YILDIZ TEKNİK ÜNİVERSİTESİ

ELEKTRİK MÜHENDİSLİĞİ BÖLÜMÜ

DEVRE TEORİSİ

Ders Notu-4

DC DEVRELERDE YILDIZ-ÜÇGEN DÖNÜŞÜMLERİ

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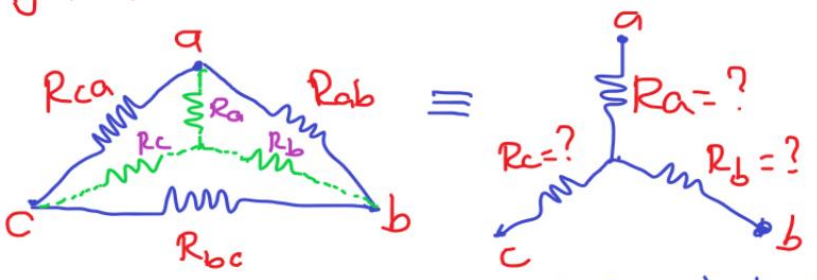
YILDIZ – ÜÇGEN DÖNÜŞÜMLERİ

Dirençlerin Yıldız-Üçgen Dönüşümü ($\lambda \rightleftarrows \Delta$)

a) Üçgen → Yıldız Dönüşümü ($\Delta \rightarrow \lambda$)



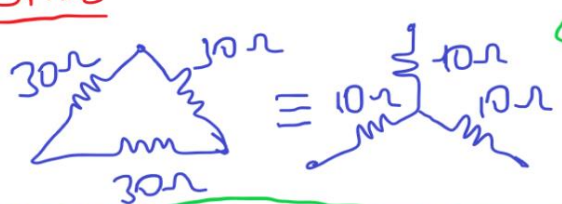
* Δ bağlı dirençlerin eşdeğeri olan λ bağlı dirençler?

$$R_a = \frac{R_{ab} \cdot R_{ca}}{R_{ab}+R_{bc}+R_{ca}} \qquad R_b = \frac{R_{ab} \cdot R_{bc}}{R_{ab}+R_{bc}+R_{ca}} \qquad R_c = \frac{R_{ca} \cdot R_{bc}}{R_{ab}+R_{bc}+R_{ca}}$$

Özel durum: $R_{ab}=R_{bc}=R_{ca}=R_\Delta$ olsun

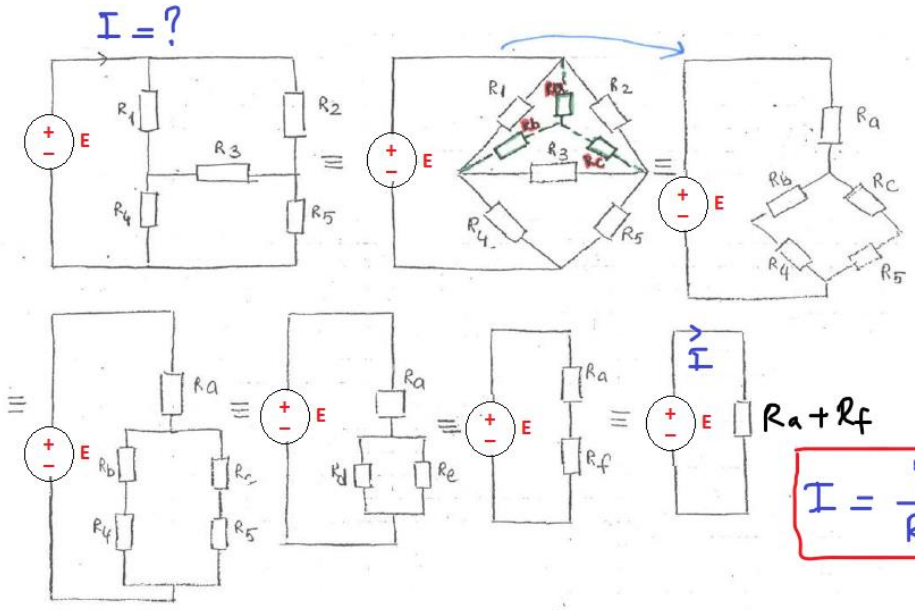
$$R_a = R_b = R_c = \frac{R_\Delta \cdot R_\Delta}{3 \cdot R_\Delta} = R_\lambda \Rightarrow \boxed{R_\lambda = \frac{R_\Delta}{3}}$$

