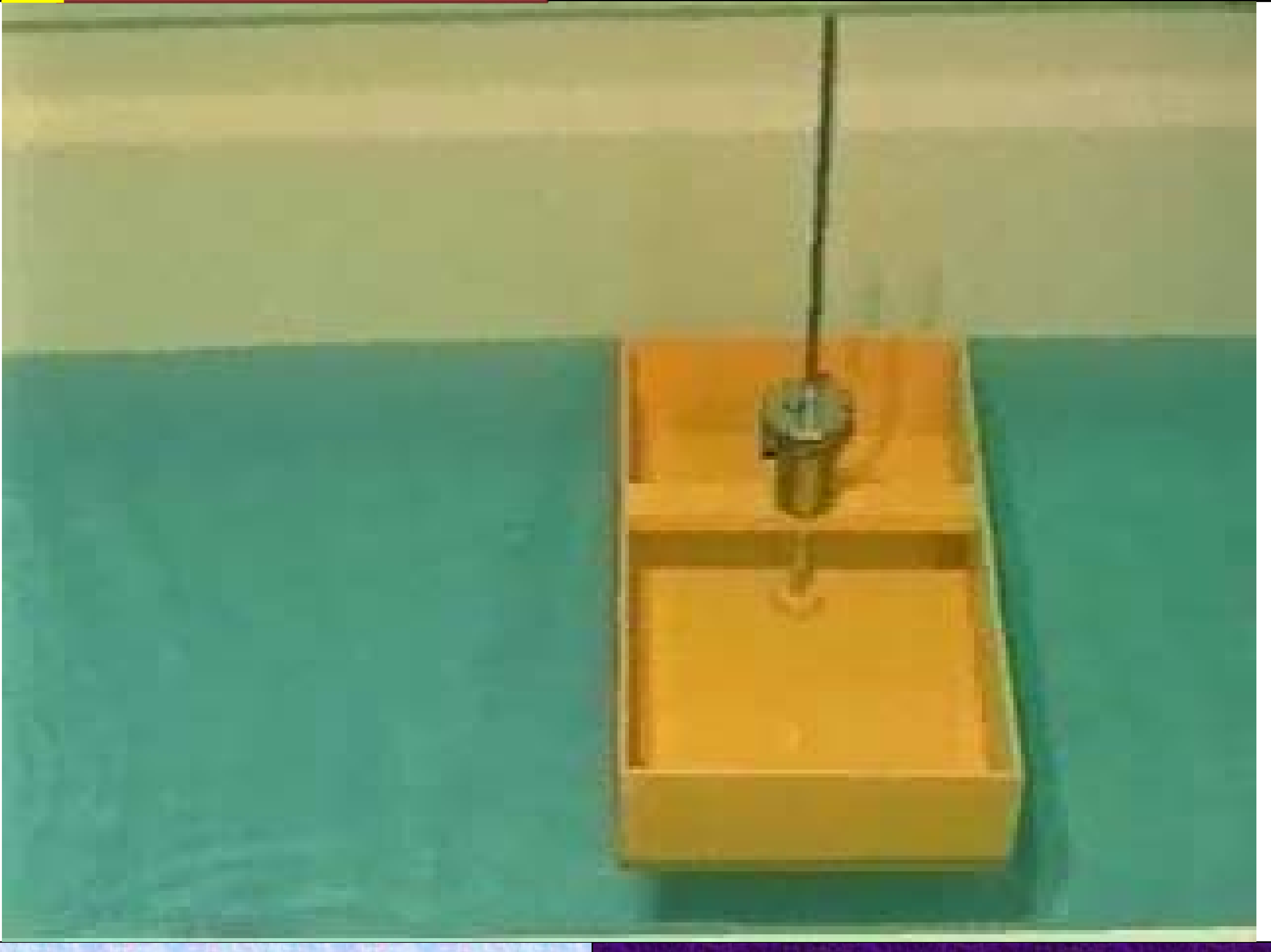
(*b*) Neutrally stable



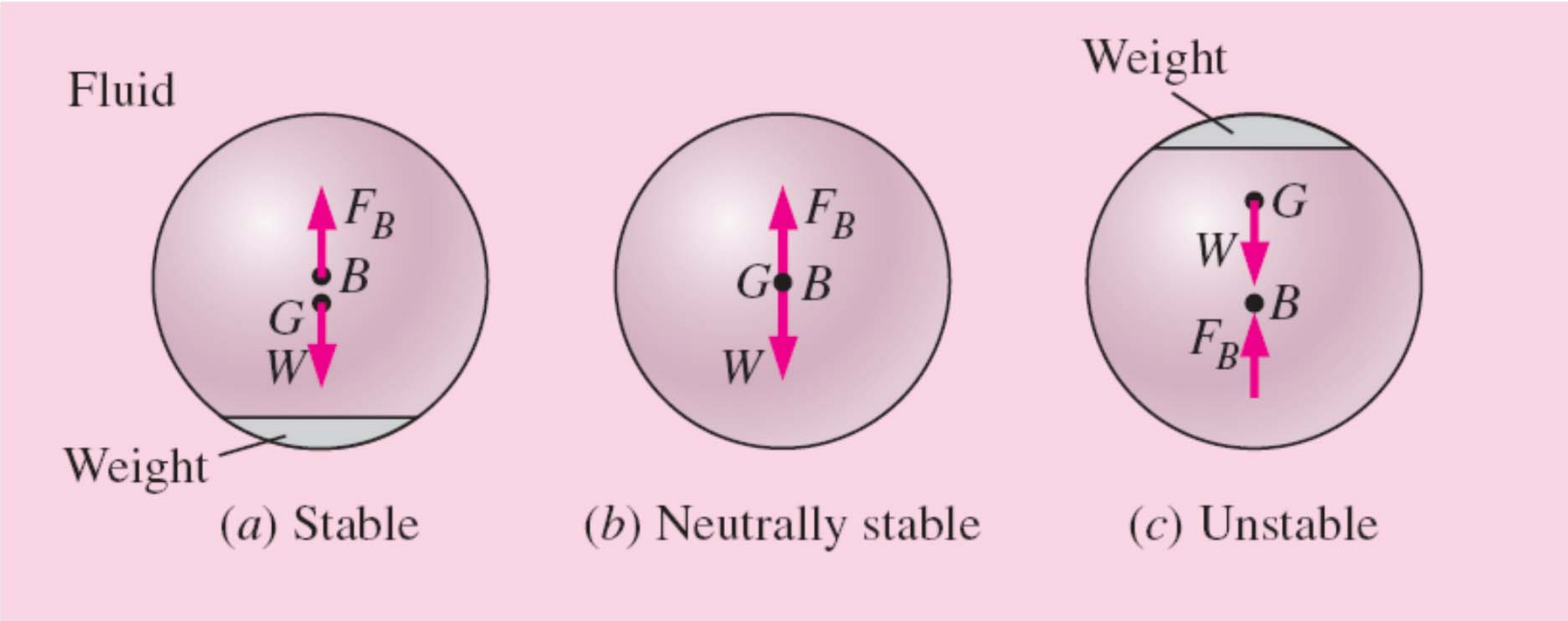
(*c*) Unstable

Stability is easily understood by analyzing a ball on the floor.

Stability of a Model Barge

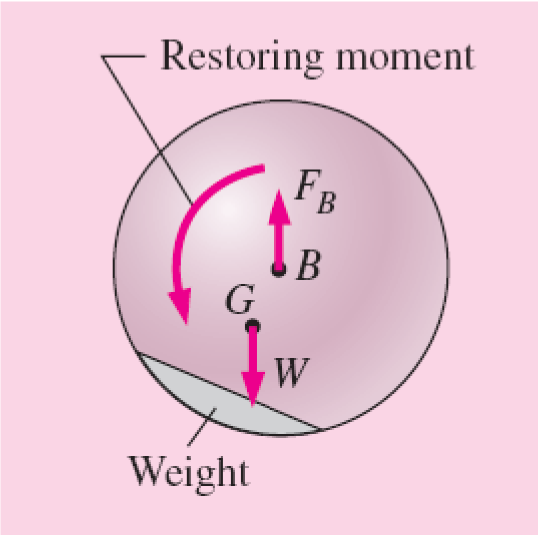


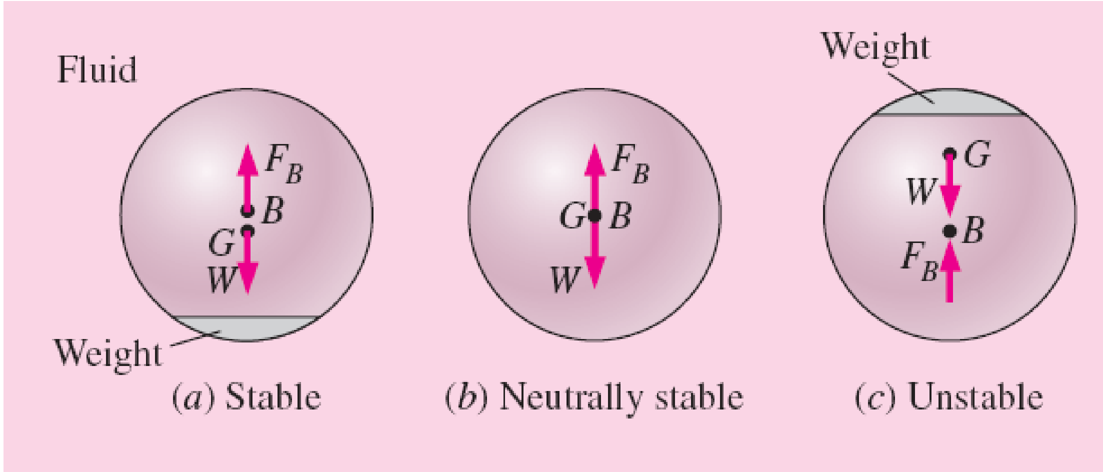
A floating body possesses vertical stability, while an immersed neutrally buoyant body is neutrally stable since it does not return to its original position after a disturbance.



An immersed neutrally buoyant body is (*a*) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body, (*b*) neutrally stable if *G* and *B* are coincident, and (*c*) unstable if *G* is directly above *B*.

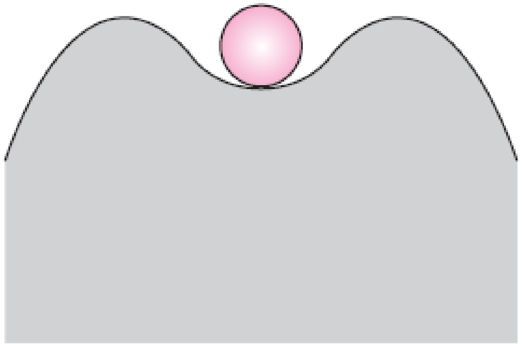
[redacted]

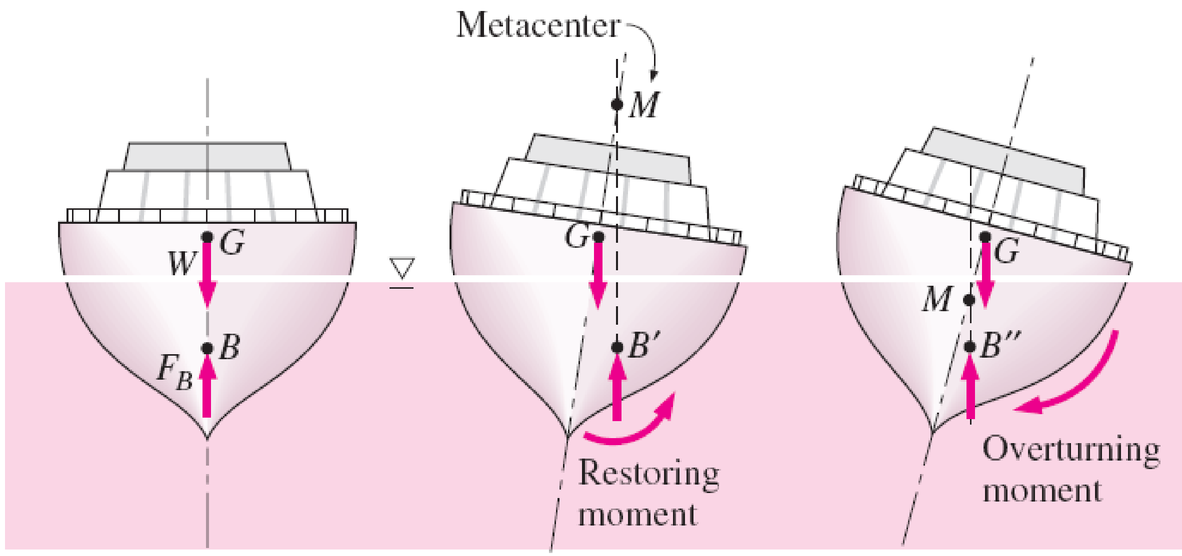




When the center of gravity *G* of an immersed neutrally buoyant body is not vertically aligned with the center of buoyancy *B* of the body, it is not in an equilibrium state and would rotate to its stable state, even without any disturbance.

An immersed neutrally buoyant body is (*a*) stable if the center of gravity *G* is directly below the center of buoyancy *B* of the body, (*b*) neutrally stable if *G* and *B* are coincident, and (*c*) unstable if *G* is directly above *B*.





(*a*) Stable (*b*) Stable (*c*) Unstable

A ball in a trough between two hills is stable for small disturbances, but unstable for large disturbances.

A floating body is *stable* if the body is bottom-heavy and thus the center of gravity *G* is below the centroid *B* of the body, or if the metacenter *M* is above point *G*. However, the body is *unstable* if point *M* is below point *G*.

Metacentric height *GM*: The distance between the center of gravity *G* and the metacenter *M*—the intersection point of the lines of action of the buoyant force through the body before and after rotation.

The length of the metacentric height *GM* above *G* is a measure of the stability: the larger it is, the more stable is the floating body.

