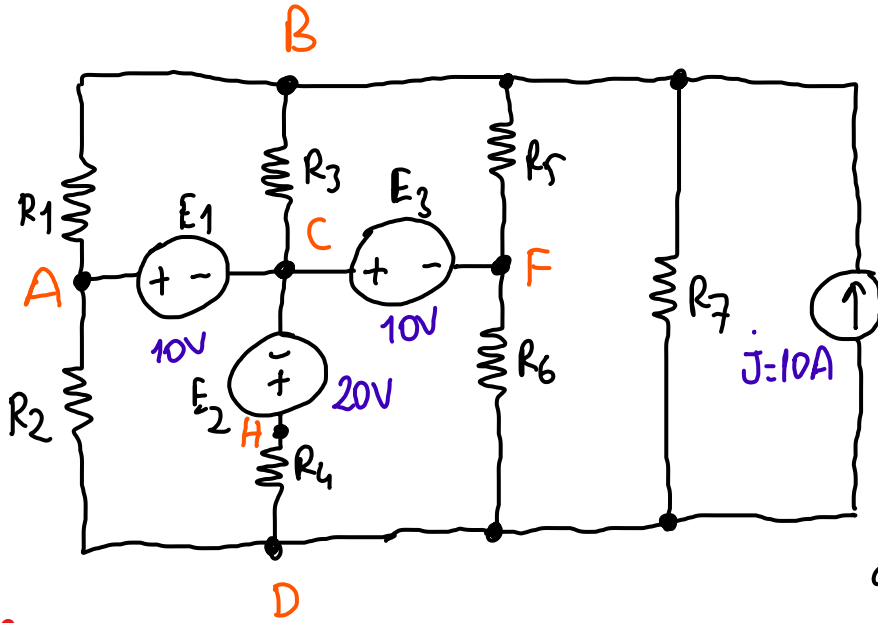


Örnek (Düğüm Gerilimleri Yöntemi) (DÜGY)

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Bölümü



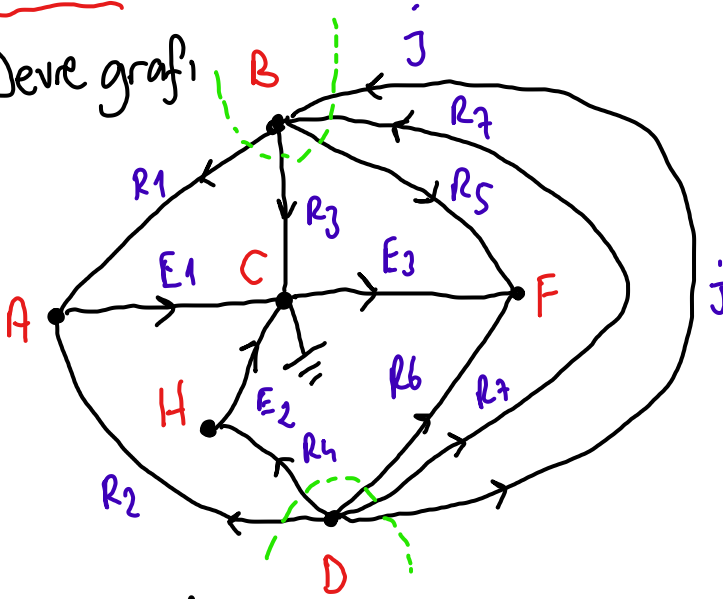
Devrede tüm dirençler $0,5 \Omega$ 'dır. (1)

a) DÜGY ile devre denklemlerini adım-adım matris biçiminde elde ediniz.

b) Denklemleri çözünüz ve $P_{R7}=?$, $R7$ 'nin akımını?, $P_{E1}=?$

Gözüm

a) Devre grafi



Düğüm gerilimleri

$V_C = 0$ (Referans düğüm)