Örnek



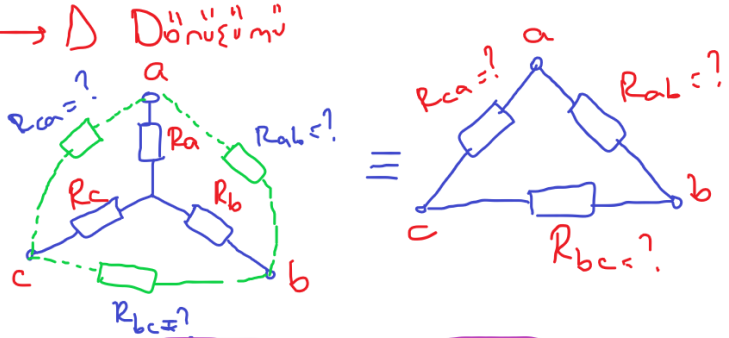
$R_\Delta = 30\Omega$

$R_\lambda = \frac{R_\Delta}{3} = \frac{30}{3} = 10\Omega$



$$I = \frac{E}{R_a + R_f}$$

Y → Δ Dönüşümü



$$R_{ab} = \frac{R_a \cdot R_b + R_b \cdot R_c + R_c \cdot R_a}{R_c}$$

$$R_{bc} = \frac{R_a \cdot R_b + R_b \cdot R_c + R_c \cdot R_a}{R_a}$$

$$R_{ca} = \frac{R_a \cdot R_b + R_b \cdot R_c + R_c \cdot R_a}{R_b}$$

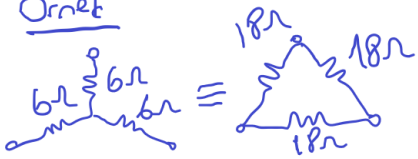
"

Özel durum

$R_a = R_b = R_c = R_Y \Rightarrow R_{ab} = R_{bc} = R_{ca} = \frac{3 \cdot R_Y \cdot R_Y}{R_Y}$

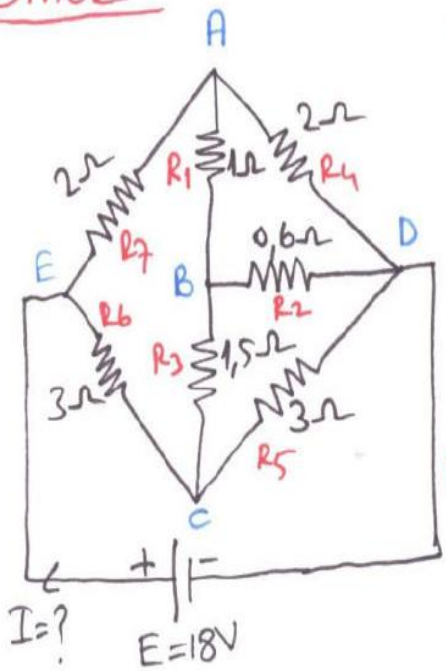
$$R_\Delta = 3 \cdot R_Y \iff R_Y = \frac{R_\Delta}{3}$$

Örnek



Örnek

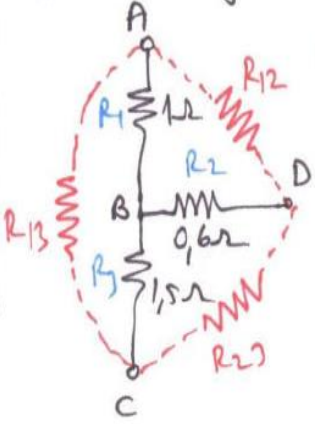
Şekildeki devrede kaynaktan çekilen akımı hesaplayınız.



Çözüm

Önce Y bağlı R_1, R_2, R_3'ü Δ'e dönüştürelim.

$$(R_1 \cdot R_2 + R_1 \cdot R_3 + R_2 \cdot R_3) = 1,0,6 + 1,1,5 + 0,6,1,5 = 3\,\Omega^2$$



$$R_{12} = \frac{3}{R_3} = \frac{3}{1,5} = 2\Omega$$

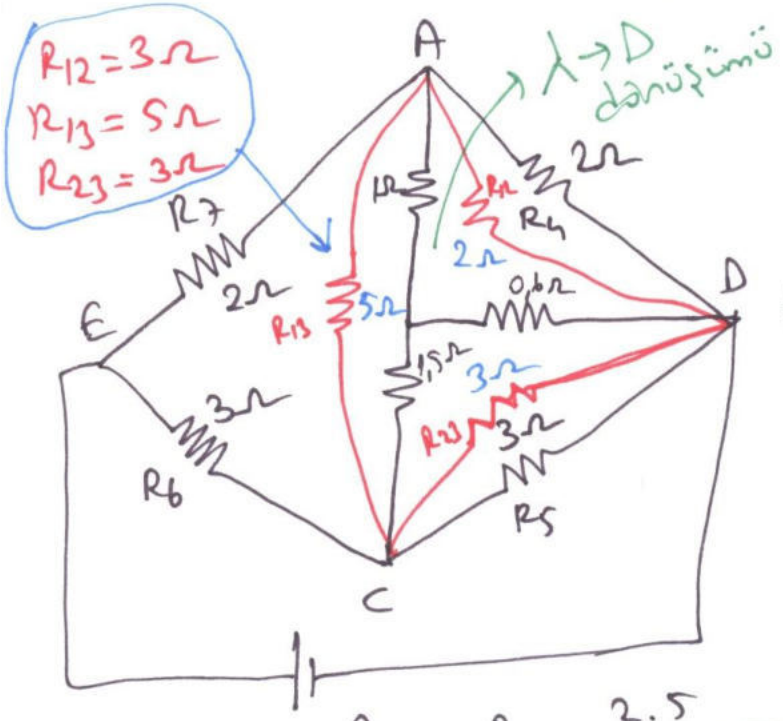
$$R_{13} = \frac{3}{R_2} = \frac{3}{0,6} = 5\Omega$$