Acceleration on a Straight Path

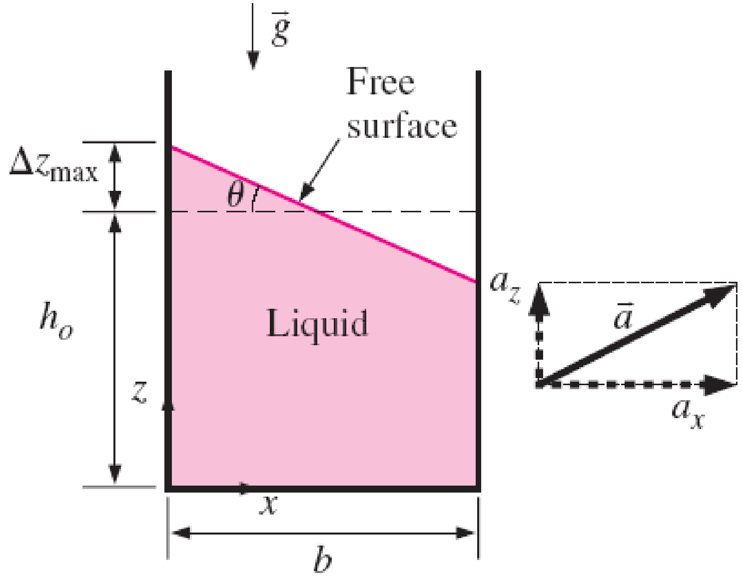
$$\boxed{\frac{\partial P}{\partial x} = -\rho a_x, \quad \frac{\partial P}{\partial y} = 0, \quad \text{and} \quad \frac{\partial P}{\partial z} = -\rho(g + a_z)}$$

$$P = P(x, z)$$

$$dP = (\partial P/\partial x)\, dx + (\partial P/\partial z)\, dz$$

$$\boxed{dP = -\rho a_x\, dx - \rho(g + a_z)\, dz}$$

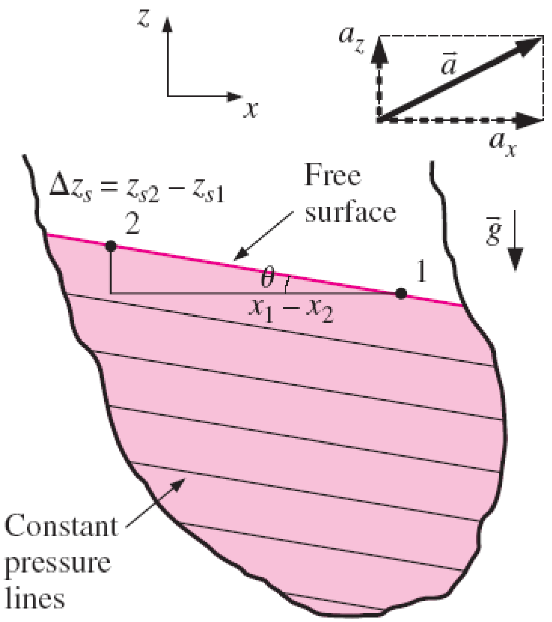
$$\boxed{P_2 - P_1 = -\rho a_x(x_2 - x_1) - \rho(g + a_z)(z_2 - z_1)}$$



Pressure variation: $P = P_0 - \rho a_x x - \rho(g + a_z)z$

Rigid-body motion of a liquid in a linearly accelerating tank.

[redacted]



Lines of constant pressure (which are the projections of the surfaces of constant pressure on the *xz*-plane) in a linearly accelerating liquid. Also shown is the vertical rise.

Vertical rise of surface:

$$\Delta z_s = z_{s2} - z_{s1} = -\frac{a_x}{g + a_z}(x_2 - x_1)$$

Surfaces of constant pressure: $$\frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = \text{constant}$$

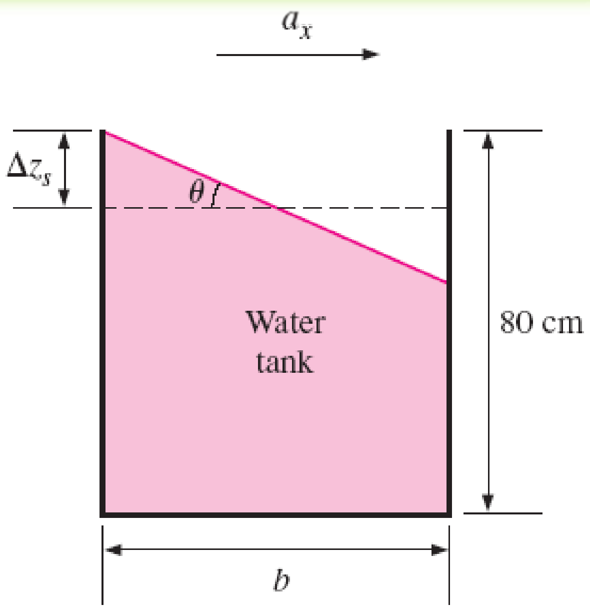
Slope of isobars: $$\text{Slope} = \frac{dz_{\text{isobar}}}{dx} = -\frac{a_x}{g + a_z} = -\tan\theta$$

EXAMPLE 3–12

An 80-cm-high fish tank of cross section 2x0.6 m that is initially filled with water is to be transported on the back of a truck. The truck accelerates from 0 to 90 km/h in 10 s. If it is desired that no water spills during acceleration, determine the allowable initial water height in the tank. Would you recommend the tank to be aligned with the long or short side parallel to the direction of motion?

Solution A fish tank is to be transported on a truck. The allowable water height to avoid spill of water during acceleration and the proper orientation are to be determined.

Assumptions **1** The road is horizontal during acceleration so that acceleration has no vertical component (a_z=0). **2** Effects of splashing, braking, driving over bumps, and climbing hills are assumed to be secondary and are not considered. **3** The acceleration remains constant.



$$a_x = \frac{\Delta V}{\Delta t} = \frac{(90-0)\text{ km/h}}{10\text{ s}}\left(\frac{1\text{ m/s}}{3.6\text{ km/h}}\right) = 2.5\text{ m/s}^2$$

$$\tan\theta = \frac{a_x}{g+a_z} = \frac{2.5}{9.81+0} = 0.255 \qquad (\text{and thus } \theta = 14.3°)$$

Case 1: The long side is parallel to the direction of motion:

$$\Delta z_{s1} = (b_1/2)\tan\theta = [(2\ \text{m})/2] \times 0.255 = 0.255\ \text{m} = \mathbf{25.5\ cm}$$

Case 2: The short side is parallel to the direction of motion:

$$\Delta z_{s2} = (b_2/2)\tan\theta = [(0.6\ \text{m})/2] \times 0.255 = 0.076\ \text{m} = \mathbf{7.6\ cm}$$

Discussion Note that the orientation of the tank is important in controlling the vertical rise. Also, the analysis is valid for any fluid with constant density, not just water, since we used no information that pertains to water in the solution.

Rotation in a Cylindrical Container

Consider a vertical cylindrical container partially filled with a liquid. The container is now rotated about its axis at a constant angular velocity of ω. After initial transients, the liquid will move as a rigid body together with the container. There is no deformation, and thus there can be no shear stress, and every fluid particle in the container moves with the same angular velocity.

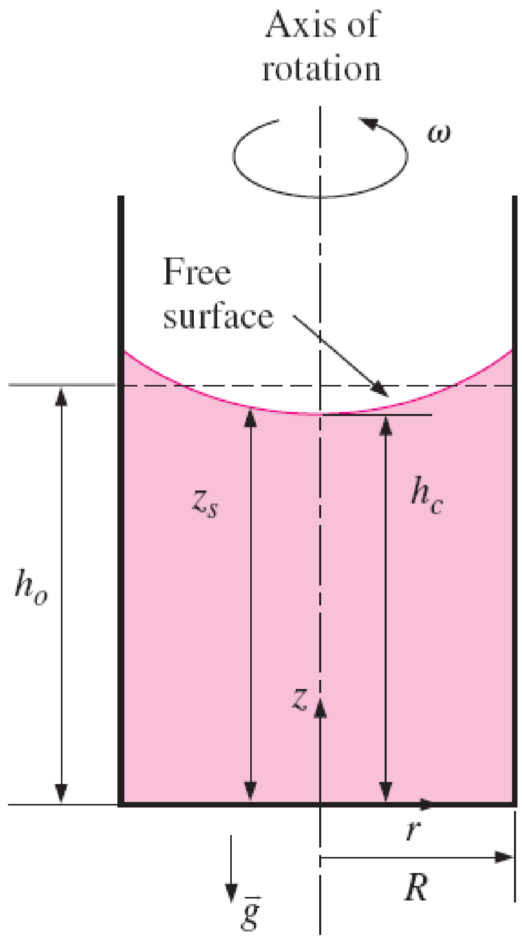
$$\frac{\partial P}{\partial r} = \rho r \omega^2, \qquad \frac{\partial P}{\partial \theta} = 0, \qquad \text{and} \qquad \frac{\partial P}{\partial z} = -\rho g$$

$$P = P(r, z) \quad dP = (\partial P/\partial r)dr + (\partial P/\partial z)dz$$

$$dP = \rho r \omega^2 \, dr - \rho g \, dz$$

$$\frac{dz_{\text{isobar}}}{dr} = \frac{r\omega^2}{g}$$

Surfaces of constant pressure: $z_{\text{isobar}} = \dfrac{\omega^2}{2g} r^2 + C_1$



Rigid-body motion of a liquid in a rotating vertical cylindrical container.

[redacted]

$$z_s = \frac{\omega^2}{2g} r^2 + h_c$$

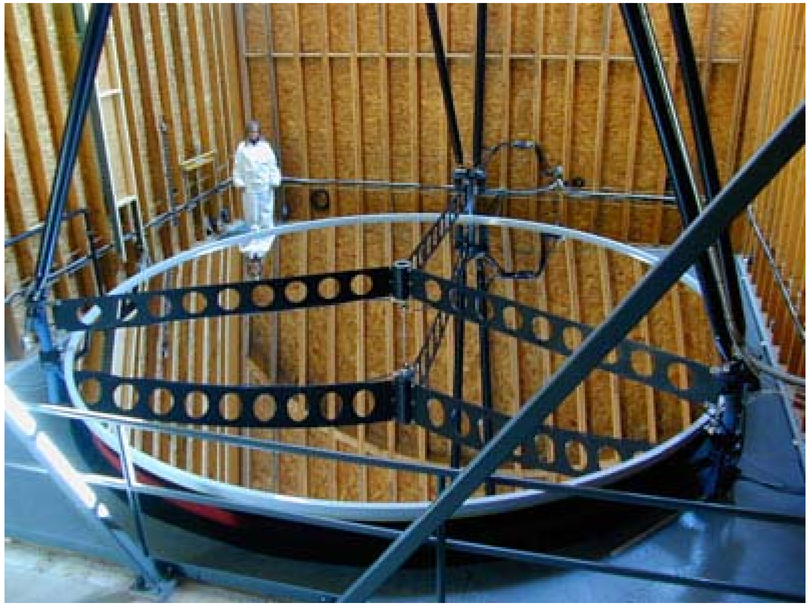
$$V = \int_{r=0}^{R} 2\pi z_s r \, dr = 2\pi \int_{r=0}^{R} \left(\frac{\omega^2}{2g} r^2 + h_c \right) r \, dr = \pi R^2 \left(\frac{\omega^2 R^2}{4g} + h_c \right)$$

$$V = \pi R^2 h_0$$

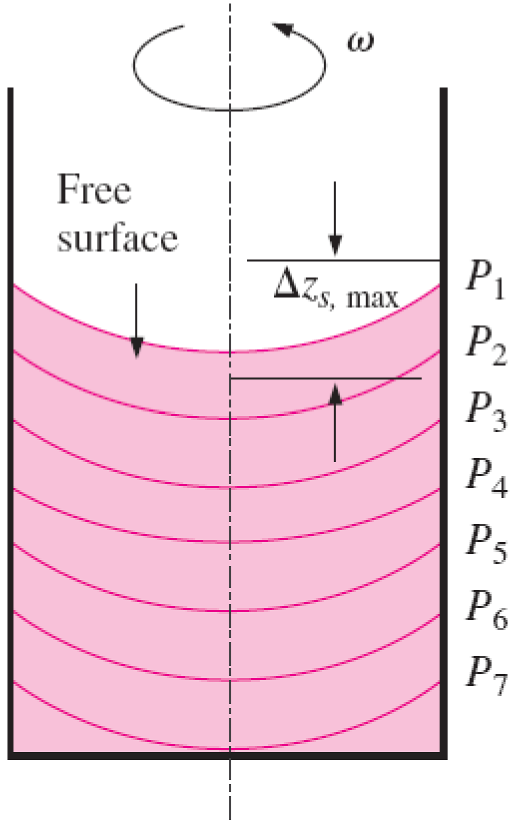
$$h_c = h_0 - \frac{\omega^2 R^2}{4g}$$

Free surface: $z_s = h_0 - \dfrac{\omega^2}{4g}(R^2 - 2r^2)$

Free Vortex



The 6-meter spinning liquid-mercury mirror of the Large Zenith Telescope located near Vancouver, British Columbia.



Surfaces of constant pressure in a rotating liquid.

[redacted]

Maximum height difference:

$$\Delta z_{s,\max} = z_s(R) - z_s(0) = \frac{\omega^2}{2g} R^2$$

$$dP = \rho r \omega^2\, dr - \rho g\, dz$$

$$P_2 - P_1 = \frac{\rho\omega^2}{2}(r_2^2 - r_1^2) - \rho g(z_2 - z_1)$$

Pressure variation:

$$P = P_0 + \frac{\rho\omega^2}{2} r^2 - \rho g z$$

Note that at a fixed radius, the pressure varies hydrostatically in the vertical direction, as in a fluid at rest.

For a fixed vertical distance *z*, the pressure varies with the square of the radial distance *r*, increasing from the centerline toward the outer edge.

In any horizontal plane, the pressure difference between the center and edge of the container of radius *R* is

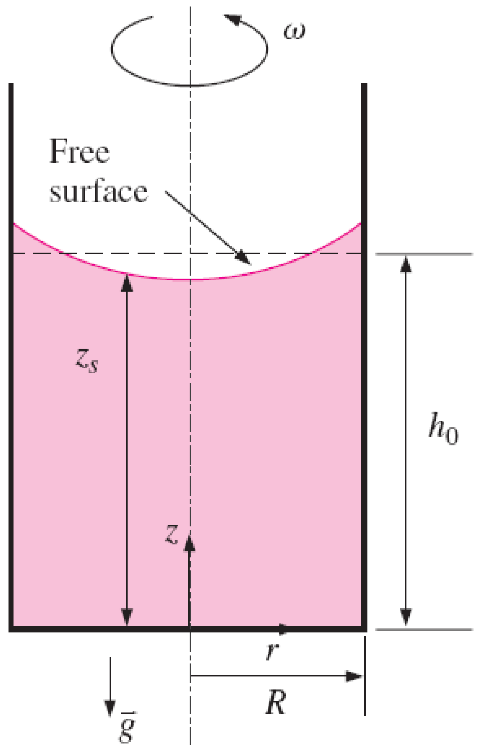
$$\boxed{\Delta P = \rho\omega^2 R^2/2}$$

EXAMPLE 3–13

A 20-cm-diameter, 60-cm-high vertical cylindrical container is partially filled with 50-cm-high liquid whose density is 850 kg/m^3. Now the cylinder is rotated at a constant speed. Determine the rotational speed at which the liquid will start spilling from the edges of the container.