$V_A = E_1 = 10V$

$V_H = E_2 = 20V$

$V_F = -E_3 = -10V$

$V_B = ?$

$V_D = ?$

Bilinenler

Bilinmeyenler

Direnç gerilimleri

$$V_{R1} = V_B - V_A = V_B - E_1$$

$$V_{R2} = V_D - V_A = V_D - E_1$$

$$V_{R3} = V_B - V_C = V_B$$

$$V_{R4} = V_D - V_H = V_D - E_2$$

$$V_{R5} = V_B - V_F = V_B - (-E_3) = V_B + E_3$$

$$V_{R6} = V_D - V_F = V_D - (-E_3) = V_D + E_3$$

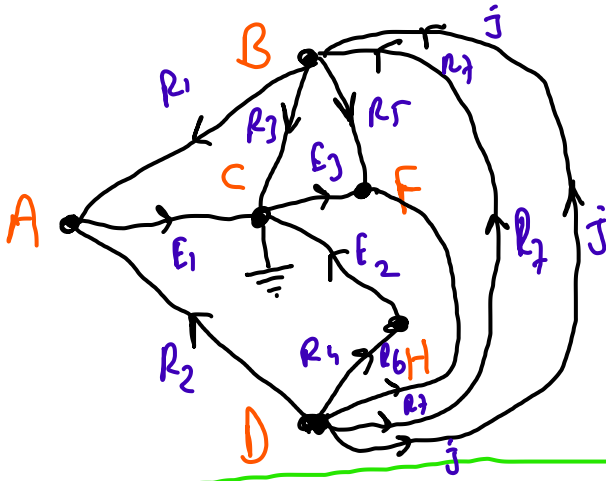
$$V_{R7} = V_D - V_B$$

B için düğüm denklemleri

$$G_1 V_{R1} + G_3 V_{R3} + G_5 V_{R5} - G_7 V_{R7} - \hat{J} = 0$$

$$G_1 (V_B - E_1) + G_3 V_B + G_5 (V_B + E_3) - G_7 (V_D - V_B) - \hat{J} = 0$$

(2)



$$(G_1 + G_3 + G_5 + G_7) \cdot V_B - G_7 \cdot V_D - G_1 \cdot E_1 + G_5 E_3 - \dot{J} = 0$$

1. denklem

D için dögüm denklemi

$$G_2 \cdot V_{k_2} + G_4 \cdot V_{k_4} + G_6 \cdot V_{k_6} + G_7 \cdot V_{k_7} + \dot{J} = 0$$

$$G_2 (V_D - E_1) + G_4 (V_D - E_2) + G_6 (V_D + E_3) + G_7 (V_D - V_B) + \dot{J} = 0$$

$$-G_7 \cdot V_B + (G_2 + G_4 + G_6 + G_7) V_D - G_2 \cdot E_1 - G_4 \cdot E_2 + G_6 \cdot E_3 + \dot{J} = 0$$

$$\begin{bmatrix} (G_1 + G_3 + G_5 + G_7) & -G_7 \\ -G_7 & (G_2 + G_4 + G_6 + G_7) \end{bmatrix} \begin{bmatrix} V_B \\ V_D \end{bmatrix} + \begin{bmatrix} -G_1 & 0 & G_5 \\ -G_2 & -G_4 & G_6 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \dot{J} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

b) Sayısal Gözüm

$$E_1 = 10V, E_2 = 20V, E_3 = 10V, \dot{J} = 10A$$

$$R_1 = R_2 = \dots = R_6 = R_7 = 0.5 \Rightarrow G_1 = G_2 = \dots = G_7 = \frac{1}{0.5} = 2 \text{ mho}$$

$$\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \cdot \begin{bmatrix} V_B \\ V_D \end{bmatrix} + \begin{bmatrix} -2 & 0 & 2 \\ -2 & -2 & 2 \end{bmatrix} \cdot \begin{bmatrix} 10 \\ 20 \\ 10 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \end{bmatrix} \cdot [10] = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \cdot \begin{bmatrix} V_B \\ V_D \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \end{bmatrix} \Rightarrow \Delta = \begin{vmatrix} 8 & -2 \\ -2 & 8 \end{vmatrix} = 64 - (-2)(-2) = 60$$

$$V_B = \frac{\begin{vmatrix} 10 & -2 \\ 30 & 8 \end{vmatrix}}{\begin{vmatrix} 8 & -2 \\ -2 & 8 \end{vmatrix}} = \frac{140}{60} \Rightarrow \boxed{V_B = 2.333V}$$

$$\begin{bmatrix} 8 & -2 \\ -2 & 8 \end{bmatrix} \begin{bmatrix} V_B \\ V_D \end{bmatrix} = \begin{bmatrix} 10 \\ 30 \end{bmatrix} \Rightarrow V_D = \frac{\begin{vmatrix} 8 & 10 \\ -2 & 30 \end{vmatrix}}{\begin{vmatrix} 8 & -2 \\ -2 & 8 \end{vmatrix}}$$

