

YILDIZ TEKNİK ÜNİVERSİTESİ

ELEKTRİK MÜHENDİSLİĞİ BÖLÜMÜ

DEVRE TEORİSİ

Ders Notu

- **R,L,C ELEMANLARININ SERİ VE PARALEL BAĞLANMASI**
- **YILDIZ-ÜÇGEN DÖNÜŞÜMLERİ**

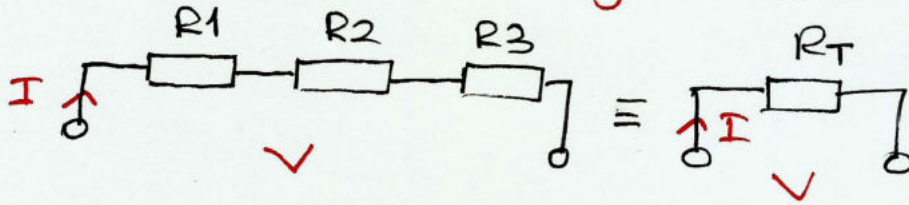
Doç. Dr. Recep YUMURTACI

R, L, C Elemanlarının Seri ve Paralel Bağlanması

1

Direnç (R)

* Dirençlerin Seri Bağlanması



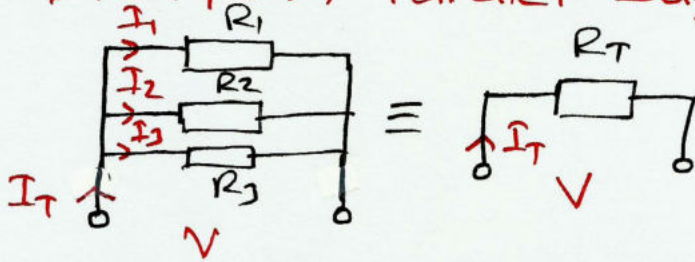
$$V = V_1 + V_2 + V_3$$

$$R_T \cdot I = (R_1 + R_2 + R_3) \cdot I \Rightarrow R_T = R_1 + R_2 + R_3$$

Özel Durum:

n adet R direnci seri bağlı ise $R_T = n \cdot R$ olur.

* Dirençlerin Paralel Bağlanması

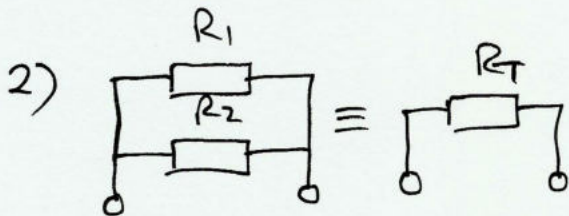


$$I_T = I_1 + I_2 + I_3$$

$$\frac{1}{R_T} \cdot V = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right) \cdot V \Rightarrow \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Özel Durum:

1) n adet R direnci paralel bağlı ise $R_T = \frac{R}{n}$ olur.



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} \Rightarrow \frac{1}{R_T} = \frac{R_1 + R_2}{R_1 \cdot R_2}$$

$$\frac{1}{R_T} = \frac{R_1 + R_2}{R_1 \cdot R_2} \Rightarrow$$

$$R_T = \frac{R_1 \cdot R_2}{R_1 + R_2}$$

Endüktans (L)

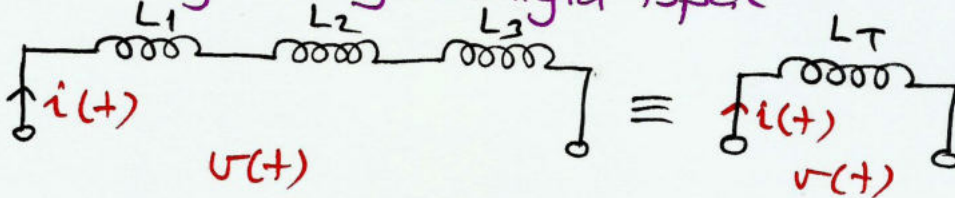
(2)

* Endüktansların Seri Bağlanması



ispatı:

a) Ani değerler yardımıyla ispat



$$U(t) = U_1(t) + U_2(t) + U_3(t)$$

$$L_T \cdot \frac{di(t)}{dt} = (L_1 + L_2 + L_3) \cdot \frac{di(t)}{dt} \Rightarrow L_T = L_1 + L_2 + L_3$$

b) Endüktif reaktanslar yardımıyla ispat

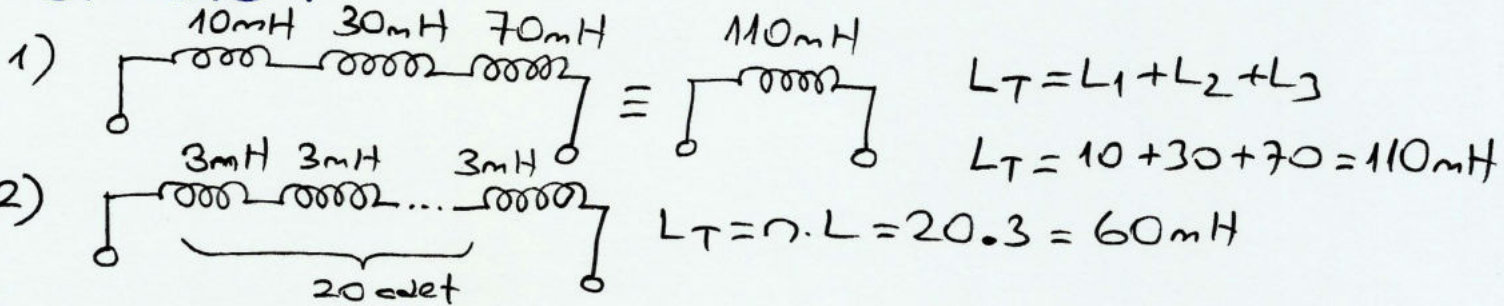
$$X_{LT} = X_{L1} + X_{L2} + X_{L3}$$

$$\omega \cdot L_T = \omega \cdot (L_1 + L_2 + L_3) \Rightarrow L_T = L_1 + L_2 + L_3$$

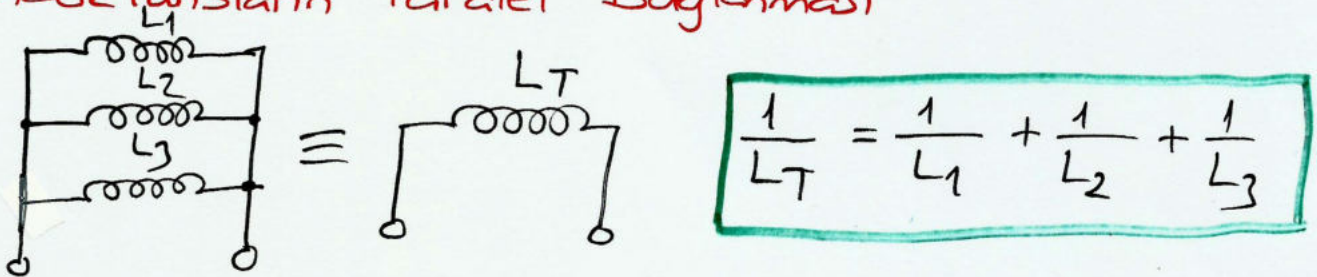
Özel durum :

n adet L endüktansı seri bağlı ise $L_T = n \cdot L$ olur.