Solution A vertical cylindrical container partially filled with a liquid is rotated. The angular speed at which the liquid will start spilling is to be determined.

Assumptions ***1*** *The increase in the rotational speed is very slow so that the* liquid in the container always acts as a rigid body. 2 The bottom surface of the container remains covered with liquid during rotation (no dry spots).



$$z_s = h_0 - \frac{\omega^2}{4g}(R^2 - 2r^2) \qquad z_s(R) = h_0 + \frac{\omega^2 R^2}{4g}$$

$$\omega = \sqrt{\frac{4g[z_s(R) - h_0]}{R^2}} = \sqrt{\frac{4(9.81\ \text{m/s}^2)[(0.6 - 0.5)\ \text{m}]}{(0.1\ \text{m})^2}} = \mathbf{19.8\ rad/s}$$

$$\dot{n} = \frac{\omega}{2\pi} = \frac{19.8\ \text{rad/s}}{2\pi\ \text{rad/rev}}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \mathbf{189\ rpm}$$

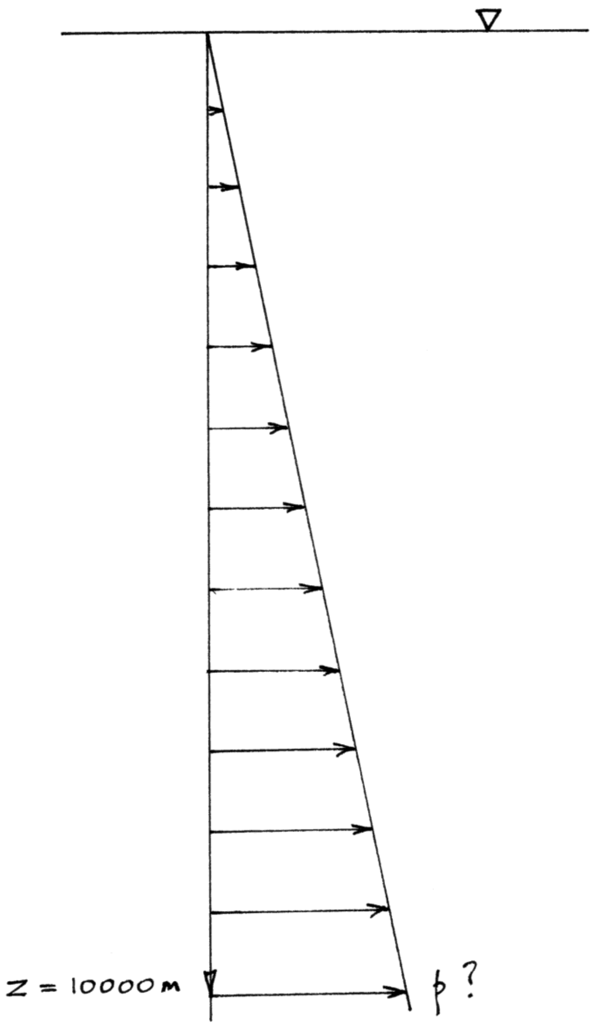
Discussion Note that the analysis is valid for any liquid since the result is independent of density or any other fluid property. We should also verify that our assumption of no dry spots is valid. The liquid height at the center is;

$$z_s(0) = h_0 - \frac{\omega^2 R^2}{4g} = 0.4\ \text{m}$$

Summary

- Pressure
- Pressure Measurement Devices
- Introduction to Fluid Statics
- Hydrostatic Forces on Submerged Plane Surfaces
- Hydrostatic Forces on Submerged Curved Surfaces
- Buoyancy and Stability
- Fluids in Rigid-Body Motion

Hydrostatics – Interactive question



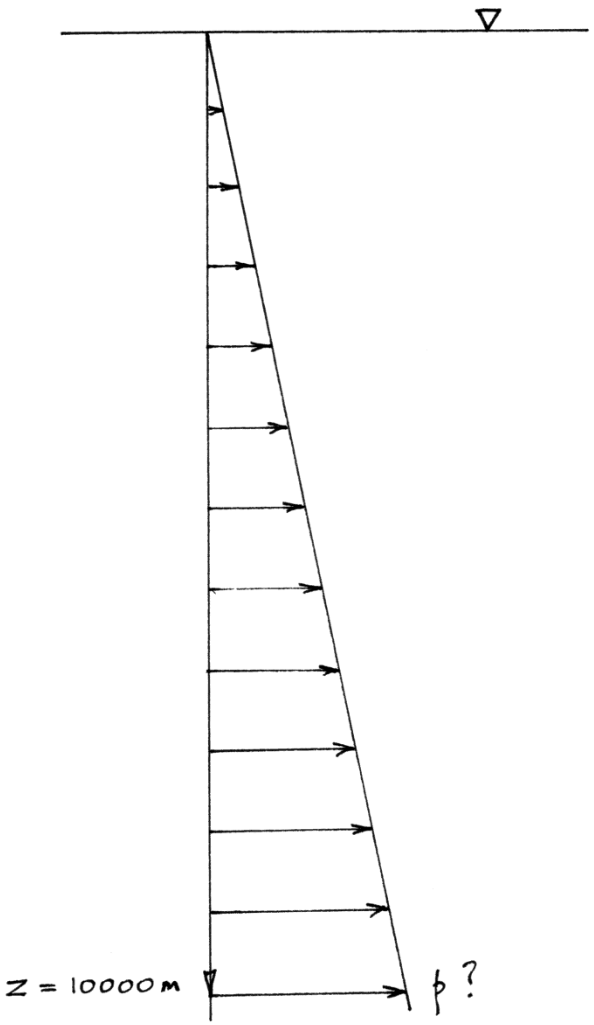
The density of seawater is 1030 kg/m^3. Which of the following values is correct for the hydrostatic pressure at a depth of 10 000 m?

(a) 101 bar

(b) 1010 bar

(c) 10101 bar

Hydrostatics – Interactive question



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(a) 101 bar

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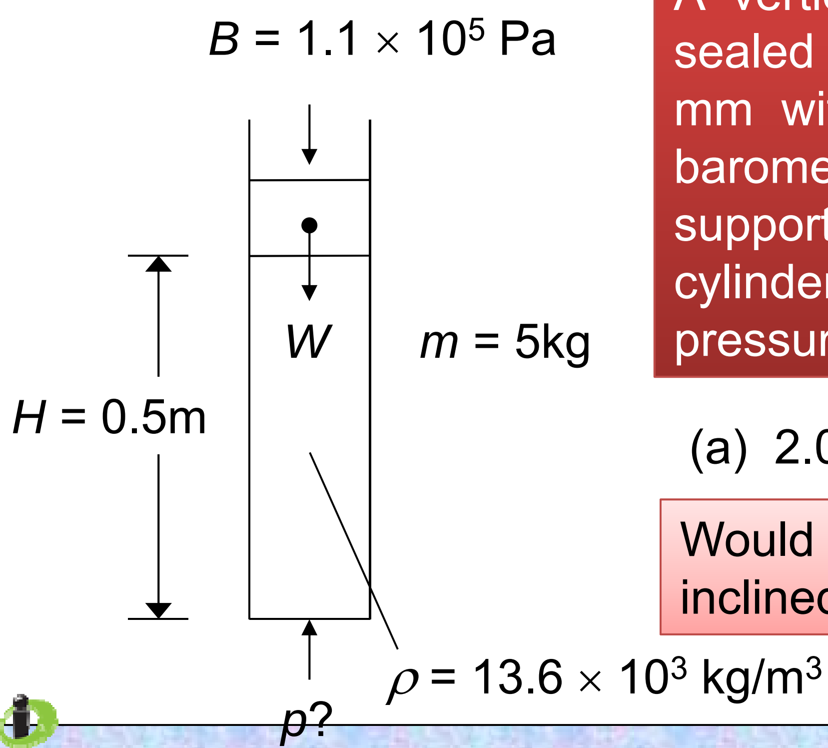
Hydrostatics – Interactive question

Principle: $\Delta p = \rho g \Delta z$

Application: Calculation of pressure due to a column of liquid supporting a weight and subject to external pressure.



A vertical cylinder of inside diameter 50 mm is sealed at the bottom and filled to a depth of 500 mm with mercury of relative density 13.6. The barometric pressure is 1.1 bar. If the mercury supports a piston of mass 5 kg (a perfect fit in the cylinder with no leakage or friction), what is the pressure at the bottom of the cylinder?

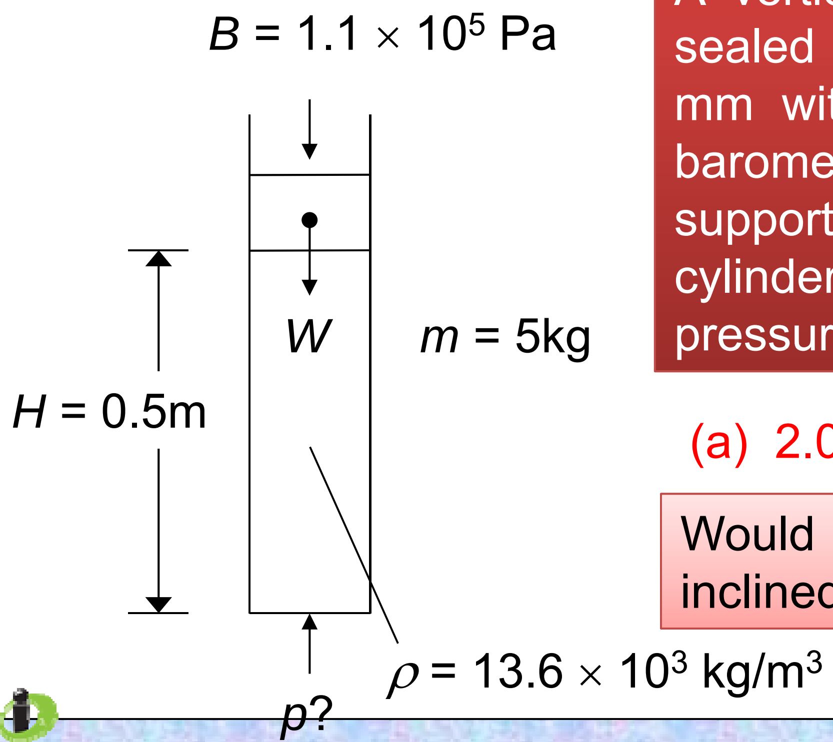
(a) 2.02 bar (b) 1.32 bar (c) 0.2 bar

Would the answer be different if the cylinder were inclined to the vertical?

(a) yes (b) no

Principle: $\Delta p = \rho g \Delta z$

Application: Calculation of pressure due to a column of liquid supporting a weight and subject to external pressure.



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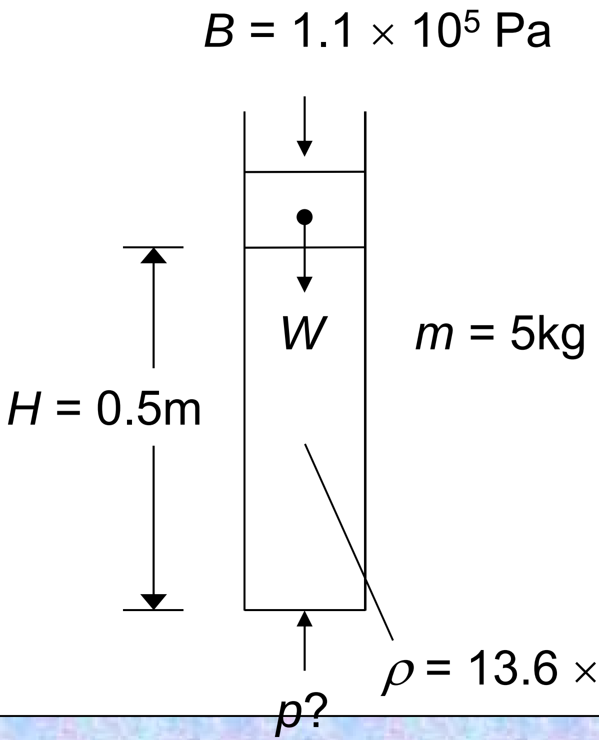

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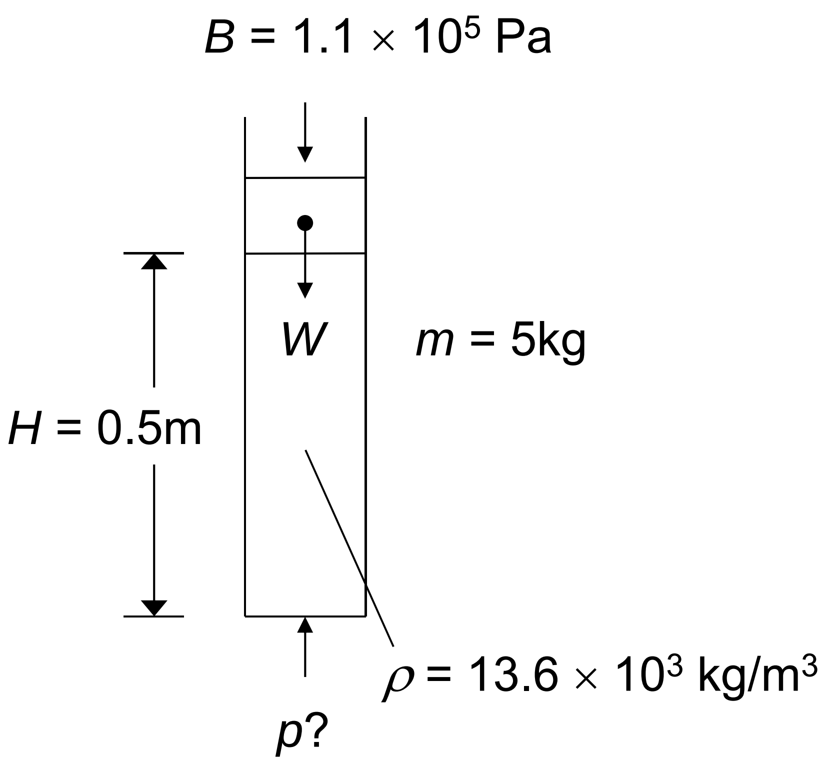
Hydrostatics – Interactive question