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$$V_D = \frac{260}{60} \Rightarrow$$

$$V_D = 4,333V$$

$$P_{R_7} = ?$$



$$\begin{aligned} V_B &= 2,777V \\ V_D &= 4,333V \end{aligned}$$

$V_D > V_B$ ise

R_7 akımı

$$D \rightarrow B$$

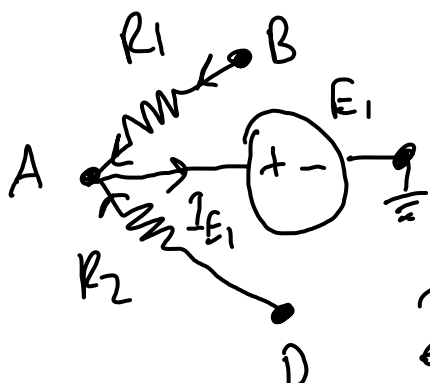
$$V_{R_7} = V_D - V_B = 4,333 - 2,777 = 2V$$

$$P_{R_7} = \frac{V_{R_7}^2}{R_7} = \frac{2^2}{0,5} \Rightarrow$$

$$P_{R_7} = 8W$$

$$P_{E_1} = ?$$

$$P_{E_1} = E_1 \cdot I_{E_1} \Rightarrow I_{E_1} = ?$$



A için düğüm denklemleri!

$$-I_{R_1} - I_{E_1} + I_{R_2} = 0 \Rightarrow I_{E_1} = I_{R_2} - I_{R_1}$$

$$I_{E_1} = G_1 \cdot (V_B - E_1) + G_2 \cdot (V_D - E_1)$$

$$I_{E_1} = 2(2,777 - 10) + 2(4,333 - 10) = -26,667A$$

$$P_{E_1} = E_1 \cdot I_{E_1} = 10 \cdot (-26,667) \Rightarrow P_{E_1} = -266,67W$$

NOT:

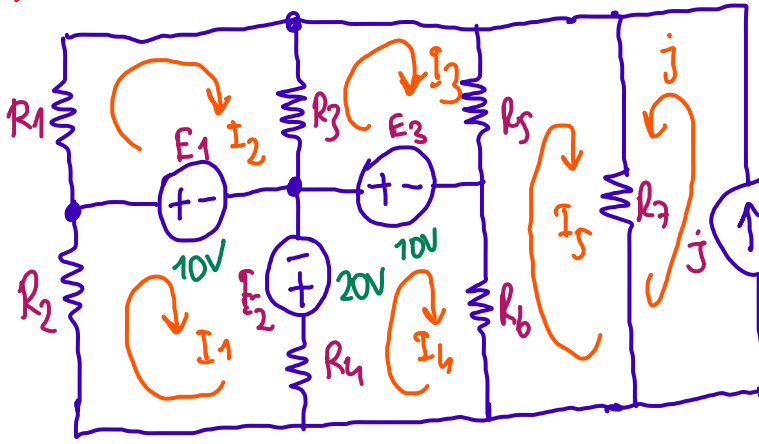
Yukarıda Düğüm Gerilimleri Yöntemi (DÜGY) ile 2 bilinmeyenli olarak çözülen devre aşağıdaki örnekte Çevre Akımları Yöntemi (ÇAY) ile 5 bilinmeyenli olarak çözülmüştür. 5 bilinmeyenli denklemler MATLAB ile çözülmüştür. P_{R_7} ve P_{E_1} yine aynı değerlerde bulunmuştur. (Kullanılan yöntem araştır, devrenin çalışmasını etkiler mi?)