Örnekler:



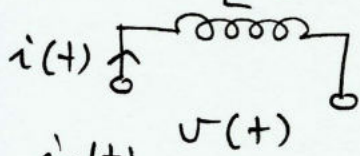
* Endüktansların Paralel Bağlanması



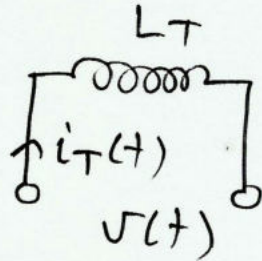
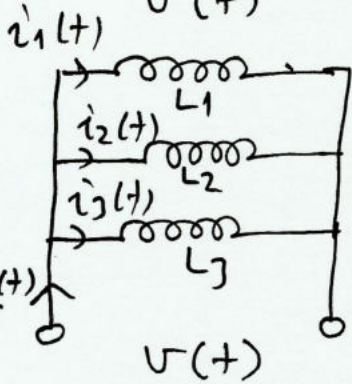
ispat:

(3)

a) Anı değerler yardımıyla ispat



$$v(t) = L \cdot \frac{di(t)}{dt} \Rightarrow i(t) = \frac{1}{L} \int v(t) dt$$



$$i_T(t) = i_1(t) + i_2(t) + i_3(t)$$

$$\frac{1}{L_T} \int v(t) dt = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right) \int v(t) dt$$

$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

b) Endüktif reaktanslar yardımıyla ispat

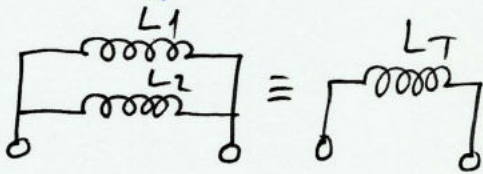
$$\frac{1}{X_{L_T}} = \frac{1}{X_{L_1}} + \frac{1}{X_{L_2}} + \frac{1}{X_{L_3}} \Rightarrow \frac{1}{\omega \cdot L_T} = \frac{1}{\omega} \cdot \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} \right)$$

$$\boxed{\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

Özel Durum:

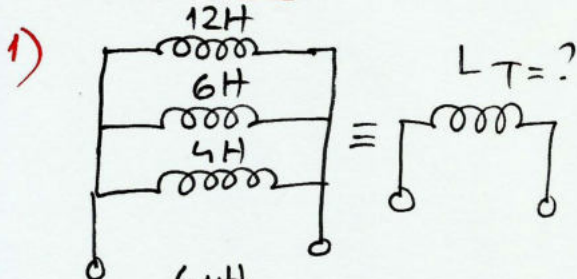
* n adet L endüktansı paralel bağlı ise $\boxed{L_T = \frac{L}{n}}$ olur.

* L1 ve L2 endüktansları paralel bağlı ise



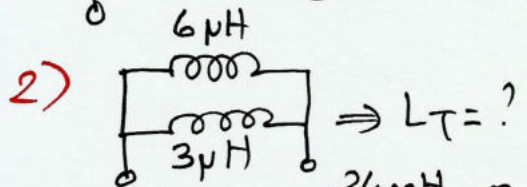
$$\boxed{L_T = \frac{L_1 \cdot L_2}{L_1 + L_2}}$$

ÖRNEKLER:

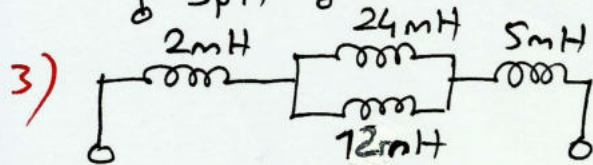


$$\frac{1}{L_T} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} = \frac{1}{12} + \frac{1}{6} + \frac{1}{4}$$

$$\frac{1}{L_T} = \frac{6}{12} = \frac{1}{2} \Rightarrow \boxed{L_T = 2H}$$



$$L_T = \frac{L_1 \cdot L_2}{L_1 + L_2} = \frac{6 \cdot 3}{6 + 3} = \frac{18}{9} = 2\mu H$$



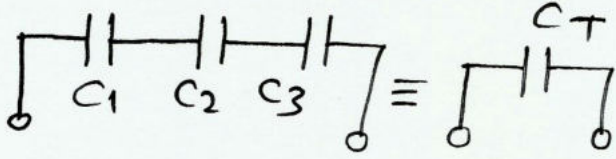
$$L_T = 2 + \left(\frac{24 \cdot 12}{24 + 12} \right) + 5 = (2 + 8 + 5) mH$$

$$\underline{L_T = 15 mH}$$

Kondansatör (C)

(4)

* Kondansatörlerin Seri Bağlanması

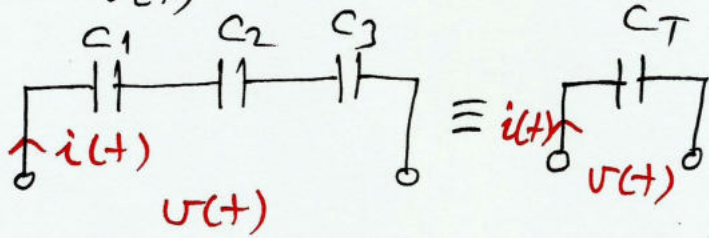


$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

ispat:

a) Ani degerler yardımıyla ispat:

$i(t) = C \cdot \frac{dv(t)}{dt} \Rightarrow v(t) = \frac{1}{C} \int i(t) dt$



$$v(t) = v_1(t) + v_2(t) + v_3(t)$$

$$\frac{1}{C_T} \cdot \int i(t) dt = \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right) \cdot \int i(t) dt$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

b) Kapasitif reaktanslar yardımıyla ispat:

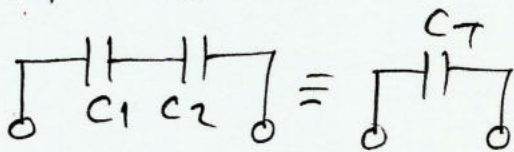
$$X_{C_T} = X_{C_1} + X_{C_2} + X_{C_3} \Rightarrow \frac{1}{\omega C_T} = \frac{1}{\omega} \cdot \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \right)$$

$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

Özel Durum:

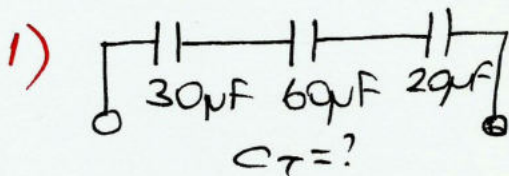
* n adet C kondansatörü seri bağlı ise $C_T = \frac{C}{n}$ olur.