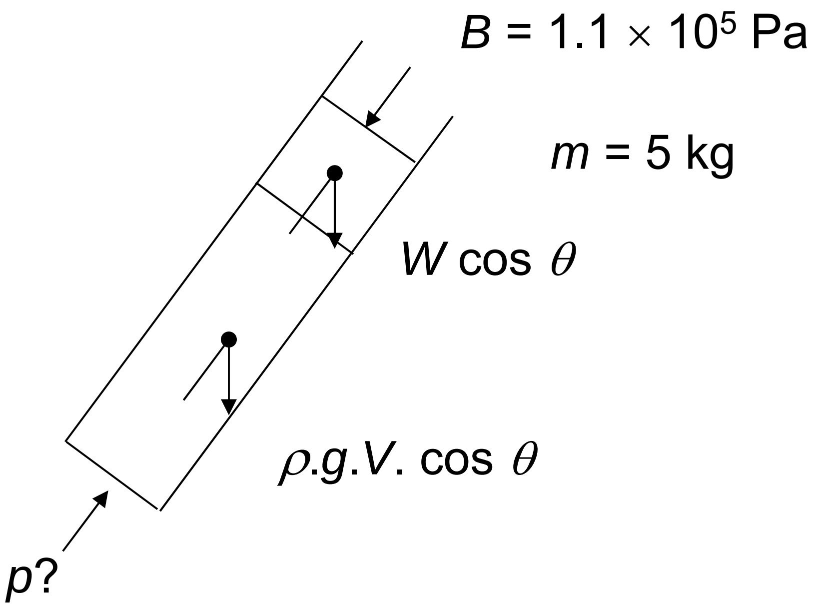
$$p\left(\frac{\pi D^2}{4}\right) - B\left(\frac{\pi D^2}{4}\right) - W - \rho g\left(\frac{\pi D^2}{4}\right)H = 0$$

$$p = B + W\left(\frac{4}{\pi D^2}\right) + \rho g H$$

$$p = 1.1x10^5 + 5\left(\frac{4}{\pi.0.05^2}\right) + 13.6x10^3x9.81x0.5$$

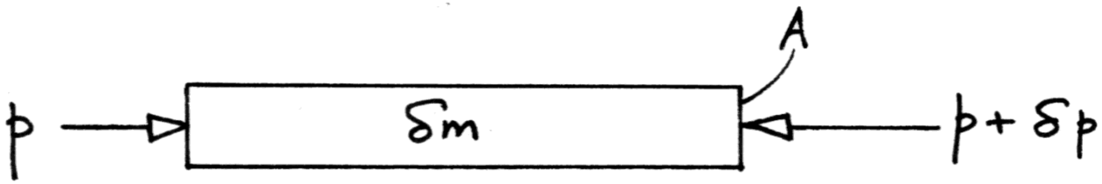
p = 2.02 bar

Hydrostatics – Interactive question



$$p = B + W\left(\frac{4}{\pi D^2}\right)\cos\theta + \rho g H \cos\theta$$

Manometry (pressure measurement)



$$p.A - (p + \delta p).A = \delta m.a = 0 \quad \Delta\delta p = 0$$

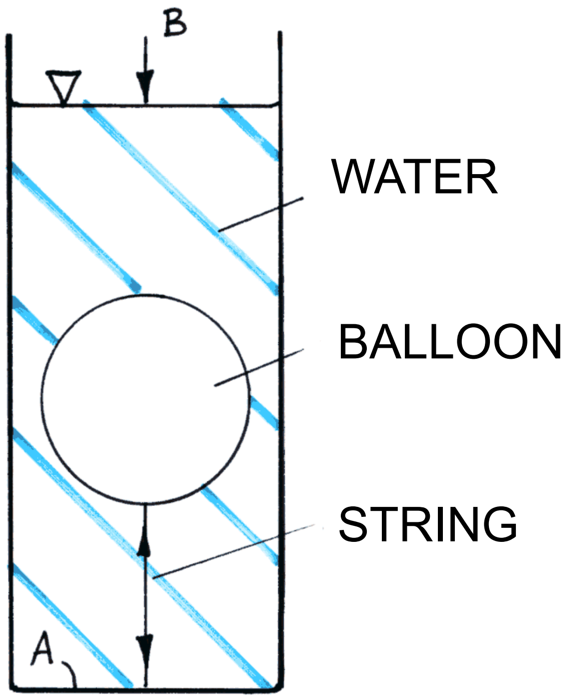
FLUID AT REST – ISOBARS HORIZONTAL

(EQUAL PRESSURES AT SAME LEVEL IN SINGLE FLUID)

Hydrostatics – Interactive question

Principle: Hydrostatic pressure increases in proportion to depth in a uniform density fluid at rest

Application: Pressure variation in a vessel containing an object submerged within a liquid.



With the balloon held under water, is the pressure at A

(a) Higher than without the balloon

(b) Unchanged

(c) Lower than without the balloon?

Hydrostatics – Interactive question

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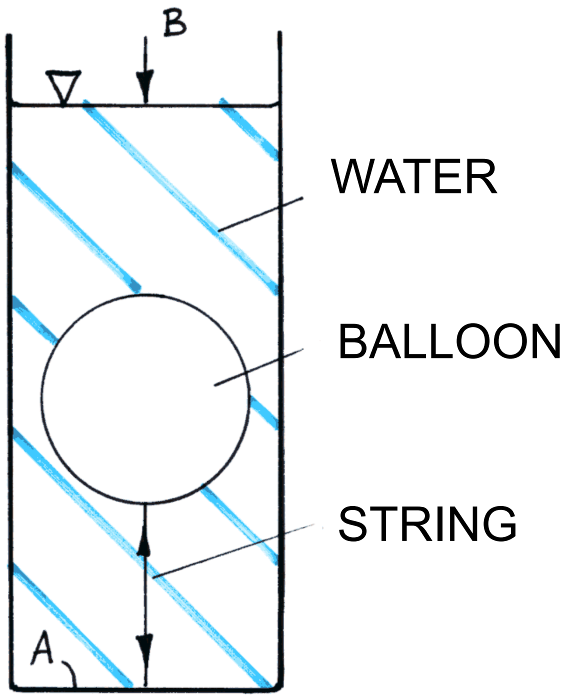

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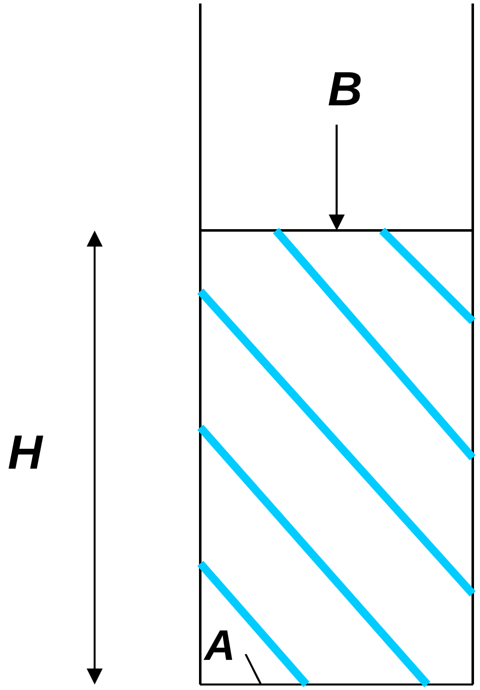
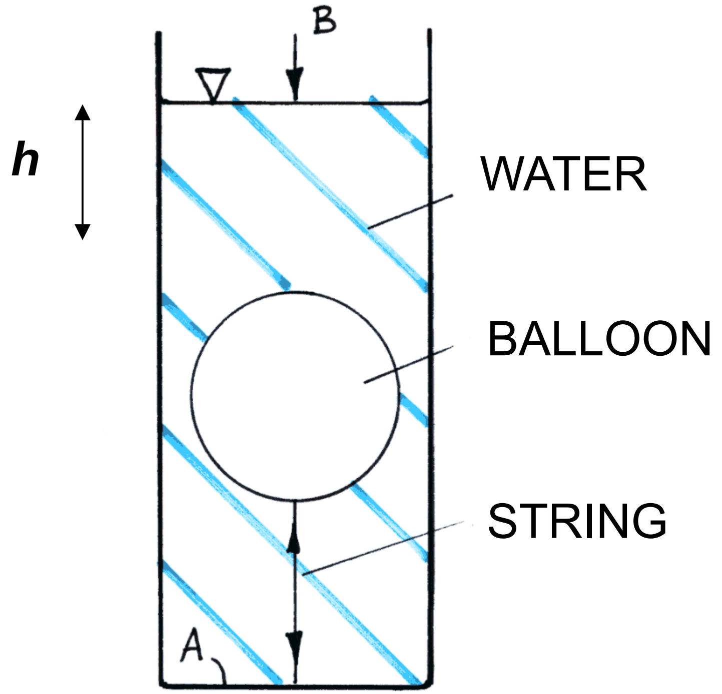
Hydrostatics – Interactive question



$p_A = \rho\, g\, H$

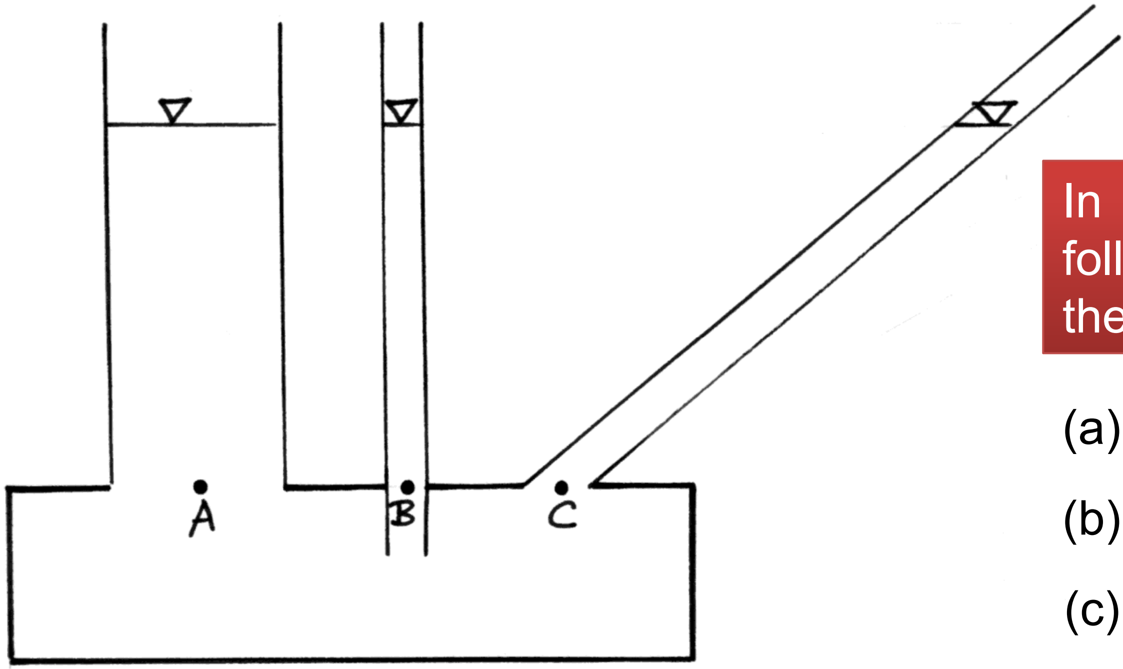


$p_A = \rho\, g\, (H + h)$

Hydrostatics – Interactive question

Principle: Lines of constant pressure (isobars) in a continuous volume of a constant-density fluid at rest are horizontal

Application: Pressure variation in vertical and inclined tubes



In the diagram, which of the following conditions applies to the pressure at A, B, and C?

(a) $p_A > p_B$ $p_B < p_C$

(b) $p_A = p_B$ $p_B < p_C$

(c) $p_A = p_B = p_C$

(d) $p_A > p_B$ $p_B > p_C$

Hydrostatics – Interactive question

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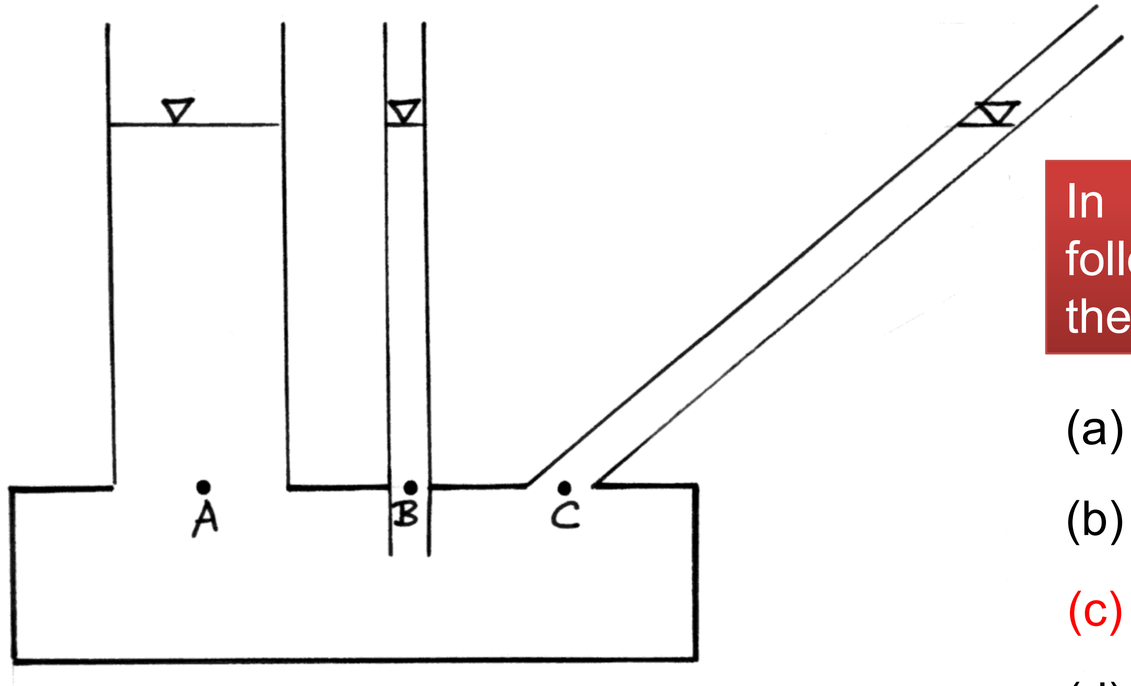