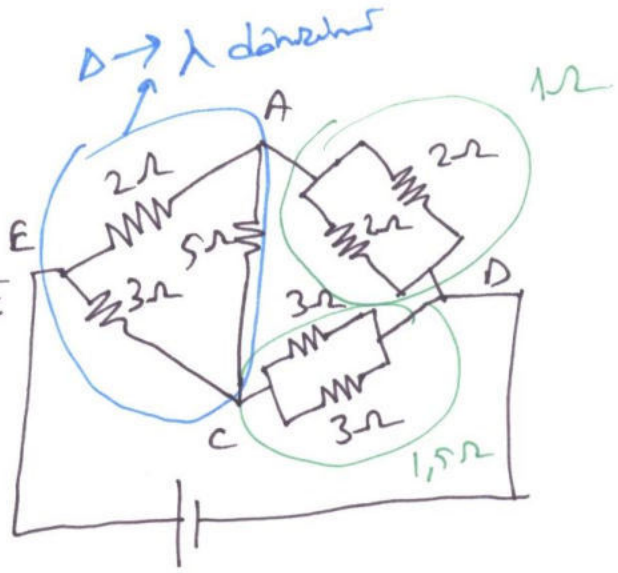
$$R_{23} = \frac{3}{R_1} = \frac{3}{1} = 3\Omega$$

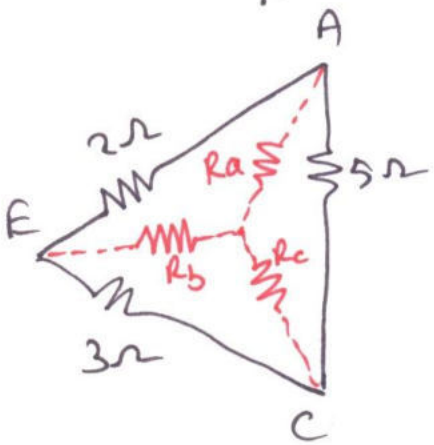
$R_{12} = 3\Omega$

$R_{13} = 5\Omega$

$R_{23} = 3\Omega$



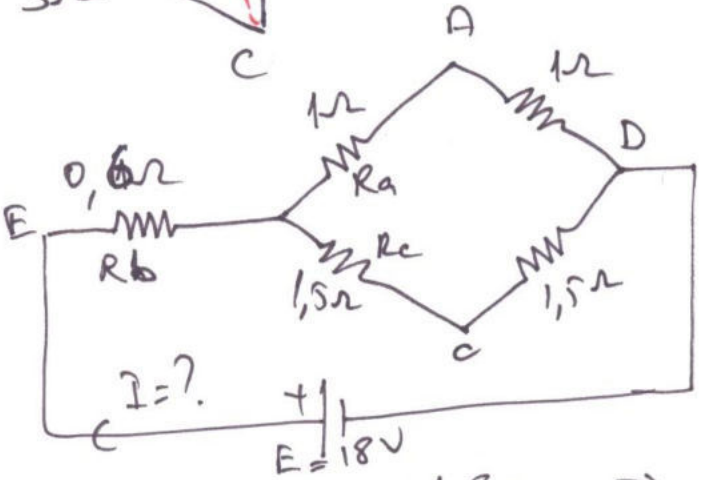




$$R_a = \frac{2.5}{2+3+5} = 1\Omega$$

$$R_b = \frac{2.3}{2+3+5} = 0{,}6\Omega$$

$$R_c = \frac{3.5}{2+3+5} = 1{,}5\Omega$$



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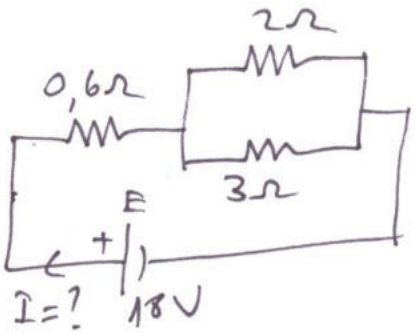
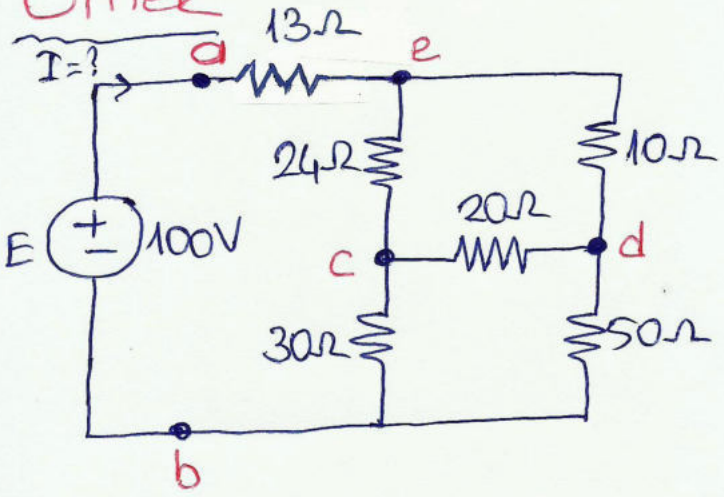


$$R_T = 0{,}6 + \frac{2.3}{2+3} = 1{,}8\Omega \Rightarrow I = \frac{E}{R_T} = \frac{18}{1{,}8} = 10A$$