* C_1 ve C_2 kondansatörleri seri bağlı ise



$$C_T = \frac{C_1 \cdot C_2}{C_1 + C_2} \text{ olur.}$$

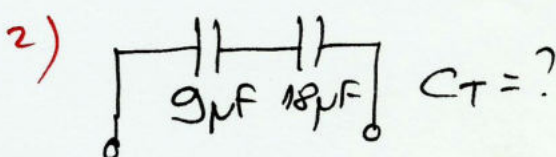
Örnekler:



$$\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{30} + \frac{1}{60} + \frac{1}{20}$$

(2) (1) (3)

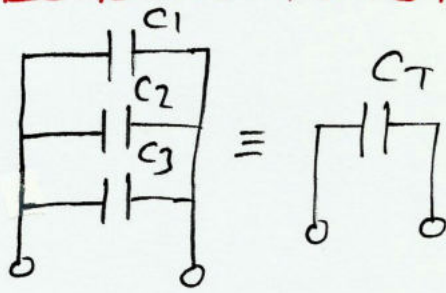
$$\frac{1}{C_T} = \frac{6}{60} = \frac{1}{10} \Rightarrow C_T = 10 \mu F$$



$$C_T = \frac{C_1 \cdot C_2}{C_1 + C_2} = \frac{9 \cdot 18}{9 + 18} = \frac{162}{27} = 6 \mu F$$

* Kondensatörlerin Paralel Bağlanması

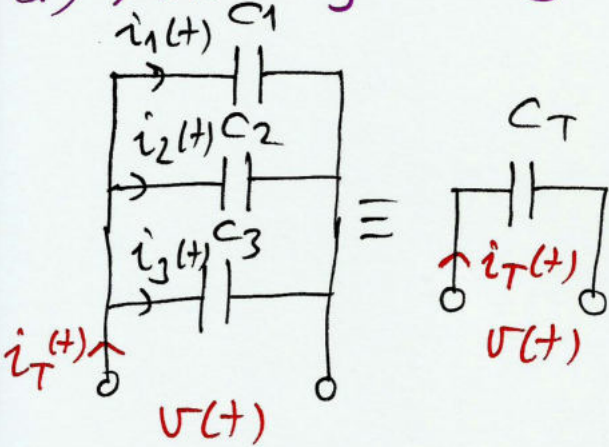
5



$$C_T = C_1 + C_2 + C_3$$

ispat:

a) Anı değerler yardımıyla ispat



$$i_T(t) = i_1(t) + i_2(t) + i_3(t)$$

$$C_T \cdot \frac{dV(t)}{dt} = (C_1 + C_2 + C_3) \cdot \frac{dV(t)}{dt}$$

$$C_T = C_1 + C_2 + C_3$$

b) Kapasitif reaktanslar yardımıyla ispat

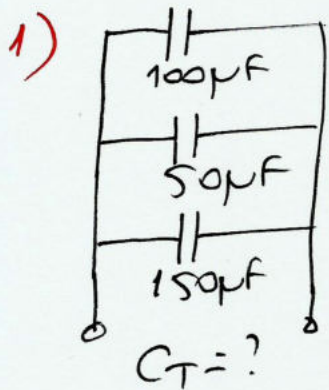
$$\frac{1}{X_{CT}} = \frac{1}{X_{C1}} + \frac{1}{X_{C2}} + \frac{1}{X_{C3}} \Rightarrow \frac{1}{\frac{1}{\omega C_T}} = \frac{1}{\frac{1}{\omega C_1}} + \frac{1}{\frac{1}{\omega C_2}} + \frac{1}{\frac{1}{\omega C_3}}$$

$$\omega C_T = \omega (C_1 + C_2 + C_3) \Rightarrow C_T = C_1 + C_2 + C_3$$

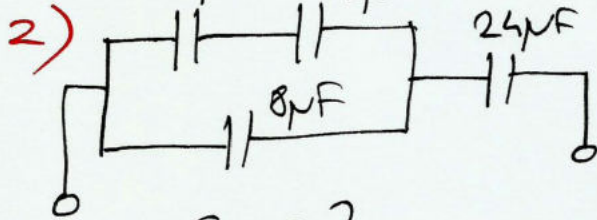
Özel durum:

n adet C kondensatörü paralel bağlarsa $C_T = n \cdot C$ olur.

Örnekler:



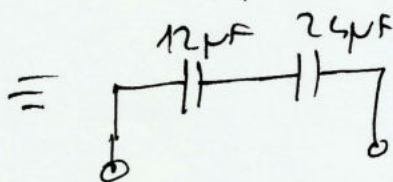
$$C_T = C_1 + C_2 + C_3 = 100 + 50 + 150 = 300 \mu F$$



$$C_{e1} = \frac{6 \cdot 12}{6 + 12} = 4 \mu F$$

$$C_{e2} = C_{e1} + 8 = 4 + 8 = 12 \mu F$$

$$C_T = ?$$



$$C_T = \frac{12 \cdot 24}{12 + 24} = \frac{12 \cdot 24}{36}$$

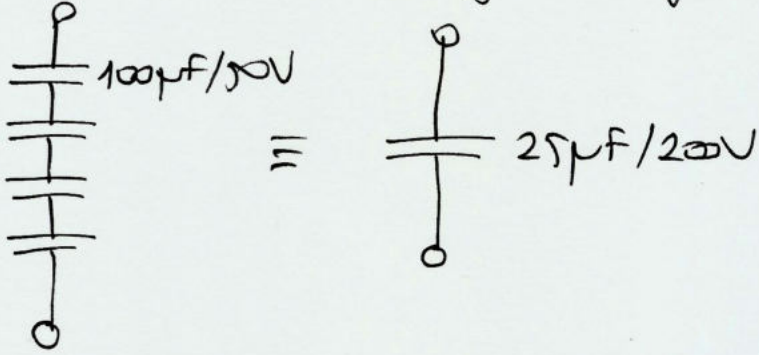
$$C_T = 8 \mu F$$

Örnek :

Malzeme depomuzda çok sayıda $100\mu\text{F } 50\text{V}$ 'lık kondensatörler bulunmaktadır. $150\mu\text{F } 200\text{V}$ 'lık kondensatör elde etmek için nasıl bir bağlantı önerirsiniz?