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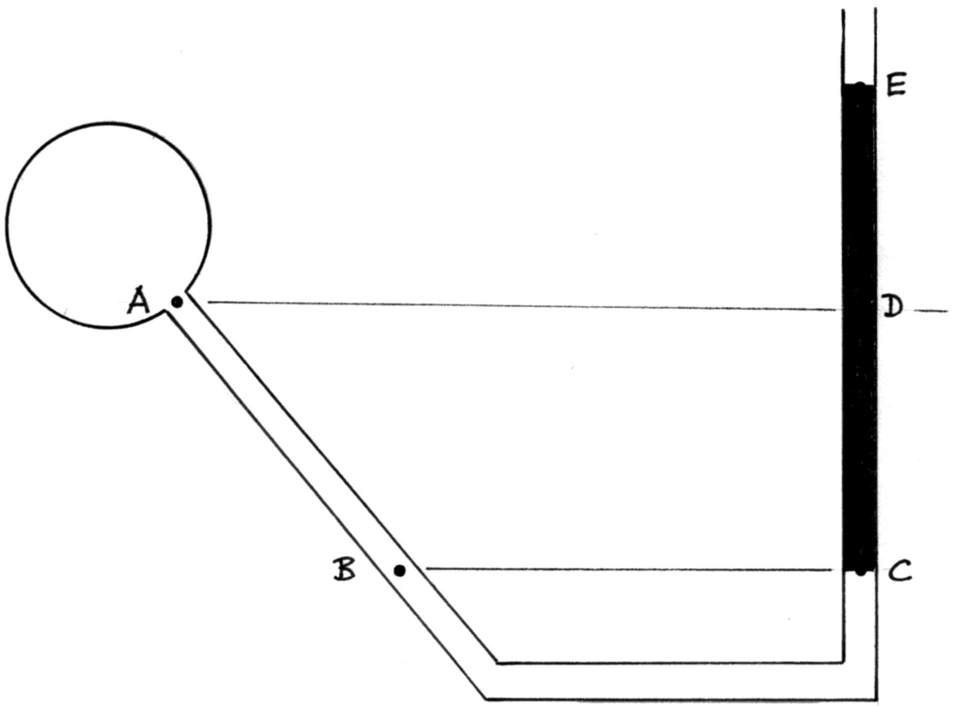
Hydrostatics – Interactive question

Principle: Lines of constant pressure (isobars) in a continuous volume of a constant-density fluid at rest are horizontal

Application: Pressure variation in a manometer



In the manometer system, the points of equal pressure are

(a) A and D

(b) A and C

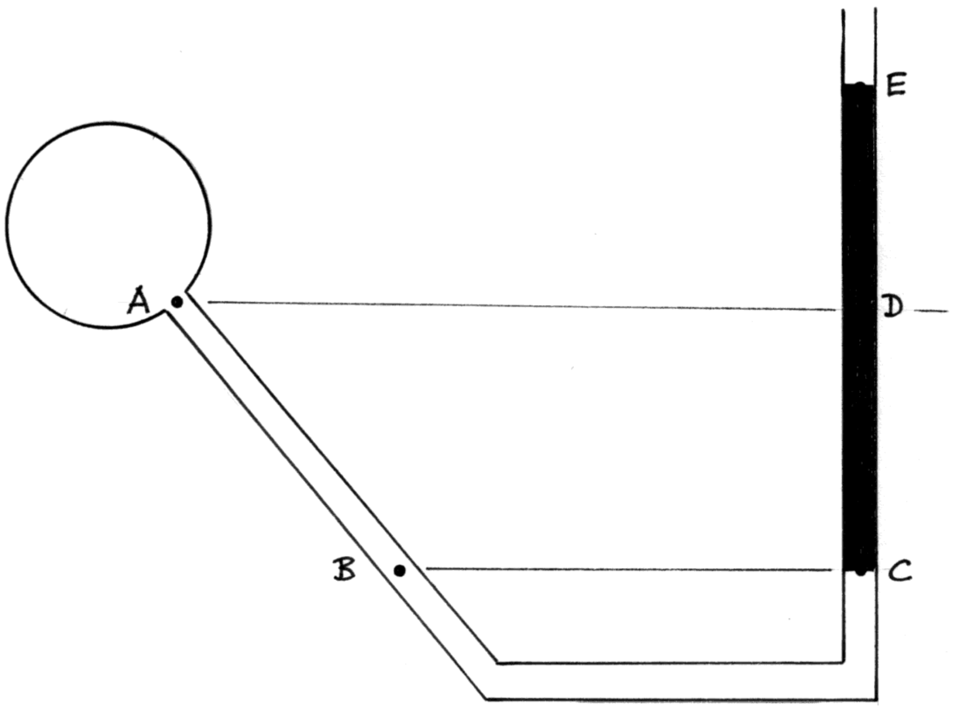
(c) B and C

(d) B and D

Hydrostatics – Interactive question

Principle: Lines of constant pressure (isobars) in a continuous volume of a constant-density fluid at rest are horizontal

Application: Pressure variation in a manometer



In the manometer system, the points of equal pressure are

(a) A and D

(b) A and C

(c) B and C

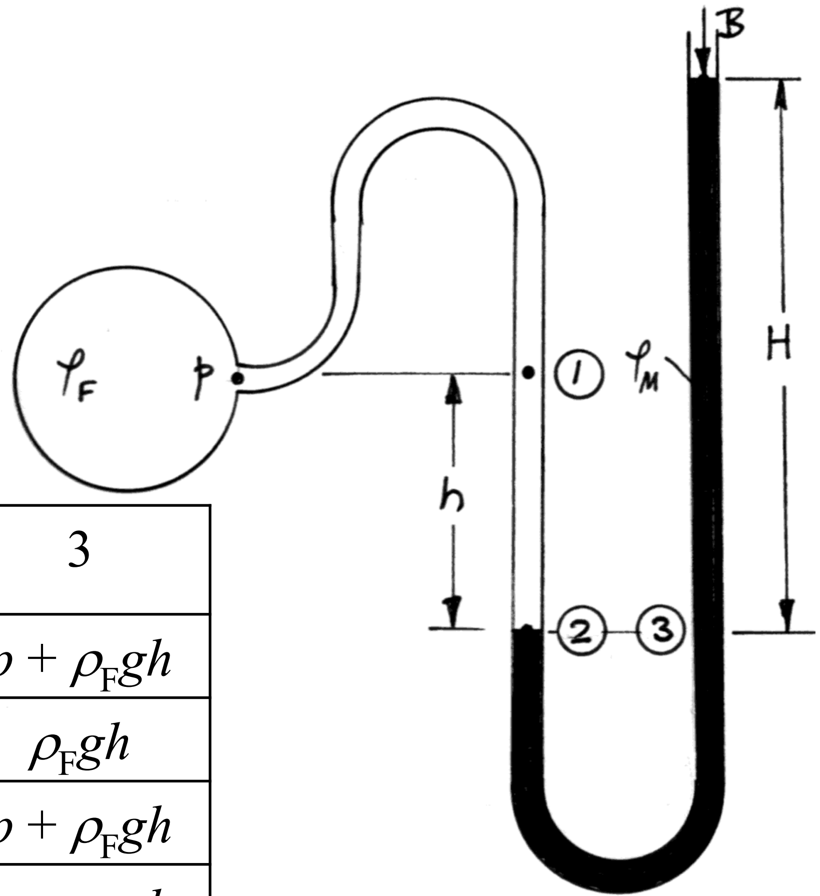
(d) B and D

Hydrostatics – Interactive question

Principle: In a fluid at rest, isobars (i.e. lines or surfaces of constant pressure are horizontal)

Application: U - tube manometer

The following table shows the pressures at locations 1, 2 and 3. Which row of the table is correct?



	1	2	3	3
(a)	P	$p - \rho_F gh$	$B - \rho_M gH$	$p + \rho_F gh$
(b)	0	$\rho_M gh$	$\rho_M gH$	$\rho_F gh$
(c)	P	$p + \rho_F gh$	$B + \rho_M gH$	$p + \rho_F gh$
(d)	p	$p + \rho_F gh$	$B - \rho_M gH$	$p - \rho_F gh$

Hydrostatics – Interactive question

Principle: In a fluid at rest, isobars (i.e. lines or surfaces of constant pressure are horizontal)

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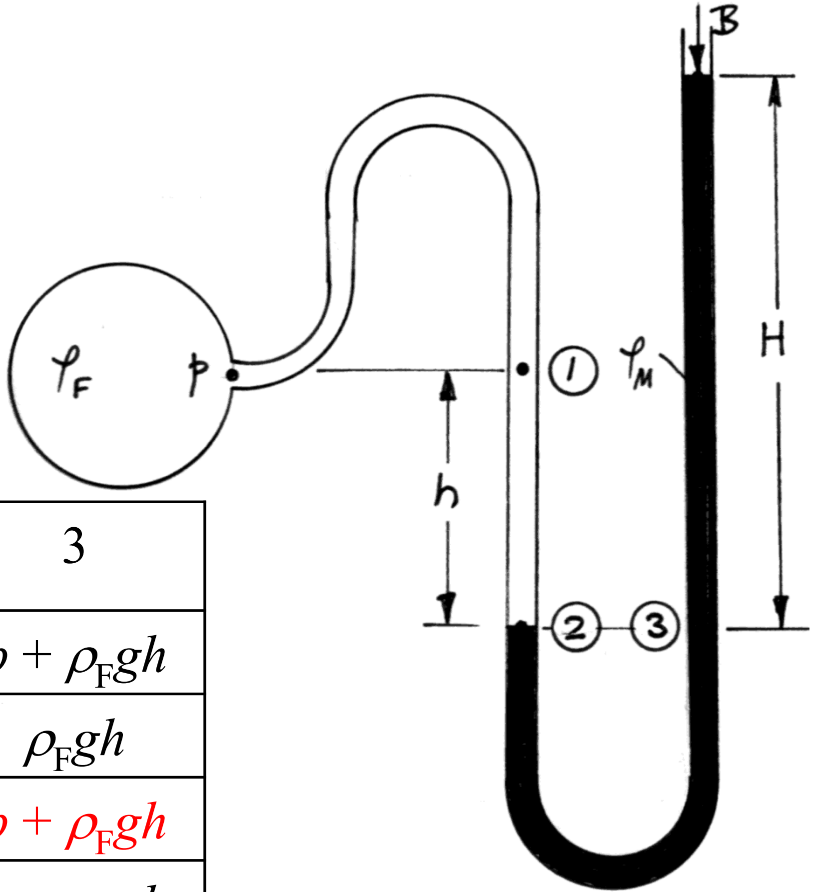
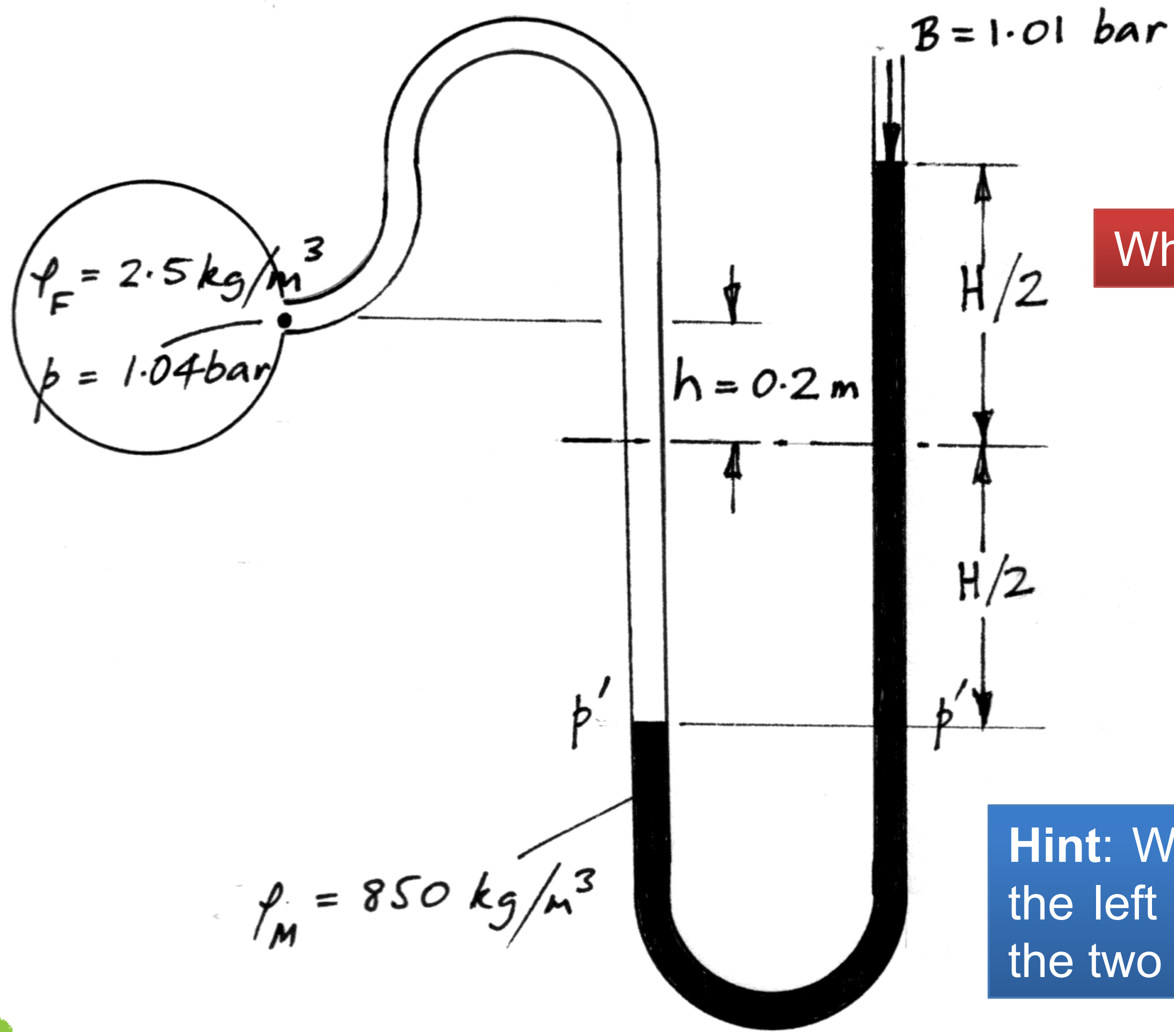


	1	2	3	3
(a)	P	$p-\rho_F gh$	$B-\rho_M gH$	$p+\rho_F gh$
(b)	0	$\rho_M gh$	$\rho_M gH$	$\rho_F gh$
(c)	P	$p+\rho_F gh$	$B+\rho_M gH$	$p+\rho_F gh$
(d)	p	$p+\rho_F gh$	$B-\rho_M gH$	$p-\rho_F gh$

Hydrostatics – Interactive question



Which is the correct value for *H*?

