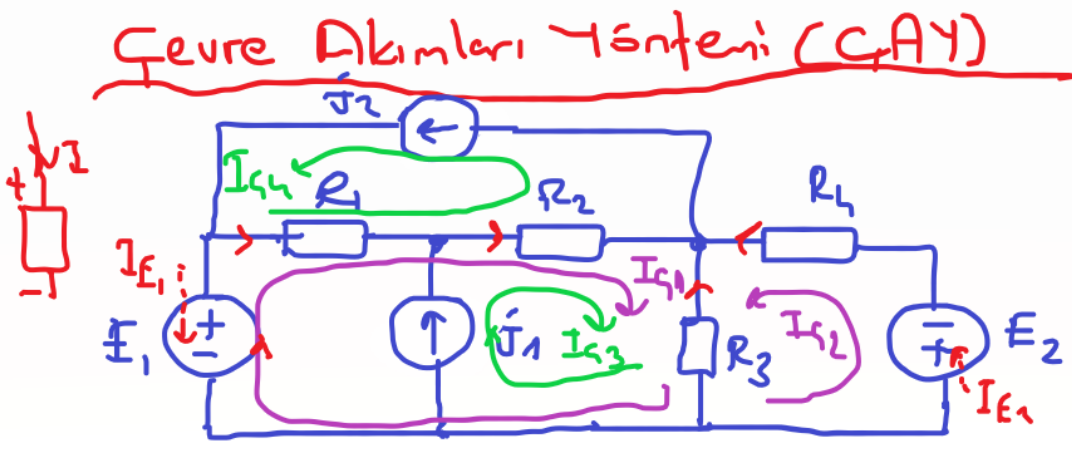
Çevre Akımları Yöntemi (ÇAY)



Çevre Akımları

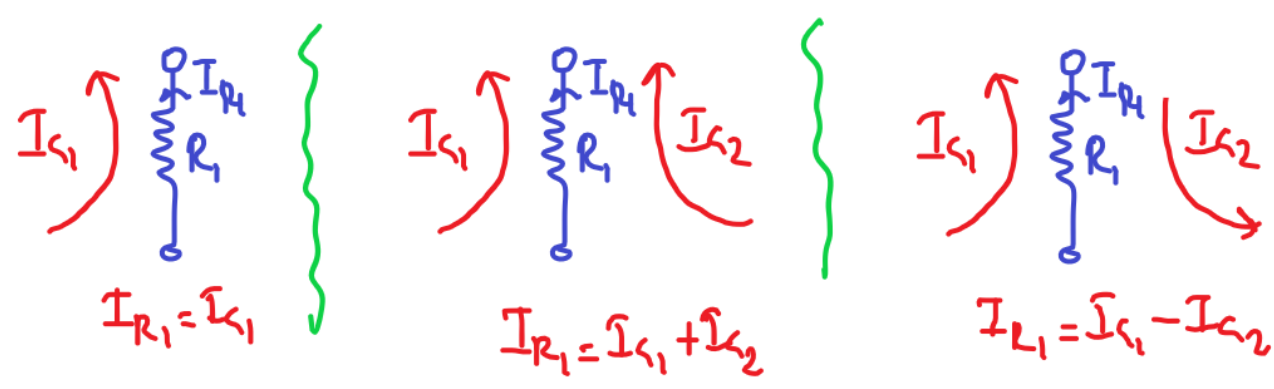
$I_{c_1} = ?$ } Bilinmiyor
$I_{c_2} = ?$ }

$I_{c_3} = J_1$ } Bilinıyor
$I_{c_4} = J_2$ }

$P_{E_1} = E_1 . I_{E_1}$

$V_{R_1} \rightarrow R I_{R_1} \rightarrow R_1 . I_{c_1}$

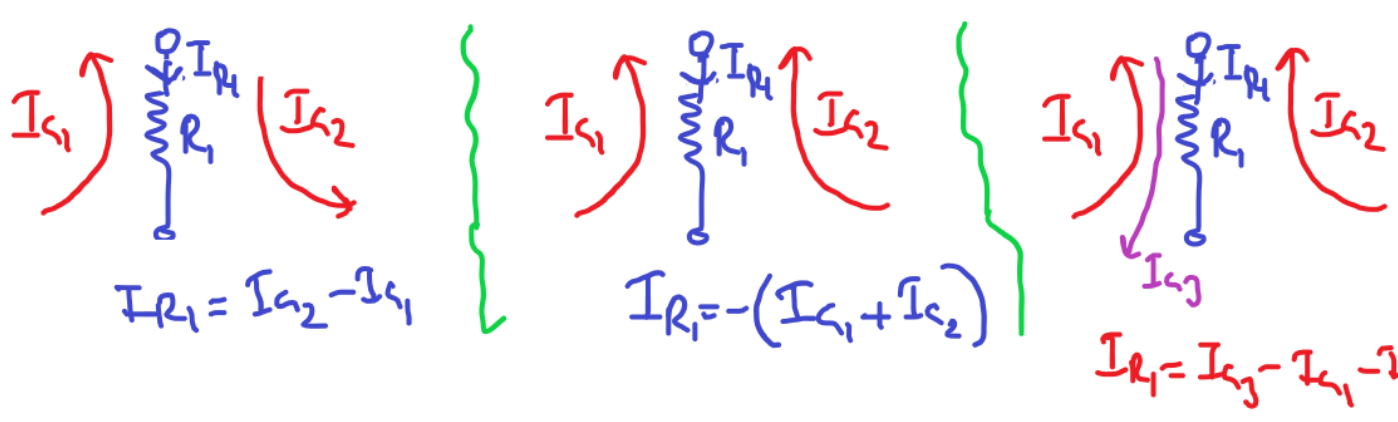
$-V_{R_3} \rightarrow -R_3 I_{R_3} \rightarrow -R_3 (-(I_{c_1} + I_{c_2} + I_{c_3}))$ ↑ $\hat{U}_1$