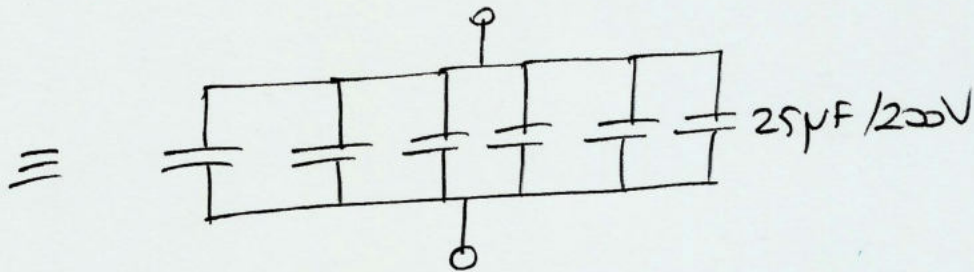
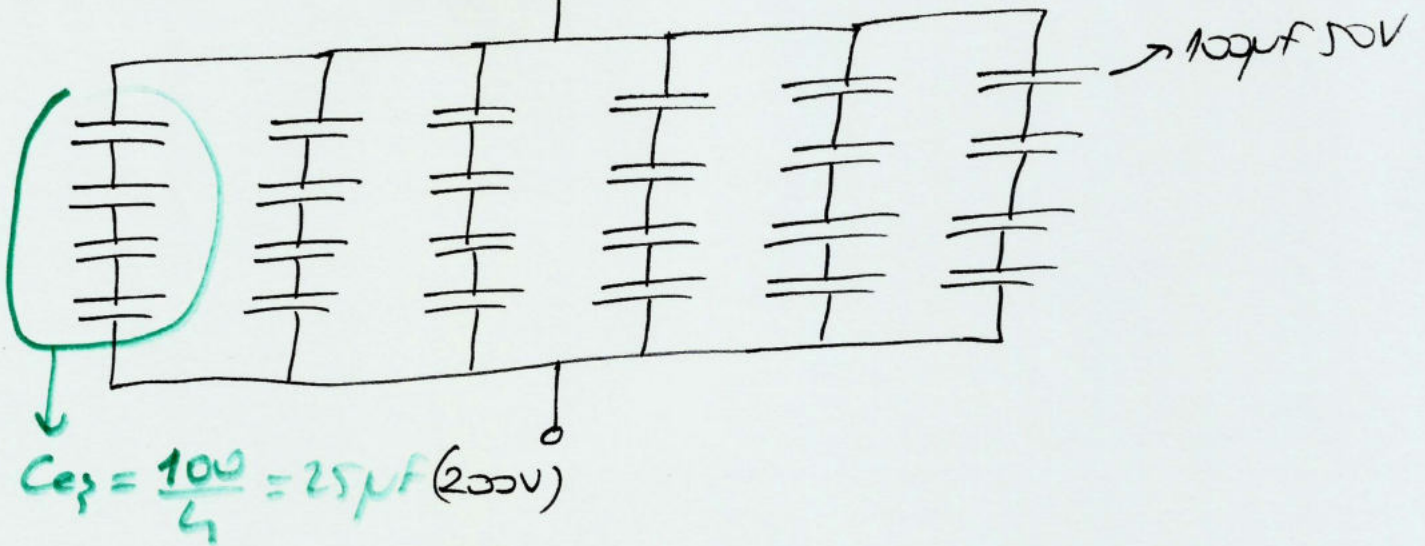
Çözüm :

200V 'a dayanabilecek yeni kalıba gerilimi 200V olan kondensatör elde etmek için $200/50 = 4$ adet $100\mu\text{F } 50\text{V}$ 'lık kondensatörü seri bağlamalıyız.



4 adet $100\mu\text{F}$ kondensatör
Seri bağlanınca eşdeğer
kapasite $C_{\text{eq}} = \frac{C}{4} = \frac{100}{4} = 25\mu\text{F}$
olur.

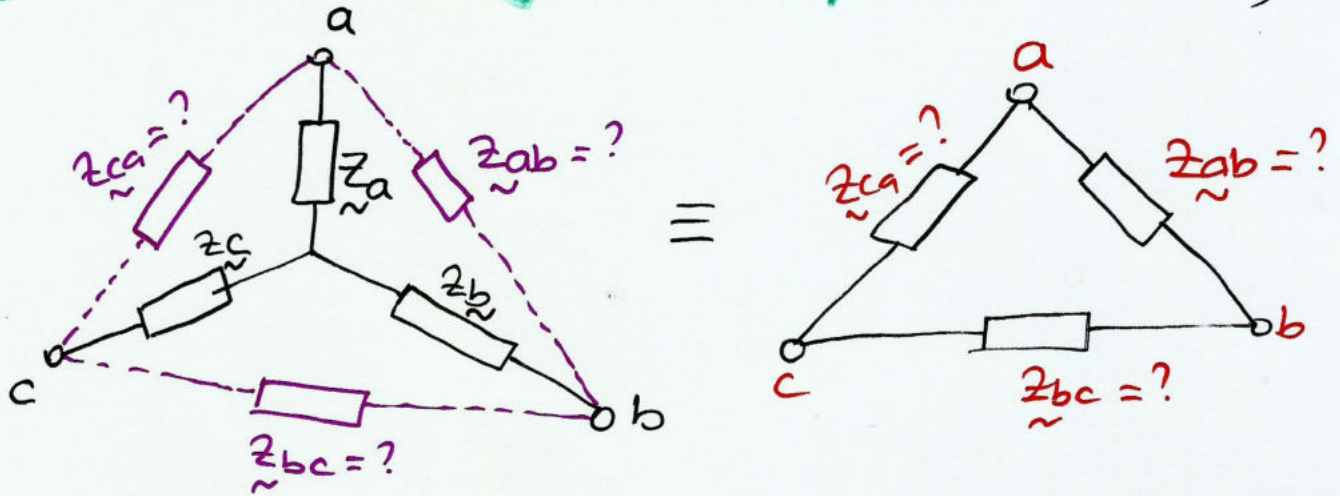
İstenen eşdeğer kapasite $150\mu\text{F}$ olduğu için $\frac{150}{25} = 6$ adet 4'lü seri kondensatör grubunu paralel bağlamalıyız.