(a) 0.361 m

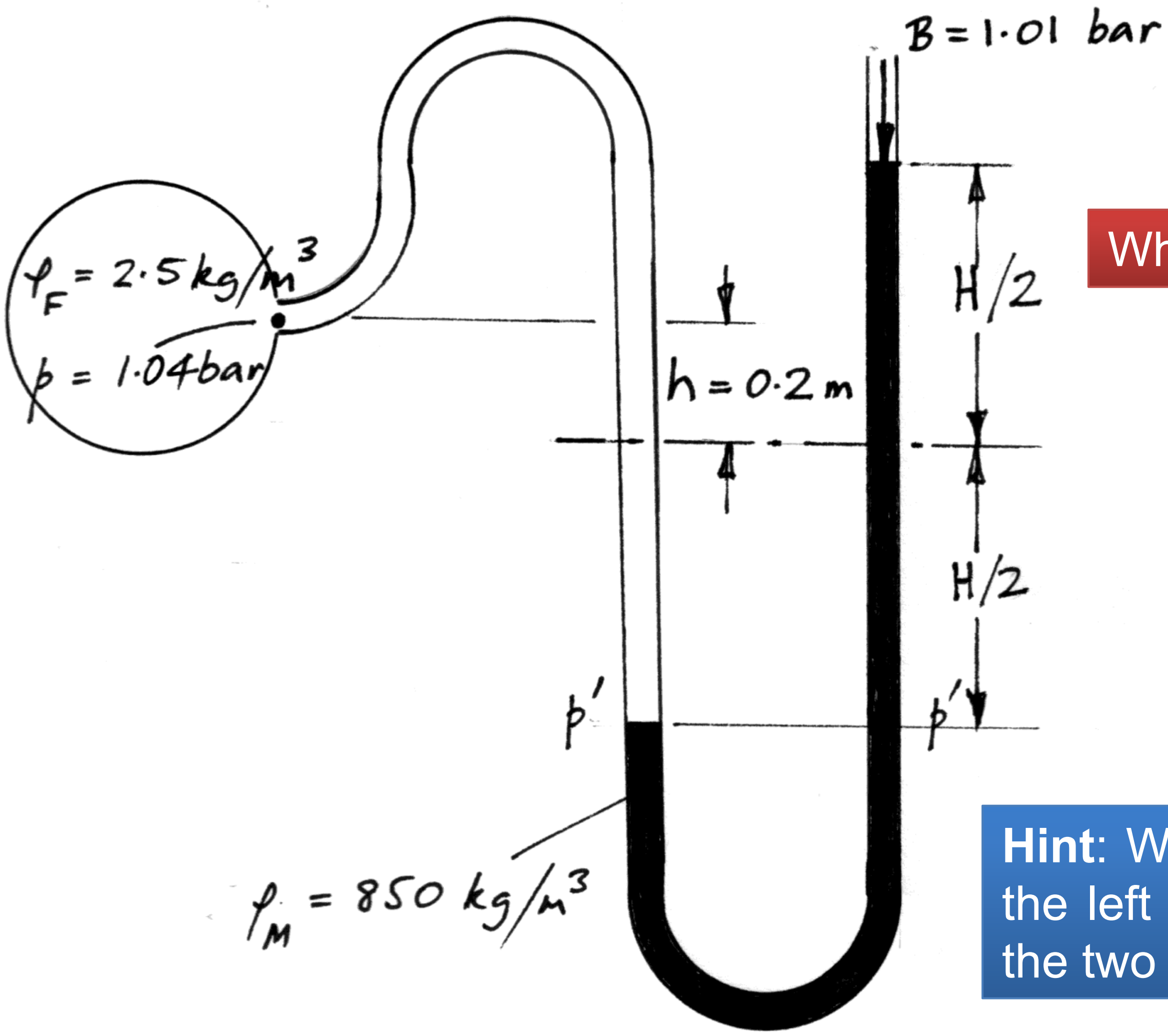
(b) 0.365 m

(c) 0.371 m

(d) 0.376 m

Hint: Write down equations for *p′* on the left and right-hand sides, equate the two and solve for *H*.

Hydrostatics – Interactive question



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Hint: Write down equations for *p′* on the left and right-hand sides, equate the two and solve for *H*.

Hydrostatics – Interactive question

LHS $$p' = p + \rho_F g\left(h + \frac{H}{2}\right)$$ RHS $$p' = B + \rho_M gH$$

$$p = B + \rho_M gH + \rho_F g\left(h + \frac{H}{2}\right)$$

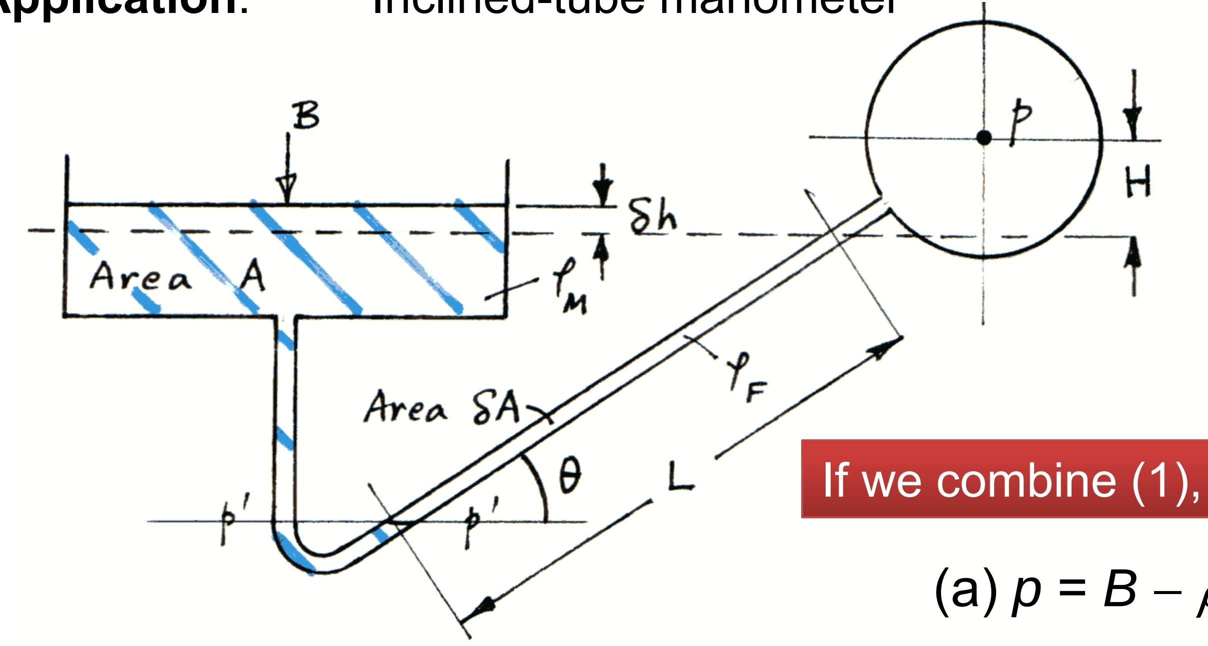
$$H = \frac{(1.04 - 1.01)10^5 + (2.5x9.81x0.2)}{\left(850 - \frac{2.5}{2}\right)x9.81}$$

$$H = 0.361\ m$$

Hydrostatics – Interactive question

Principles: Equal volumes and hydrostatic pressure variation

Application: Inclined-tube manometer



If we combine (1), (2) and (3), which is correct?

(a) $p = B - \rho_F gH + \rho_M gL\left(\sin\theta + \frac{\delta A}{A}\right)$

(b) $p = B + \rho_M gL \sin\theta$

(c) $p = B + \rho_F gH + \rho_M g\, L\frac{\delta A}{A}$

(d) $p = B - \rho_F gH + \rho_M g\, L\frac{\delta A}{A} + (\rho_M - \rho_F)gL\sin\theta$

$L.\delta A = \delta h.A$ (1)

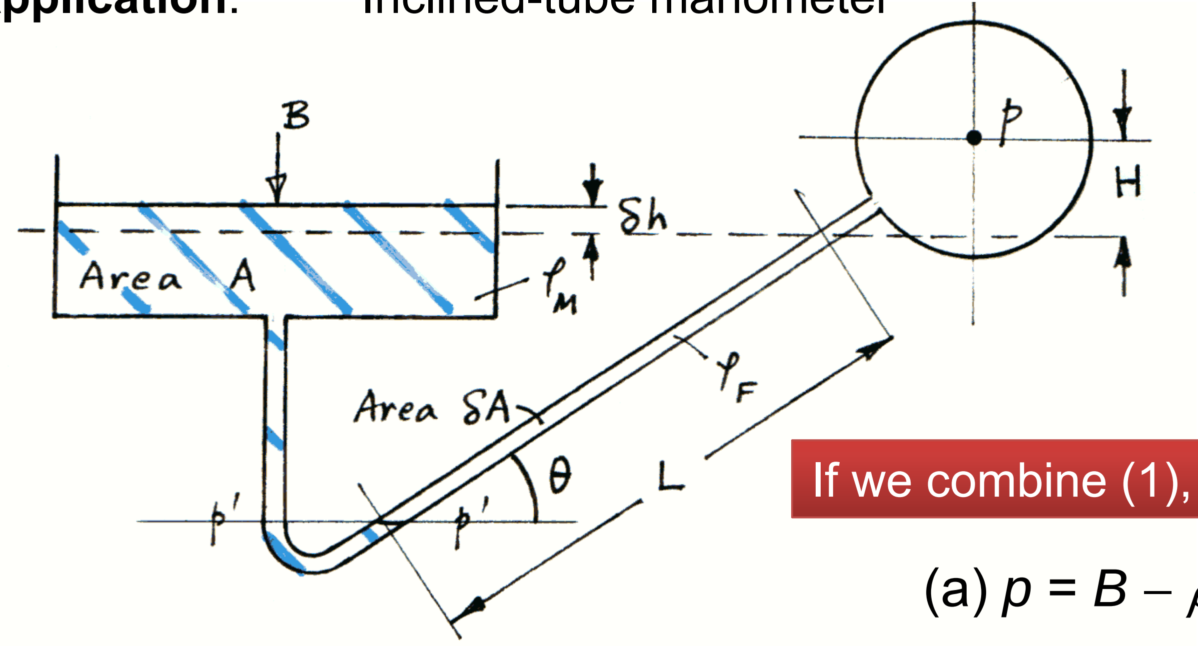
LHS $p' = B + \rho_M g\,(\delta h + L\sin\theta)$ (2)

RHS $p' = p + \rho_F g\,(H + L\sin\theta)$ (3)

Hydrostatics – Interactive question

Principles: Equal volumes and hydrostatic pressure variation

Application: Inclined-tube manometer



If we combine (1), (2) and (3), which is correct?

(a) $p = B - \rho_F gH + \rho_M gL \left(\sin\theta + \dfrac{\delta A}{A}\right)$

(b) $p = B + \rho_M gL \sin\theta$

(c) $p = B + \rho_F gH + \rho_M g \; L\dfrac{\delta A}{A}$

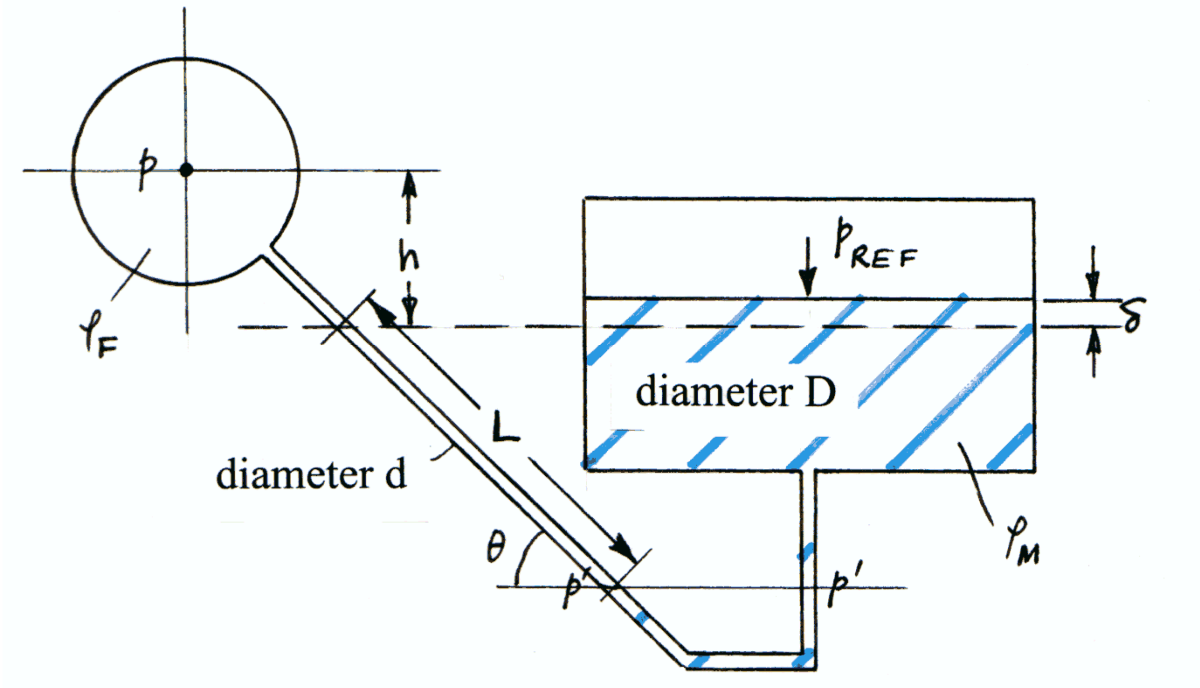
(d) $p = B - \rho_F gH + \rho_M g\, L\dfrac{\delta A}{A} + (\rho_M - \rho_F)gL\sin\theta$

$L.\delta A = \delta h.A$ (1)

LHS $p' = B + \rho_M g\,(\delta h + L\sin\theta)$ (2)

RHS $p' = p + \rho_F g\,(H + L\sin\theta)$ (3)