$I_{R_1} = I_{c_1}$

$I_{R_1} = I_{c_1} + I_{c_2}$

$I_{R_1} = I_{c_1} - I_{c_2}$



$I_{R_1} = I_{c_2} - I_{c_1}$

$I_{R_1} = -(I_{c_1} + I_{c_2})$

$I_{R_1} = I_{c_3} - I_{c_1} - I_{c_2}$

Ek Bilgi (Lineer cebir)

$a . x_1 + b . x_2 = e$
$c . x_1 + d . x_2 = f$
$x_1 = ?\quad x_2 = ?$

Matris biçiminde

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix}$$

$A . \underset{\sim}{X} = \underset{\sim}{B}$ ~ : Sütun matris

Çözüm

Cramer Yöntemi ile

A matrisi: $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ve A'nın determinantı: $\Delta = \det(A)$

$$\Delta = \det(A) = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = (a.d - b.c)$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix} \Rightarrow x_1 = \frac{\begin{vmatrix} e & b \\ f & d \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{(e.d - b.f)}{(a.d - b.c)}$$

$$x_2 = \frac{\begin{vmatrix} a & e \\ c & f \end{vmatrix}}{\begin{vmatrix} a & b \\ c & d \end{vmatrix}} = \frac{(a.f - c.e)}{(a.d - b.c)}$$

x_1 ve x_2'nin matris tersinden bulunması

$A . \underset{\sim}{X} = \underset{\sim}{B}$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} e \\ f \end{bmatrix} \Rightarrow$$

A, X, B

A^{-1} : A matrisinin tersi

$A . \underset{\sim}{X} = B \Rightarrow \underset{\sim}{X} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = ?$

$\underline{A . A^{-1}} . \underset{\sim}{X} = A^{-1} . \underset{\sim}{B}$

Birim matris $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \Rightarrow A^{-1} = ?$

$$\boxed{\underset{\sim}{X} = A^{-1} . \underset{\sim}{B}}$$

$$A^{-1} = \frac{1}{\det A} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{(a.d - b.c)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$$\underset{\sim}{X} = A^{-1} . \underset{\sim}{B} \Rightarrow \boxed{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{(a.d - b.c)} \cdot \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \cdot \begin{bmatrix} e \\ f \end{bmatrix}}$$

ÖRNEK

$\begin{bmatrix} 20 & -10 \\ 40 & 30 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 260 \end{bmatrix}$ ise $x_1 = ?$, $x_2 = ?$

ÇÖZÜM

a) Cramer Yöntemi ile çözüm

$$\Delta = \begin{vmatrix} 20 & -10 \\ 40 & 30 \end{vmatrix} = 20.30 - (-10).40 = 600 + 400 \Rightarrow \Delta = \det A = 1000$$

$\Delta = 1000$

$$x_1 = \frac{\begin{vmatrix} 80 & -10 \\ 260 & 30 \end{vmatrix}}{\begin{vmatrix} 20 & -10 \\ 40 & 30 \end{vmatrix}} = \frac{80.30 - (-10).260}{1000} = \frac{2400 + 2600}{1000} = \frac{5000}{1000}$$

$\boxed{x_1 = 5}$

$$x_2 = \frac{\begin{vmatrix} 20 & 80 \\ 40 & 260 \end{vmatrix}}{\begin{vmatrix} 20 & -10 \\ 40 & 30 \end{vmatrix}} = \frac{20.260 - 80.40}{1000} = \frac{2000}{1000} \Rightarrow \boxed{x_2 = 2}$$

Sonuç : $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$

b) Matris tersi yardımıyla çözüm

$\begin{bmatrix} 20 & -10 \\ 40 & 30 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 80 \\ 260 \end{bmatrix}$ ise $x_1 = ?$, $x_2 = ?$

A, $\underset{\sim}{X}$, $\underset{\sim}{B}$

$\Rightarrow A . \underset{\sim}{X} = \underset{\sim}{B} \Rightarrow \boxed{\underset{\sim}{X} = A^{-1} . \underset{\sim}{B}}$

$\det(A) = \Delta = \begin{vmatrix} 20 & -10 \\ 40 & 30 \end{vmatrix} = 1000$

$$A = \begin{bmatrix} 20 & -10 \\ 40 & 30 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{\Delta} \cdot \begin{bmatrix} 30 & 10 \\ -40 & 20 \end{bmatrix} = \frac{1}{1000} \cdot \begin{bmatrix} 30 & 10 \\ -40 & 20 \end{bmatrix}$$

$\underset{\sim}{X} = A^{-1} . \underset{\sim}{B}$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{1000} \cdot \begin{bmatrix} 30 & 10 \\ -40 & 20 \end{bmatrix} \cdot \begin{bmatrix} 80 \\ 260 \end{bmatrix} = \frac{1}{1000} \cdot \begin{bmatrix} 30.80 + 10.260 \\ -40.80 + 20.260 \end{bmatrix}$$

Sonuç :

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \frac{1}{1000} \cdot \begin{bmatrix} 5000 \\ 2000 \end{bmatrix} \Rightarrow \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 5 \\ 2 \end{bmatrix} \Rightarrow \boxed{x_1 = 5} \quad \boxed{x_2 = 2}$$