$$C_{\text{eq}} = 6 \cdot 25 = 150\mu\text{F} / 200\text{V}$$

YILDIZ $\longleftrightarrow$ ÜÇGEN DÖNÜŞÜMLERİ

a) Yıldız $\rightarrow$ Üçgen Dönüşümü ($\lambda \rightarrow \Delta$) (7)



* Burada yapılan işlemler; λ bağlı Z_a, Z_b ve Z_c empedanslarının a, b, c düğümlerinden "aynı akımları" ve "aynı güçleri" geçirecek Δ bağlı eşdeğerlerinin hesaplanmasıdır. Bu işleme $\lambda \rightarrow \Delta$ dönüşümü denir.

$$\underline{Z}_{ab} = \frac{\underline{Z}_a \cdot \underline{Z}_b + \underline{Z}_b \cdot \underline{Z}_c + \underline{Z}_c \cdot \underline{Z}_a}{\underline{Z}_c}$$

$$\underline{Z}_{bc} = \frac{\underline{Z}_a \cdot \underline{Z}_b + \underline{Z}_b \cdot \underline{Z}_c + \underline{Z}_c \cdot \underline{Z}_a}{\underline{Z}_a}$$

$$\underline{Z}_{ca} = \frac{\underline{Z}_a \cdot \underline{Z}_b + \underline{Z}_b \cdot \underline{Z}_c + \underline{Z}_c \cdot \underline{Z}_a}{\underline{Z}_b}$$

Özel Durum:

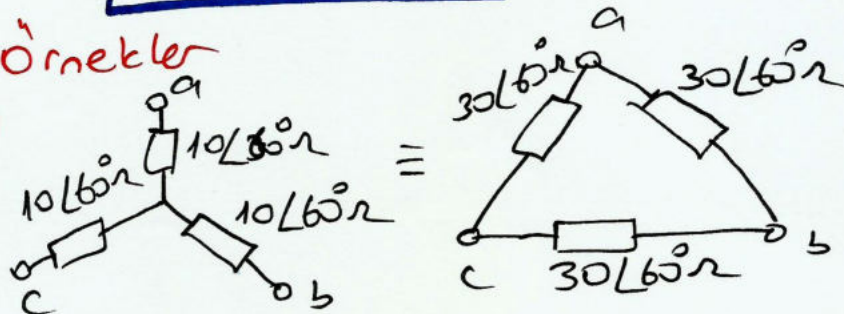
Üç fazlı sistemde λ bağlı empedanslar "denge"li yani birbirine eşit ise

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_\lambda \text{ ise } \underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = \underline{Z}_\Delta = \frac{3 \cdot \underline{Z}_\lambda \cdot \underline{Z}_\lambda}{\underline{Z}_\lambda}$$

$$\underline{Z}_\Delta = 3 \cdot \underline{Z}_\lambda \text{ olur.}$$

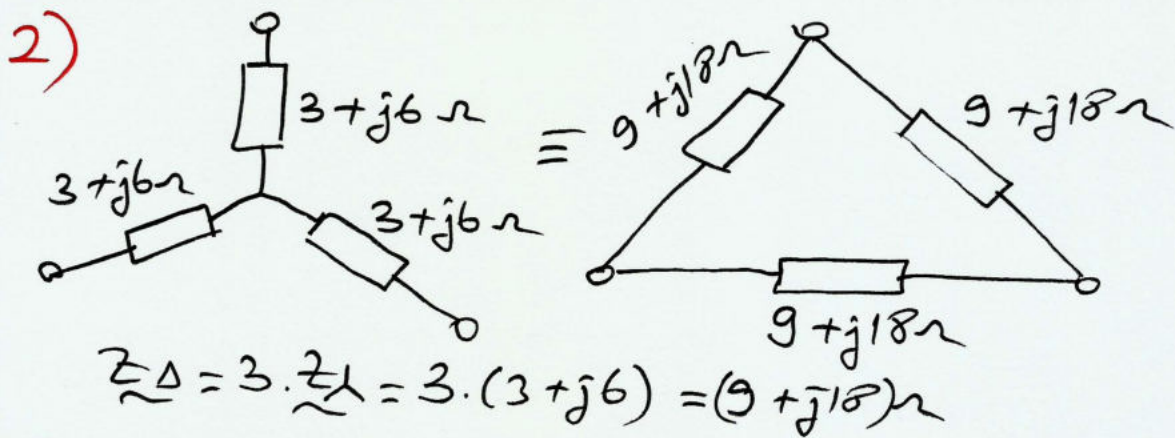
Örnekler

1)