$\boxed{I = 10A}$

DC DEVRELERDE YILDIZ-ÜÇGEN (λ-Δ) DÖNÜŞÜMLERİ

Örnek



a) R_{ab} = ? ($\Delta \rightarrow \lambda$ dönüşümüyle bulunur)

b) I = ?

c) 13Ω'luk direncin gücünü ve kWh olarak bir ayda harcadığı elektrik enerjisini hesaplayınız.

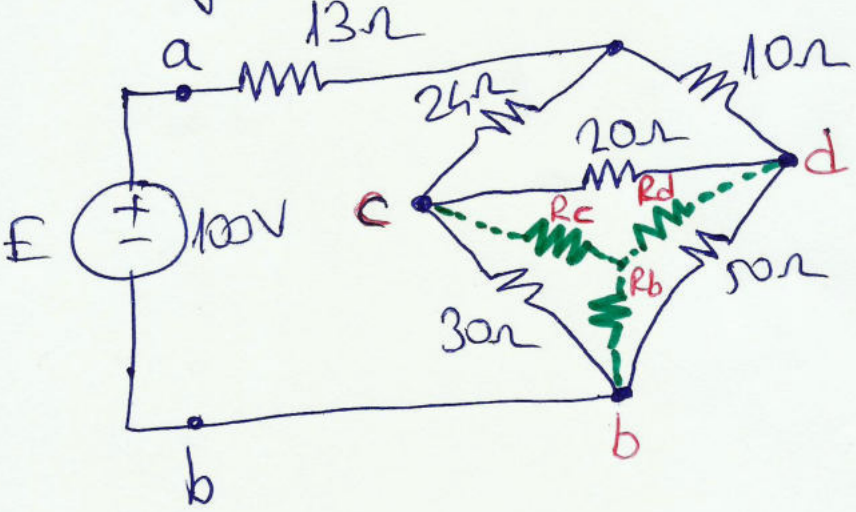
d) Rab direncini 20Ω, 10Ω ve 50Ω ($\lambda_{bağlı}$) dirençlerini Δ'e çevirerek $\Delta \rightarrow \lambda$ dönüşümüyle bulunuz

Çözüm

a) a-b uçları arasındaki eşdeğer direnci bulmak için aşağıdaki λ-Δ dönüşümü seçeneklerinden birini bulabiliriz.

- Δ bağlı olan 20Ω, 30Ω ve 50Ω'u λ'a çevirmek
- " " " 24Ω, 20Ω ve 10Ω'u " "
- λ bağlı olan 20Ω, 10Ω ve 50Ω'u Δ'e çevirmek
- λ " " 24Ω, 20Ω ve 30Ω'u " "

Biz Δ bağlı olan alttaki 20Ω, 30Ω ve 50Ω'luk dirençlere $\Delta \rightarrow \lambda$ dönüşümü uygulayıp bunlara eşdeğerlerini bulalım:

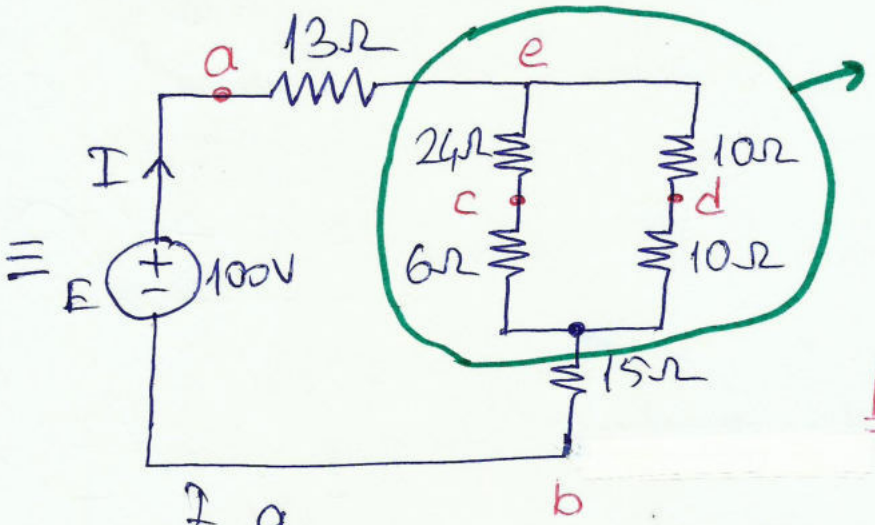


$\sum R_\Delta = 20+30+50 = 100\Omega$

$R_c = \frac{30 \cdot 20}{100} = 6\Omega$

$R_d = \frac{20 \cdot 50}{100} = 10\Omega$

$R_b = \frac{30 \cdot 50}{100} = 15\Omega$

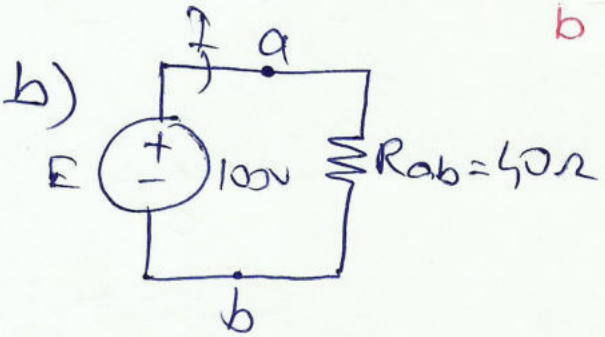


$$Re_1 = \frac{30.20}{30+20} = 12\Omega$$

$$R_{ab} = 13+12+15$$

$$\boxed{R_{ab} = 40\Omega}$$

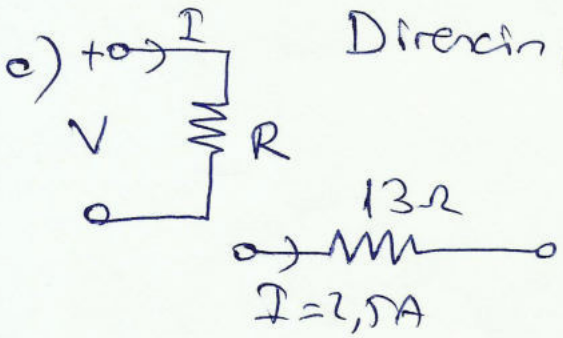
b)



$$I = \frac{E}{R_{ab}} = \frac{100}{40}$$

$$\boxed{I = 2,5A}$$

c)



Direnç için: $\boxed{P = V.I} = (R.I).I \Rightarrow \boxed{P = RI^2}$

$$P = V.I = V.\left(\frac{V}{R}\right) \Rightarrow \boxed{P = \frac{V^2}{R}}$$

$$P = RI^2 = 13.2,5^2$$

$$\boxed{P = 81,25W}$$

Enerji (kWh) olarak

$E = P.t$

↓ kwh ↓ kw ↳ h

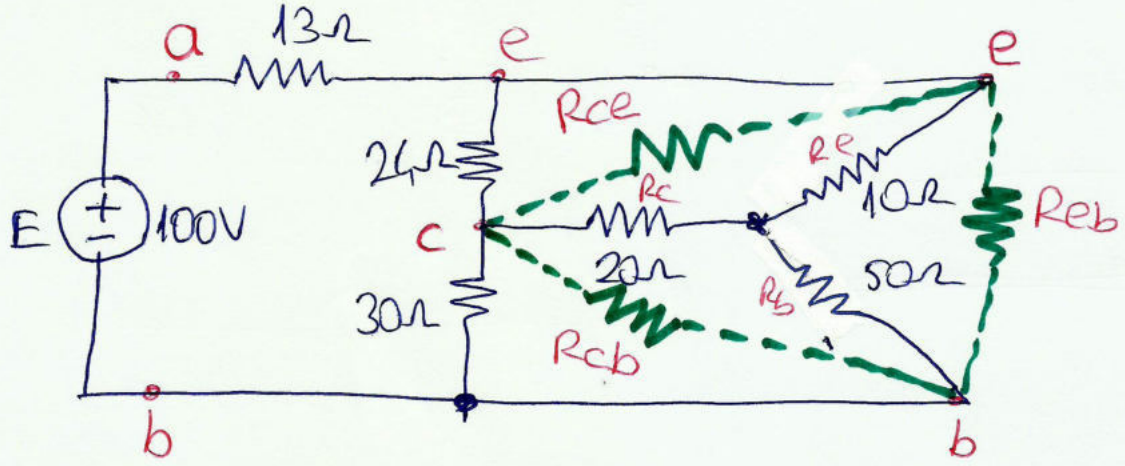
$P = 81,25W = 81,25.10^{-3}kW$

$t = 1ay = 30.24 = 720h$ (saat)

$E = P.t = 81,25.10^{-3}.720$

$$\boxed{E = 58,5\ kWh}$$

d) R_{ab}'nin $\lambda \rightarrow \Delta$ dönüşümüyle bulunması



İkizerli bağlı dirençlerin çarpımlarının toplamı:

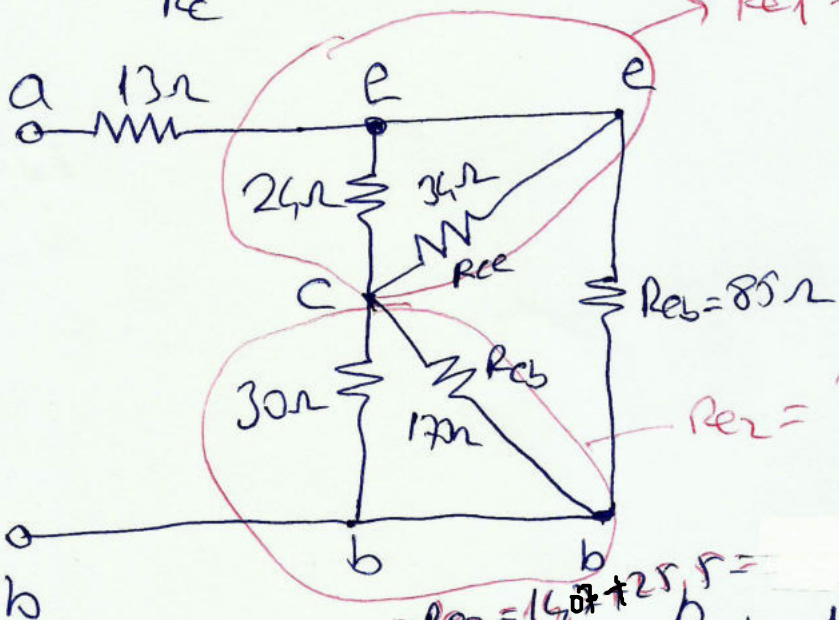
$$R_c.R_b + R_c.R_e + R_b.R_e = 20.50 + 20.10 + 50.10 = 1700$$

$$R_{ce} = \frac{R_c.R_b + R_c.R_e + R_b.R_e}{R_b} = \frac{1700}{50} = 34\Omega$$