ÖRNEK (ÇAY, 5 Bilinmeyenli)

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Tüm dirençler $0,5\Omega$ 'dır.



a) ÇAY ile devre denklemlerini elde ediniz ve matris biçiminde yazınız.

b) $P_{R7}=?$, $P_{E1}=?$

Çözüm

I_1 devresi için KVL denklemleri

$$(R_2 + R_4)I_1 - R_4I_4 + E_1 - E_2 = 0 \Rightarrow (R_2 + R_4)I_1 - R_4I_4 = -E_1 + E_2 \quad (1)$$

I_2 devresi için G.D.

$$(R_1 + R_3)I_2 - R_3I_3 - E_1 = 0 \Rightarrow (R_1 + R_3)I_2 - R_3I_3 = E_1 \quad (2)$$

I_3 devresi için G.D.

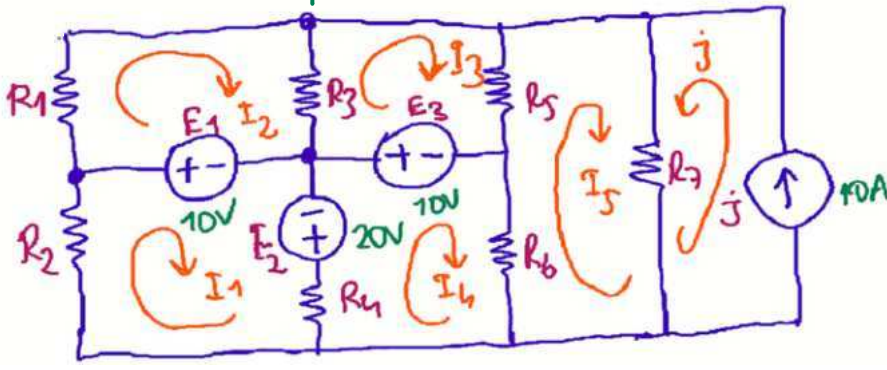
$$(R_3 + R_5)I_3 - R_5I_5 - E_3 = 0 \Rightarrow (R_3 + R_5)I_3 - R_5I_5 = E_3 \quad (3)$$

I_4 devresi için G.D.

$$(R_4 + R_6)I_4 - R_6I_5 + E_2 + E_3 = 0 \Rightarrow (R_4 + R_6)I_4 - R_6I_5 = -E_2 - E_3 \quad (4)$$

I_5 devresi için G.D.

$$(R_5 + R_6 + R_7)I_5 - R_5I_3 - R_6I_4 + R_7j = 0 \Rightarrow -R_5I_3 - R_6I_4 + (R_5 + R_6 + R_7)I_5 = -R_7j \quad (5)$$



$$(R_2 + R_4)I_1 - R_4I_4 = -E_1 + E_2 \quad (1) \quad (5)$$

$$(R_1 + R_3)I_2 - R_3I_3 = E_1 \quad (2)$$

$$(R_3 + R_5)I_3 - R_5I_5 = E_3 \quad (3)$$

$$(R_4 + R_6)I_4 - R_6I_5 = -E_2 - E_3 \quad (4)$$

$$-R_5I_3 - R_6I_4 + (R_5 + R_6 + R_7)I_5 = -R_7J \quad (5)$$

Denklemleri matris biçiminde yazalım:

$$\begin{bmatrix} R_2 + R_4 & 0 & 0 & -R_4 & 0 \\ 0 & R_1 + R_3 & -R_3 & 0 & 0 \\ 0 & -R_3 & R_3 + R_5 & 0 & -R_5 \\ -R_4 & 0 & 0 & R_4 + R_6 & -R_6 \\ 0 & 0 & -R_5 & -R_6 & R_5 + R_6 + R_7 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} -E_1 + E_2 \\ E_1 \\ E_3 \\ -E_2 - E_3 \\ -R_7J \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 & -0.5 & 0 \\ 0 & 1 & -0.5 & 0 & 0 \\ 0 & -0.5 & 1 & 0 & -0.5 \\ -0.5 & 0 & 0 & 1 & -0.5 \\ 0 & 0 & -0.5 & -0.5 & 1.5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \end{bmatrix} = \begin{bmatrix} 10 \\ 10 \\ 10 \\ -30 \\ -5 \end{bmatrix}$$