Hydrostatics – Interactive question

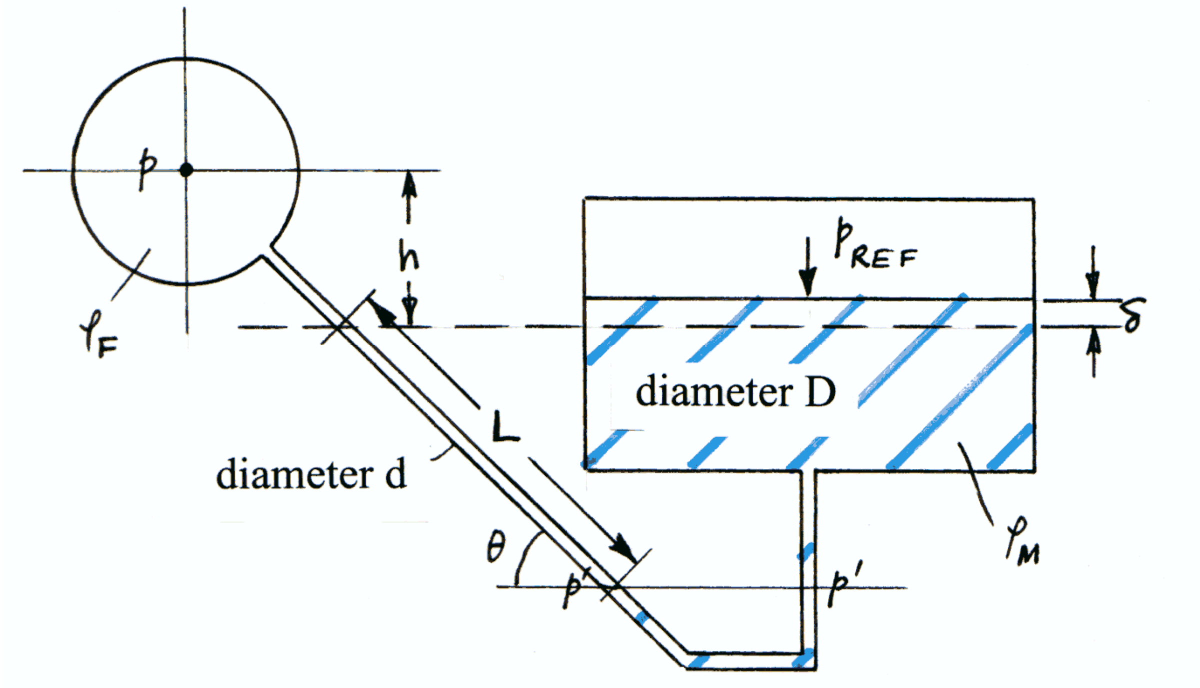


Principle: Equal volumes

If L = 400 mm, d = 5 mm and D = 100 mm, which is the correct value for δ?

(a) 20 mm (b) 1 mm (c) 0.2 mm

Hydrostatics – Interactive question



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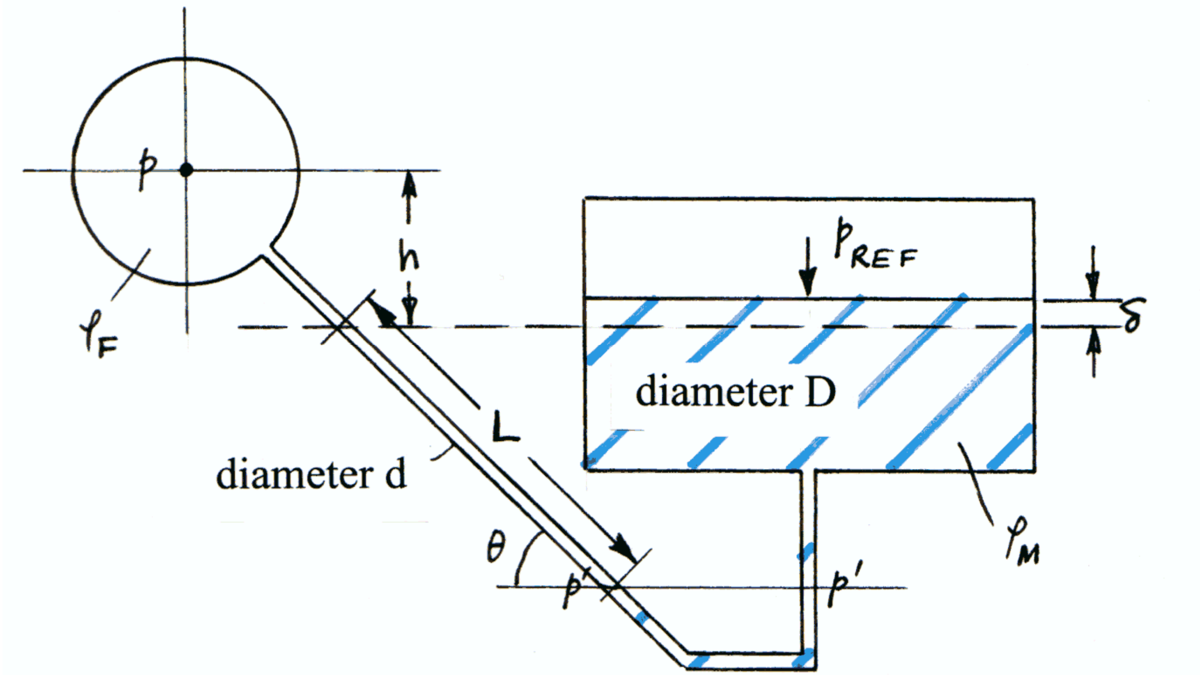
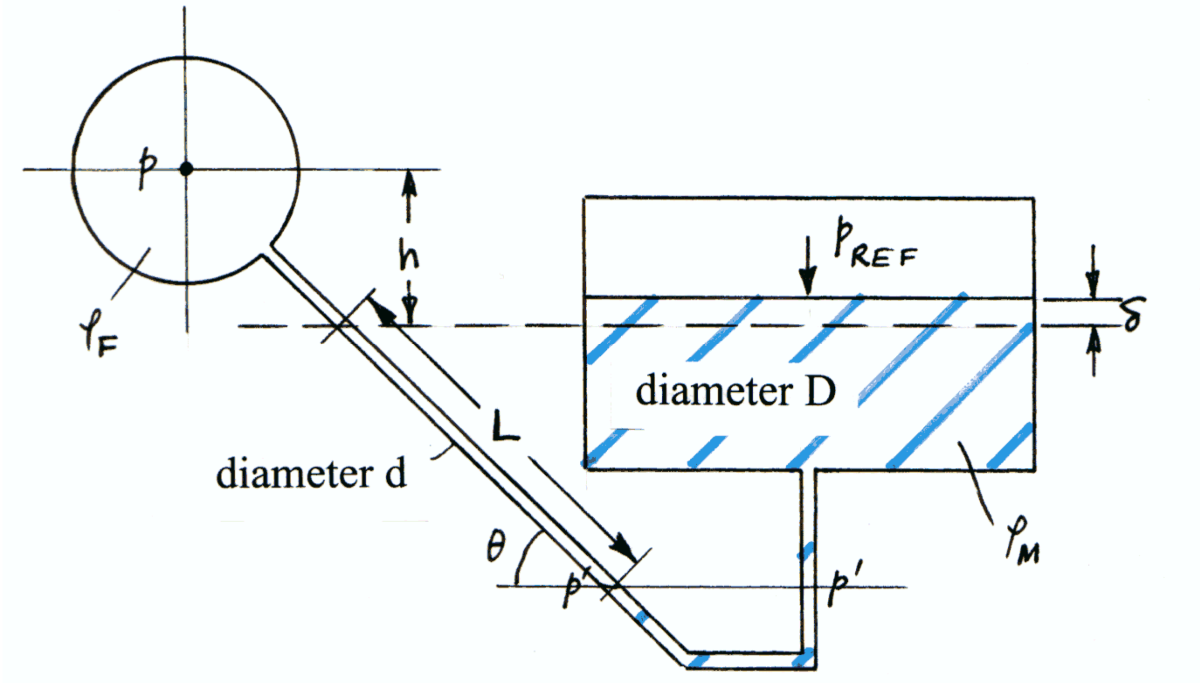


Principle: Hydrostatic pressure variation

If p_{REF} = 0.015 bar, θ = 30°, ρ_M = 850 kg/m^3, ρ_F = 2 kg/m^3 and h = 0.2 m, which is the correct value for p?

(a) 0.3 bar (b) 713 Pa (c) 3168 Pa (d) 1.7 bar

Hydrostatics – Interactive question



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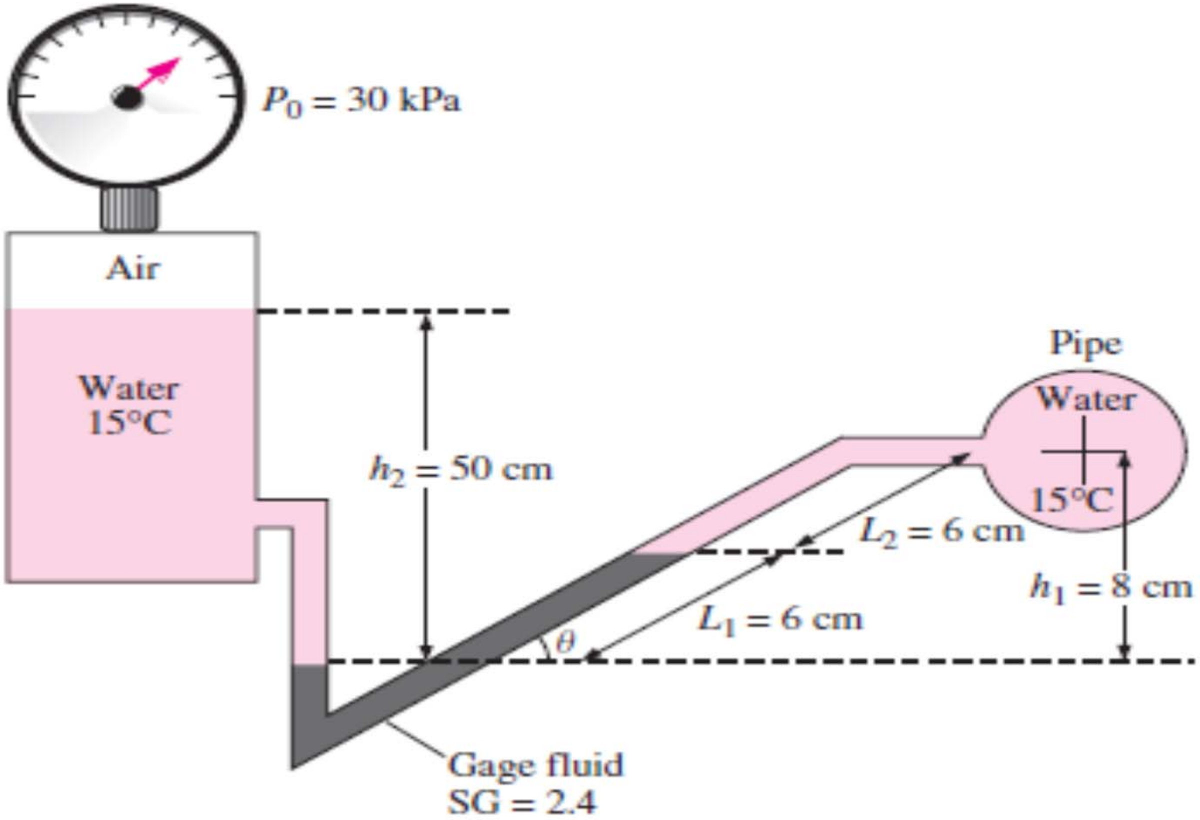
Hint: calculate p' for the right-and left-hand side, then find p from the equation

The pressure of water flowing through a pipe is measured by the arrangement shown in Figure. For the values given, calculate the pressure in the pipe. (MT1 G(1) 2012)



[redacted]

$$P_{gage} + P_w g h_{w1} - \rho_{gage} g h_{gage} - \rho_w g h_{w2} = P_{water}$$

$$P_{water} = P_{gage} + \rho_w g\left(h_{w1} - SG_{gage} h_{gage} - h_{w2}\right)$$

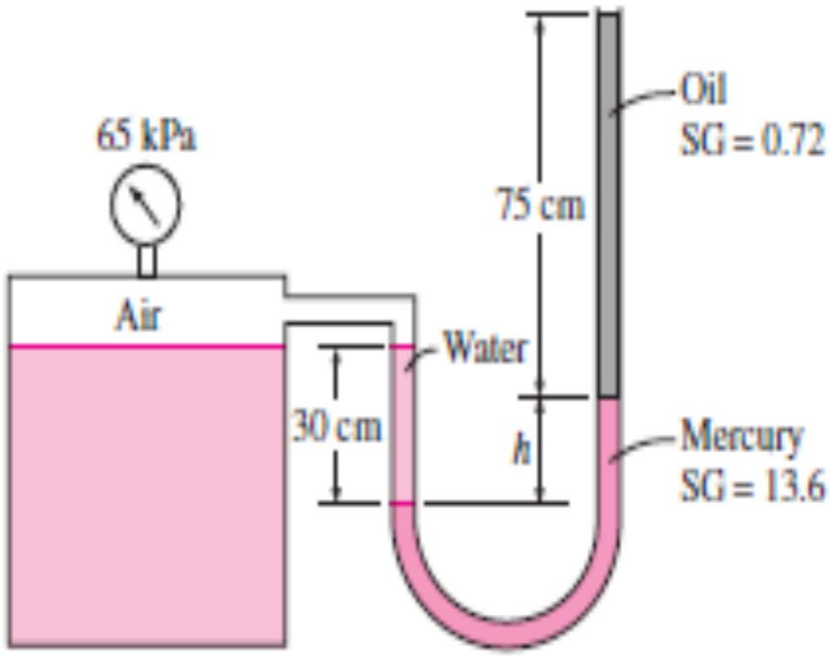
$$= P_{gage} + \rho_w g\left(h_2 - SG_{gage} L_1 Sin\theta - L_2 Sin\theta\right)$$

θ =8/12= 0.6667

$$\mathrm{P_{water}} = 30 + 1000 \cdot 9.81 \cdot (0.50 - 2.4 \cdot 0.06 \cdot 0.6667 - 0.06 \cdot 0.6667)/1000$$

P_{water}=33.6 kPa

The gage pressure of the air in the tank shown in Figure is measured to be 65 kPa. Determine the differential height *h* of the mercury column.