$$R_{cb} = \frac{1700}{R_e} = \frac{1700}{10} = 170\Omega$$

$$R_{eb} = \frac{1700}{R_c} = \frac{1700}{20} = 85\Omega$$

$$R_{e1} = \frac{24.34}{24+34} = 14{,}07\Omega$$

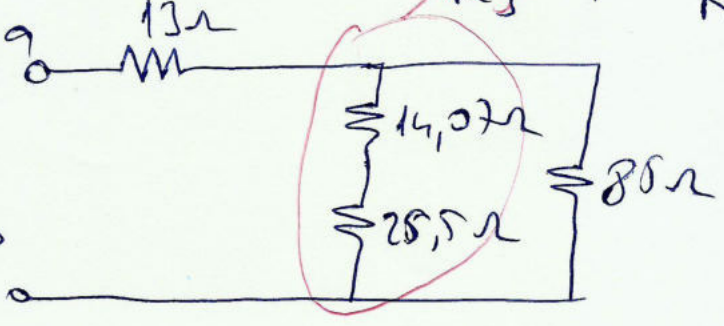


$$R_{e2} = \frac{30.170}{30+170} = 25{,}5\Omega$$

$$R_{eq} = 14{,}07 + 25{,}5 =$$

$$R_{ab} = 13 + \left(\frac{39{,}57 \cdot 85}{39{,}57 + 85}\right)$$

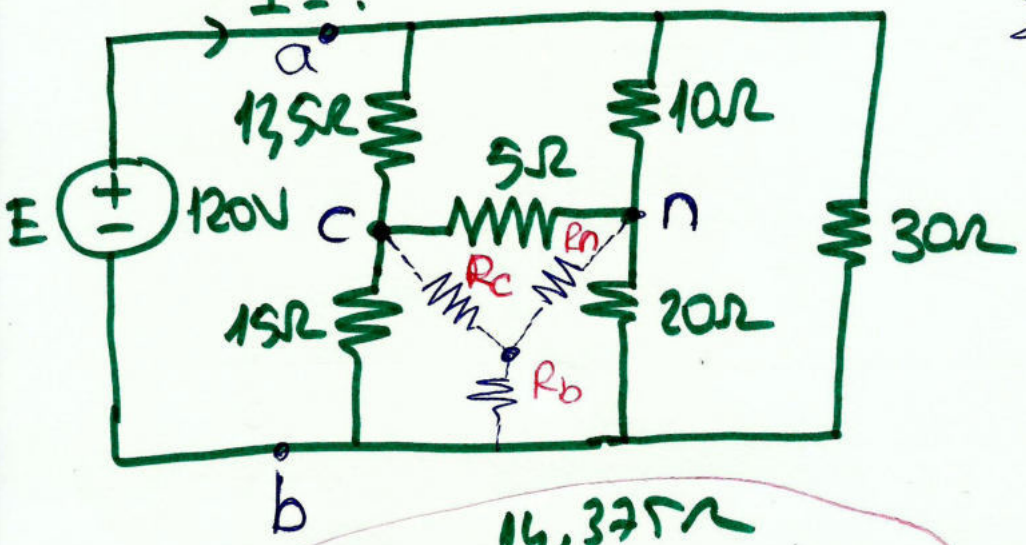
27



$$\boxed{R_{ab} = 40\Omega}$$

Örnek

Devrede $R_{ab}=?$ $I=?$

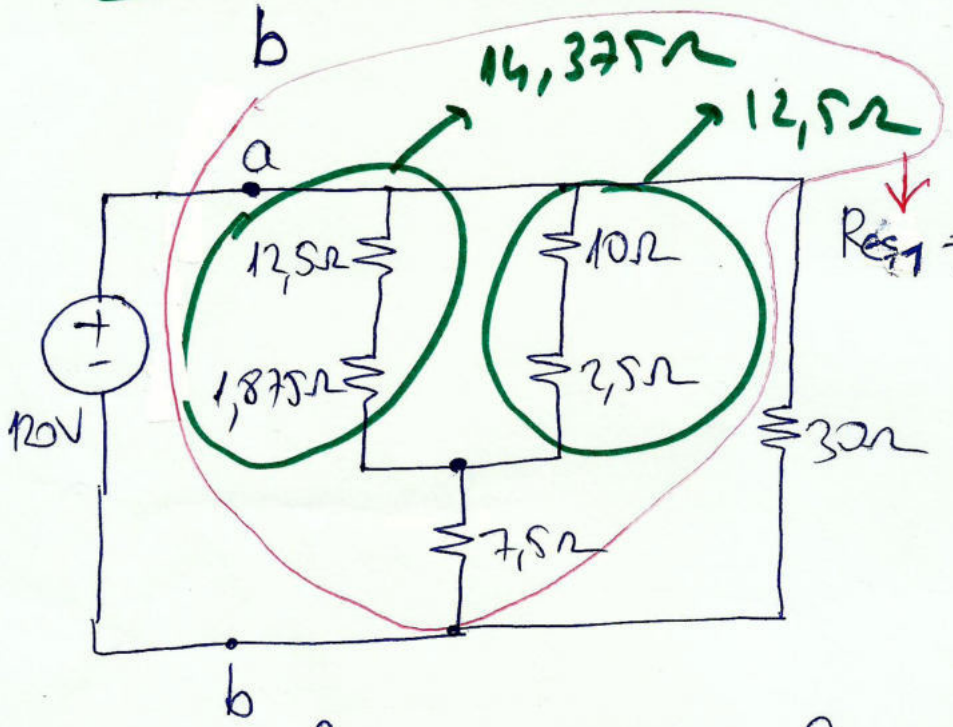


$$\Sigma R_D = 5+15+20 = 40\Omega$$

$$R_c = \frac{15.5}{\Sigma R_D} = \frac{15.5}{40} = 1,375\Omega$$

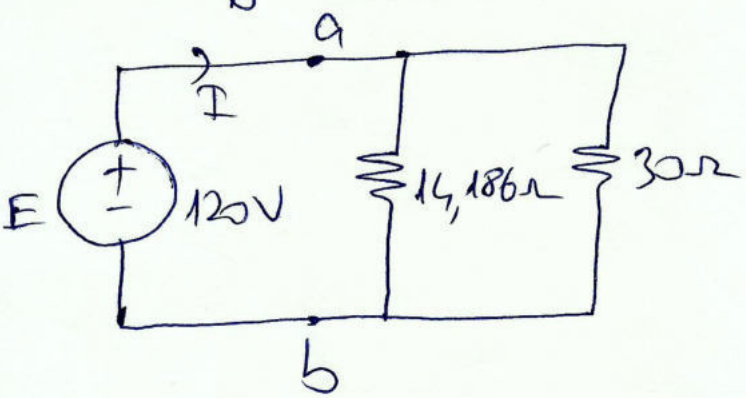
$$R_n = \frac{5.20}{40} = 2,5\Omega$$

$$R_b = \frac{15.20}{40} = 7,5\Omega$$



$$Res_1 = \frac{14,375.12,5}{(14,375+12,5)} + 7,5$$

$$Res_1 = 14,186\Omega$$

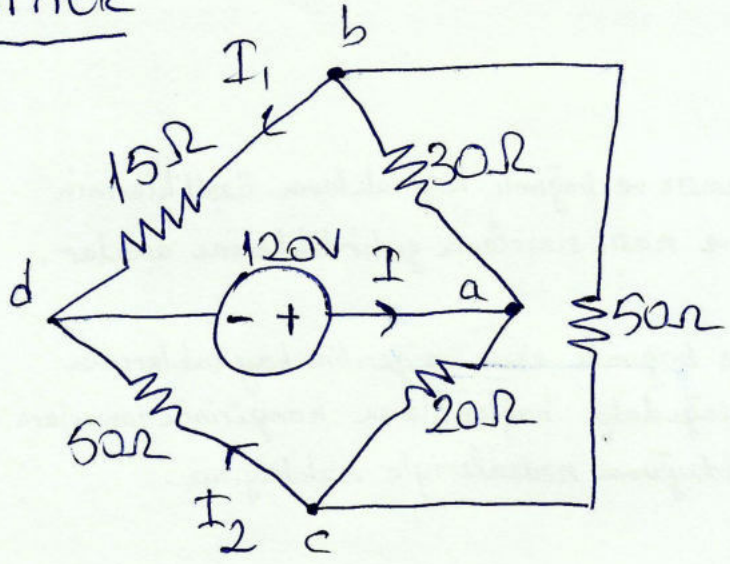


$$R_{ab} = \frac{14,186.30}{(14,186+30)}$$

$$\boxed{R_{ab} = 9,632\Omega}$$

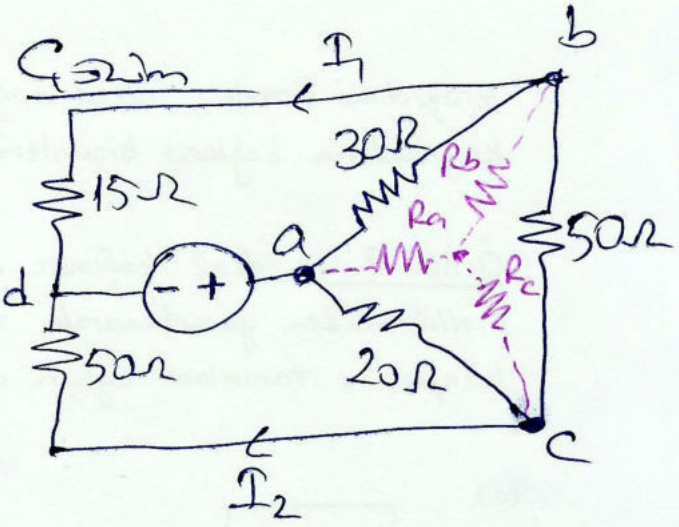