5 Bilinmeyenli denklemin çözümü aşağıda MATLAB ile yapılmıştır.

MATLAB İLE 5 BİLİNMEYENLİ ÇEVRE AKIMLARI DENKLEM ÇÖZÜMÜ

```
>> R=0.5;E1=10;E2=20;E3=10;J=10;
>> A=[1 0 0 -0.5 0;0 1 -0.5 0 0;0 -0.5 1 0 -0.5;-0.5 0 0 1 -0.5;0 0
-0.5 -0.5 1.5]
```

A =

```
1.0000    0    0 -0.5000    0
    0    1.0000 -0.5000    0    0
    0 -0.5000    1.0000    0 -0.5000
-0.5000    0    0    1.0000 -0.5000
    0    0 -0.5000 -0.5000    1.5000
```

$$A \cdot \underline{X} = \underline{B}$$

$$\underline{X} = \underline{A}^{-1} \cdot \underline{B}$$

```
>> B=[-E1+E2;E1;E3;-E2-E3;-R*J]
B =
```

```
10
10
10
-30
-5
```

```
>> Ic=inv(A)*B
```

```
Ic =
-11.3333
15.3333
10.6667
-42.6667
-14.0000
```

```
>> IE1=Ic(1)-Ic(2)
```

```
IE1 =
-26.6667
```

```
>> PE1=E1*IE1
```

```
PE1 =
-266.6667
```

```
>> R7=R;IR7=Ic(5)+J
```

```
IR7 =
-4.0000
```

```
>> VR7=R7*IR7
```

```
VR7 =
-2.0000
```

```
>> PR7=R7*IR7^2
```

```
PR7 =
8.0000
```

MATLAB sonuçları

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$$\underline{X} = \underline{A}^{-1} \cdot \underline{B} \quad (\text{MATLAB'de } \underline{I}_C = \text{inv}(\underline{A}) * \underline{B})$$

$$\underline{I}_1 = \underline{I}_C(1) = -11,3333 \text{ A}$$

$$\underline{I}_2 = \underline{I}_C(2) = 15,3333 \text{ A}$$

$$\underline{I}_3 = \underline{I}_C(3) = 10,6667 \text{ A}$$

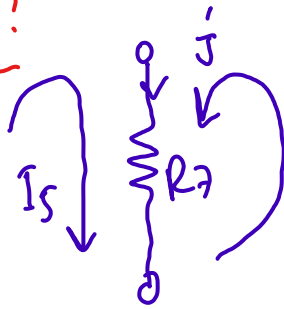
$$\underline{I}_4 = \underline{I}_C(4) = -42,6667 \text{ A}$$

$$\underline{I}_5 = \underline{I}_C(5) = -14 \text{ A}$$

} Devrede istenenler

$$P_{R_7} = ? \text{ ve } P_{E_1} = ?$$

$$P_{R_7} = ?$$

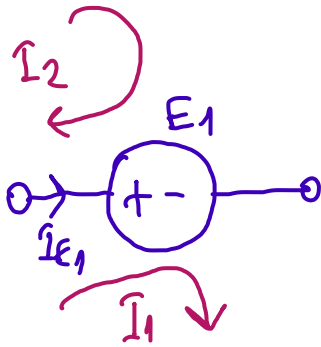


$$\underline{I}_{R_7} = \underline{I}_5 + \underline{I}_7 = -14 + 10 = -4 \text{ A}$$

$$P_{R_7} = R_7 \cdot \underline{I}_{R_7}^2 = 0,5 \cdot 4^2 \Rightarrow \boxed{P_{R_7} = 8 \text{ W}}$$

$$P_{E_1} = ?$$

$$P_{E_1} = E_1 \cdot \underline{I}_{E_1} \Rightarrow \underline{I}_{E_1} = ?$$



$$\underline{I}_{E_1} = \underline{I}_1 - \underline{I}_2 = -11,3333 - 15,3333 = \underline{-26,6667 \text{ A}}$$

$$P_{E_1} = E_1 \cdot \underline{I}_{E_1} = 10 \cdot (-26,6667) \text{ A}$$

$$\boxed{P_{E_1} = -266,67 \text{ W}}$$