$$P_1 + \rho_w g h_w - \rho_{Hg} g h_{Hg} - \rho_{oil} g h_{oil} = P_{atm}$$

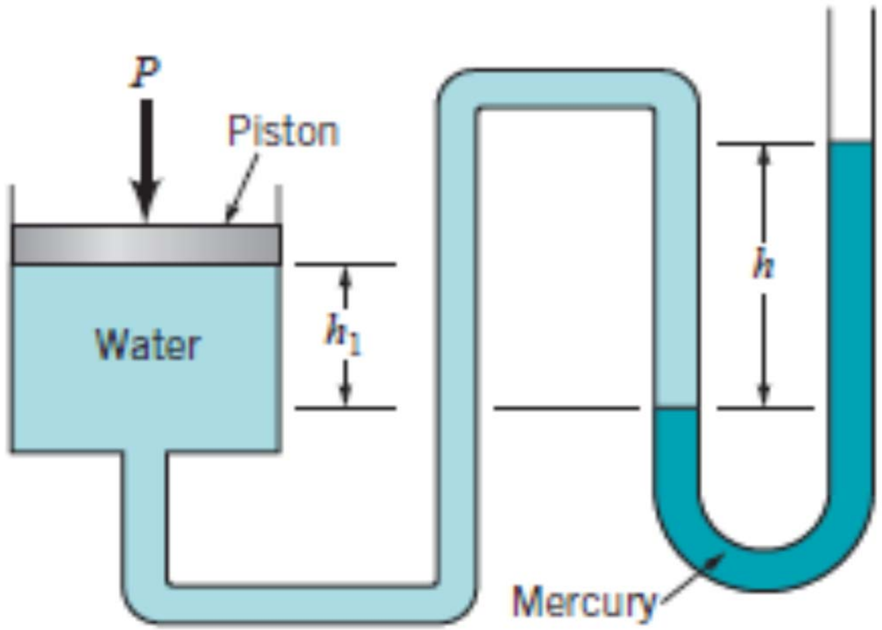
$$P_1 - P_{atm} = \rho_{oil} g h_{oil} + \rho_{Hg} g h_{Hg} - \rho_w g h_w$$

$$\frac{P_{1,gage}}{\rho_w g} = \rho_{s,oil} h_{oil} + \rho_{s,Hg} h_{Hg} - h_w$$

$$\left(\frac{65}{1000 \cdot 9.81}\right) \cdot 1000 = 0.72 \cdot 0.75 + 13.6 \cdot h_{Hg} - 0.3$$

$$\Rightarrow \; h_{Hg} = 0.47\; m$$

A piston having a cross-sectional area of 0.07 m^2 is located in a cylinder containing water as shown in Figure. An open U-tube manometer is connected to the cylinder as shown. For h_1=60 mm and h_2=100 mm, what is the value of the applied force, P, acting on the piston? The weight of the piston is negligible.



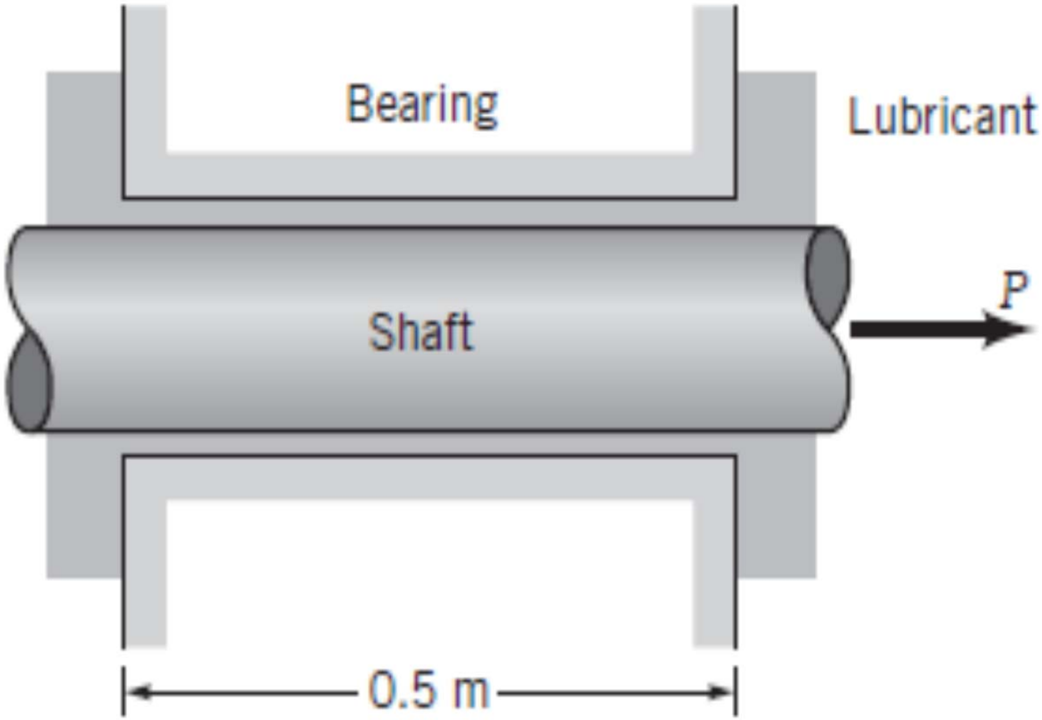
[redacted]

$$F = PA \quad P + \gamma_{water} h_1 - \gamma_{Hg} h = 0$$

$$P = \gamma_{Hg} h - \gamma_{water} h_1 = 133 * 0.1 - 9.90 * 0.06 = 1.27\ kN/m^2$$

$$F = (1.27x10^3)(0.07) = 889\ N$$

A 25-mm-diameter shaft is pulled through a cylindrical bearing as shown in Figure. The lubricant that fills the 0.3-mm gap between the shaft and bearing is an oil having a kinematic viscosity of 8.0 10^{-4} m^2/s and a specific gravity of 0.91. Determine the force *P* required to pull the shaft at a velocity of 3 m/s. Assume the velocity distribution in the gap is linear.



[redacted]

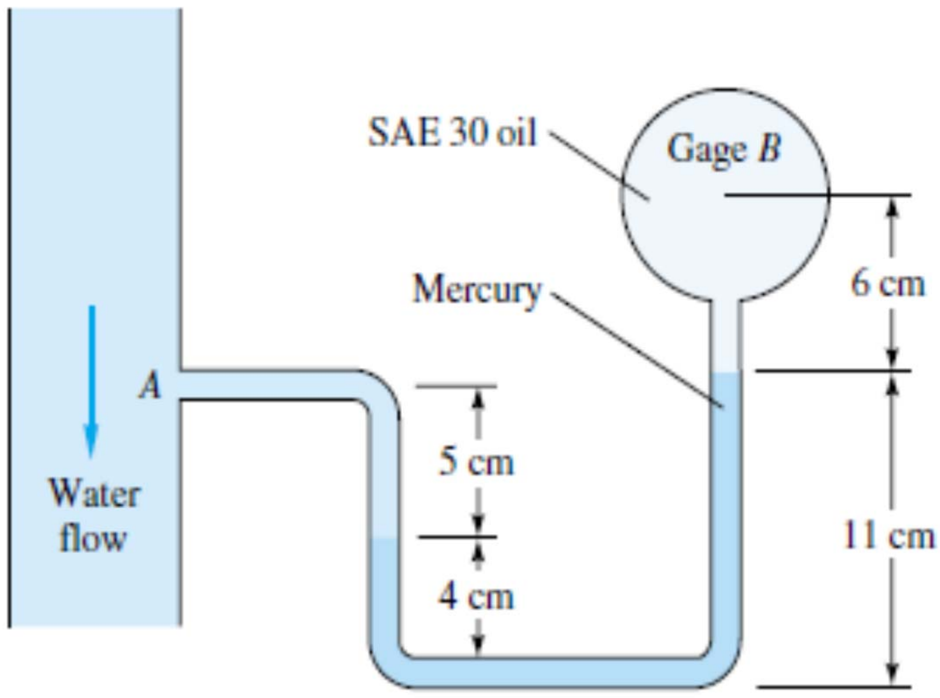
$$\sum F_x = 0 \qquad F = \tau A \qquad A = \pi D L$$

$$\tau = \mu \frac{V}{b}, \qquad so\ that, \quad F = \mu \frac{V}{b} \pi D L \qquad \mu = \nu \rho$$

$$F = \frac{(8.10^{-4}) * \ 0.91x\,10^3 * 3\pi * 0.025 * 0.5}{0.0003} = 285.74\ N$$

Pressure gage *B* is to measure the pressure at point *A* in a water flow. If the pressure at *B* is 87 kPa, estimate the pressure at *A*, in kPa. Assume all fluids are at 20°C.

$(\gamma_w = 9790\,\frac{N}{m^3}, \gamma_{mercury} = 133\,100\,\frac{N}{m^3}, \gamma_{oil} = 8720\,\frac{N}{m^3})$



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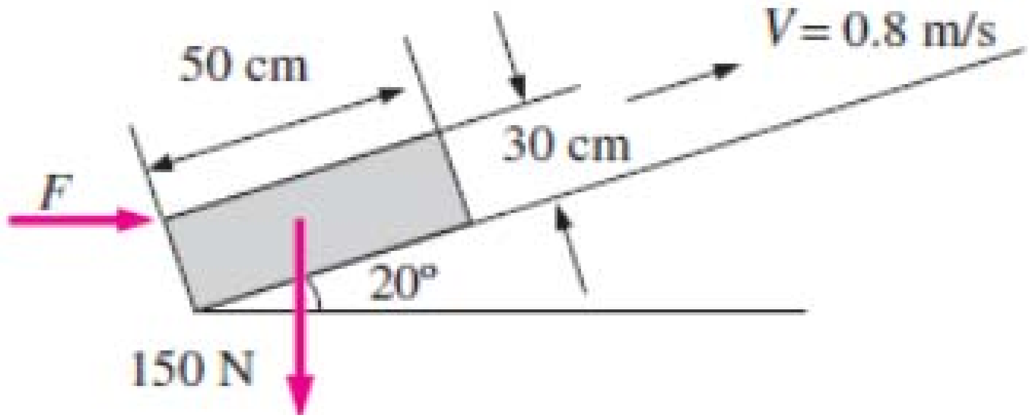
$$p_A - \gamma_W(\Delta z)_W - \gamma_M(\Delta z)_M - \gamma_O(\Delta z)_O = p_B$$

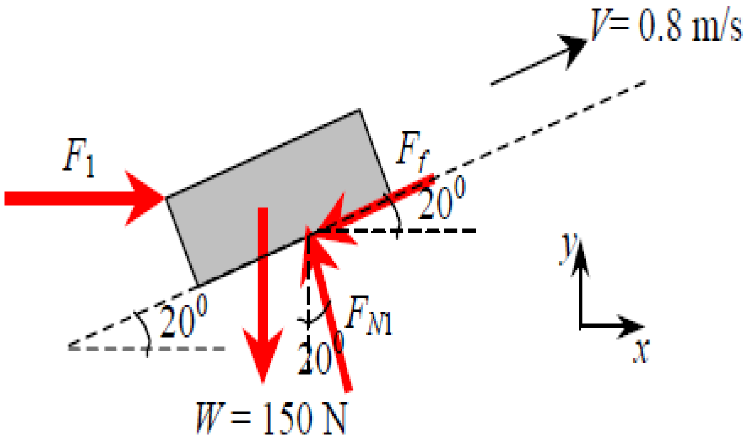
$$p_A - (9790\ \mathrm{N/m^3})(-\,0.05\ \mathrm{m}) - (133{,}100\ \mathrm{N/m^3})(0.07\ \mathrm{m}) - (8720\ \mathrm{N/m^3})(0.06\ \mathrm{m})$$

$$= p_A + 489.5\ \mathrm{Pa} - 9317\ \mathrm{Pa} - 523.2\ \mathrm{Pa} = p_B = 87{,}000\ \mathrm{Pa}$$

$$p_A = 96{,}351\ \mathrm{Pa} = 96.4\ \mathrm{kPa}$$

A 50x30x20-cm block weighing 150 N is to be moved at a constant velocity of 0.8 m/s on an inclined surface with a friction coefficient of 0.27. (*a*) Determine the force *F* that needs to be applied in the horizontal direction. (*b*) If a 0.4-mm-thick oil film with a dynamic viscosity of 0.012 Pa.s is applied between the block and inclined surface, determine the percent reduction in the required force.





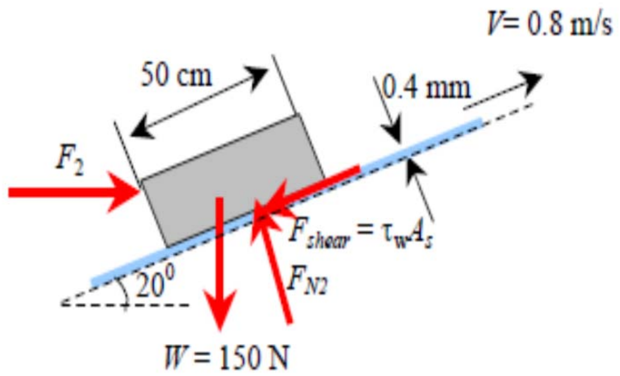
$$\sum F_x = 0: \qquad F_1 - F_{shear}\cos 20 - F_{N1}\sin 20 = 0$$

$$\sum F_y = 0: \qquad F_{N1}\cos 20 - F_{shear}\sin 20 - W = 0$$

Friction force: $F_f = fF_{N1}$

$$F_{N1} = \frac{W}{\cos 20 - f Sin\, 20} = \frac{150}{\cos 20 - 0.27\ Sin\, 20} = 177.0\ N$$

$$F_1 = F_f\cos 20 + F_{N1}\sin 20 = (0.27x177)\cos 20 + 177\sin 20 = 105.5\ N$$



$$F_{shear} = \tau_w A_s = \mu A_s \frac{V}{h} = 0.012 \cdot \frac{(0.5x0.2)0.8}{4x10^{-4}} = 2.4\ N$$

$$\sum F_x = 0: \qquad F_2 - F_{shear}\cos 20 - F_{N2}\sin 20 = 0$$

$$\sum F_y = 0: \qquad F_{N2}\cos 20 - F_{shear}\sin 20 - W = 0$$

$$F_{N2} = \frac{F_{shear}\sin 20 + W}{\cos 20} = \frac{(2.4\sin 20 + 150)}{\cos 20} = 160.5\ N$$

$$F_2 = F_{shear}\cos 20 + F_{N2}\sin 20 = 2.4\cos 20 + 160.5\sin 20 = 57.2\ N$$

Percentage reduction in the required force: $\frac{F_1 - F_2}{F_1}100 = \frac{105.5-57.2}{105.5}100 = 45.8\ \%$