$$I = \frac{E}{R_{ab}} = \frac{120}{9,632} \Rightarrow \boxed{I = 12,458A}$$

Örnek



$I_t = ?,\ I_1 = ?,\ I_2 = ?$

Çözüm



Δ–Y Dönüşümü

$$\sum R_\Delta = 30+20+50 = 100\Omega$$

$$R_a = \frac{30\cdot 20}{100} = 6\Omega$$

$$R_b = \frac{30\cdot 50}{100} = 15\Omega$$

$$R_c = \frac{20\cdot 50}{100} = 10\Omega$$

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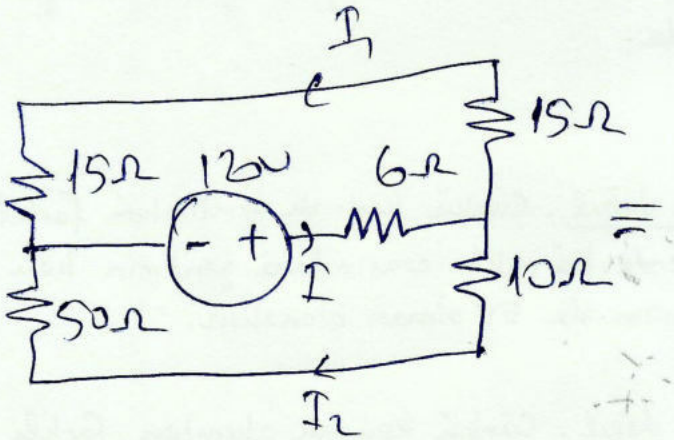

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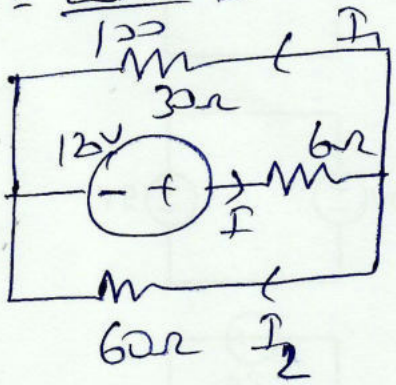

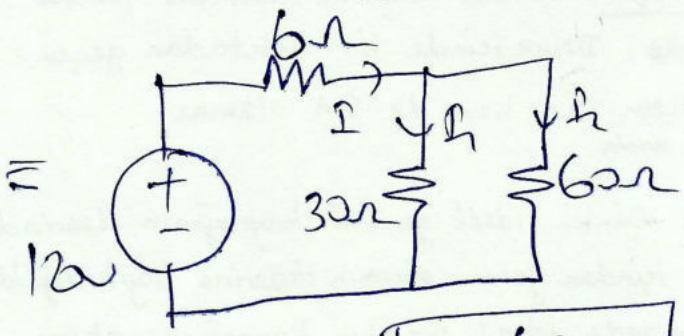

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$$R_T = 6 + \frac{30\cdot 60}{30+60} = 26\Omega$$

$$\boxed{R_T = 26\Omega}$$

$$I = \frac{120}{26} \Rightarrow \boxed{I = \frac{60}{13}\,A}$$

Akım bölücüden: $I_1 = \frac{60}{30+60}\cdot I = \frac{60}{90}\cdot\frac{60}{13} \Rightarrow \boxed{I_1 = \frac{40}{13}\,A}$

$$I_2 = \frac{30}{90}\cdot\frac{60}{13} \Rightarrow \boxed{I_2 = \frac{20}{13}\,A}$$