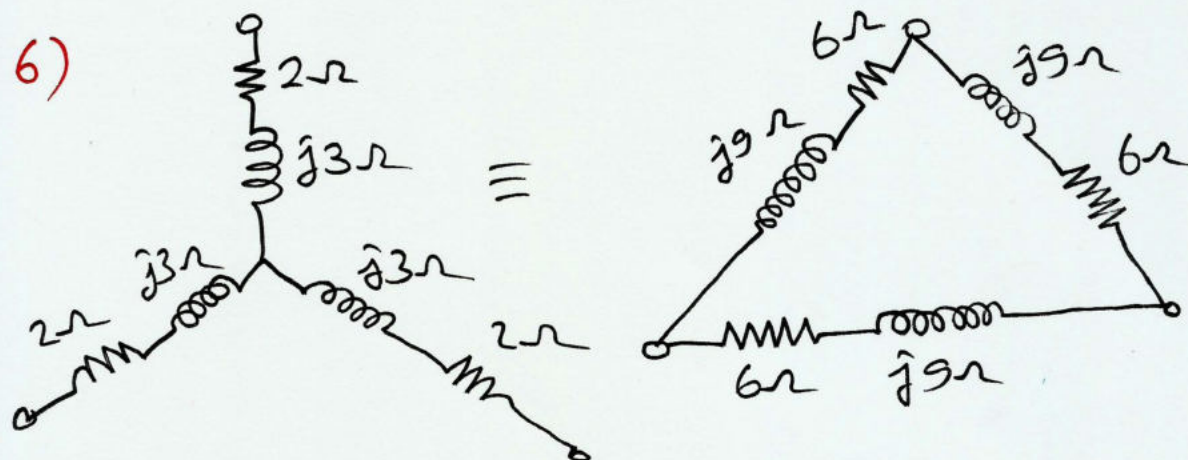
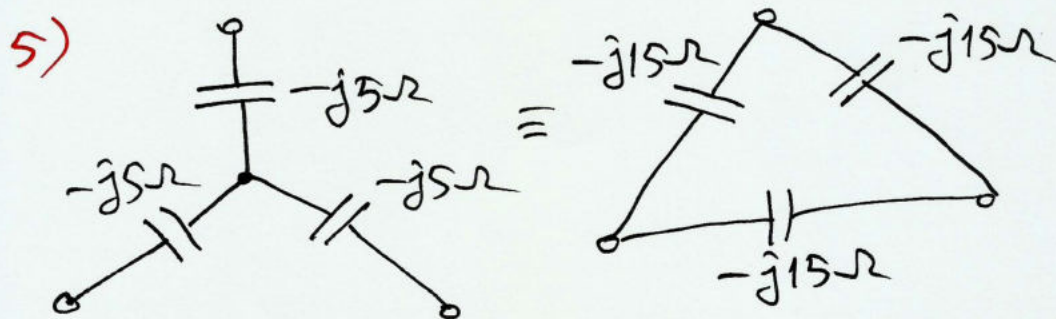
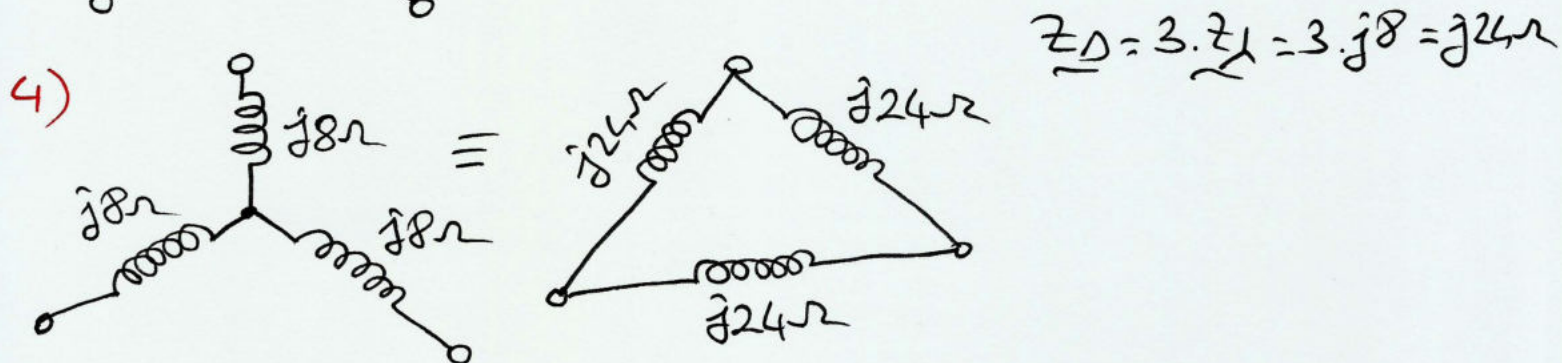
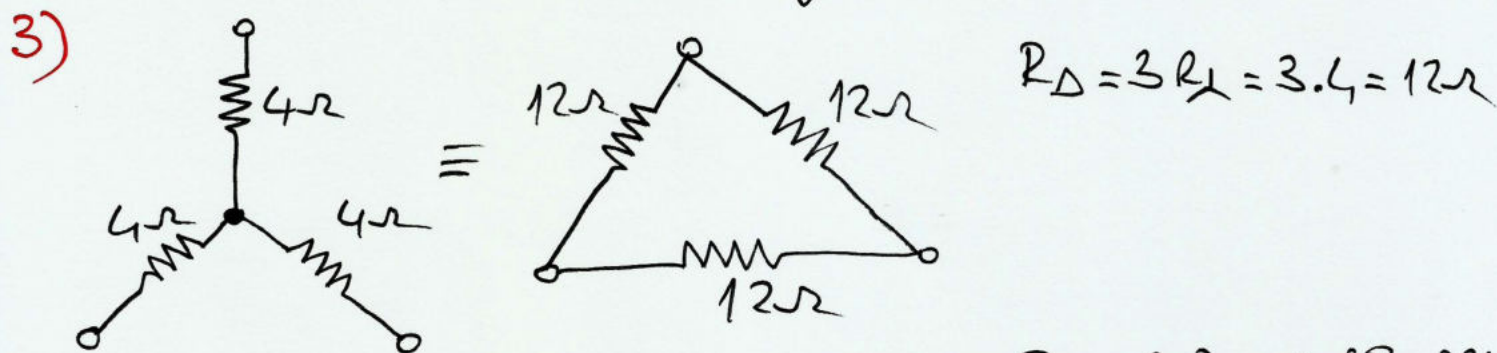


$$\underline{Z}_\Delta = 3 \underline{Z}_\lambda = 3 \cdot 10 \angle 60^\circ \Omega$$

$$\underline{Z}_\Delta = 30 \angle 60^\circ \Omega$$

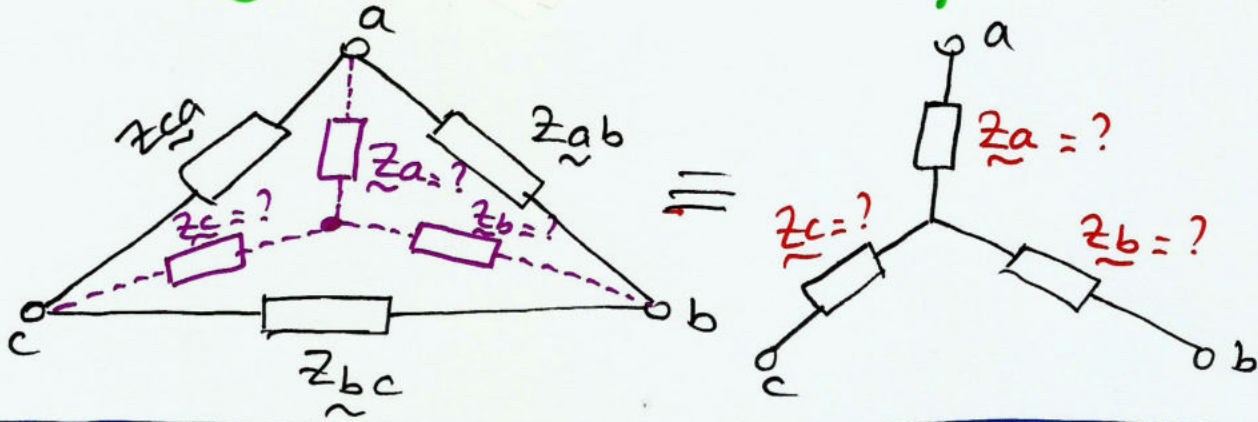


8



$Z_{\Delta} = 3 \cdot Z_Y = 3(2 + j3) = (6 + j9) \Omega$

b) Üçgen → Yıldız Dönüşümü ($\Delta \rightarrow Y$) 9



$$\underline{Z}_a = \frac{\underline{Z}_{ab} \cdot \underline{Z}_{ca}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}$$

$$\underline{Z}_b = \frac{\underline{Z}_{ab} \cdot \underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}$$

$$\underline{Z}_c = \frac{\underline{Z}_{ca} \cdot \underline{Z}_{bc}}{\underline{Z}_{ab} + \underline{Z}_{bc} + \underline{Z}_{ca}}$$

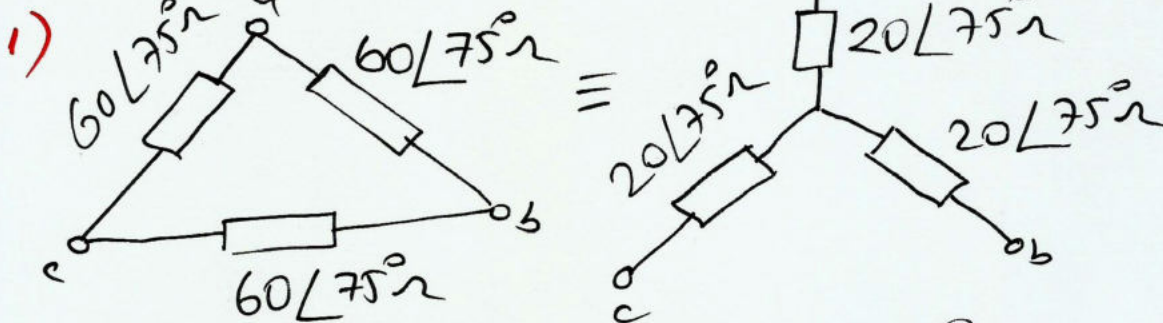
Özel Durum :

Δ bağlı 3 farklı empedanslar dengeli ise (eşit ise)

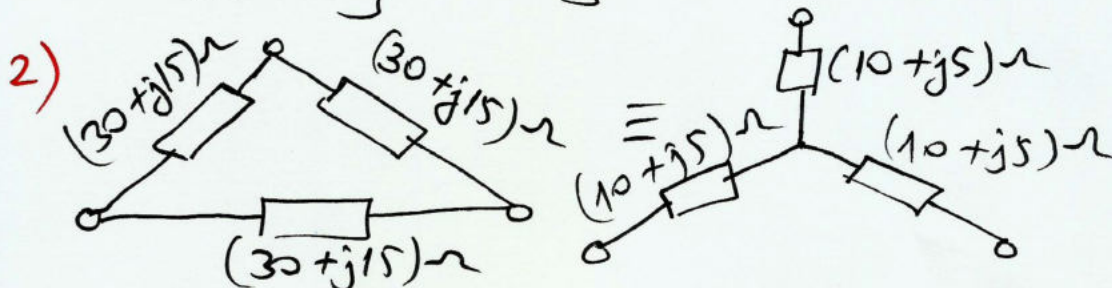
$\underline{Z}_{ab} = \underline{Z}_{bc} = \underline{Z}_{ca} = \underline{Z}_{\Delta}$ olur.

$$\underline{Z}_a = \underline{Z}_b = \underline{Z}_c = \underline{Z}_{\Delta} = \frac{\underline{Z}_{\Delta} \cdot \underline{Z}_{\Delta}}{3 \cdot \underline{Z}_{\Delta}} \Rightarrow \underline{Z}_Y = \frac{\underline{Z}_{\Delta}}{3} \text{ olur.}$$

Örnekler:

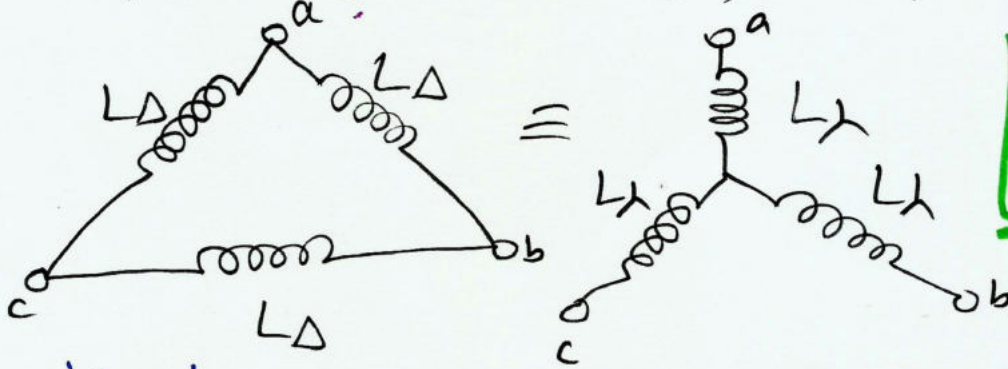


$$\underline{Z}_Y = \frac{\underline{Z}_{\Delta}}{3} = \frac{60 \angle 75^\circ}{3} = 20 \angle 75^\circ$$



Endüktanslarda $\Delta \rightarrow Y$ Dönüşümü

Δ bağlı endüktansların (L_{Δ}) yerine, a, b ve c düğümlerinden aynı akımları ve aynı güçleri geçirecek Y bağlı **eşdeğerlerinin** (L_Y) hesaplanması



$$L_Y = \frac{L_{\Delta}}{3}$$

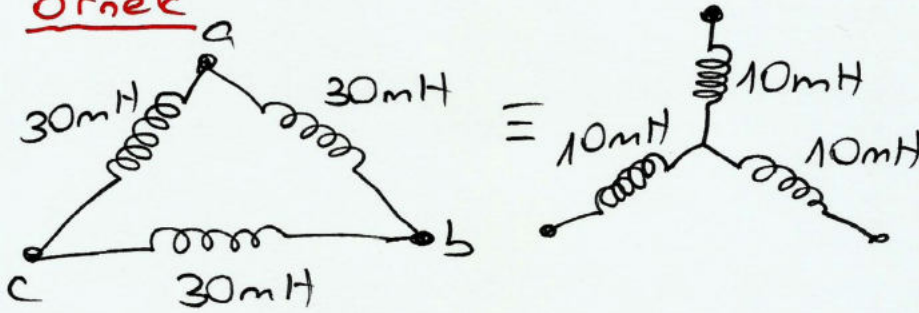
ispat:

$$X_{L_Y} = \frac{X_{L_{\Delta}}}{3} \Rightarrow \omega L_Y = \frac{\omega L_{\Delta}}{3} \Rightarrow L_Y = \frac{L_{\Delta}}{3}$$

veya reaktif güçlerin eşitliğinden;

$$\frac{3 \cdot U^2}{X_{L_{\Delta}}} = 3 \cdot \frac{U^2}{X_{L_Y}} \Rightarrow \frac{3 \cdot \cancel{U^2}}{\cancel{\omega} L_{\Delta}} = \frac{\cancel{U^2}}{\cancel{\omega} L_Y} \Rightarrow L_Y = \frac{L_{\Delta}}{3}$$

örnek



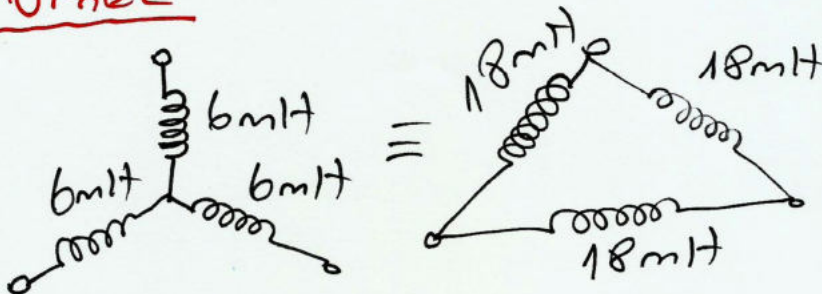
$$L_Y = \frac{L_{\Delta}}{3} = \frac{30}{3} = 10 \text{ mH}$$

Endüktanslarda $Y \rightarrow \Delta$ Dönüşümü

Yukarıda $\Delta \rightarrow Y$ dönüşümü için $L_Y = \frac{L_{\Delta}}{3}$ bulundu

$$L_Y = \frac{L_{\Delta}}{3} \Rightarrow L_{\Delta} = 3 \cdot L_Y$$

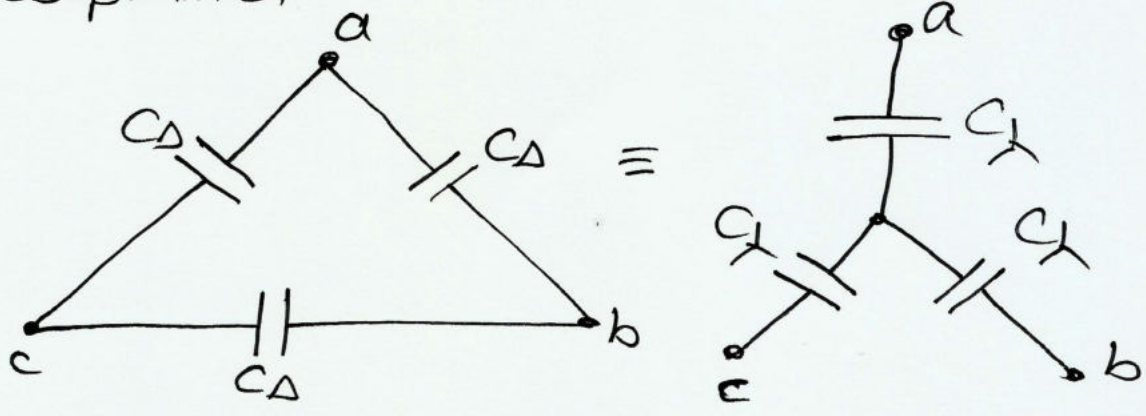
örnek



$$L_{\Delta} = 3 \cdot L_Y = 3 \cdot 6 = 18 \text{ mH}$$

Kondensatörlerde $\Delta \rightarrow Y$ Dönüşümü

Δ bağlı kondensatörlerin a, b, c düğümlerinden aynı akımlar ve aynı güçleri geçecek Y bağlı eşdeğerlerinin hesaplanması



$$C_Y = 3 \cdot C_{\Delta}$$

İspat :

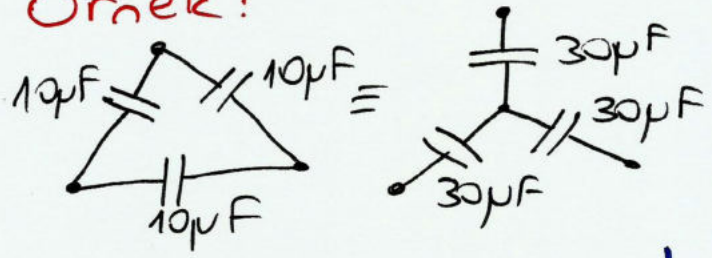
$$X_{C_Y} = \frac{X_{C_{\Delta}}}{3} \Rightarrow \frac{1}{\omega C_Y} = \frac{\frac{1}{\omega C_{\Delta}}}{3} \Rightarrow \frac{1}{\omega C_Y} = \frac{1}{\omega 3 C_{\Delta}}$$

$$C_Y = 3 \cdot C_{\Delta}$$

veya reaktif güçlerin eşitliğinden; U^2

$$\frac{3 \cdot U^2}{X_{C_{\Delta}}} = 3 \cdot \frac{U^2}{X_{C_Y}} \Rightarrow \frac{3 \cdot U^2}{\frac{1}{\omega C_{\Delta}}} = \frac{U^2}{\frac{1}{\omega C_Y}} \Rightarrow C_Y = 3 \cdot C_{\Delta}$$

Örnek :

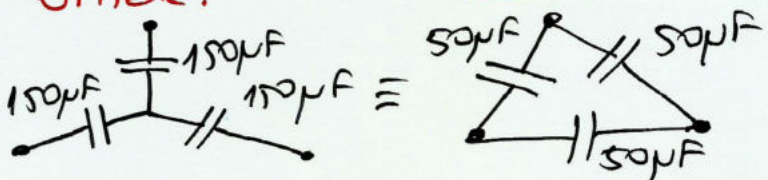


$$C_Y = 3 \cdot C_{\Delta} = 3 \cdot 10 = 30 \mu F$$

Kondensatörlerde $Y \rightarrow \Delta$ Dönüşümü

$$C_Y = 3 \cdot C_{\Delta} \Rightarrow C_{\Delta} = \frac{C_Y}{3}$$

Örnek :



$$C_{\Delta} = \frac{C_Y}{3} = \frac{150}{3}$$

$$C_{\Delta} = 50 \mu F$$